Phenomenological aspects of magnetized brane models

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4. Massive modes

   Hamada, T.K, arXiv:1207.6867
5. Summary

also based on several collaborations with H.Abe, K.S.Choi, Y.Hamada, H.Ohki, A.Oikawa, K.Sumita

1 Introduction

(Type IIA) intersecting D-brane models
and (type IIB) magnetized D-brane models
are T-dual each others.

These string models are quite interesting
from phenomenological viewpoints.

Indeed, many models with (semi-) realistic spectra
have been constructed, in particular
within intersecting D-brane models.
(See e.g. Ibanez-Uranga's textbook
and references therein.)

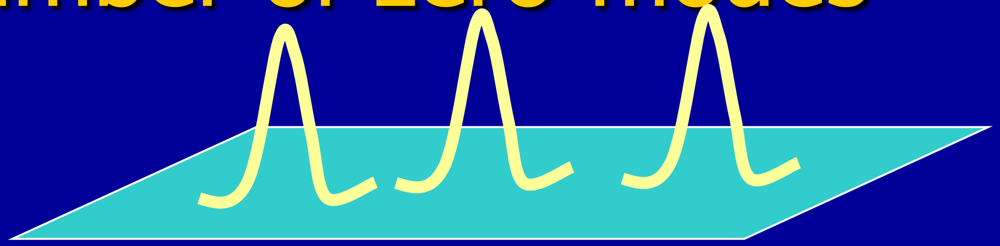
Zero-modes

When we start with higher dimensional field theory, the 4D massless modes correspond to the solutions of zero-mode equation

$$i\gamma^m D_m \psi = 0$$

⇒ non-trival zero-mode profile

the number of zero-modes



4D effective theory

Higher dimensional Lagrangian (e.g. 10D)

$$L_{10} = g\int d^4x d^6y\, \overline{\lambda}(x,y) A(x,y) \lambda(x,y)$$

integrate the compact space ⇒ 4D theory

$$L_4 = Y\int d^4x \overline{\chi}(x)\varphi(x)\chi(x)$$

$$Y = g\int d^6y \overline{\psi}(y)\phi(y)\psi(y)$$

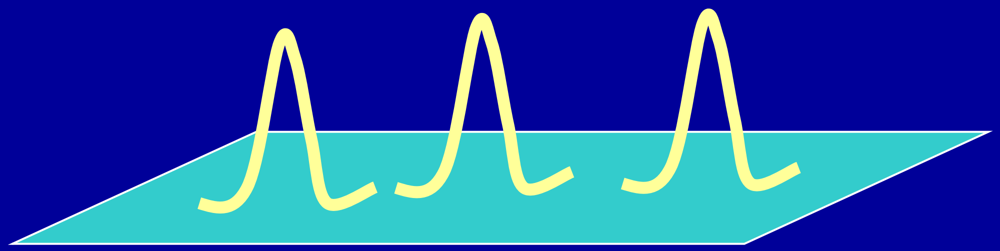
Coupling is obtained by the overlap integral of wavefunctions

Couplings in 4D

Zero-mode profiles are quasi-localized

far away from each other in compact space

⇒ suppressed couplings



Chiral theory

When we start with extra dimensional field theories,
how to realize chiral theories is one of important issues from the viewpoint of particle physics.

$$i\gamma^m D_m \psi = 0$$

Zero-modes between chiral and anti-chiral fields are different from each other on certain backgrounds,
e.g. CY, toroidal orbifold, warped orbifold, magnetized extra dimension, etc.

Magnetic flux

$$i\gamma^m D_m \psi = 0$$

The limited number of solutions with
non-trivial backgrounds are known.

Generic CY is difficult.

Toroidal/Wapred orbifolds are well-known.

Background with magnetic flux is
one of interesting backgrounds.

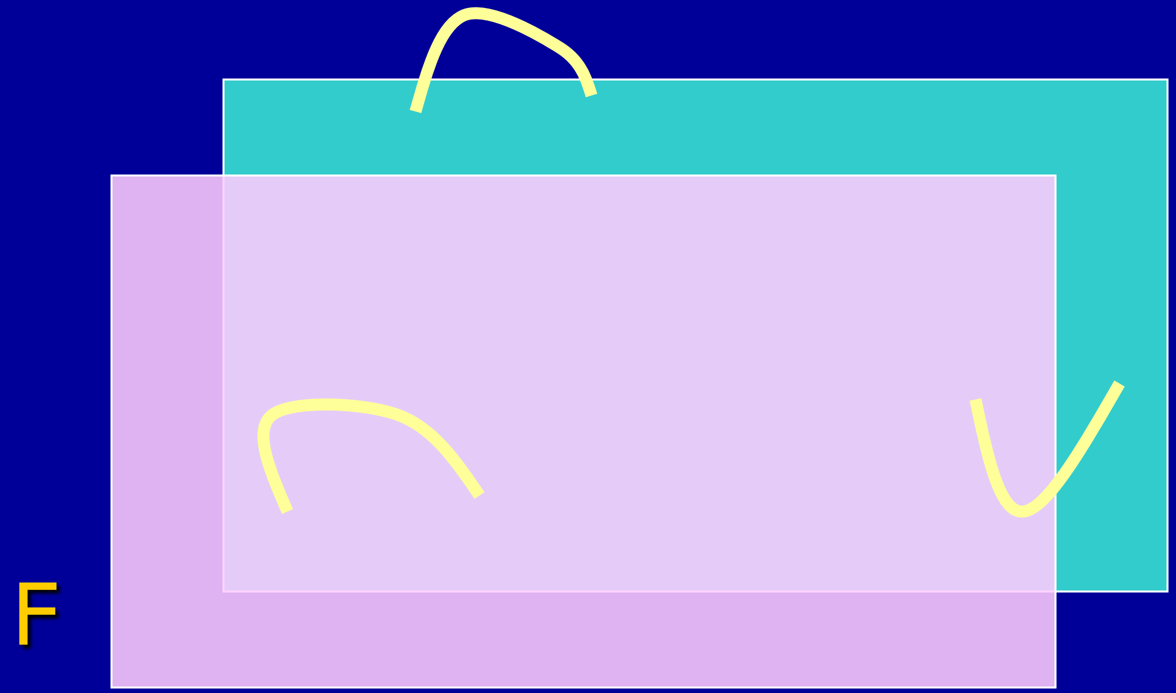
Phenomenology of magnetized brane models

It is important to study phenomenological aspects of magnetized brane models such as Yukawa couplings and higher order n-point couplings among massless modes
in 4D effective theory,
their symmetries like flavor symmetries, etc.

