Flux and Chiral Matter in SU(5)xU(1) F-Theory GUT Models and their IIB Analogue

joint work with S. Krause and T. Weigand: arXiv:1109.3454 & arXiv:1202.3138

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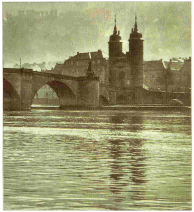

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Possibility of **exceptional groups** and **locality** are the crucial reasons to consider F-theory!

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Type IIB interpretation

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- F-theory basically 'book-keeping' device to describe vacua of IIB;
- From duality via M-theory and assumptions on non-compact 4d space, we find that compact space Y_4 has to be elliptically fibred CY$_4$;

Basics of F-theory: Non-abelian gauge symmetries

- Describe elliptic fibration with Weierstraß equation,

$$y^2 = x^3 + f(y_i)\, x\, z^4 + g(y_i)\, z^6\, ;$$

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- Obtain also E_6, E_7 and E_8 gauge groups beside of A_n, C_n and D_n;

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- Give Weierstraß equ. in Tate form:

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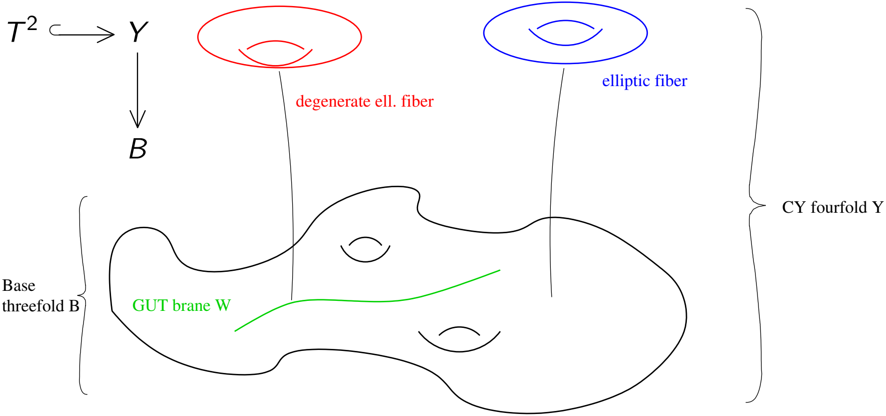
- At codimension 3, Δ enhances further (P vanishes to a higher order):

  $$E_6:\ a_1 = a_{2,1} = 0\,, \qquad SO(12):\ a_1 = a_{3,2} = 0\,;$$

Picture of elliptically fibred CY_4



by J.K.

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- Related to the Cartan generators of gauge symmetry;

	x	y	z	e_1	e_2	e_3	e_4	e_0
W	·	·	·	·	·	·	·	1
c_1	2	3	·	·	·	·	·	·
Z	2	3	1	·	·	·	·	·
E_1	-1	-1	·	1	·	·	·	-1
E_2	-2	-2	·	·	1	·	·	-1
E_3	-2	-3	·	·	·	1	·	-1
E_4	-1	-2	·	·	·	·	1	-1

cf. Tops over $\mathbb{P}_{231}$

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- $U(1)$ since restriction of c.s. induces additional section; [Morrison, Vafa '97]

