Global SU(5) F Theory model

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arXiv:1206.6132 (hep-th)

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Outline

- Local vs. Global model building
- Local SU(5) GUT + matter
  - Gauge coupling unification
  - R parity
- Summary
- Local model → Global model
- Conclusions

Local vs. Global

F-theory compactifications

- Type IIB has non-perturbative $Sl(2,\mathbb{Z})$ symmetry rotating $\tau = C_0 + ie^{-\phi}$

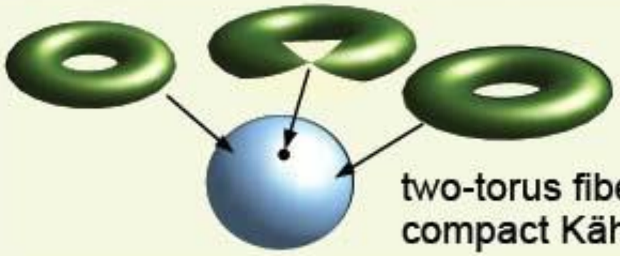
  $\Rightarrow$ interpret τ as complex structure of a two-torus (2 auxiliary dimensions)

  [Vafa] [Morrison,Vafa]

- minimally supersymmetric F-theory compactifications:
  - F-theory on torus fibered Calabi-Yau 4-fold Y_4

    $\Rightarrow$ 4 dim, N=1 supergravity theory

    $\Rightarrow$ base B_3 is a Kähler manifold



- singularities of the fibration are crucial to encode 7-brane physics

  $\Rightarrow$ pinching torus indicates presence of 7-branes magn. charged under τ