It is also important to study couplings including massive modes, because massive modes would be important, e.g. for the proton decay, right-handed neutrinos, FCNC, etc.

2. Magnetized D-branes

We consider torus compactification
with magnetic flux background.



Type IIB magnetized D-brane models

D9, D7, D5, D3

D9: wrapping on T2xT2xT2 with magnetic fluxes

D7: wrapping on T2xT2 with magnetic fluxes

D5: wrapping on T2 with magnetic fluxes

LEEFT of magnetized D-branes

Low-energy effective field theory

of D-brane models

= higher dimensional super Yang-Mills theory

e.g.

D9-brane models

⇒ 10D SYM (gauge bosons, gauginos)

↓ KK decomposition

4D LEEFT

Higher Dimensional SYM theory with flux

Cremades, Ibanez, Marchesano, '04

4D Effective theory <= dimensional reduction

$$\mathcal{L}_{SYM} = -\frac{1}{4g^2} Tr\{F^{MN} F_{MN}\} + \frac{i}{2g^2} Tr\{\bar{\lambda}\Gamma^M D_M \lambda\}$$

$$\begin{aligned} \lambda(x^\mu, y^m) &= \sum_n \chi_n(x^\mu) \times \psi_n(y_m), \\ A_M(x^\mu, y^m) &= \sum_n \varphi_{n,M}(x^\mu) \times \phi_{n,M}(y_m) \end{aligned}$$

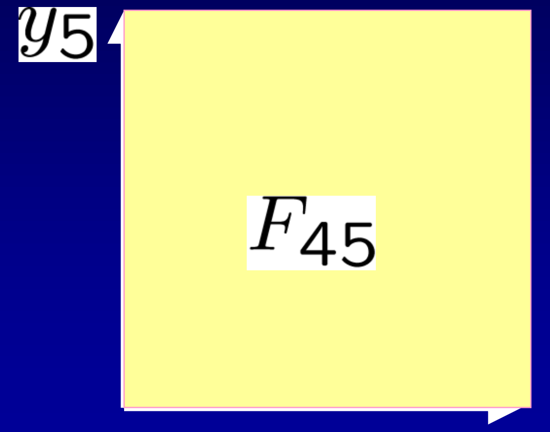
$\Rightarrow$

$$\begin{aligned} i\Gamma_m D^m \psi_n(y) &= m_n \psi_n, \\ \Delta_6 \phi_{n,M}(y) &= M^2_{n,M} \phi_{n,M} \end{aligned}$$

The wave functions $\longrightarrow$ eigenstates of corresponding internal Dirac/Laplace operator.

Higher Dimensional SYM theory with flux $U(1)$

Abelian gauge field on magnetized torus T^2



$$y_4 \sim y_4 + 1, \quad y_5 \sim y_5 + 1$$

Constant magnetic flux $F_{45} = b,$

gauge fields of background $\begin{cases} A_4 = 0, \\ A_5 = b y_4 \end{cases}$

$$\Downarrow \quad \frac{b}{2\pi} = M \in Z$$

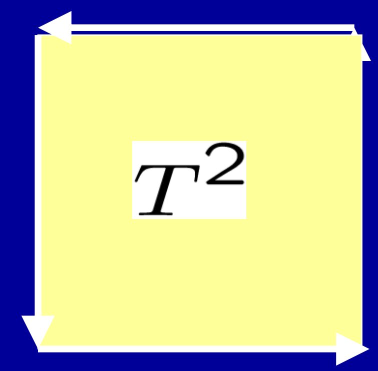
The boundary conditions on torus (transformation under torus translations)

$$\begin{cases} A_m(y_4 + 1, y_5) = A_m(y_4, y_5) + \partial_m \chi_4, & \chi_4 = b y_5, \\ A_m(y_4, y_5 + 1) = A_m(y_4, y_5) + \partial_m \chi_5, & \chi_5 = 0, \end{cases}$$

Higher Dimensional SYM theory with flux $U(1)$

We now consider a complex field $\psi(y_4, y_5)$ with charge Q (+/-1)

$$
\begin{cases}
\psi(y_4+1, y_5) = e^{iQ\chi_4}\psi(y_4, y_5) = e^{iQby_5}\psi(y_4, y_5), \\
\psi(y_4, y_5+1) = e^{iQ\chi_5}\psi(y_4, y_5) = \psi(y_4, y_5),
\end{cases}
$$



Consistency of such transformations under a contractible loop in torus which implies Dirac's quantization conditions.

$$\frac{b}{2\pi} = M \in Z$$

Dirac equation on 2D torus

ψ is the two component spinor.

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

U(1) charge Q=1

$$\begin{cases} \left[\bar{\partial} + 2\pi M y_4\right]\psi_+(y) = 0 \\ \left[\partial - 2\pi M y_4\right]\psi_-(y) = 0 \end{cases}$$

$$\partial = \partial_4 + i\partial_5, \qquad \bar{\partial} = \partial_4 - i\partial_5$$

with twisted boundary conditions (Q=1)

$$\begin{aligned} \psi(y_4+1, y_5) &= e^{2\pi i M y_5}\psi(y_4, y_5), \\ \psi(y_4, y_5+1) &= \psi(y_4, y_5), \end{aligned}$$

Dirac equation and chiral fermion

|M| independent zero mode solutions in Dirac equation.

$$\Theta^j(y_4, y_5) = N_j e^{-M\pi y_4^2} \cdot \vartheta \begin{bmatrix} j/M \\ 0 \end{bmatrix} (M(y_4 + iy_5), Mi)$$

$$(j = 0, 1, \cdots, |M| - 1)$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) \equiv \sum_n e^{\pi i (n+a)^2 \tau} e^{2\pi (a+n)(\nu+b)}$$

(Theta function)