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W	·	·	·	·	·	·	·	·	1
c_1	2	3	·	·	·	·	·	·	·
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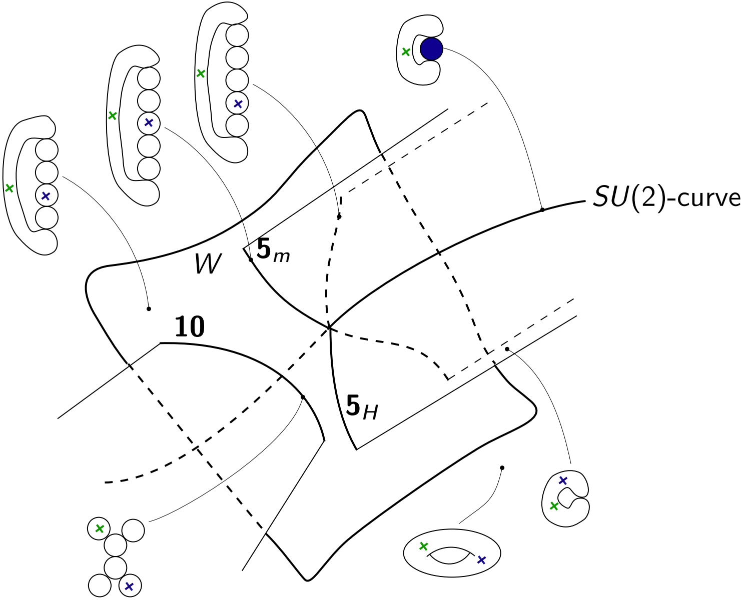
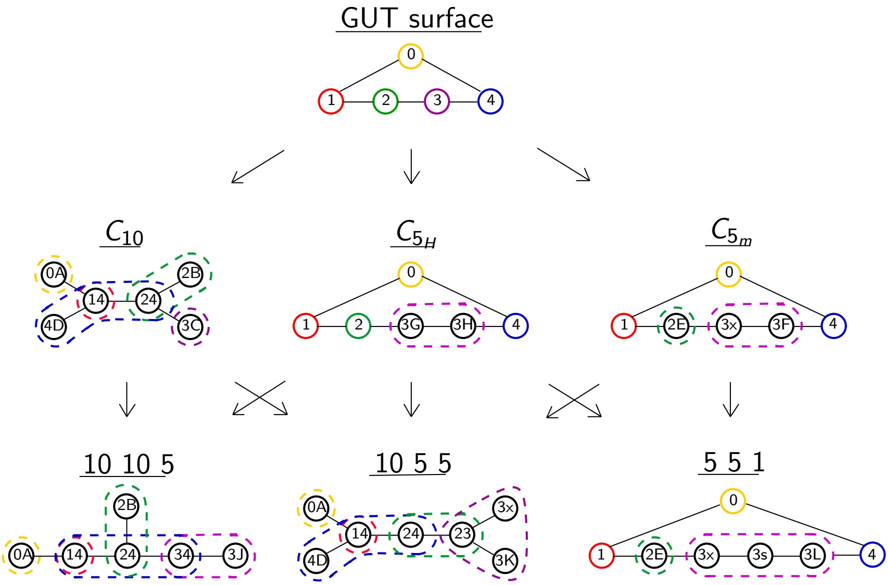
cf. Tops over $\mathcal{B}_s\mathbb{P}_{231}$

Resolved geometry



Some details on $SU(5) \times U(1)$ case: matter & Yukawas I



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  2. M2-branes wrapping one (simple roots) or several $\mathbb{P}^1$'s $\leftrightarrow$ positive roots; inverse orientation $\leftrightarrow$ negative roots;

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- Similar things happen at $\mathbf{10}$-curve;
- The E_6-point differs from naïve expectations—e.g. not extended [Esolé, Yau '11]; However, Yukawas still exist [Marsano, Schäfer-Nameki '11];

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- Generic case $\dim(H^{2,2}(Y_4) \cap H^4(Y_4, \mathbb{Z})) \sim (\dim H^{1,1})^2$; $G_4 = \omega \wedge \nu$ violates Poincaré invariant; Remedy: tuning complex structure to obtain appropriate G_4 without introducing new divisors [Braun, Collinucci, Valandro '11]

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- Generic combination of this fluxes breaks $SU(5)$
- Only $G_4 = F_2^{(X)} \wedge w_X$ with

$$w_X = 5([S] - [Z] - [\bar{K}]) + (2,4,6,3)_i\,[E_i]$$

leaves $SU(5)$ invariant; (Shioda map [Morrison, Park '12])

Chirality

- Type IIB: chirality along curve of intersecting branes given by

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- Matter surfaces, C_{R_q}, consist of linear combinations of blow-up $\mathbb{P}^1$'s fibred over enhancement curve $\mathcal{C}_{R_q}$;
- Recall: linear combination such that in dual M-theory picture, M2-brane wrapping this combination is in one of the states of R_q ;

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- Want to know chirality for $G_4^{(X)}$

$$\int_{C_{R_q}} G_4 = \int_{C_{R_q}} w_X \wedge F_X \quad \Rightarrow \quad q_R \int_{\mathcal{C}_{R_q}} F_X \, ;$$