- brane and bulk physics encoded by <u>complex</u> geometry

T. Grimm

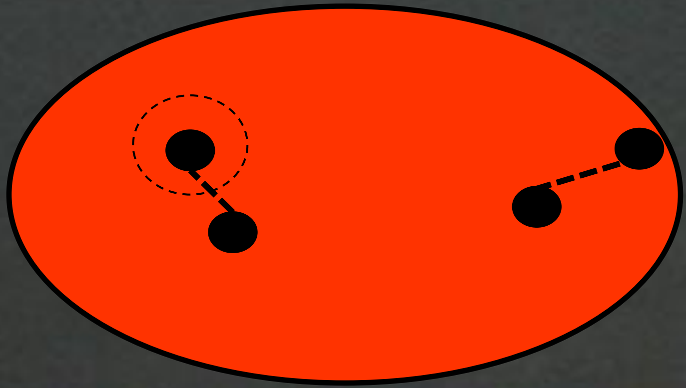
Elliptic curve = torus

Weierstrass function

$$y^2 = x^3 + f\,v^4\,x + g\,v^6$$

$y = \sqrt{x^3 + f\,x + g} = 0$ has 3 solutions

$v = 1$ + one at ∞



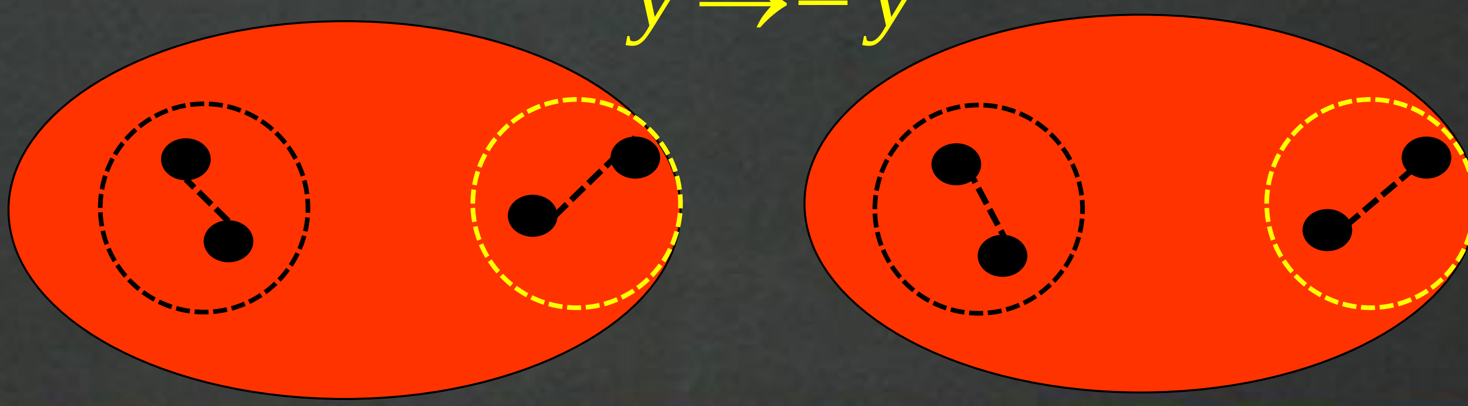
Weierstrass function

$$y^2 = x^3 + f\,v^4\,x + g\,v^6 \;\Leftrightarrow\; \text{torus}$$

$$(v, x, y) \rightarrow (\lambda v, \lambda^2 x, \lambda^3 y)$$

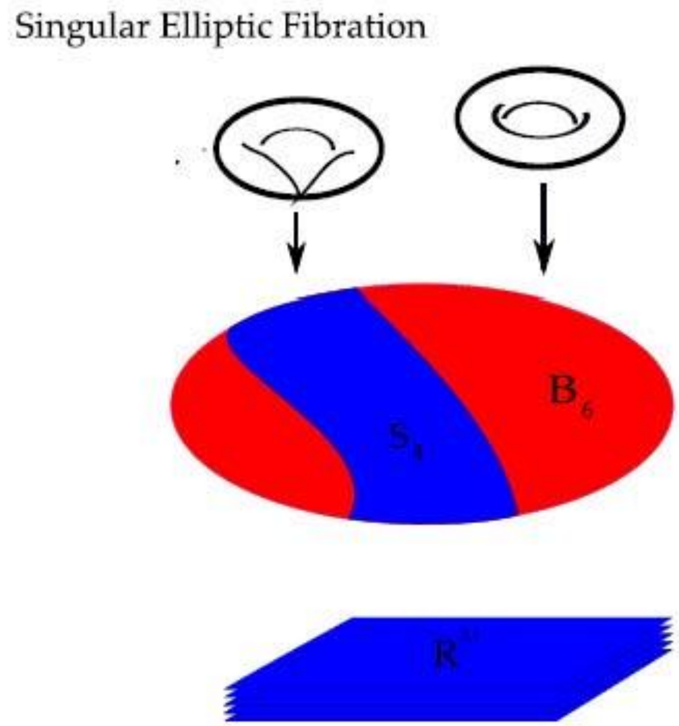
$$\mathbb{P}_{(1,2,3)}(v, x, y) \rightarrow \lambda^6\,\mathbb{P}_{(1,2,3)}(v, x, y)$$

$$y \rightarrow -y$$



Singular elliptic fibration

Gauge degrees of freedom on 7-branes realized in terms of ADE singularities, in codim 1 in the base B_3: divisor S_{GUT}



Geometrically: elliptically fibered CY4 with Weierstrass form

$$y^2 = x^3 + fx + g$$

f and g are global sections of $\mathcal{O}(-4K_B)$ and $\mathcal{O}(-6K_B)$, resp.

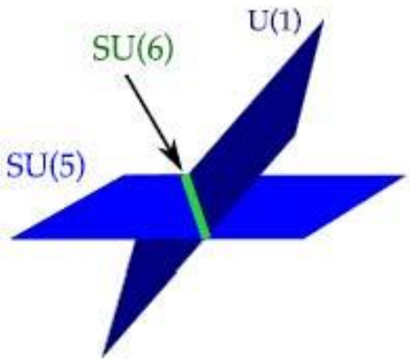
Gauge degrees of freedom: discriminant locus