Properties of theta functions

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m, \tau) = e^{2\pi i m a} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu + m\tau, \tau) = e^{-\pi m^2 \tau - 2\pi i m (\nu + b)} \cdot \vartheta \begin{bmatrix} a \\ b \end{bmatrix}$$

chiral fermion

$M \gtrless 0 \Rightarrow$

$\psi_{+/-}$: zero-modes

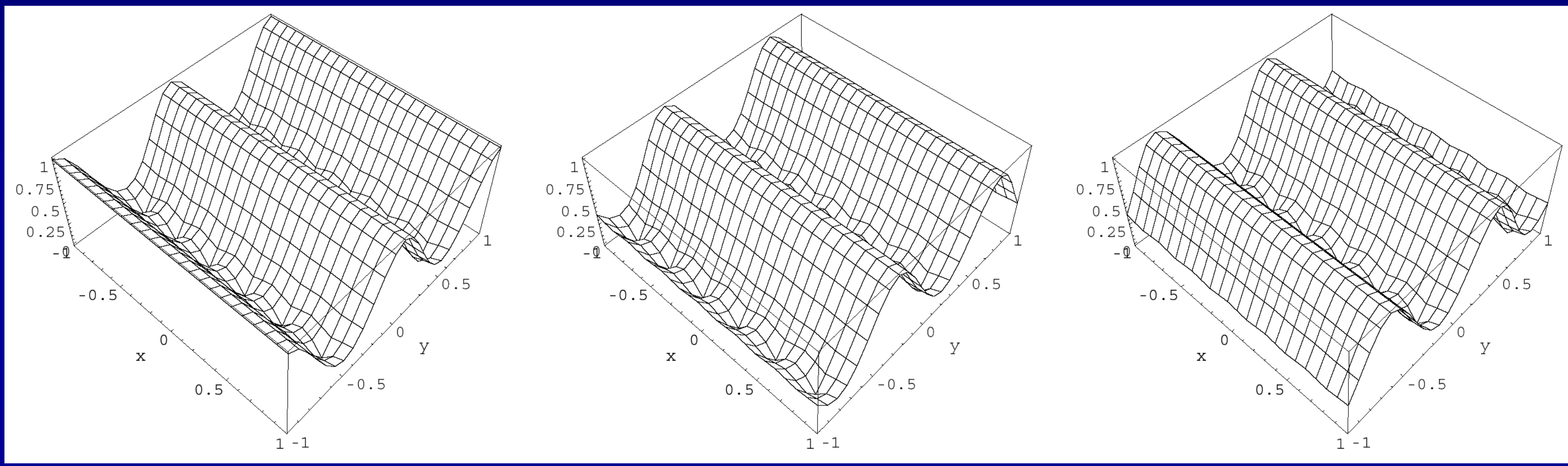
$\psi_{-/+}$: no zero-mode

By introducing magnetic flux, we can obtain chiral theory.

Wave functions

For the case of M=3

$|\Theta^0(y)|$ $|\Theta^1(y)|$ $|\Theta^2(y)|$



Wave function profile on toroidal background

Zero-modes wave functions are quasi-localized far away each other in extra dimensions. Therefore the hierarchirally small Yukawa couplings may be obtained.

Fermions in bifundamentals $(N = N_a + N_b)$

$$F_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a \times N_a} & 0 \\ 0 & M_b \mathbf{1}_{N_b \times N_b} \end{pmatrix}.$$

Breaking the gauge group $U(N) \to U(N_a) \times U(N_b)$

(Abelian flux case $M_a, M_b \in Z$

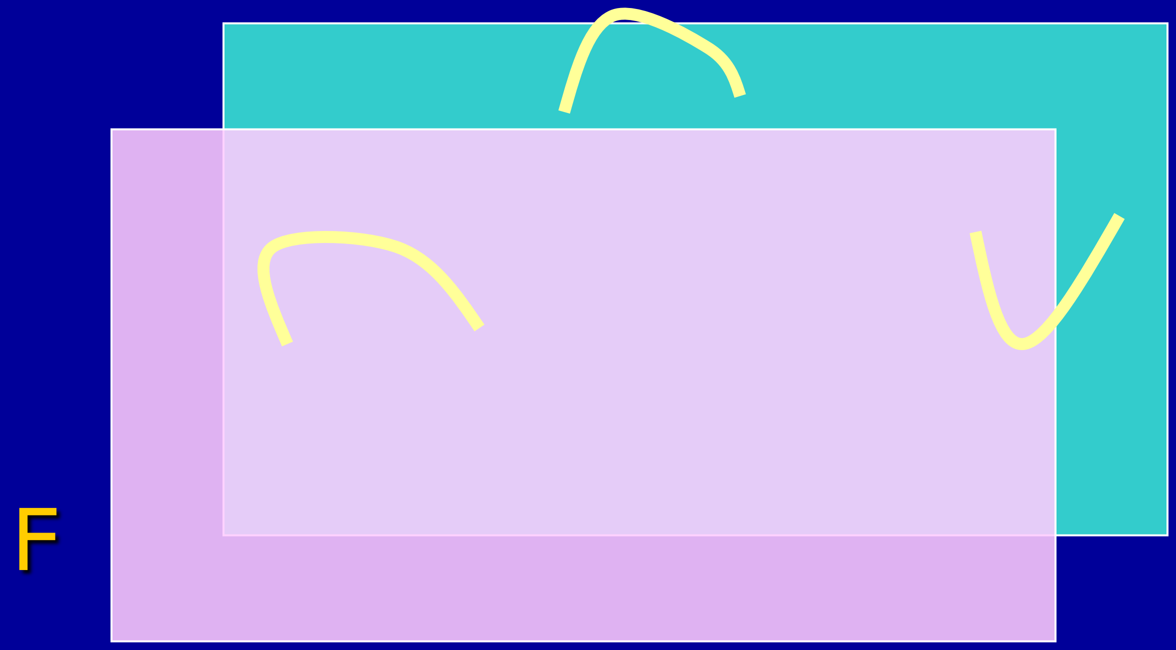
The gaugino fields $$\lambda(x,y) = \begin{pmatrix} \lambda^{aa}(x,y) & \lambda^{ab}(x,y) \\ \lambda^{ba}(x,y) & \lambda^{bb}(x,y) \end{pmatrix}.$$

⇨ λ^{aa} and λ^{bb} gaugino of unbroken gauge $\text{Adj } N_a, \text{ Adj } N_b.$

λ^{ab} and λ^{ba} bi-fundamental matter fields $(N_a, \bar{N}_b), (\bar{N}_a, N_b).$

Bi-fundamental

Gaugino fields in off-diagonal entries correspond to bi-fundamental matter fields and the difference M= m-m' of magnetic fluxes appears in their Dirac equation.



