F-theory vs Type IIB Orientifolds
Some global aspects

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Oct. 5, 2012

Based on work with:

A. Collinucci, arXiv: 1011.6388, 1203.4542

M. Esole, arXiv: 1209.1633

Motivations

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FW anomaly cancellation

- → prescribes the quantization rules of brane fluxes → G-flux quantization
- → constrains the topological type of bulk fluxes → G-flux on instantons

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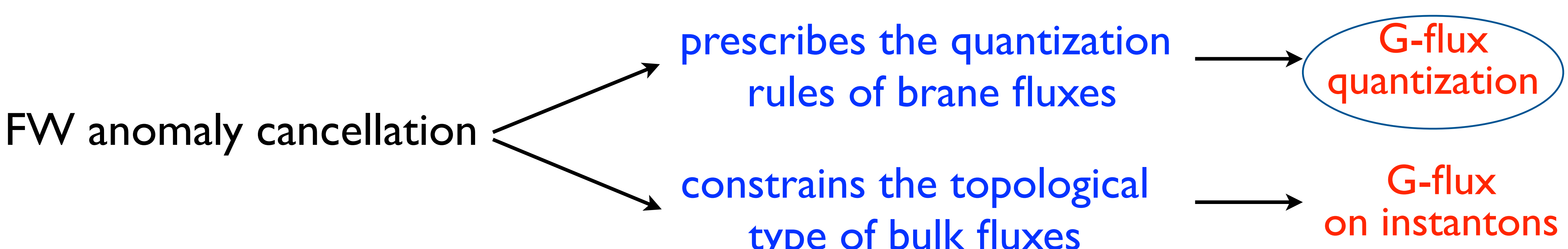

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- Summary and outlook.

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F-theory on $\mathbb{R}^{1,3} \times \mathrm{CY}_4$ "defined as" M-theory on elliptic CY_4 in the limit of vanishing fiber volume

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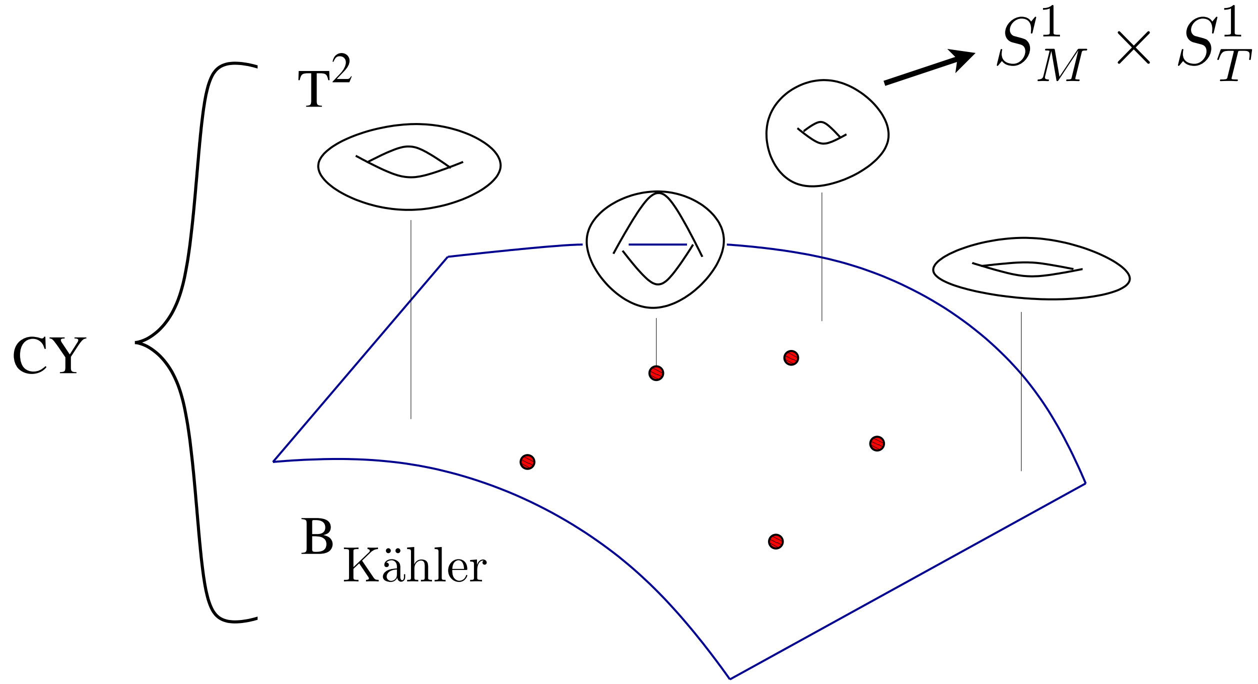