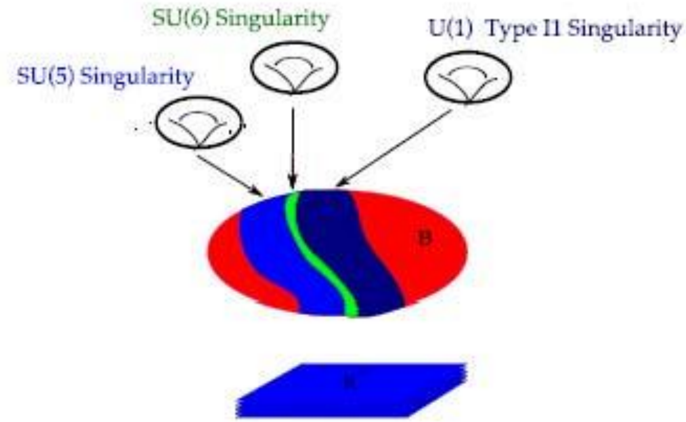
$$\Delta = 4f^3 + 27g^2 = 0 \quad \supset \quad S_{\text{GUT}}$$

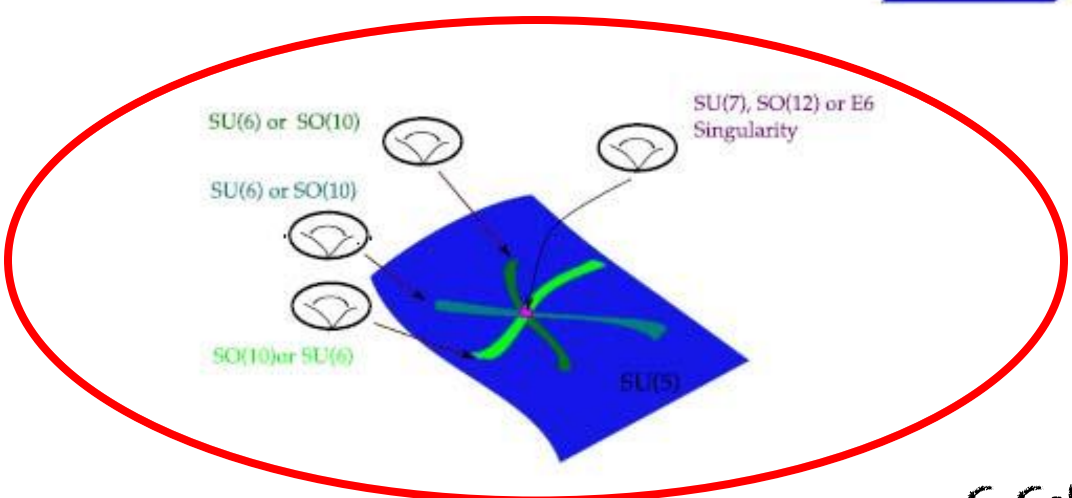
Kodaira classification ADE

Higher codimension singularities

"Physics intuition": Matter fields and Yukawas arise from intersecting branes, i.e. higher codimension singularity enhancements:







Calabi - Yau 4 fold Y_4

Build Y_4 = hypersurface in 5 fold

$$W_5 - \mathbb{P}\left(\mathcal{O} \oplus K_{B_3}^{-2} \oplus K_{B_3}^{-3}\right) = \begin{array}{ccc} \mathbb{P}^2 & \longrightarrow & W_5 \\ & & \downarrow \\ & & B_3 \end{array}$$

Given by **Tate form** of Weierstrass function

$$vy^2 = x^3 + \hat{b}_0 z^5 v^3 + \hat{b}_2 z^3 v^2 x + \hat{b}_3 z^2 v^2 y + \hat{b}_4 z v x^2 + \hat{b}_5 v x y$$

$$vy^2 = x^3 + \hat{b}_0 z^5 v^3 + \hat{b}_2 z^3 v^2 x + \hat{b}_3 z^2 v^2 y + \hat{b}_4 z v x^2 + \hat{b}_5 v x y$$