Zero-mode Dirac equations

$$\begin{pmatrix} \bar{\partial}\psi_{+}^{aa} & \left[\bar{\partial} + 2\pi(M_a - M_b)y_4\right]\psi_{+}^{ab} \\ \left[\bar{\partial} + 2\pi(M_b - M_a)y_4\right]\psi_{+}^{ba} & \bar{\partial}\psi_{+}^{bb} \end{pmatrix} = 0.$$

$$\begin{pmatrix} \partial\psi_{-}^{aa} & \left[\partial - 2\pi(M_a - M_b)y_4\right]\psi_{-}^{ab} \\ \left[\partial - 2\pi(M_b - M_a)y_4\right]\psi_{-}^{ba} & \partial\psi_{-}^{bb} \end{pmatrix} = 0.$$

No effect due to magnetic flux for adjoint matter fields, λ^{aa} and λ^{bb}

Total number of zero-modes of λ^{ab} $\Rightarrow$ $I_{ab} = |M_a - M_b|.$

$M_a - M_b > 0 \Rightarrow$

$\psi_{+}^{ab}, \psi_{-}^{ba}$: Zero-modes

$\psi_{-}^{ab}, \psi_{+}^{ba}$: No zero-mode

Illustrating model: U(8) SYM theory on T6

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & m_2 \mathrm{I}_{N_2} & \\ 0 & & m_3 \mathrm{I}_{N_3} \end{pmatrix}$$

$$N_1 = 4,\ N_2 = 2,\ N_3 = 2$$

$$U(4) \times U(2)_L \times U(2)_R$$

Pati-Salam group up to U(1) factors

$$(m_1 - m_2) = (m_3 - m_1) = 3 \text{ for the first } T^2$$
$$(m_1 - m_2) = (m_3 - m_1) = 1 \text{ for the other tori}$$

Three families of matter fields

$$(4,2,1) + (\bar{4},1,2)$$

with many Higgs fields

$$(1,2,2)$$

Wilson lines

Cremades, Ibanez, Marchesano, '04,
Abe, Choi, T.K. Ohki, '09

torus without magnetic flux

constant Ai → mass shift

every modes massive

magnetic flux

$$\left[\bar{\partial} + 2\pi(My + a)\right]\psi_+ = 0$$

$$\left[\partial - 2\pi(My + a)\right]\psi_- = 0$$

the number of zero-modes is the same.

the profile: f(y) → f(y +a/M)

with proper b.c.

U(1)a*U(1)b theory

magnetic flux, Fa=2πM, Fb=0

Wilson line, Aa=0, Ab=C

matter fermions with U(1) charges, (Qa,Qb)

chiral spectrum,

for Qa=0, massive due to nonvanishing WL

when MQa >0, the number of zero-modes is MQa.

zero-mode profile is shifted depending on Qb,

$$f(z) \Rightarrow f(z + CQ_b/(MQ_a))$$

Illustrating model: Pati-Salam → SM model

$$F_{z\bar{z}} = 2\pi i \begin{pmatrix} m_1 \mathrm{I}_{N_1} & & 0 \\ & m_2 \mathrm{I}_{N_2} & \\ 0 & & m_3 \mathrm{I}_{N_3} \end{pmatrix}$$

$$N_1 = 4,\ N_2 = 2,\ N_3 = 2$$

Pati-Salam group $U(4) \times U(2)_L \times U(2)_R$

$$(m_1 - m_2) = (m_3 - m_1) = 3 \text{ for the first } T^2$$
$$(m_1 - m_2) = (m_3 - m_1) = 1 \text{ for the other tori}$$

WLs along a U(1) in U(4) and a U(1) in U(2)R

=> Standard gauge group up to U(1) factors

$$U(3)_C \times U(2)_L \times U(1)^3$$

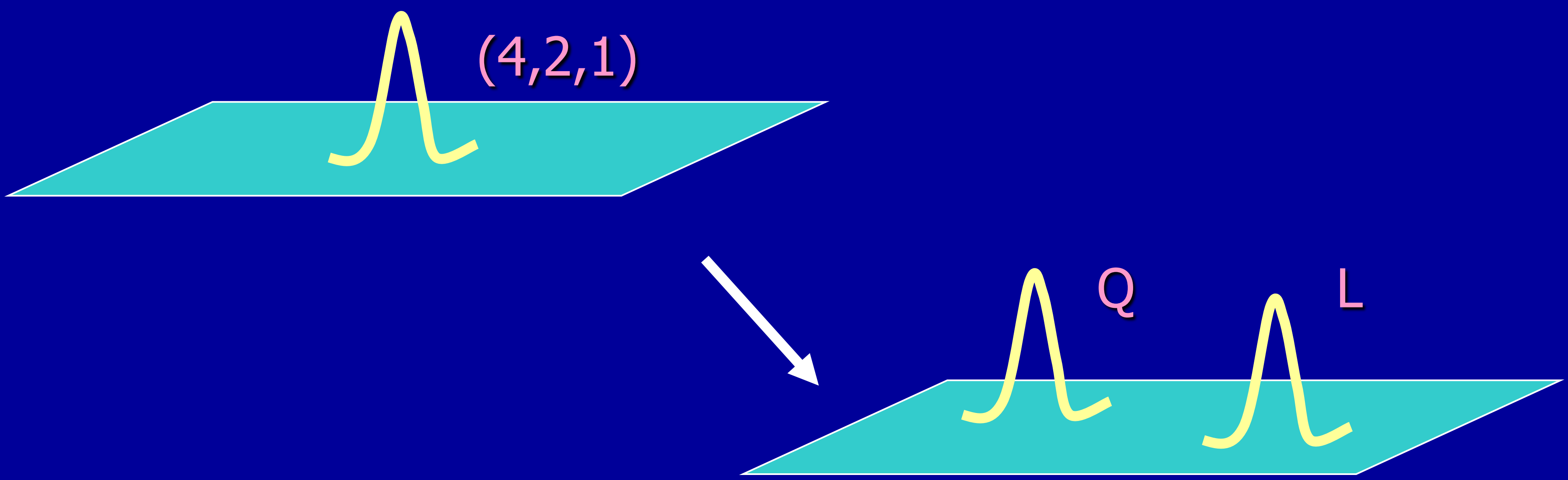
U(1)Y is a linear combination.

