

Naturalness in SUSY models and its implications.

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- Outline

- I. Review of naturalness.

- II. Constrained likelihood accounts for naturalness.

- III. Numerical results for naturalness in SUSY Models

- IV. Naturalness beyond MSSM: NMSSM, GNMSSM, ...

I. Review of Naturalness:

L. Susskind PRD 20 (1979), 2619

- why SM EW scale $v \sim M_Z \ll M_P$, stable under quantum corrections. $\left[\frac{G_f h^2}{G_N c^2} = 1.7 \times 10^{33} \right]$

$$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j) \Lambda^2, \text{ if } \Lambda \sim M_P: \text{ tune coupling: } 1 : 10^{33} (!).$$

Hierarchy Problem $\Leftrightarrow$ Fine tuning. Unnatural. Ways out?

A physical parameter $\rho(\mu)$ is naturally v.small if $\rho(\mu) = 0$ increases the symmetry.

“Naturalness dogma”: 't Hooft (1979)

- 3 options:

a). Scale/conformal symmetry. Or:

see for example Bardeen 1995

b). SUSY: $\delta m_h^2 \sim m_S^2 \ln \Lambda/m_S$, $m_S \sim \text{TeV} \dots$ no SUSY seen, $m_S \gg \text{TeV} \rightarrow$ back to SM fine-tuning

.....why worry? worse (unrelated?) fine tunings: cosmological const:

$$\left[\frac{\rho_v}{\rho} \approx \frac{(2.3 \times 10^{-12} \text{ GeV})^4}{(10^{19} \text{ GeV})^4} \right]$$

c). Forget it. Use an EFT approach, ignore the UV completion.

- Fine tuning in SUSY models:

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (B_0 \mu_0 H_1 \cdot H_2 + h.c.) + \lambda_1/2 |H_1|^4 + \lambda_2/2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 \\ + \lambda_4 |H_1 \cdot H_2|^2 + \left[\lambda_5/2 (H_1 \cdot H_2)^2 + \lambda_6 |H_1|^2 (H_1 \cdot H_2) + \lambda_7 |H_2|^2 (H_1 \cdot H_2) + h.c. \right]$$

$$m^2 \equiv m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - B_0 \mu_0 \sin 2\beta, \quad \text{UV : } m_{1,2}^2 = m_0^2 + \mu_0^2$$

$$\lambda \equiv \frac{\lambda_1}{2} \cos^4 \beta + \frac{\lambda_2}{2} \sin^4 \beta + \frac{\lambda_{345}}{4} \sin^2 2\beta + \sin 2\beta (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta)$$

$$\mathbf{2 \ constraints:} \quad v^2 + \frac{m^2}{\lambda} = 0, \quad \frac{\partial m^2}{\partial \beta} - m^2 \frac{\partial \lambda}{\partial \beta} = 0,$$

- Problem: scales vs. couplings “tension” (tuning):

$$v^2 = -m^2/\lambda, \quad \text{with } v = \mathcal{O}(100 \text{ GeV}), \quad \lambda < 1, \quad \mathbf{but} \quad m_{1,2}, B_0 \text{ and } m \sim \mathcal{O}(1 \text{ TeV}).$$

- for $m_h > m_Z$ large loop effects needed i.e. large $m_{1,2}, B_0 \dots$; also a problem of couplings (λ small)

$\Rightarrow$ **Solution: increase λ by** 1.- quantum corrections

2.- corrections from “new physics” beyond MSSM.

- Measures of fine tuning. Fine-tuning vs likelihood.

$$\Delta_{max} \equiv \max_{\gamma} |\Delta_{\gamma}|, \quad \Delta_{\gamma} \equiv \frac{\partial \ln v^2}{\partial \ln \gamma^2}, \quad \gamma = \{m_0, m_{1/2}, \mu_0, A_0, B_0\}, \quad v = \text{EW scale}$$

Ellis, Enqvist, Nanopoulos, Zwirner (1986)

Barbieri, Giudice (1988)

- based on physical grounds/intuition. Mathematical support?
- small Δ preferable (?) what is “small” Δ ? usually ≤ 20 ? ≤ 100 ?, 1000? sufficient/necessary?*
- many other definitions for $\Delta \Rightarrow$ different results?

see: Anderson, Castano, hep-ph/9409419

$$\Delta_q = \left[\sum_{\gamma} \Delta_{\gamma}^2 \right]^{1/2}, \quad \text{“quadrature } \Delta \text{”}$$

- local in $\{\gamma\}$ space; to compare models, go global: $\int d\gamma$? $\int$ measure? $\Rightarrow$ Use Bayesian probability.
- *Take 2 models: model A: v. good $\chi^2/\text{d.o.f.}$ but large $\Delta = 1000 \rightarrow$ **model v. likely but...unnatural!**
 model B: good $\chi^2/\text{d.o.f.}$ but small $\Delta = 10. \rightarrow$ **model less likely, but ...natural!**
 $\Rightarrow$ **So what model is better?** same problem in Bayesian approach!
 $\Rightarrow$ **Fine-tuning (likelihood): tune γ_i to respect EW min (observables); they are part of same problem!**

II. Constrained likelihood accounts for naturalness:

D.G., G.G. Ross, arXiv:1208:0837.

Likelihood to fit the EW observables: O_j ($\tilde{\gamma}_i$ parameters). Usually:

$$L(\text{data}|\tilde{\gamma}_i; v, \beta) = \prod_{j \geq 1} L(O_j|\tilde{\gamma}_i; v, \beta), \quad \tilde{\gamma}_i = \gamma_i, y_t, y_b \dots; \quad \gamma_i = m_0, m_{1/2}, A_0, B_0, \mu_0 \dots (\text{susy})$$

and maximize it. $\chi^2 = -2 \ln L + \text{constant}$, minimize $\chi^2/\text{d.o.f.}$ Is this correct?

Observables are independent, parameters are not **! EW min: $f_1 = f_2 = 0$ (theor), $m_Z = m_Z^0$ (exp).

$$f_1(\tilde{\gamma}_i, v, \beta) = v - (-m^2/\lambda)^{1/2} = 0,$$

$$f_2(\tilde{\gamma}_i, v, \beta) = \tan \beta - \tan \beta_0(\tilde{\gamma}_i) = 0.$$

$$\begin{aligned} L(\text{data}|\tilde{\gamma}_i) &= m_Z^0 \int dv \, d(\tan \beta) \, \delta(m_Z - m_Z^0) \, \delta(f_1(\tilde{\gamma}_i; v, \beta)) \, \delta(f_2(\tilde{\gamma}_i; v, \beta)) \, L(\text{data}|\tilde{\gamma}_i; v, \beta) \\ &= v_0 \left[L(\text{data}|\tilde{\gamma}_i; v_0, \beta) \, \delta[f_1(\tilde{\gamma}_i; v_0, \beta)] \right]_{\beta=\beta_0(\tilde{\gamma}_i)}, \quad v_0 = 246 \text{GeV}, m_Z^0 = 91.18 \text{GeV}. \end{aligned}$$

with $m_Z = gv/2$; $m_Z^0 = gv_0/2$, $g^2 = g_1^2 + g_2^2$.