Tate form

v, y, x = elliptic fiber (torus)

z = 0 ⇒ gauge brane

b_m holomorphic sections of line bundles in B_3

Section	Bundle
v	$\mathcal{O}(\sigma)$
x	$\mathcal{O}(\sigma + 2c_{1,B})$
y	$\mathcal{O}(\sigma + 3c_{1,B})$
z	$\mathcal{O}(S_2)$
$\hat{b}_m$	$\mathcal{O}([6-m]c_{1,B} - [5-m]S_2)$

b_ms determine adjoint Higgs vev breaking E_8 to $SU(5)$

Defines - Spectral Cover

$$b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 \sim b_0 \prod_{i=1}^{5} (s + t_i)$$

eg. $$b_1 \sim \sum_{i=1}^{5} t_i \equiv 0, \quad b_5 \sim \prod_{i=1}^{5} t_i$$

b_m fcns. of coordinates restricted to S_2

In addition G flux – $G_4 = dC_3$

(M theory on resolved CY_4)

gives non-Abelian gauge fields

C_3 given by $C_3 = A \wedge \omega_{1,1}$

with $\omega_{1,1}$ vol. of blow up P^1

& A gives U(1) gauge boson

M_2s wrapping blow up P^1s gives W's

of visible non-Abelian gauge group

see Mayrhofer talk

In addition G flux - $G_4 = dC_3$ needed for chirality on gauge and matter branes

see Mayrhofer talk

SU(5) breaking and Gauge coupling unification

$$G_4 = \langle F_Y \rangle \wedge \omega$$

On heterotic side - gives photon mass

On F theory side - allowed but requires clever global constructions

SU(5) -> SM + doublet-triplet splitting

Large threshold corrections at the compactification scale

Donagi & Wijnholt, Conlon & Palti
Leontaris & (Tracas; Vlachos; Tracas & Tsamis)
Dolan, Marsano & Schafer-Nameki

4*D* GUT

$$\alpha_i^{-1}(\mu) \approx \alpha_G^{-1} + \frac{b_i^{MSSM}}{2\pi} \ln\left(\frac{M_G}{\mu}\right) - \frac{\alpha_G^{-1}\,\varepsilon_3}{1+\varepsilon_3}\,\delta_{i3}, \quad \mu \sim M_G$$

F theory

$$\alpha_i^{-1}(\mu) \approx \alpha^{-1}(\Lambda) + \frac{b_i^{MSSM}}{2\pi} \ln\left(\frac{\Lambda}{\mu}\right) + \frac{b_i^{KK}}{2\pi} \ln\left(\frac{\Lambda}{M_C}\right) + \Delta_i, \quad \mu \sim M_C$$

4D GUT

$$\alpha_i^{-1}(\mu) \approx \alpha_G^{-1} + \frac{b_i^{MSSM}}{2\pi} \ln\left(\frac{M_G}{\mu}\right) - \frac{\alpha_G^{-1}\,\varepsilon_3}{1+\varepsilon_3}\delta_{i3}, \quad \mu \sim M_G$$

F theory

$$\alpha_i^{-1}(\mu) \approx \alpha^{-1}(\Lambda) + \frac{b_i^{MSSM}}{2\pi} \ln\left(\frac{\Lambda}{\mu}\right) + \frac{b_i^{KK}}{2\pi} \ln\left(\frac{\Lambda}{M_C}\right) + \Delta_i, \qquad \mu \sim M_C$$

Δ_i finite corrections depending on geometry of S_2

$$b_i^{KK} = b_i^{(3,\bar{2})_{-5/3} + \ cc} = (5, 3, 2)$$

$$b_i^{MSSM} = \left(\frac{33}{5}, 1, -3\right)$$

match at $\mu = M_C$

$$\left\{\alpha_G, M_G, \varepsilon_3\right\} \Rightarrow \left\{\alpha(\Lambda), \Lambda, M_C\right\}$$

$$2-1: \frac{\left[(b_2-b_1)^{MSSM} + (b_2-b_1)^{KK}\right]}{2\pi}\ln\left(\frac{\Lambda}{M_G}\right) = \frac{\left[(b_2-b_1)^{KK}\right]}{2\pi}\ln\left(\frac{M_C}{M_G}\right) - (\Delta_2-\Delta_1)$$