PS => SM

$$(4,2,1)+(\bar{4},1,2)$$

Zero modes corresponding to three families of matter fields remain after introducing WLs, but their profiles split

$$(4,2,1)=(3,2,1)+(1,2,1)$$
$$(\bar{4},1,2)=(\bar{3},1,1)+(\bar{3},1,1)+(1,1,1)+(1,1,1)$$



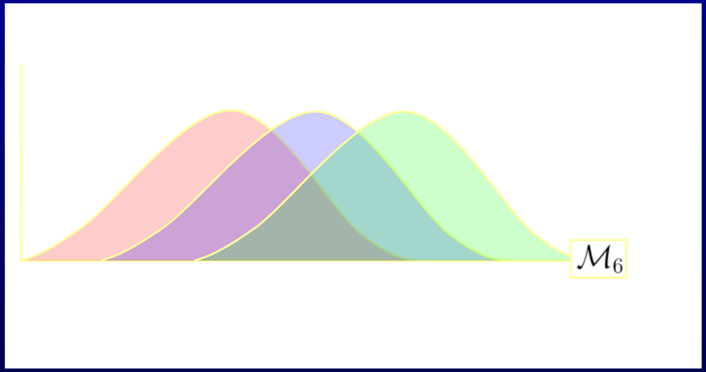
3. N-point couplings among massless modes

3.1 N-point couplings of zero-modes

The N-point couplings are obtained by overlap integral of their zero-mode w.f.'s.

$$Y = g\int d^2z\,\psi^i_M(z)\psi^j_N(z)\ldots\psi^k_P(z)$$

$$z = y_4 + iy_5$$



Moduli

Torus metric

$$ds^2 = 2(2\pi R)^2 dz d\bar{z}$$

$$z = x + \tau\, y$$

Area $A = 4\pi^2 R^2 \,\mathrm{Im}\,\tau$

We can repeat the previous analysis.

Scalar and vector fields have the same wavefunctions.

Wilson moduli

shift of w.f.

$$\alpha = M\zeta$$

$$\psi(z) \Rightarrow \psi(z+\zeta)$$

Zero-modes

Cremades, Ibanez, Marchesano, '04

$$\psi_M^j(z) = N_M \exp[i\pi M z \,\mathrm{Im}(z)/\mathrm{Im}\,\tau] \cdot \vartheta\begin{bmatrix} j/M \\ 0 \end{bmatrix}(Mz, \tau M)$$

$$N_M = \left(2M\,\mathrm{Im}\,\tau / A^2\right)^{1/4}, \qquad j = 1, \cdots, M$$

Zero-mode w.f. = gaussian x theta-function

Product of zero-mode wavefunctions

$$\psi_M^i(z) \cdot \psi_N^j(z) = \sum_{m=1}^{M+N} y_{ijm} \psi_{M+N}^{i+j+Mm}(z),$$

$$y_{ijm} = \frac{N_N N_M}{N_{N+M}} \vartheta\begin{bmatrix} (Ni - Mj + MNm)/(MN(M+N)) \\ 0 \end{bmatrix}(0, \tau MN(M+N))$$

Products of wave functions: Hint to understand

$$\left[\bar{\partial} + 2\pi M y\right]\psi_{M+} = 0,$$
$$\left[\bar{\partial} + 2\pi N y\right]\psi_{N+} = 0,$$
$$\left[\bar{\partial} + 2\pi (M+N) y\right]\psi_{(M+N)+} = 0,$$

$$\psi_{(M+N)+} = \psi_{M+} \times \psi_{N+}$$

products of zero-modes = zero-modes

3-point couplings

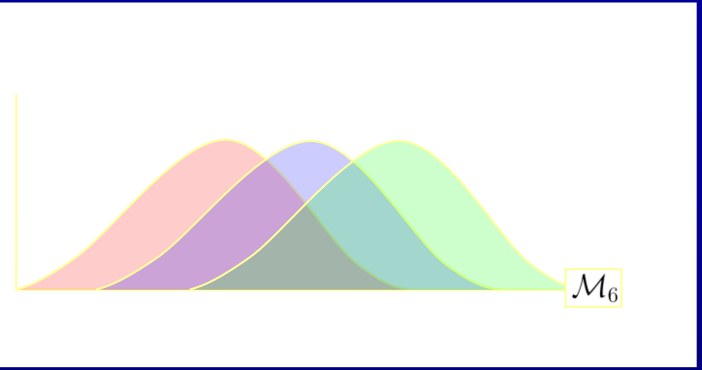
Cremades, Ibanez, Marchesano, '04

The 3-point couplings are obtained by overlap integral of three zero-mode w.f.'s.

$$Y_{ijk} = \int d^2 z\, \psi^i_M(z) \psi^j_N(z) \left(\psi^k_{M+N}(z)\right)^*$$

$$\int d^2 z\ \psi^i_M(z) \left(\psi^k_M(z)\right)^* = \delta^{ik}$$

$$Y_{ijk} = \sum_{m=1}^{M+N} \delta_{i+j+mM,k}\, y_{ijm}$$



4-point couplings

Abe, Choi, T.K., Ohki, '09

The 4-point couplings are obtained by overlap integral of four zero-mode w.f.'s.

$$Y_{ijkl} = \int d^2 z \, \psi_M^i(z) \psi_N^j(z) \psi_P^k(z) \left(\psi_{M+N+P}^l(z) \right)^*$$

split

$$\int d^2 z d^2 z' \psi_M^i(z) \psi_N^j(z) \delta(z - z') \psi_P^k(z') \left(\psi_{M+N+P}^l(z') \right)^*$$

insert a complete set

$$\delta(z - z') = \sum_{\text{all modes}} \left(\psi_K^n(z) \right)^* \psi_K^n(z')$$

$$Y_{ijk\bar{l}} = \sum_{s=\text{only zero-modes}} y_{ij\bar{s}} \, y_{sk\bar{l}}$$