$\Rightarrow$ ** consequence: (constrained L) = (unconstrained L) $\times$ (Dirac delta, to respect EW min!).

Further

$$\delta(f_1(\vec{z})) = \frac{1}{|\nabla_z f_1|_o} \delta\left[\vec{n} \cdot (\vec{z} - \vec{z}^0)\right], \quad z_j \rightarrow \ln \tilde{\gamma}_j, \quad n_j = \frac{\partial_{z_j} f_1}{|\nabla f_1|_o}$$

where $\vec{n}$ is the normal to the EW min surface $f_1 = 0$. So:

$$\Rightarrow L(\text{data}|\tilde{\gamma}_i) = \frac{1}{\Delta_q} \delta\left(\sum_{j \geq 1} n_j (\ln \tilde{\gamma}_j - \ln \tilde{\gamma}_j^0)\right) L(\text{data}|\tilde{\gamma}_i; v_0, \beta) \Big|_{\beta=\beta_0(\tilde{\gamma}_i)}$$

where

$$\Delta_q = \left(\sum_j \Delta_{\tilde{\gamma}_j}^2\right)^{1/2}, \quad \Delta_{\tilde{\gamma}_j} = \frac{\partial \ln \tilde{v}}{\partial \ln \gamma_j}, \quad \tilde{v} = (-m^2/\lambda)^{1/2}$$

$$\Rightarrow L(\text{data}|\tilde{\gamma}_i^0) = \frac{1}{\Delta_q} L(\text{data}|\tilde{\gamma}_i; v_0, \beta) \Big|_{\beta=\beta_0(\tilde{\gamma}_i); \tilde{\gamma}_i=\tilde{\gamma}_i^0}$$

$\Rightarrow$ Naturalness is part of the (constrained) likelihood (not put in by hand).

$\Rightarrow$ Large “constrained” L demands to maximize the ratio of old/unconstrained L and Δ_q !

$\Rightarrow$ answers how to compare (likely but ... unnatural) and (unlikely but ...natural) models!

With $\chi^2 = -2 \ln L + \text{constant}$:

D.G., G.G. Ross, arXiv:1208:0837.

$$\chi_{\text{new}}^2(\tilde{\gamma}_i) = \left[\chi_{\text{old}}^2(\tilde{\gamma}_i) + \underbrace{2 \ln \Delta_q(\tilde{\gamma}_i)}_{\delta\chi^2} \right]_{f_1=0, f_2=0}$$

$\Rightarrow$ current $\chi^2/\text{d.o.f.}$ in SUSY models: worse than thought.

$\Rightarrow \chi^2/\text{d.o.f.} \approx 1. \Rightarrow$ Model independent bound: $\Delta_q \ll \exp(\text{d.o.f.}/2)$.

- Can also compute the profile likelihood.

- if y_t, y_b also fixed by m_t^0, m_b^0 , then $L \rightarrow L \delta(m_t - m_t^0) \delta(m_b - m_b^0)$, so $\tilde{\gamma}_i \rightarrow \gamma_i$.

$\Rightarrow$ Implications for χ^2 fits in SUSY models (fine tuning cost of the likelihood):

$$\Delta_q \approx 10 \Rightarrow \delta\chi^2/\text{d.o.f} \approx 4.6/9$$

$$\Delta_q \approx 100 \Rightarrow \delta\chi^2/\text{d.o.f} \approx 9.2/9$$

$$\Delta_q \approx 500 \Rightarrow \delta\chi^2/\text{d.o.f} \approx 12.5/9$$

$\Rightarrow$ for $\chi^2/\text{d.o.f.} \approx 1$ need small $\Delta_q \leq \mathcal{O}(10) \Rightarrow$ rule out models in probabilistic sense using only Δ_q !

• Can we have $\Delta_q \sim 10$ in SUSY models?

- Naturalness in the Bayesian approach.

$$p(a|b) p(b) = p(a \cap b) = p(b|a) p(a)$$

Bayes theorem:

[initial belief + data $\rightarrow$ updated belief].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\mathbf{data}) = \frac{L(\mathbf{data}|\gamma) p(\gamma)}{p(\mathbf{data})}, \quad p(\mathbf{data}) = \int L(\mathbf{data}|\gamma) p(\gamma) d\gamma, \quad \gamma: \{m_0, m_{1/2}, \mu_0, A_0, B_0; y_t, y_b, \dots\}.$$

- $p(\mathbf{data})$: “evidence”. Models 1, 2: $p_1(\mathbf{data})/p_2(\mathbf{data})$. $p(\gamma)$ =priors.
- EW constraints: $f_1(\gamma; v, \beta) = f_2(\gamma; v, \beta) = 0, \Rightarrow v(\gamma, \dots), \tan \beta_0(\gamma \dots)$.

$$\begin{aligned} p(\mathbf{data}) &= \int d\gamma p(\gamma) dv d(\tan \beta) \delta(m_Z - m_Z^0) \delta(f_1(\gamma; v, \beta)) \delta(f_2(\gamma; v, \beta)) L(\mathbf{data}|\gamma; \beta, v), \\ &= \int_{f_{1,2}=0} dS_\gamma \frac{L(\mathbf{data}|\gamma)}{\Delta_q(\gamma)} p(\gamma), \quad \Delta_q \equiv \left[\sum_\gamma \Delta_\gamma^2 \right]^{1/2}, \quad d\gamma \equiv \prod_i d\gamma_i, \quad p(\gamma) = p(\gamma_1, \dots, \gamma_i). \end{aligned}$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow \Delta_q$ induced by theor constraints (EW min), independent of priors $p(\gamma)$; large L/Δ_q needed !

$$\int_{R^n} h(z_1, \dots, z_n) \delta(g(z_1, \dots, z_n)) dz_1 \dots dz_n = \int_{S_{n-1}} dS_{n-1} h(z_1, \dots, z_n) \frac{1}{|\nabla_{z_i} g|},$$