$$\ln\left(\frac{M_C}{M_G}\right) = \frac{\left[(b_2-b_1)^{MSSM} + (b_2-b_1)^{KK}\right]}{(b_2-b_1)^{KK}}\ln\left(\frac{\Lambda}{M_G}\right) + 2\pi\frac{(\Delta_2-\Delta_1)}{(b_2-b_1)^{KK}}$$

$$\Rightarrow \qquad \left(\frac{M_C}{\Lambda}\right) = \exp\left(-\pi(\Delta_2-\Delta_1)\right)\left(\frac{\Lambda}{M_G}\right)^{14/5} \ll 1$$

Fine - tuned ??

Donagi & Wijnholt

0808.2223 (hep-th)

finite corrections can work

Dolan, Marsano, Schafer-Nameki

1109.4958 (hep-ph)

exotics contribute to thresholds below M_c

Non-local discrete Wilson line breaking may solve this problem

Need gauge brane with non-trivial fundamental group

$$S_{GUT} = K3 \big/ \mathbb{Z}_2 = \text{Enriques surface}$$

$$\pi_1\left(S_{GUT}\right) = \mathbb{Z}_2$$

Define CY_4

Defined as hypersurface in 5 fold W_5

$$W_5 = \mathbb{P}\left(\mathrm{O} \oplus K_{B_3}^{-2} \oplus K_{B_3}^{-3}\right)$$

$$vy^2 = x^3 + \hat{b}_0 z^5 v^3 + \hat{b}_2 z^3 v^2 x + \hat{b}_3 z^2 v^2 y + \hat{b}_4 zvx^2 + \hat{b}_5 vxy$$

Section	Bundle
v	$\mathcal{O}(\sigma)$
x	$\mathcal{O}(\sigma + 2c_{1,B})$
y	$\mathcal{O}(\sigma + 3c_{1,B})$
z	$\mathcal{O}(S_2)$
$\hat{b}_m$	$\mathcal{O}([6-m]c_{1,B} - [5-m]S_2)$

Tate divisor – spectral cover

$$vy^2 = x^3$$

$$C_{Tate}:\ \hat{b}_0 z^5 v^2 + \hat{b}_2 z^3 vx + \hat{b}_3 z^2 vy + \hat{b}_4 zx^2 + \hat{b}_5 xy = 0$$

$$z \to U, \quad x/v = V^2, \quad y/v = V^3 \qquad \text{restricted to } S_2$$

$$C_{spec}:\ b_0 U^5 + b_2 U^3 V^2 + b_3 U^2 V^3 + b_4 U V^4 + b_5 V^5$$

$$\phi = \ \mathrm{diag}(t_1, t_2, t_3, t_4, t_5) \quad \text{Higgs in } SU(5)^{\perp}$$

$$b_1 \approx b_0 \sum_{i=1}^{5} t_i = 0, \quad b_5 \approx b_0 \prod_{i=1}^{5} t_i, \quad \ldots$$

Tate divisor – spectral cover

$$C_{spec}:\ b_0U^5+b_2U^3V^2+b_3U^2V^3+b_4UV^4+b_5V^5=0$$

$\mathbb{Z}_2 = \mathbb{Z}_2^{(Enriques)}$ $(K3/(-1)$ $=$ *freely acting* $\mathbb{Z}_2$ *involution on* $S_2)$

* $\mathbb{Z}_2$ *center of* $U(1)_Y$ $(y\rightarrow -y \Rightarrow V\rightarrow -V =$ $\mathbb{Z}_2$ *involution on fiber*$)$