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- Rearranging of states in terms of $SO(10)$ representations

Matter curve $\mathcal{C}_R$	R_q	$SO(10)$ origin	GUT interpretation
$\{a_1 = w = 0\}$	$\mathbf{10}_{-1}$	$\mathbf{16}$	(Q_L, U_R^c, e_R^c)
$\{a_{3,2} = w = 0\}$	$\bar{\mathbf{5}}_3$	$\mathbf{16}$	(D_R^c, L)
$\{a_1 \, a_{4,3} - a_{2,1} \, a_{3,2} = w = 0\}$	$\mathbf{5}_2 + \bar{\mathbf{5}}_{-2}$	$\mathbf{10}$	Higgs
$\{a_{3,2} = a_{4,3} = 0\}$	$\mathbf{1}_{-5}$	$\mathbf{16}$	N_R^c

D3 tadpole

- Flux induces D3-tadpole

$$\frac{1}{2}\int_{\hat{Y}_4} G_4^{(X)} \wedge G_4^{(X)} = \int_{B_3} F_X \wedge F_X \wedge (15\,[W] - 25\,c_1(B_3))$$

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- Tadpole contribution differs from local constructions via spectral cover;
- Has to be cancelled by D3-branes and geometric induced tadpole:

$$N_{D3} + \frac{1}{2}\int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{\chi(\hat{Y}_4)}{24};$$

Brane recombination I

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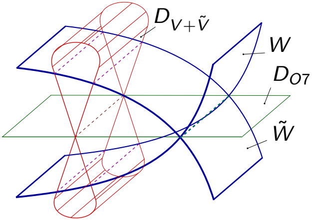
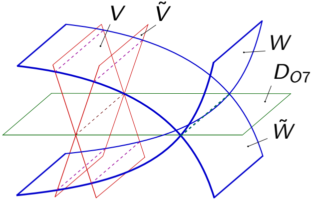


(c) Non-restricted Models



(d) $U(1)$-Restricted Models

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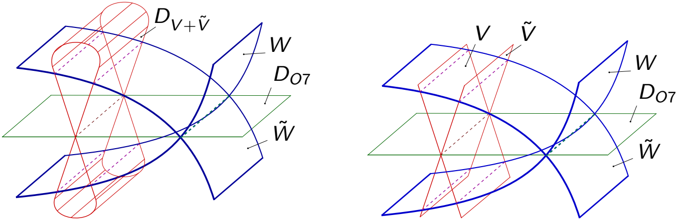

(e) Non-restricted Models (f) $U(1)$-Restricted Models

- Change in the Euler numbers:

$$\Delta^{SU_n \times U_1}_{SU_n} \chi(Y_4) = 3\,\chi(C_{34})$$

Brane recombination II

- Conservation of D3-charge $\Rightarrow$ change in G_4

$$G_4^{su_n} = \tilde{G}_4^X(\mathcal{P}) + \tilde{G}_4^\lambda, \qquad G_4^{su_n \times u_1} = G_4^X(\mathcal{F}) + G_4^{\lambda'}$$

with

$$\mathcal{P} - \mathcal{F} = -\tfrac{1}{2} c_1(C_{34}), \qquad \lambda - \lambda' = \tfrac{1}{2}$$

and

$$G_4^\lambda = \lambda \left(E_2 \wedge E_4 + \tfrac{1}{5}(2, -1, 1, -2)_i E_i \wedge \bar{K} \right);$$

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- In agreement with observation:
$[\mathcal{F}] = [\mathcal{P}] - \frac{1}{2}[a_{6,5}]$ & $0 \leq [\mathcal{P}] \leq [a_{6,5}]$

Sen/weak coupling limit

- Following [Sen '96 '97][Donagi & Wijnholt '09], one possible weak coupling limit:

$$a_3 \to \epsilon\, a_3, \qquad a_4 \to \epsilon\, a_4, \qquad a_6 \to \epsilon^2\, a_6;$$

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with $b_8 = \frac{1}{4}(b_2(a_3^2 + 4\, a_6) - (a_1\, a_3 + 2a_4)^2)$ and $b_2 = a_1^2 + 4\, a_2$