III. Numerical results for naturalness in SUSY models:

D.G., H. M. Lee, M. Park, arXiv:1203:0569

- 1) CMSSM (constrained MSSM)
- 2) NUHM1 (non universal Higgs Mass)
- 3) NUHM2 (non universal Higgs Mass)
- 4) NUGM (non universal gaugino masses)

$$\begin{aligned}\gamma &= \{m_0, \mu_0, m_{1/2}, A_0, B_0\} \\ \gamma &= \{m_0, \mu_0, m_{H_1}^{uv} = m_{H_2}^{uv}, m_{1/2}, A_0, B_0\} \\ \gamma &= \{m_0, \mu_0, m_{H_1}^{uv}, m_{H_2}^{uv}, m_{1/2}, A_0, B_0\} \\ \gamma &= \{m_0, \mu_0, m_{\lambda_{1,2,3}}, A_0, B_0\}\end{aligned}$$

• Experimental Constraints:

- SUSY masses:
- muon magnetic moment:
- $b \rightarrow s \gamma$
- $B_s \rightarrow \mu^+ \mu^-$
- ρ -parameter
- dark matter
- ATLAS/CMS: $m_h \approx 126 \text{ GeV}$ and δa_μ not imposed.

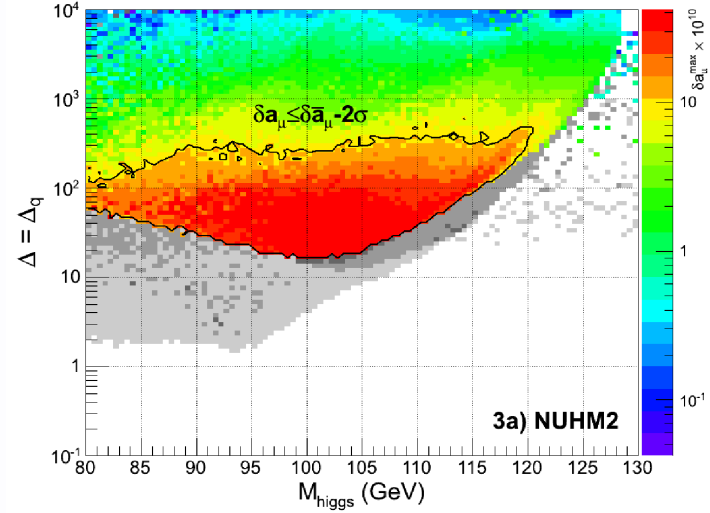
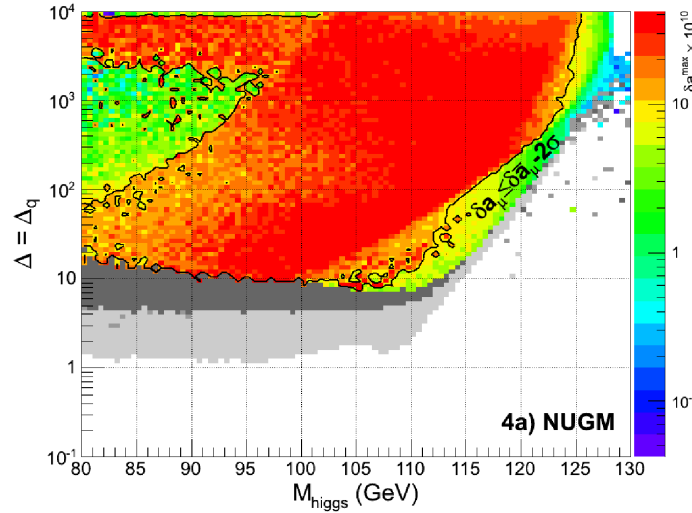
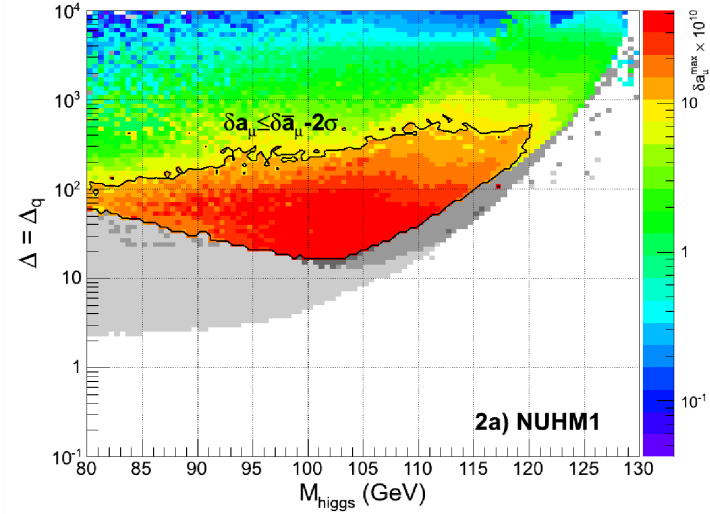
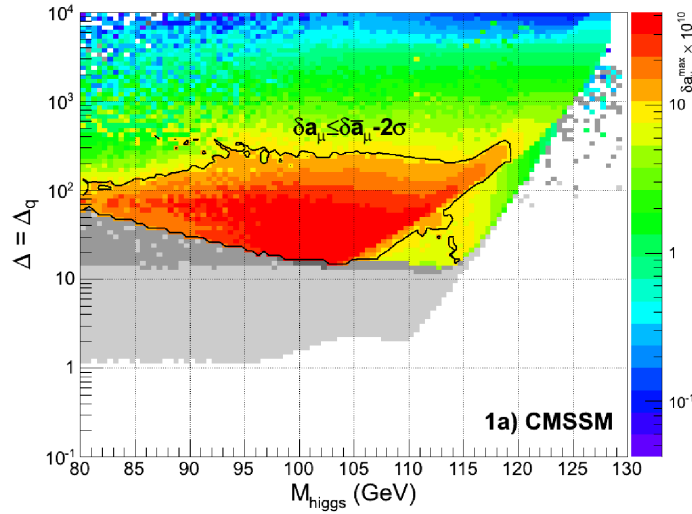
micrOmegas 2.4.5, "MSSM/masslim.c"

$$\begin{aligned}\delta a_\mu &= (25.5 \pm 2 \times 8) \times 10^{-10} \text{ at } 2\sigma \\ 3.03 &< 10^4 \text{ Br}(b \rightarrow s \gamma) < 4.07 \text{ at } 2\sigma \\ \text{Br}(B_s \rightarrow \mu^+ \mu^-) &< 1.08 \times 10^{-8} \text{ at } 2\sigma \\ -0.0007 &< \delta \rho < 0.0033 \text{ at } 2\sigma \\ \Omega h^2 &= 0.1099 \pm 3 \times 0.0062 \text{ at } 3\sigma\end{aligned}$$

- Tools: micrOMEGAs 2.4.5, SoftSUSY 3.2.4. Random scan. $\Rightarrow \Delta_q, \Delta_{max}$ at 2-loop LL.