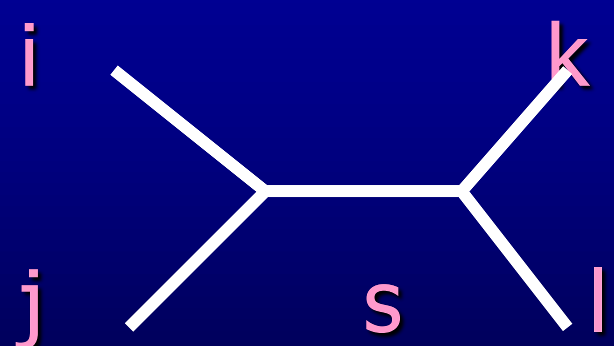
for K=M+N

4-point couplings: another splitting

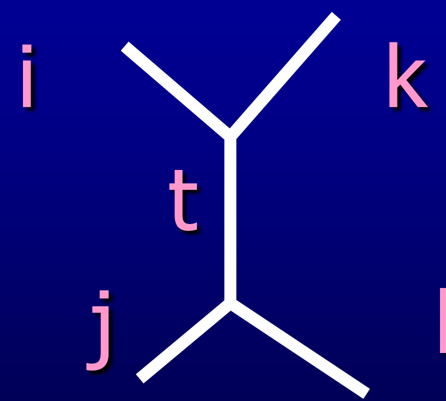
$$\int d^2 z\, d^2 z'\, \psi_M^i(z)\, \psi_P^k(z)\, \delta(z-z')\, \psi_N^j(z') \left(\psi_{M+N+P}^l(z')\right)^*$$

$$Y_{ijk\bar{l}} = \sum_t y_{ik\bar{t}}\, y_{tj\bar{l}}$$

$$Y_{ijk\bar{l}} = \sum_s y_{ij\bar{s}}\, y_{sk\bar{l}}$$



$$Y_{ijk\bar{l}} = \sum_t y_{ik\bar{t}}\, y_{tj\bar{l}}$$



N-point couplings

Abe, Choi, T.K., Ohki, '09

We can extend this analysis to generic n-point couplings.

N-point couplings = products of 3-point couplings
= products of theta-functions

This behavior is non-trivial. (It's like CFT.)
Such a behavior would be satisfied
not for generic w.f.'s, but for specific w.f.'s.

However, this behavior could be expected
from T-duality between magnetized
and intersecting D-brane models.

T-duality

The 3-point couplings coincide between magnetized and intersecting D-brane models.
explicit calculation

Cremades, Ibanez, Marchesano, '04

Such correspondence can be extended to 4-point and higher order couplings because of CFT-like behaviors, e.g.,

$$Y_{ijk\bar{l}} = \sum_{s} y_{ij\bar{s}} y_{sk\bar{l}}$$

Abe, Choi, T.K., Ohki, '09

Non-Abelian discrete flavor symmetry

Effective field theory has non-Abelian discrete flavor symmetires such as D4 and Δ(27).

Abe, Choi, T.K, Ohki, '09

Berasatuce-Gonzalez, Camara, Marchesano, Regalado, Uranga, '12

Cf. heterotic orbifolds, T.K. Raby, Zhang, '04

T.K. Nilles, Ploger, Raby, Ratz, '06

3.2 Applications of couplings

We can obtain quark/lepton masses and mixing angles.

Yukawa couplings depend on volume moduli,
complex structure moduli and Wilson lines.

By tuning those values, we can obtain semi-realistic results.

Ratios depend on complex structure moduli
and Wilson lines.

Quark/lepton masses and mixing angles

Abe, T.K., Ohki, Oikawa, Sumita, work in progress

Example

$$M_t = 164 \quad GeV, \qquad M_b = 11 \quad GeV$$
$$M_c = 1.4 \quad GeV, \qquad M_s = 240 \quad MeV$$
$$M_u = 3 \quad MeV, \qquad M_d = 3 \quad MeV$$

$$V_{us} = 0.21, \qquad V_{cb} = 0.03, \qquad V_{ub} = 0.09$$

$$M_\tau = 5 \quad GeV,$$
$$M_\mu = 97 \quad MeV,$$
$$M_e = 0.9 \quad MeV,$$

Flavor is still a challenging issue.

4. Massive modes

Hamada, T.K. ‘12

Massive modes play an important role in 4D LEEFT such as the proton decay, FCNCs, etc.

It is important to compute mass spectra of massive modes and their wavefunctions.
Then, we can compute couplings among massless and massive modes.

Fermion massive modes

Two components are mixed.

$$\begin{pmatrix} \bar{D}D & 0 \\ 0 & D\bar{D} \end{pmatrix} \begin{pmatrix} \psi_{+,n} \\ \psi_{-,n} \end{pmatrix} = m_n^2 \begin{pmatrix} \psi_{+,n} \\ \psi_{-,n} \end{pmatrix}$$

2D Laplace op.

$$\Delta = \{\bar{D}, D\}/2$$

algebraic relations

$$[D, \bar{D}] = 4\pi M / A$$

$$[\Delta, \bar{D}] = 4\pi M\bar{D} / A, \qquad [\Delta, D] = -4\pi M D / A$$

It looks like the quantum harmonic oscillator

Fermion massive modes

Creation and annhilation operators

$$a = D\sqrt{A/4\pi M}, \qquad a^{+} = \bar{D}\sqrt{A/4\pi M},$$

$$[a, a^{+}] = 1$$

mass spectrum

$$m_n^2 = 4\pi M n / A$$

wavefunction

$$\psi_n^{j,M} = (1/\sqrt{n!})(a^{+})^n \psi_0^{j,M}$$

Fermion massive modes

explicit wavefunction

$$\psi_n^{j,M} = \frac{(2M \,\mathrm{Im}\,\tau)^{1/4}}{(2^n n! A)^{1/2}} \sum_k \Theta_k^{j,M}(z+\zeta,\tau) \times H_n\left(\sqrt{2\pi M \,\mathrm{Im}\,\tau}\left(k + j/M + \mathrm{Im}(z+\zeta)/\mathrm{Im}\,\tau\right)\right)$$

$$\Theta_k^{j,M}(z,\tau) = \exp[-\pi M \,\mathrm{Im}\,\tau (k + j/M + \mathrm{Im}\,z/\mathrm{Im}\,\tau)^2 + i\pi M \,\mathrm{Re}\,z(2k + 2j/M + \mathrm{Im}\,z/\mathrm{Im}\,\tau) + i\pi M \,\mathrm{Re}\,\tau (k + j/M)$$

Hn: Hermite function

Orthonormal condition:

$$\int d^2 z\, \psi_n^{j,M} (\psi_\ell^{k,M})^* = \delta_{jk}\delta_{n\ell}$$

Scalar and vector modes

The wavefunctions of scalar and vector fields are the same as those of spinor fields.