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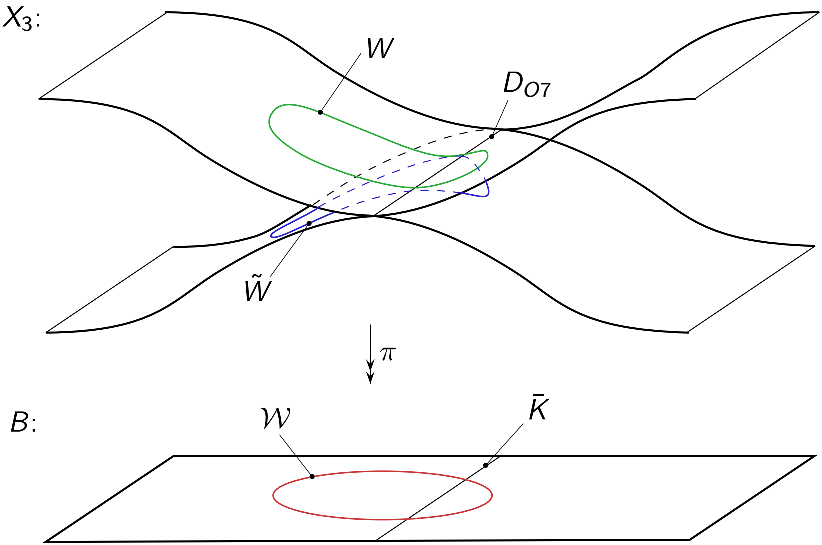
- Define a double cover of B (CY_3):

$$X_3 : \xi^2 = b_2,$$

  with orientifold involution

$$\sigma : \xi \longrightarrow -\xi.$$



X_3:
W
D_{O7}
$\tilde{W}$
π
B:
$\mathcal{W}$
$\bar{K}$

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- Consider on the IIB side D5-tadpole cancelling flux configurations (F_a, F_b) (a $\cong$ GUT & b $\cong$ additional)

		$n = 2k+1$	$n = 2k$
F_X	$:=$	$\left(\frac{1}{2n}F, -\frac{1}{2}F\right)$	$\left(0, \frac{1}{n}F\right)$
F_λ	$:=$	$\left(\frac{2\lambda}{n}D_{O7}, 0\right)$	$\left(\frac{2\lambda}{n}D_{O7}, 0\right)$

with $F \in H^2_+(X_3)$ and $B^- \in H^2_-(X_3)$;

Identification of the fluxes II:

- Calculate also the induced chiralities:

State	Chirality under F_λ	Chirality under F_X
$\mathbf{10}_{(2,0)}$	$\frac{\lambda}{5}\int_{X_3} D_{O7} W_+^2$	$\frac{1}{10}\int_{X_3} D_{O7} W_+ F$
$\mathbf{5}_{(1,-1)}$	$-\frac{\lambda}{10}\int_{X_3} D_{O7} W_+^2$	$\frac{1}{10}\int_{X_3} \left(9D_{O7} W_+ - 6W_+^2\right) F$
$\mathbf{5}_{(1,1)}$	$-\frac{\lambda}{10}\int_{X_3} D_{O7} W_+^2$	$\frac{1}{10}\int_{X_3} -\left(10D_{O7} W_+ - 6W_+^2\right) F$
$\mathbf{1}_{(0,2)}$	0	$\frac{1}{10}\int_{X_3} -5\left(12D_{O7}^2 - 17D_{O7} W_+ + 6W_+^2\right) F$

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- Comparison of chirality and induced D3-brane charge shows:

$$G_4^\lambda \leftrightarrow F_\lambda, \qquad G_4^X(\mathcal{F}) \leftrightarrow F_X = \left(\tfrac{1}{10}F, -\tfrac{1}{2}F\right) \quad \text{with} \quad F = \pi^*(\mathcal{F});$$

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- $U(1)_X$ charges q agree with spectral cover construction;
- Computed induced D3-tadpole; Correction to spectral cover construction
- Found (convincing arguments for) IIB interpretation of fluxes;

Thank you for your attention!