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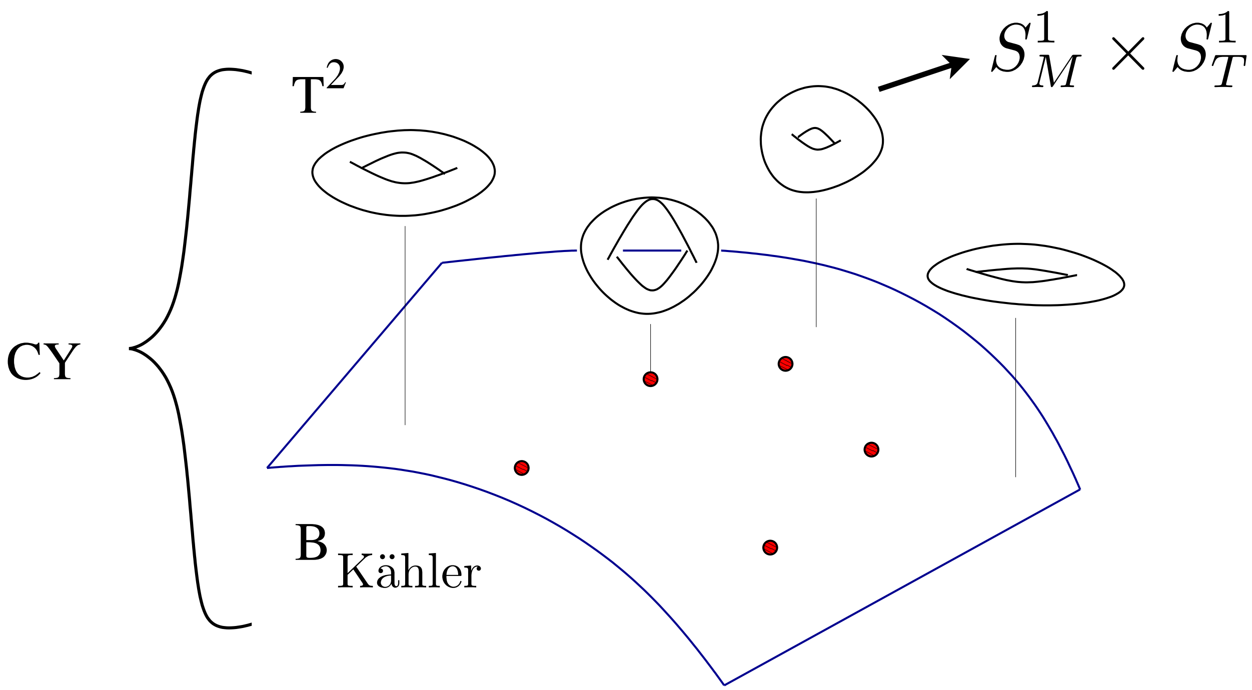