- Impact of m_h , δa_μ on Δ_q . [2-loop, all $\{\gamma, \tan \beta\}$ all values]

D.G., H. M. Lee, M. Park, arXiv:1203:0569

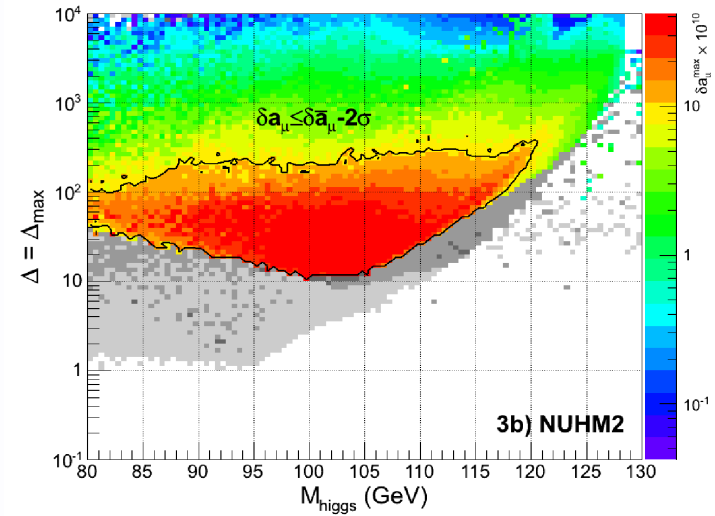
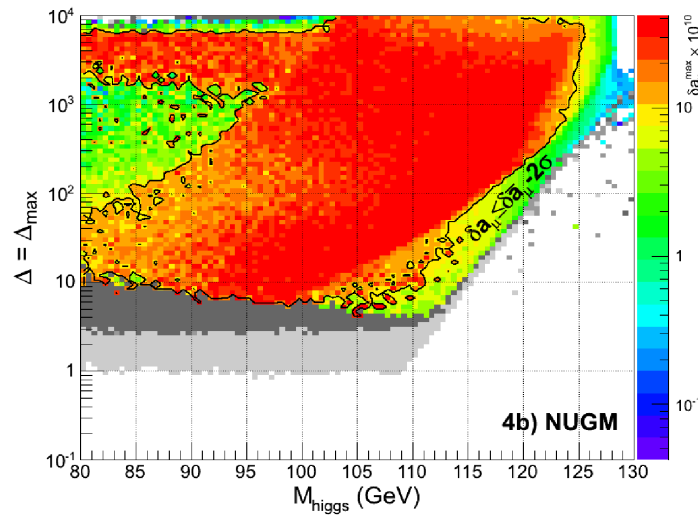
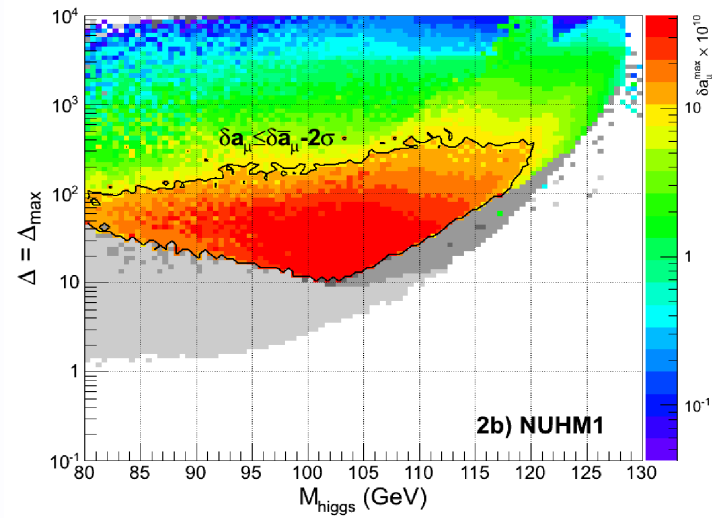
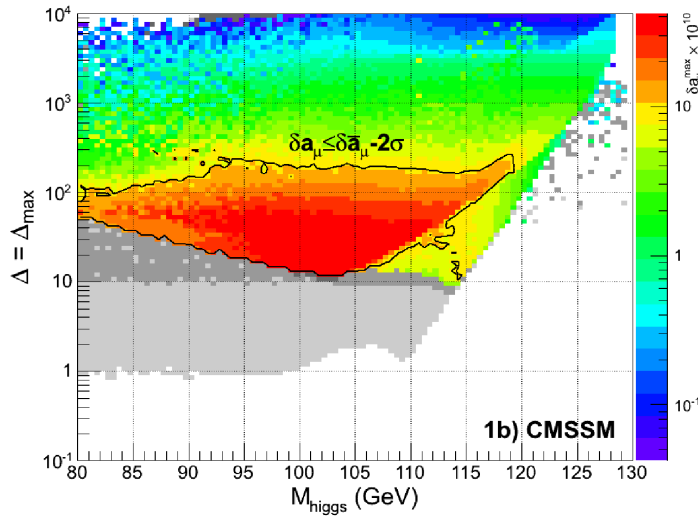


- grey 0: excluded by SUSY; grey 1: $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

$\Rightarrow m_h$ strongest impact: $\Delta_q \sim e^{m_h}$. $\Delta_q \sim 1000$ (!) near 125 GeV. NUGM better behaviour.