Invariance of C_{spec} under Z_2 requires

$$\mathbb{Z}_2:\ b_m\rightarrow(-1)^m\,b_m$$

R parity

Matter curves intersect gauge brane

$$\Sigma_{\mathbf{10},\downarrow}: \quad 0 = b_5 \sim \prod_{i=1}^{5} t_i \qquad \to SO(10) \text{ on curve}$$

$$\Sigma_{\bar{\mathbf{5}},\downarrow}: \quad 0 = b_0 b_5^2 - b_2 b_3 b_5 + b_3^2 b_4 \sim \prod_{i<j} (t_i + t_j)$$

$$\Sigma_{\mathbf{10},\downarrow} \Rightarrow 10 \text{ and } \overline{10}$$

$$\Sigma_{\bar{\mathbf{5}},\downarrow} \Rightarrow 5 \text{ and } \bar{5}$$

R parity

Intersection of 3 matter curves

= cubic coupling

$$10_m\ ,\ \overline{5}_m\ ,\ \ 5_h + \overline{5}_h$$

$$10_m\ \overline{5}_m\ \ \overline{5}_h\ ,\ 10_m\, 10_m\, 5_h \quad \text{but NOT} \ \ 10_m\ \overline{5}_m\ \ \overline{5}_m$$

4+1 split and $U(1)_{B-L}$

$$(a_4V^4 + a_3V^3U + a_2V^2U^2 + \alpha U^3[e_1V - e_0U])(e_1V + e_0U) = 0$$

R parity

$$\mathbf{10}^{(4)} \times \mathbf{10}^{(4)} \times \mathbf{5}^{(44)}$$

$$\mathbf{10}^{(4)} \times \overline{\mathbf{5}}^{(44)} \times \overline{\mathbf{5}}^{(41)}$$

~~$$\mathbf{10}^{(4)} \times \mathbf{10}^{(1)} \times \mathbf{5}^{(41)}$$~~

~~$$\mathbf{10}^{(1)} \times \overline{\mathbf{5}}^{(44)} \times \overline{\mathbf{5}}^{(44)}$$~~

$$\mathbf{10}^{(4)} \leftrightarrow \mathbf{10}_M , \quad \overline{\mathbf{5}}^{(44)} \leftrightarrow \mathbf{5}_H + \overline{\mathbf{5}}_H , \quad \overline{\mathbf{5}}^{(41)} \leftrightarrow \overline{\mathbf{5}}_M$$

Global vs. Local Model

Construct K3 inside B_3 with Z_2 involution

- B_3 must be able to serve as the base of an elliptically fibered Calabi-Yau 4-fold with section that exhibits an A_4 singularity along an effective anti-canonical divisor S_2=K3
- B_3 must admit an Enriques involution that acts freely on S_2=K3

$$\mathbb{Z}_2 : \; b_m \to (-1)^m b_m$$

$$h^0(B_3,\, O_{B_3}([6-m]c_{1,B} - [5-m]S_2)) > 0 \;\; \text{for } m = 0, 2, 3, 4, 5$$

$$h^{0,+}(B_3, O_{B_3}(S_2)) \geq 4 \qquad h^{0,-}(B_3, O_{B_3}(S_2)) \geq 2$$

Need for global model

$$U(1)\ \text{D term} \sim \int_{Y_4} \omega \wedge J \wedge G_4$$

$$n_{D3,\ \text{induced}} = \frac{1}{2}\int_{Y_4} G \wedge G - \frac{\chi(Y_4)}{24}$$

D_3 brane tadpole equation

A complete global model allows for the evaluation of these quantities

The Global Model
Define B_3 in terms of intersection of two quadrics in $\mathbb{P}^5$

$\mathbb{P}^5 : [u_1, u_2, u_3, v_1, v_2, v_3]$ homogeneous coordinates

$\mathbb{Z}_2^{(Enriques)}: \; u_i \to u_i \qquad v_i \to -v_i \qquad i = 1, 2, 3$

Fixed point locus of $\mathbb{Z}_2^{(Enriques)}$ given by

$\mathbb{P}_u^2 : [u_1, u_2, u_3, 0, 0, 0] \qquad \mathbb{P}_v^2 : [0, 0, 0, v_1, v_2, v_3]$

$$\mathbb{P}^2 = \mathbb{P}_u^2 \bigcup \mathbb{P}_v^2$$

$\mathbb{P}^5 : [u_1, u_2, u_3, v_1, v_2, v_3]$ homogeneous coordinates

$\mathbb{Z}_2^{(Enriques)}$: $u_i \to u_i \qquad v_i \to -v_i \qquad i = 1, 2, 3$