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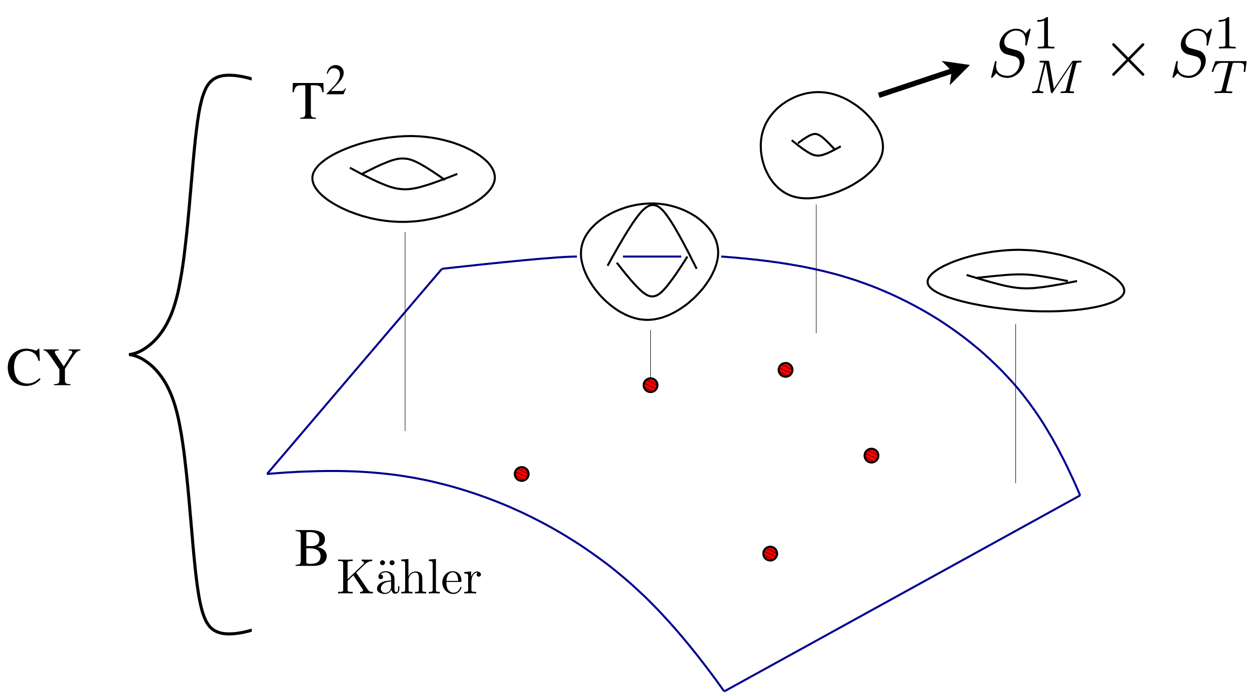

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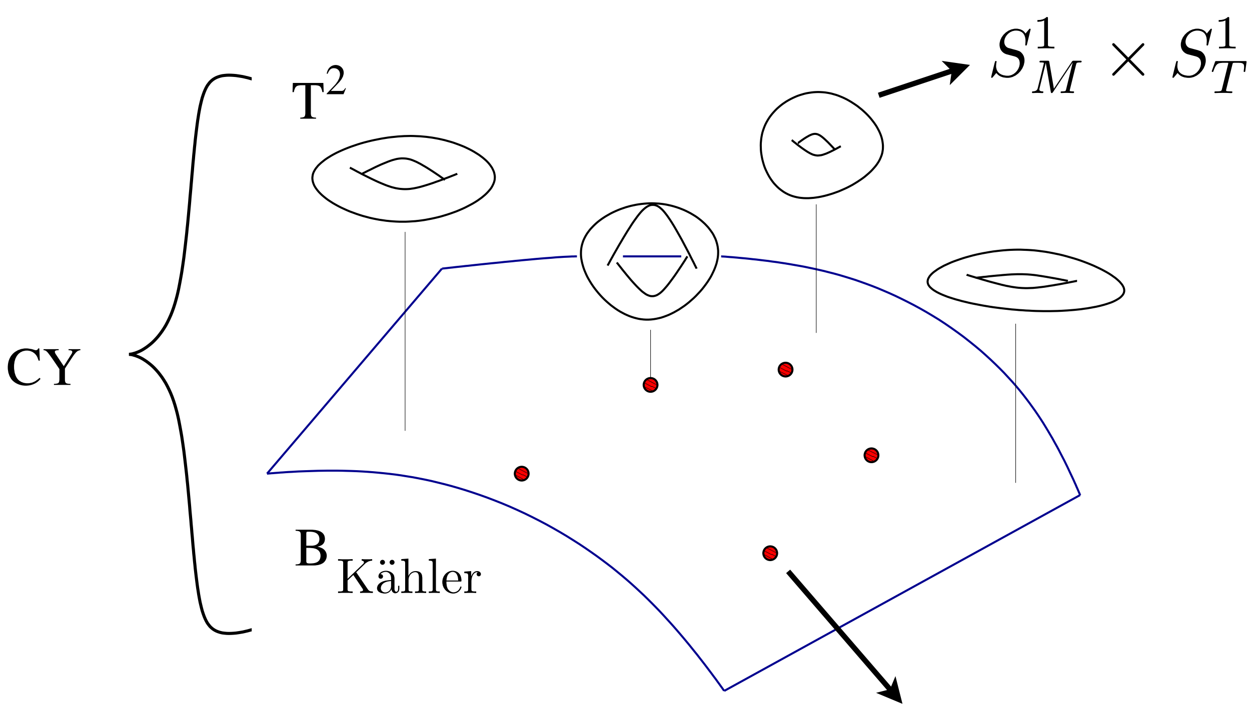