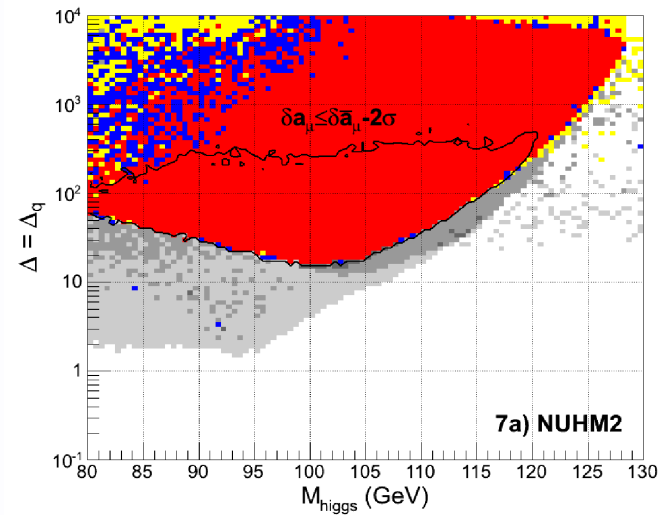
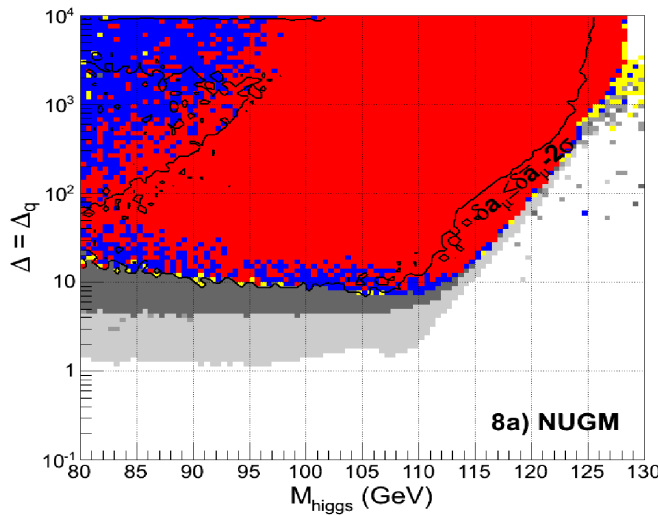
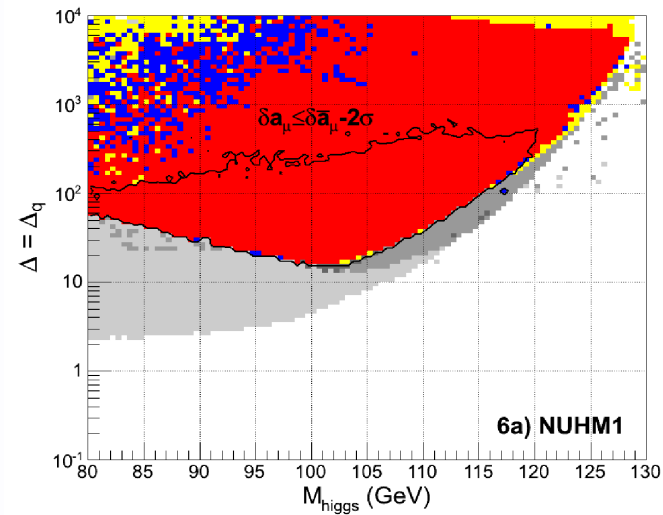
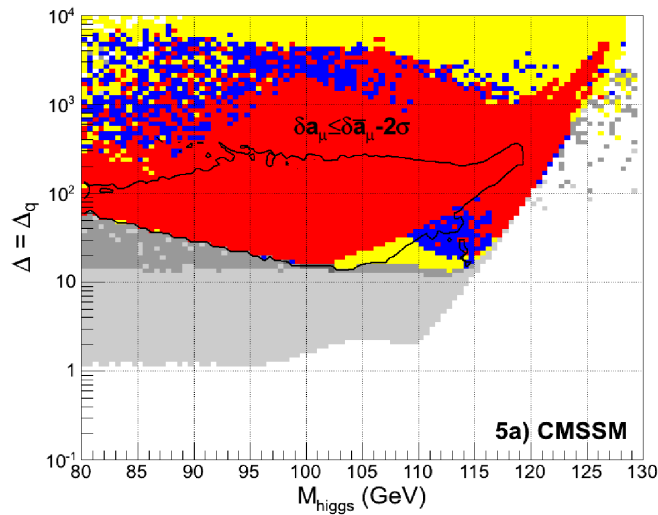
- Impact of m_h , δa_μ on Δ_{max} . [2-loop, all $\{\gamma, \tan \beta\}$ all values].

D.G., H. M. Lee, M. Park, arXiv:1203:0569



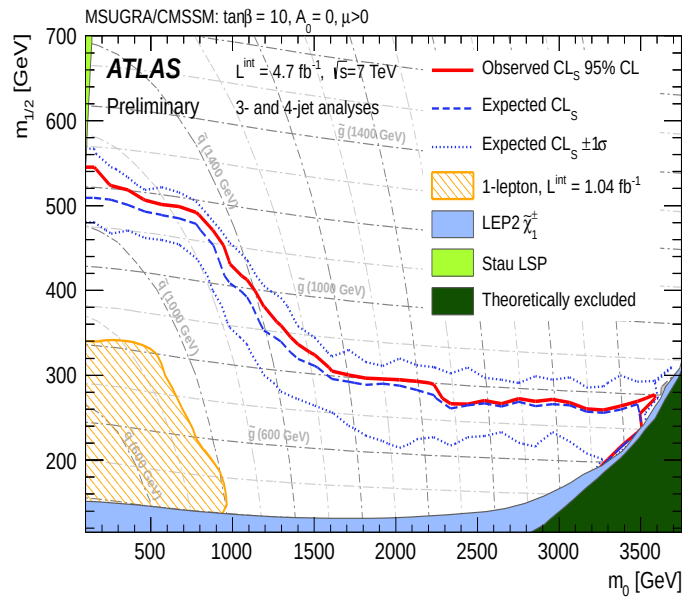
- δa_μ : 2σ contour (red) [smaller δa_μ outside]. $\Rightarrow \Delta_{max}$ identical behaviour to Δ_q (marginally smaller)

- Impact of m_h , dark matter on Δ_q : [2-loop, $\{\gamma, \tan \beta\}$ all values]. D.G., H. M. Lee, M. Park, arXiv:1203:0569

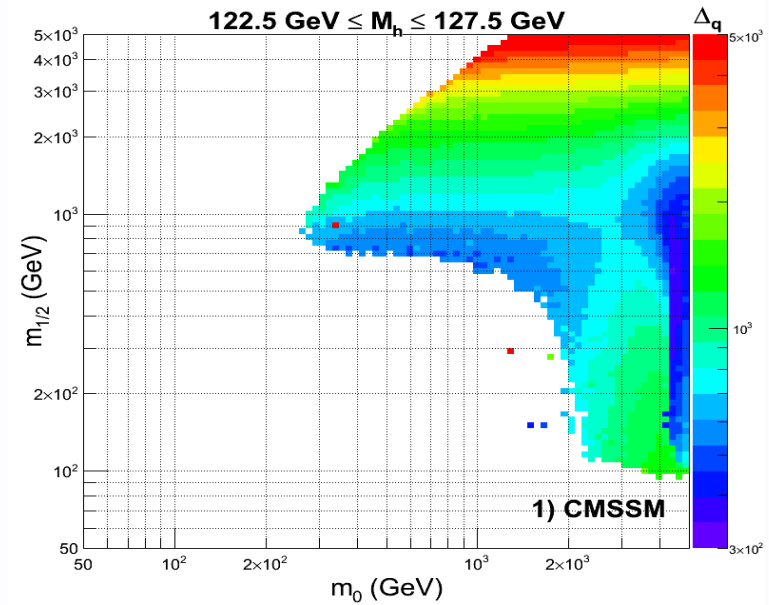


- blue: consistent with Ωh^2 ; Red: saturate Ωh^2 within 3σ . Grey areas ruled out by data.
- $\Rightarrow \Delta_{max}$ (not shown) nearly identical plots to Δ_q , (marginally smaller for same Higgs mass).