Fixed point locus of $\mathbb{Z}_2^{(Enriques)}$ given by

$\mathbb{P}_u^2 : [u_1, u_2, u_3, 0, 0, 0] \qquad \mathbb{P}_v^2 : [0, 0, 0, v_1, v_2, v_3]$

$$\mathbb{P}^2 = \mathbb{P}_u^2 \bigcup \mathbb{P}_v^2$$

$K3$ given by intersection of 3 $\mathbb{Z}_2^{(Enriques)}$ invariant quadrics in $\mathbb{P}^5$

$$Q_2^{(a)} = f_2^{(a)}(u_1, u_2, u_3) - g_2^{(a)}(v_1, v_2, v_3) \qquad a = 1, 2, 3$$

require no solutions in $\mathbb{P}^2$ (locus of fixed points) of

$$f_2^{(1)} = f_2^{(2)} = f_2^{(3)} = 0, \qquad g_2^{(1)} = g_2^{(2)} = g_2^{(3)} = 0$$

Summary: B_3 in terms of intersection of two quadrics in $\mathbb{P}^5$

$\mathbb{P}^5 : [u_1, u_2, u_3, v_1, v_2, v_3]$ homogeneous coordinates

$\mathbb{Z}_2^{(Enriques)}: \; u_i \to u_i \qquad v_i \to -v_i \qquad i = 1, 2, 3$

B_3 given by intersection of 2 $\mathbb{Z}_2^{(Enriques)}$ invariant quadrics in $\mathbb{P}^5$

$$Q_2^{(a)} = f_2^{(a)}(u_1, u_2, u_3) - g_2^{(a)}(v_1, v_2, v_3) \qquad a = 2, 3$$

$$z = Q_2^{(1)}(u_1, u_2, u_3, v_1, v_2, v_3)$$

$$h_+^0(B_3, O_{B_3}(S_2)) = 10 \qquad h_-^0(B_3, O_{B_3}(S_2)) = 9$$

$$u_i u_j, \; v_i v_j \qquad\qquad u_i v_j$$

sections of holomorphic line bundles in B_3

Just the beginning -
Unfortunately not so simple

Need to increase the Picard group of B_3

$$B_3 = (2H)^2 \qquad S_2 = (2H)^3$$

$$H = \text{ hyperplane bundle of } \mathbb{P}^5$$

$$Q_2^{(1)} = u_1 f_1^{(1)}(u_1, u_2, u_3) - v_1 g_1^{(1)}(v_1, v_2, v_3)$$

$$Q_2^{(2)} = u_2 f_1^{(2)}(u_1, u_2, u_3) - v_2 g_1^{(2)}(v_1, v_2, v_3)$$

$$Q_2^{(3)} = f_2^{(3)}(u_1, u_2, u_3) - g_2^{(3)}(v_1, v_2, v_3)$$

Then blow up $\mathbb{P}^5$ along $u_1 = v_1 = 0$ and $u_2 = v_2 = 0$

$$u_1 = \tilde{u}_1\delta_1, \quad v_1 = \tilde{v}_1\delta_1, \quad u_2 = \tilde{u}_2\delta_2, \quad v_2 = \tilde{v}_2\delta_2$$

Introduces 2 new exceptional divisors $E_{1,2}$

$$B_3 = (2H - E_2)\bullet(2H) \qquad S_2 = (2H - E_1)\bullet(2H - E_2)\bullet(2H)$$

Check that enough sections for b_m's

G flux obtained in fully resolved CY fourfold

Marsano, Saulina & Schafer-Nameki

1006.0483 (hep-th), 1107.1718 (hep-th), 1108.1794 (hep-th)

Krause, Mayrhofer & Weigand

1109.3454 (hep-th)