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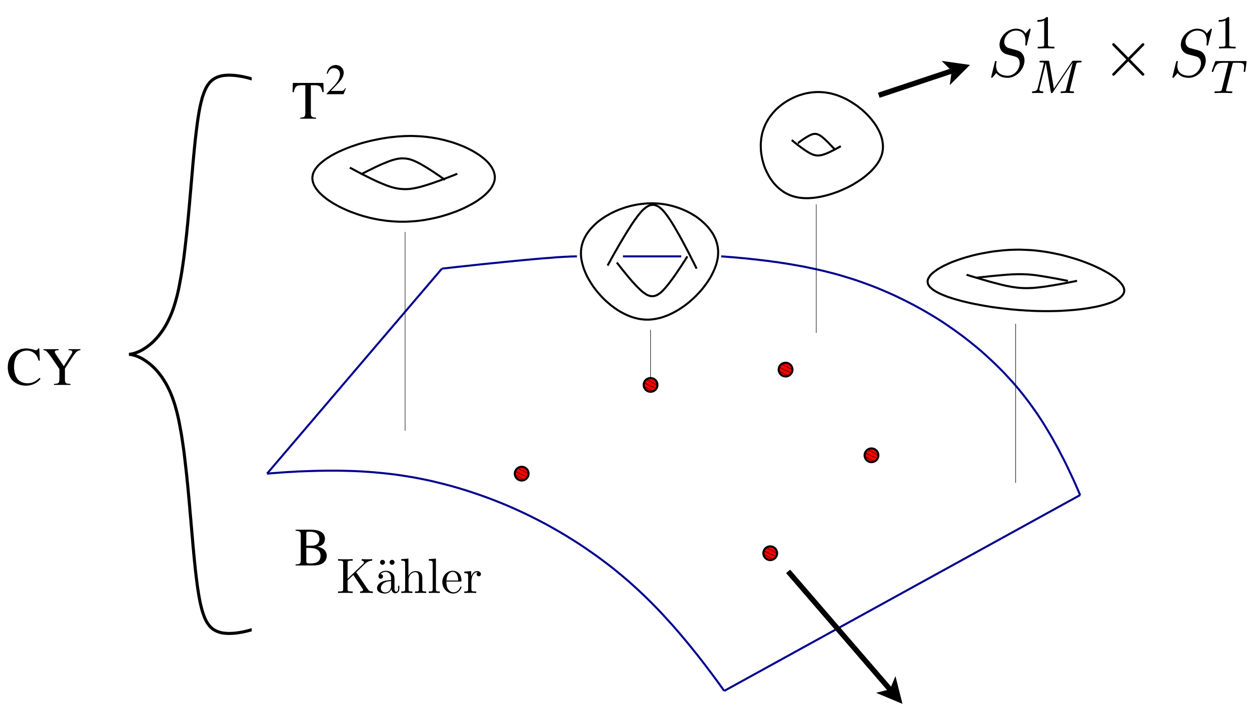