- SUSY searches from ATLAS and impact on parameter space (CMSSM only):



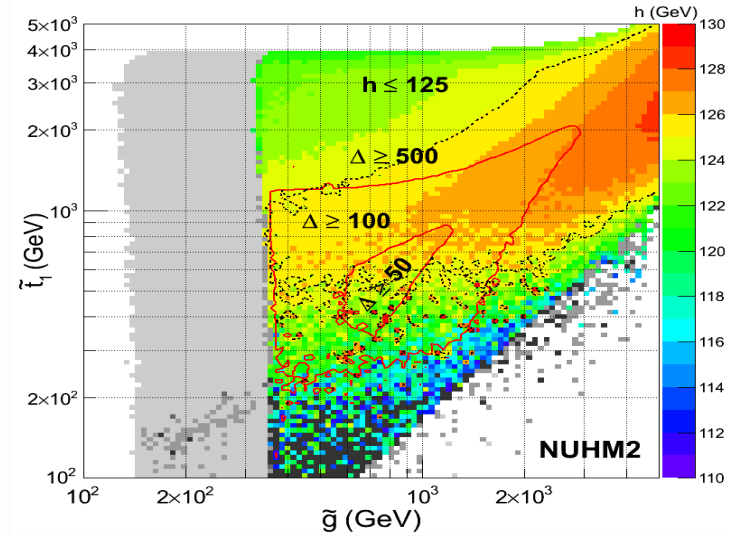
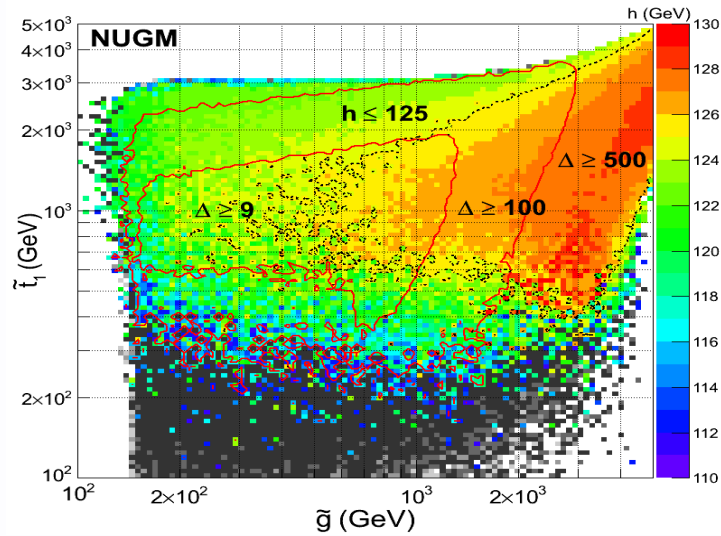
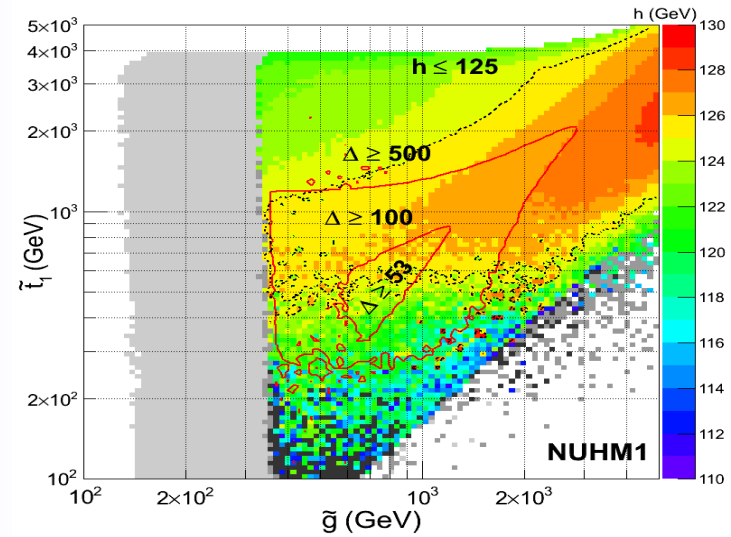
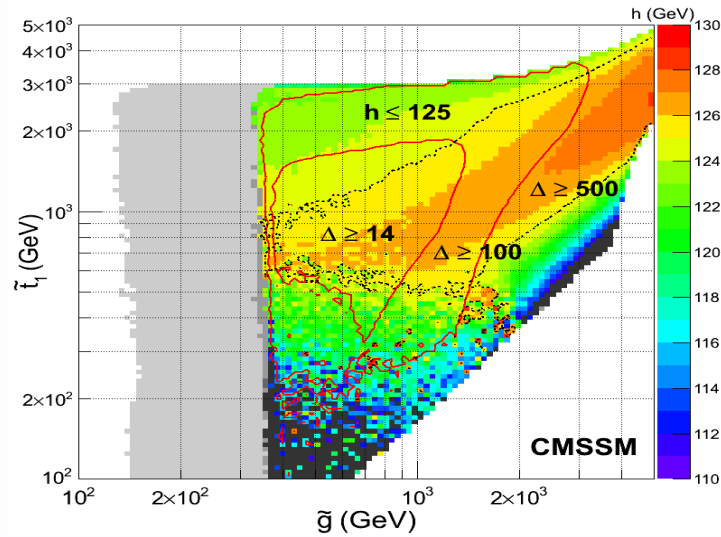
ATLAS-CONF-2012-041



CMSSM, $122.5 \leq m_h \leq 127.5 \text{ GeV}$ and Δ_q cost.
[all values for γ , $\tan\beta$]. $\Delta_q > 500$.