Fully resolved Y_4 after successive blow-ups

$$vy^2 = x^3 - c_1 e_0^2 h_0 v^3 z^5 + (c_1 h_0 e_1^2 + c_4 e_0 h_0 + d_2 e_0 h_1) x v^2 z^3$$

$$+ (c_4 h_0 e_1 + c_3 e_0 h_0 + d_2 h_1 e_1 + d_3 e_0 h_1 - d_2 e_0 h_2) y v^2 z^2$$

$$+ (c_3 h_0 e_1 + c_4 e_0 h_0 + d_3 h_1 e_1 - d_2 h_2 e_1 - d_3 e_0 h_2) x^2 vz + e_1 (c_4 h_0 - d_3 h_2) xyv$$

G flux obtained in fully resolved CY fourfold

$$vy^2 = x^3 - c_1 e_0^2 h_0 v^3 z^5 + (c_1 h_0 e_1^2 + c_4 e_0 h_0 + d_2 e_0 h_1) x v^2 z^3$$
$$+ (c_4 h_0 e_1 + c_3 e_0 h_0 + d_2 h_1 e_1 + d_3 e_0 h_1 - d_2 e_0 h_2) y v^2 z^2$$
$$+ (c_3 h_0 e_1 + c_4 e_0 h_0 + d_3 h_1 e_1 - d_2 h_2 e_1 - d_3 e_0 h_2) x^2 v z + e_1 (c_4 h_0 - d_3 h_2) x y v$$

Section	Bundle	$\mathbb{Z}_2^{(\text{Enriques})}$
v	$\mathcal{O}(\sigma)$	$+$
x	$\mathcal{O}(\sigma + 2[2H - E_1])$	$+$
y	$\mathcal{O}(\sigma + 3[2H - E_1])$	$-$
$\hat{h}_0$	$\mathcal{O}(E_2)$	$+$
$\hat{h}_1$	$\mathcal{O}(H - E_1)$	$+$
$\hat{h}_2$	$\mathcal{O}(H - E_1)$	$-$
$\hat{c}_3$	$\mathcal{O}(2H - E_1 - E_2)$	$-$
$\hat{c}_1, \hat{c}_4$	$\mathcal{O}(2H - E_1 - E_2)$	$+$
$\hat{d}_2$	$\mathcal{O}(H)$	$+$
$\hat{d}_3$	$\mathcal{O}(H)$	$-$
$\hat{e}_0$	$\mathcal{O}$	$+$
$\hat{e}_1$	$\mathcal{O}$	$-$

Chiral matter

G flux is obtained in terms of complete intersections of two divisors in the resolved CY four fold

Chiral matter is obtained by Knowledge of gauge fields (G flux) on matter branes

The Bottom Line
The Good, the Bad and the Ugly

Good:

- 3 families of quarks and leptons
  NO anti-families
- $D_{U(1)} = 0$
- 4 D_3 branes needed to cancel
  geometric and gauge tadpole cond.

The Bottom Line
The Good, the Bad and the Ugly

Bad:

- 4 pairs of Higgs doublets
  But NO triplets ☺

Ugly:

- vector-like exotics

$$(3, \bar{2})_{-5/3} + cc$$

$$SU(5)_{\text{adjoint}} = \begin{pmatrix} (8,1)_0 & (3,\bar{2})_{-5} \\ (\bar{3},2)_{+5} & (1,3)_0 \end{pmatrix}$$

V gauge multiplet

Φ chiral multiplet

$$\mathbb{Z}_2 = \mathbb{Z}_2^{(Enriques)} \otimes \mathbb{Z}_2^{U(1)_Y}$$

$$V = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \qquad \Phi = \begin{pmatrix} - & + \\ + & - \end{pmatrix}$$

Conclusions

- Constructed the first Global SU(5) F theory model with Wilson line breaking
- Model has good, bad and one ugly feature
- Perhaps S_2 with $Z_2 \times Z_2$ isometry or combination of Flux and Wilson line breaking might solve this ugliness
- 4 Higgs pairs ? Understand μ problem
- $\mathbb{Z}_4^R$ symmetry ?
- SUSY breaking & moduli stabilization ?