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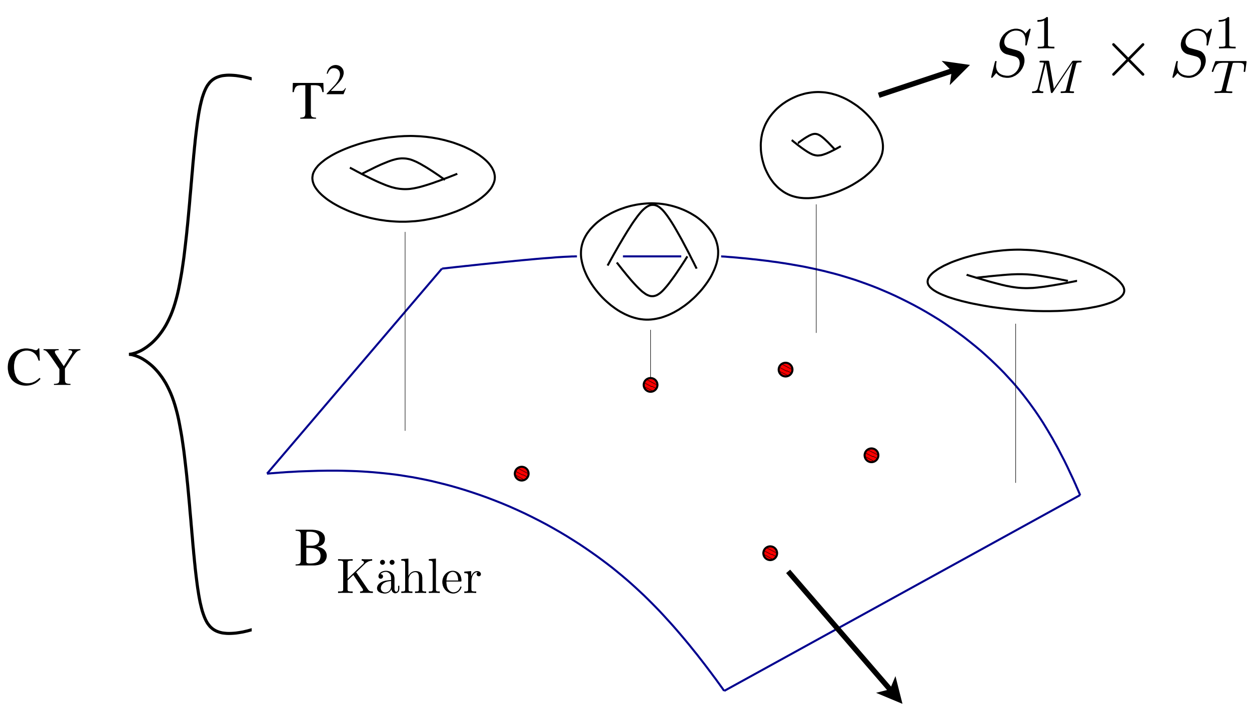

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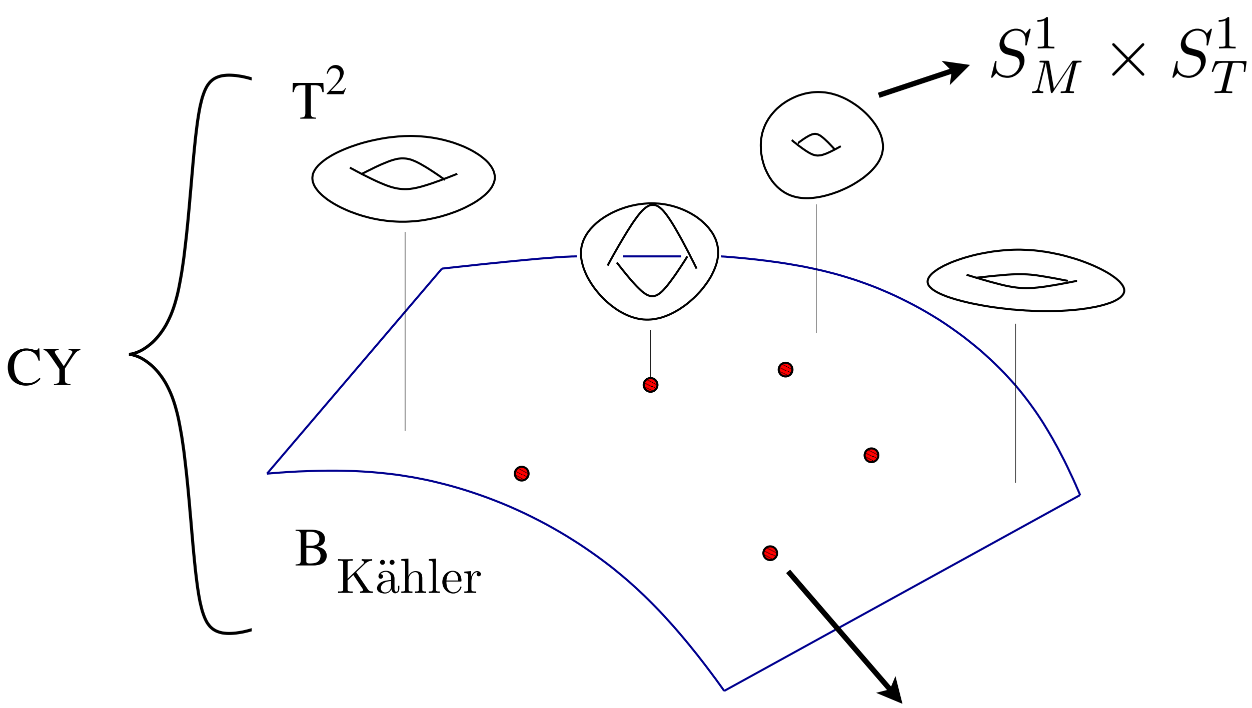