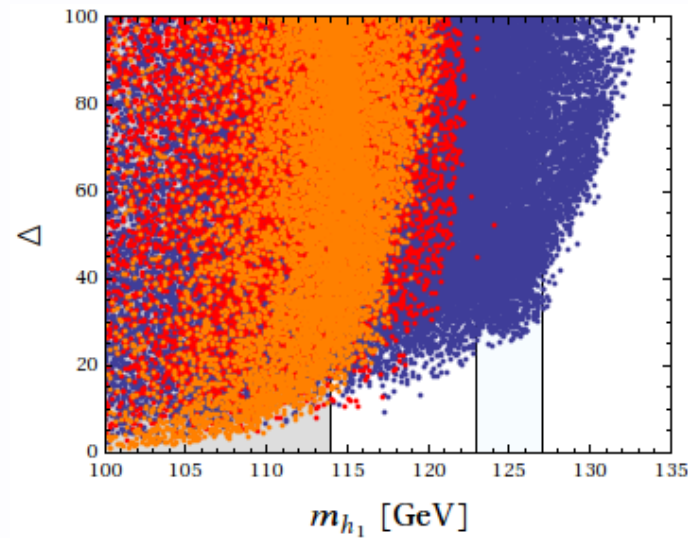
D.G., Hyun Min Lee, Myeonghun Park (2012)

- Stop vs Gluino with largest m_h and min Δ_q . $[\{\gamma, \tan \beta\} \text{ all values}]$ D.G., H. M. Lee, M. Park, arXiv:1203:0569



$\Rightarrow$ constraints on m_h strongly reduce the viable regions.

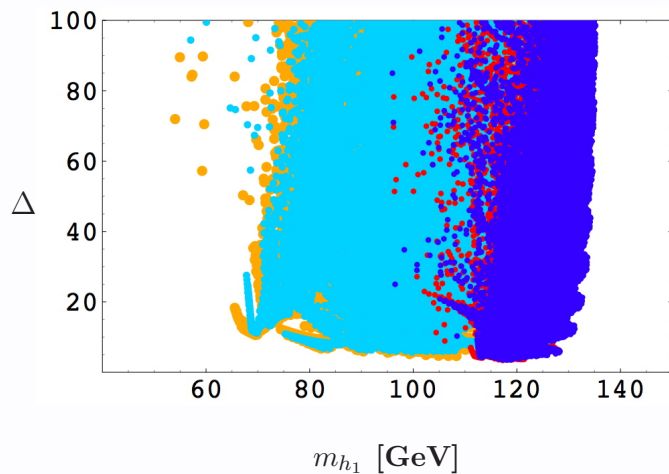
IV. Fine tuning beyond MSSM: NMSSM, GNMSSM, ...



MSSM
 NMSSM: $W = W_Y + \lambda S H_1 H_2 + \kappa S^3$
 GNMSSM: $W = W_Y + (\mu + \lambda S) H_1 H_2 + M_* S^2 + \kappa S^3$
 $\Delta < 30$ for $m_h \leq 126 \text{ GeV}$.

G.G. Ross et al, arXiv:1205.1509

U. Ellwanger et al arXiv:1107.2472



Decoupling limit: MSSM with a massive gauge singlet:

⇒ MSSM + d=5 operator:

$$W = W_Y + \mu H_1 H_2 + \lambda S H_1 H_2 + M_* S^2$$

$$\Rightarrow W = W_Y + \mu H_1 H_2 + \zeta_0 (H_1 H_2)^2, \quad \zeta_0 \sim \lambda/M_*$$

$$\Delta < 20 \text{ for } m_h \leq 126 \text{ GeV}.$$

S. Cassel, D.G., G.G. Ross, NPB 825(2010)

⇒ massive gauge singlet ⇒ reduces fine tuning considerably, to $\Delta \sim 20$.

• Fine tuning beyond MSSM: MSSM + d=6 operators

$$\begin{aligned}
\mathcal{O}_j &= \int d^4\theta \mathcal{Z}_j (H_j^\dagger e^{V_j} H_j)^2, \quad (j = 1, 2). & \mathcal{O}_3 &= \int d^4\theta \mathcal{Z}_3 (H_1^\dagger e^{V_1} H_1) (H_2^\dagger e^{V_2} H_2) \\
\mathcal{O}_4 &= \int d^4\theta \mathcal{Z}_4 (H_2 H_1) (H_2 H_1)^\dagger, & \mathcal{O}_k &= \int d^4\theta \mathcal{Z}_k (H_k^\dagger e^{V_k} H_k) H_2 H_1 + h.c., \quad (k=5,6) \\
\mathcal{O}_7 &= \int d^2\theta \mathcal{Z}_7 \text{Tr} W_i^\alpha W_{i,\alpha} (H_2 H_1) + h.c.,
\end{aligned}$$

where $\mathcal{Z}_j(S, S^\dagger) = \alpha_{j0} + \alpha_{j1} S + \alpha_{j1}^* S^\dagger + \alpha_{j2} m_0^2 S S^\dagger$, $\alpha_{jk} \sim 1/M_*^2$, $S = m_0 \theta\theta$

$\mathcal{O}_{1,2,3}$: generated by massive T, U(1); $\mathcal{O}_4$: singlet, T. $\mathcal{O}_{5,6}$: 2 D, singlet.

$$\begin{aligned}
\Rightarrow \delta m_h^2 &= -2 v^2 [(\alpha_{30} + \alpha_{40}) \mu_0^2 - \alpha_{20} m_Z^2] - \frac{(2 \zeta_0 \mu_0)^2 v^4}{m_A^2 - m_Z^2} + \frac{v^2 \cot \beta}{m_A^2 - m_Z^2} [4 m_A^2 \mu_0^2 (2\alpha_{50} + \alpha_{60}) \\
&\quad - (2\alpha_{60} - 3\alpha_{70}) m_A^2 m_Z^2 - (2\alpha_{60} + \alpha_{70}) m_Z^4] + \mathcal{O}(1/(M_*^2 \tan^2 \beta))
\end{aligned}$$

$\Rightarrow \alpha_{j0}$ (choice?) $\Rightarrow$ increase m_h , reduce fine-tuning by:

D.G. et al, NPB 848(2011), NPB 831(2010),

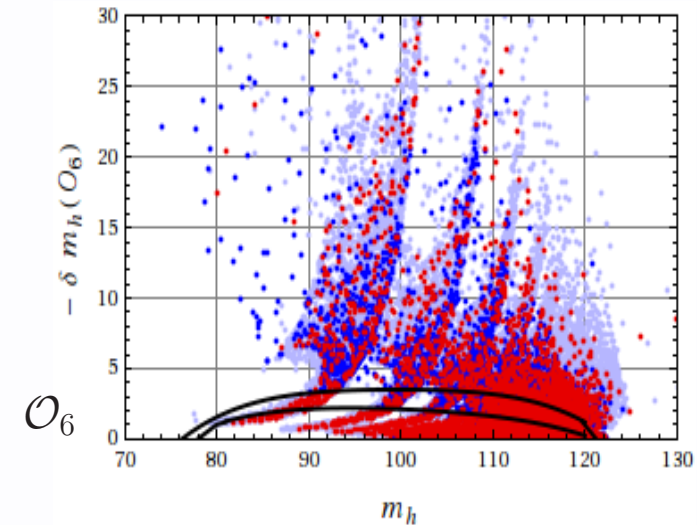
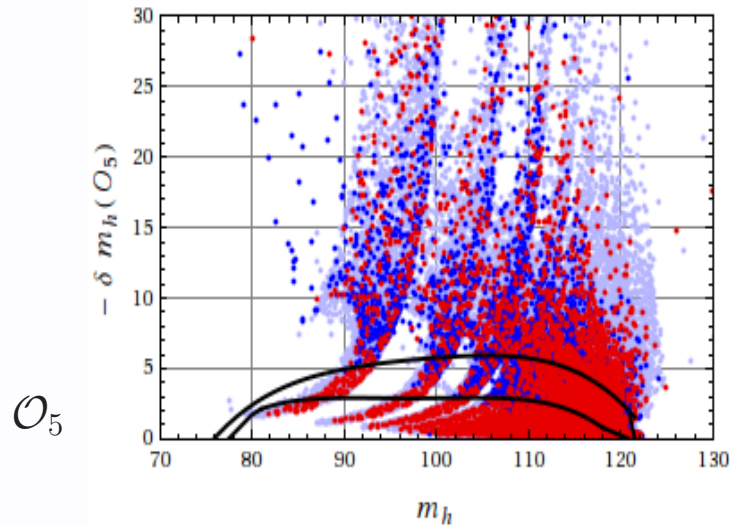
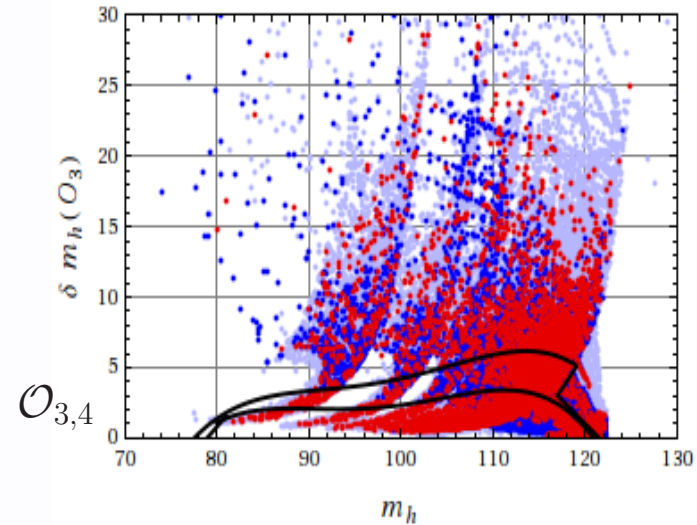
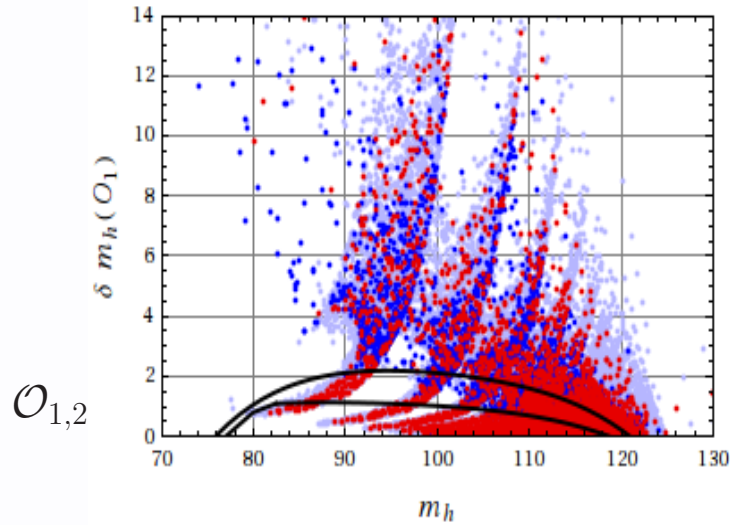
M. Carena et al, PRD 85(2012), PRD 81, 82(2010),

F. Boudjema et al, PRD 85 (2012)

$$\Delta_q(m_h) \approx \exp(-\delta m_h / \text{GeV}) \Delta_q(m_h) \Big|_{\text{CMSSM}}$$

- Corrections to m_h from MSSM + d=6 operators

D.G., I. Antoniadis, E. Dudas, P. Tziveloglou, NPB B848(2011)1.



$$\delta m_h = (m_h^2 + \delta m_h^2)^{1/2} - m_h, \quad m_h: \text{2-loop LL CMSSM.} \quad M_* = 8 \text{ TeV.} \quad \text{take: } \alpha_{j0} \leq 1/4$$

$$\Rightarrow \text{top curve: } \Delta < 200: \quad m_h < 122 \text{ GeV}, \quad \delta m_h < 6 \text{ GeV.} \quad \pm 1 \text{ GeV } (\delta m_h) \leftrightarrow \mp 1 \text{ TeV } (\delta M_*).$$

$$\Rightarrow \text{largest } \delta m_h: \mathcal{O}_3, \mathcal{O}_4. \quad \Delta_q(128 \text{ GeV}) \approx e^{-6} \Delta_q(128 \text{ GeV})_{\text{MSSM}} = \mathcal{O}(10).$$

• Conclusions:

- ⇒ Naturalness (Δ_q): intrinsic part of the constrained likelihood! (correlations of γ_i).
- ⇒ large $L_{new} = L_{old}/\Delta_q$ required, or equivalently $\chi^2_{new} = \chi^2_{old} + 2 \ln \Delta_q$.
- ⇒ current $\chi^2/\text{d.o.f.}$ in SUSY models: worse than thought. $\Delta_q \ll \exp(\text{d.o.f.})$. (~ 10)
- ⇒ Numerical results: Δ_q : of all EW data, m_h strongest constraint in CMSSM, NUHM1,2, NUGM.
- ⇒ $\Delta_q, \Delta_{max} \sim \exp(m_h/\text{GeV}) \sim 500 - 1000$ in these models, for $m_h \sim 126$ GeV.
- ⇒ GNMSSM preferable, $\Delta_q = \mathcal{O}(10)$.
- ⇒ Beyond MSSM: with d=5, d=6 operators. ⇒ increase m_h , reduce Δ by $\exp(\delta m_h/\text{GeV})$.
 - ⇒ best scenario: extra massive singlet, extra U(1).