Mass spectrum

scalar $$m_n^2 = 2\pi M(2n+1)/A$$

vector $$m_n^2 = 2\pi M(2n-1)/A$$

Scalar modes are always massive on T2.
The lightest vector mode along T2,
i.e. the 4D scalar, is tachyonic on T2.

Such a vector mode can be massless on T4 or T6.

$$m^2 = 2\pi(M_1/A_1 + M_2/A_2 - M_3/A_3)$$

T-duality ?

Mass spectrum

spinor $$m_n^2 = 2\pi M(2n)/A$$

scalar $$m_n^2 = 2\pi M(2n+1)/A$$

vector $$m_n^2 = 2\pi M(2n-1)/A$$

the same mass spectra as excited modes (with oscillator excitations) in intersecting D-brane models, i.e. “gonions”

Aldazabal, Franco, Ibanez, Rabadan, Uranga, ‘01

Products of wavefunctions

explicit wavefunction

$$\psi_{n_1}^{i,M}(z)\cdot\psi_{n_2}^{j,N}(z)=\sum_{m=1}^{M+N}\sum_{\ell=0}^{n_1}\sum_{s=0}^{n_2} y_{\ell+s}^{ijm}\psi_{\ell+s}^{i+j+Mm,N+M}(z),$$

$$y_{\ell+s}^{ijm}={}_{n_1}C_{\ell}\ {}_{n_2}C_{s}(-1)^{n_2-s}N^{(n_2+\ell-s)/2}M^{(n_1-\ell+s)/2}(N+M)^{-(n_1+n_2+1)/2}$$
$$\times\sqrt{(\ell+s)!(n_1+n_2-\ell-s)!/(n_1!n_2!)}$$
$$\times\psi_{n_1+n_2-\ell-s}^{Mi-Nj+NMm,\ NM(N+M)}(0,\tau)$$

See also Berasatuce-Gonzalez, Camara, Marchesano, Regalado, Uranga, '12

$$\psi^{i,M}(z)\cdot\psi^{j,N}(z)=\sum_{m=1}^{M+N}y_{ijm}\psi^{i+j+Mm,M+N}(z),$$

Derivation:

products of zero-mode wavefunctions

We operate creation operators on both LHS and RHS.

3-point couplings including higher modes

The 3-point couplings are obtained by overlap integral of three wavefunctions.

$$Y^{ij\bar{k}}_{n_1 n_2 n_3} = \int d^2z\, \psi^{i,N}_{n_1}(z)\psi^{j,M}_{n_2}(z)\left(\psi^{k,M+N}_{n_3}(z)\right)^*$$

$$\int d^2z\ \psi^{i,M}_{\ell}(z)\left(\psi^{j,M}_{s}(z)\right)^* = \delta^{ik}\delta_{\ell s}$$

$$Y^{ij\bar{k}}_{n_1 n_2 n_3} = \sum_{m=1}^{M+N}\sum_{\ell=0}^{n_1}\sum_{s=0}^{n_2} \delta_{i+j+mM,k}\,\delta_{\ell+s,n_3}\, y^{ijm}_{\ell+s}$$

(flavor) selection rule $i+j=k \mod M$

is the same as one for the massless modes.

(mode number) selection rule $n_3 \le n_1 + n_2$

3-point couplings:
2 zero-modes and one higher mode

$$n_3 \le n_1 + n_2$$

3-point coupling $n_1 = n_3 = 0$

$$Y = N^{n_2/2}(N+M)^{-(n_2+1)/2}\psi_{n_2}^{Mk-(M+N)j,\ NM(N+M)}(0,\tau)$$

Higher order couplings including higher modes

Similarly, we can compute higher order couplings including zero-modes and higher modes.

$$Y = \int d^2 z \, \psi_{n_1}^{i,N}(z) \psi_{n_2}^{j,M}(z) \cdots \left(\psi_{n_m}^{k,P}(z) \right)^*$$

They can be written by the sum over products of 3-point couplings.

3-point couplings including massive modes only due to Wilson lines

Massive modes appear only due to Wilson lines without magnetic flux

$$\psi^{(W)}_{n_R n_I} = A^{-1/2} \exp[i\pi(2n_R + \mathrm{Im}\,\alpha / \mathrm{Im}\,\tau)\,\mathrm{Re}\,z + i\pi(-\mathrm{Re}\,\alpha + 2(n_I - n_R\,\mathrm{Re}\,\tau))\,\mathrm{Im}\,z / \mathrm{Im}\,\tau]$$

We can compute the 3-point coupling

$$Y^{j\bar{k}}_{(W)n_R n_I} = \int d^2 z\, \psi^{(W)}_{n_R n_I}(z)\, \psi^{j,M}_0(z+\zeta_1) \left(\psi^{k,M}_0(z+\zeta_2)\right)^*$$

e.g. $|Y^{j\bar{k}}_{(W)n_R=0 n_I=0}| = A^{-1/2} \exp[-\pi |\zeta_2 - \zeta_1|^2 / (2\,\mathrm{Im}\,\tau)]$

Gaussian function for the Wilson line.

$$M\zeta_1 + \alpha = M\zeta_2$$

3-point couplings including massive modes only due to Wilson lines

$$|Y^{j\bar{k}}_{(W)n_R=0n_I=0}| = A^{-1/2}\exp[-\pi|\zeta_2-\zeta_1|^2/(2\,\mathrm{Im}\,\tau)]$$

For example, we have

$$|Y^{j\bar{k}}_{(W)n_R=0n_I=0}| \approx \exp[-\pi] \approx 0.04$$

for

$$|\zeta_2-\zeta_1|^2/(2\,\mathrm{Im}\,\tau) = 1$$

Several couplings

Similarly, we can compute the 3-point couplings including higher modes

$$Y^{j\bar{k}}_{n_1 n_2 (W) n_R n_I} = \int d^2 z \, \psi^{(W)}_{n_R n_I}(z) \psi^{j,M}_{n_1}(z) \left(\psi^{k,M}_{n_2}(z)\right)^*$$

Furthermore, we can compute higher order couplings including several modes, similarly.

$$Y = \int d^2 z \, \psi^{(W)}_{n_R n_I}(z) \cdots \psi^{j,M}_{n_1} \cdots (z) \left(\psi^{k,M}_{n_2}(z)\right)^*$$

4.2 Phenomenological applications

In 4D SU(5) GUT,

The heavy X boson couples with quarks and leptons
by the gauge coupling.