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M/F → **IIB**

Fluxes: G_4 → 7-brane flux
G_4 → bulk fluxes

4D Lorentz invariance
⬇
G_4 must have one and only one leg along T^2

The Tate form

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The F-theory elliptic fiber is usually described by a Weierstrass equation in $W\mathbb{P}^2_{2,3,1}(X,Y,Z)$

$$Y^2 + a_1 XYZ + a_3 YZ^3 = X^3 + a_2 X^2 Z^2 + a_4 X Z^4 + a_6 Z^6$$

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Sen `96
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In the limit $\epsilon \to 0$: Type IIB string theory on a CY_3 double cover of B_3 $\xi^2 = h \equiv a_1^2 + 4a_2$

with 7-brane content given by: $\Delta|_{\text{leading}} \sim \underbrace{h^2}_{\text{O7 plane}}\ \underbrace{s^N}_{\text{gauge stack}}\ \underbrace{\eta^2 - h\chi}_{\text{Whitney Umbrella}}$

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Connection to the Freed-Witten anomaly of the corresponding D7-stack S

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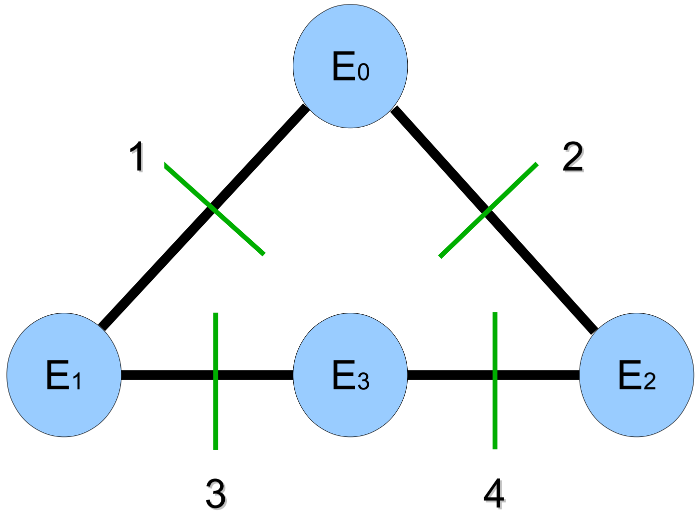
Resolved fiber over SU(4) locus ⟷ Affine Dynkin diagram of SU(4)



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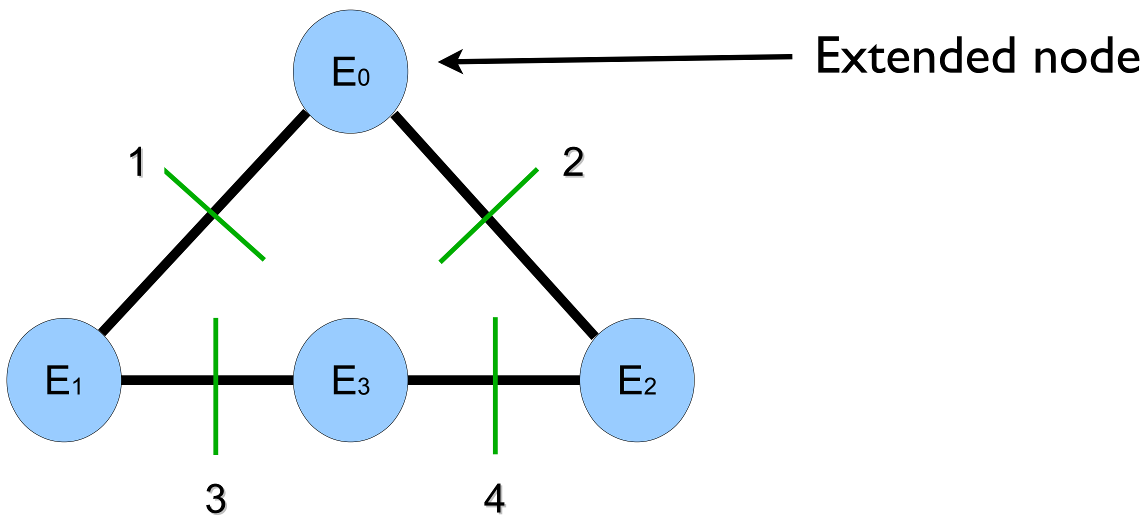

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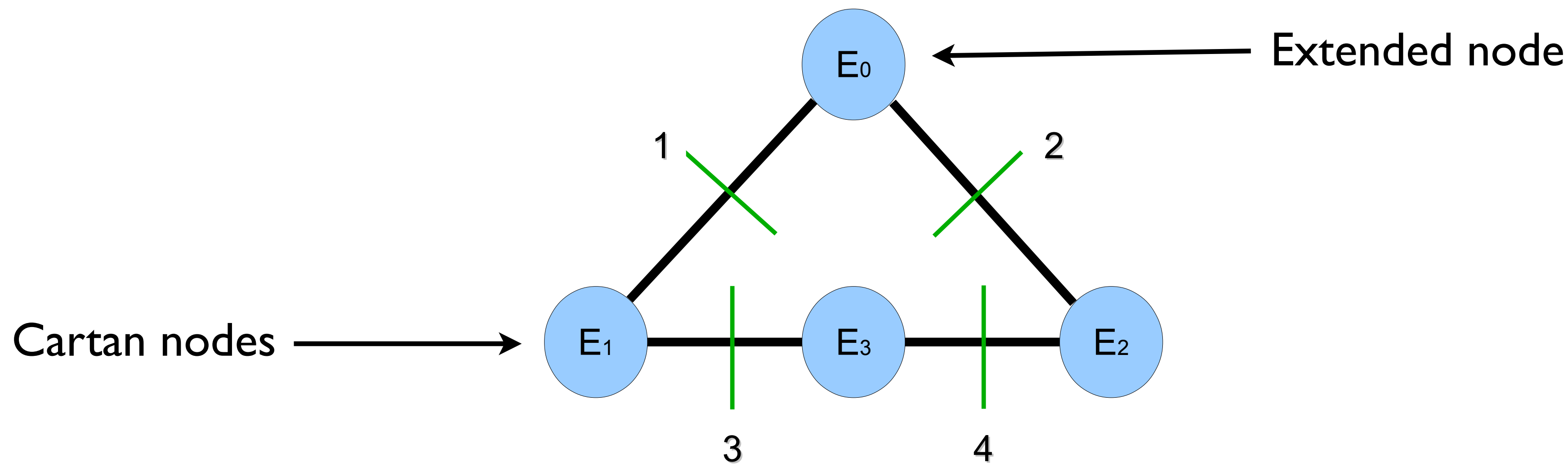