Their couplings do not change even after GUT breaking
and it is the gauge coupling.

However, that changes in our models.

Phenomenological applications

For example,
we consider the SU(5)xU(1) GUT model
and we put magnetic flux along extra U(1).

The 5 matter field has the U(1) charge q,
and the quark and lepton in 5 are quasi-localized
at the same place.
Their coupling with the X boson is given by
the gauge coupling before the GUT breaking.

SU(5) => SM

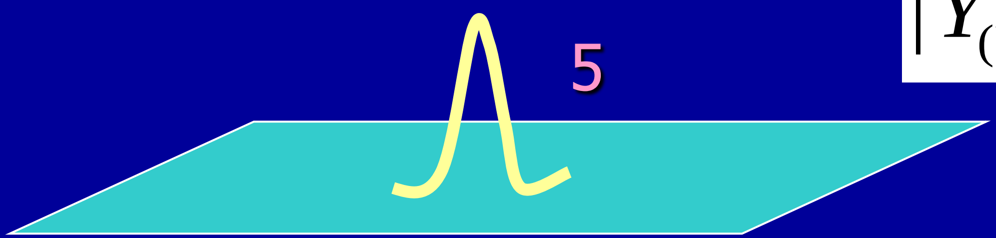
We break SU(5) by the WL along the U(1)Y direction.

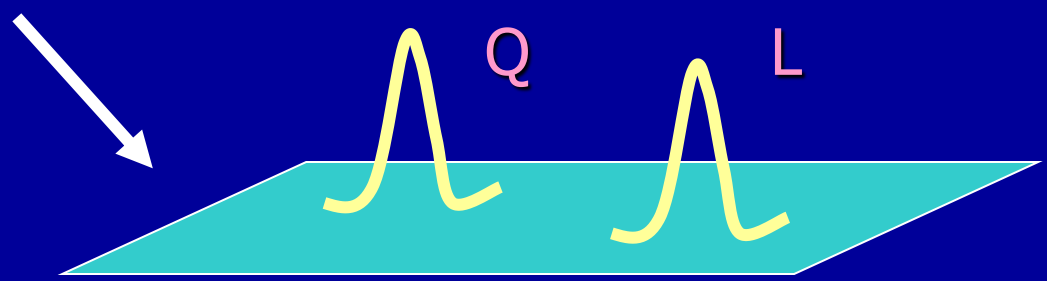
The X boson becomes massive.

The quark and lepton in 5 remain massless, but their profiles split each other.

Their coupling with X is not equal to the gauge coupling, but includes the suppression factor

$$| Y^{j\bar{k}}_{(W) n_R=0 n_I=0} | \approx \exp[-\pi] \approx 0.04$$

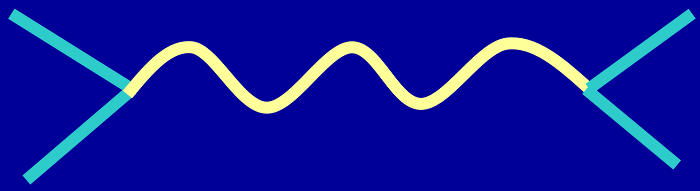




Proton decay

Similarly, the couplings of the X boson with quarks and leptons in the 10 matter fields can be suppressed.

That is important to avoid the fast proton decay.



The proton life time would drastically change by the factor,

$$O(10^4 - 10^5)$$

$\leftarrow$ $$|Y^{j\bar{k}}_{(W)n_R=0 n_I=0}| \approx \exp[-\pi] \approx 0.04$$

Other aspects

Other couplings including massless and massive modes
can be suppressed and those would be important ,
such as right-handed neutrino masses and
off-diagonal terms of Kahler metric, etc.

Threshold corrections on the gauge couplings,
Kahler potential after integrating out massive modes

Summary

We have studied phenomenological aspects of magnetized D-brane models.

We can write the 4D LEEFT of massless modes, perturbative coupling terms and their moduli dependence.

We have studied mass spectra and wavefunctions of higher modes.

We have computed couplings including higher modes.

We can write the LEEFT with the full modes.

These results have important implications.

Further works

We know that couplings among zero-modes coincide between the magnetized and intersecting D-brane models.

What about couplings including higher modes ?

Anyway, the mass spectra coincide each other.

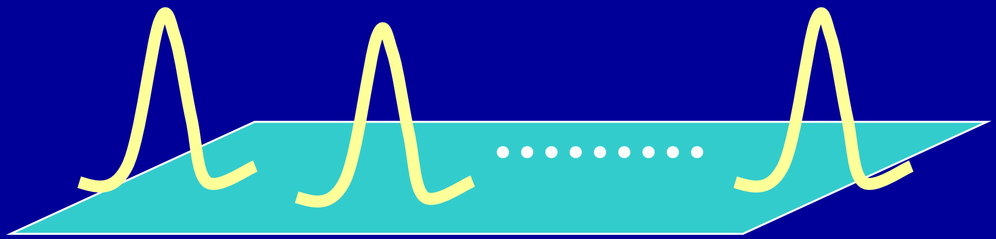
The 4D LEEFT has certain discrete (flavor) symmetries.

What about their anomalies ?

3.2 Non-Abelian discrete flavor symmetry

The coupling selection rule is controlled by Zg charges.

For M=g,　　1　　2　　.........　　g



Effective field theory also has a cyclic permutation symmetry of g zero-modes.

These lead to non-Abelian discrete flavor symmetires such as D4 and Δ(27)　　Abe, Choi, T.K, Ohki, '09

Cf. heterotic orbifolds,　T.K. Raby, Zhang, '04

T.K. Nilles, Ploger, Raby, Ratz, '06

Non-Abelian discrete flavor symm.

Recently, in field-theoretical model building, several types of discrete flavor symmetries have been proposed with showing interesting results,

e.g. S3, D4, A4, S4, Q6, Δ(27), ...

Review: e.g

Ishimori, T.K., Ohki, Okada, Shimizu, Tanimoto '10

⇒ large mixing angles

one Ansatz: tri-bimaximal

$$\begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \end{pmatrix}$$