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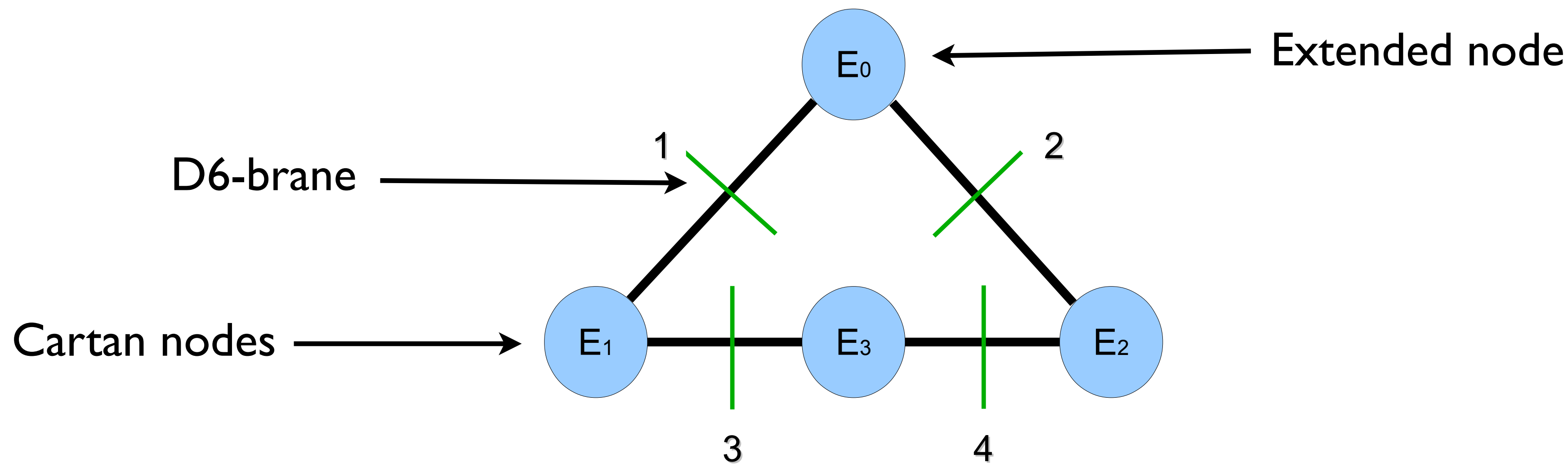

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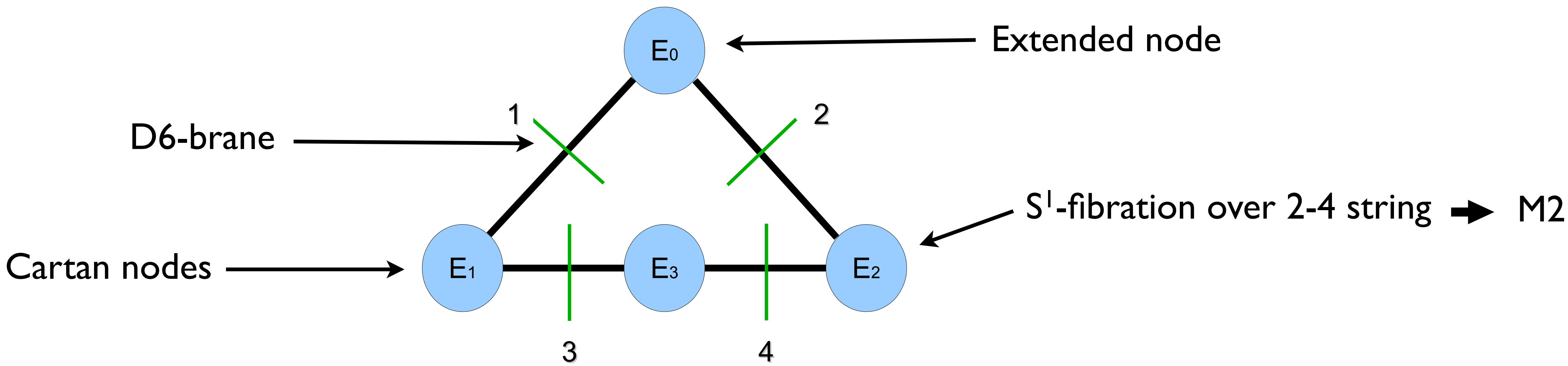

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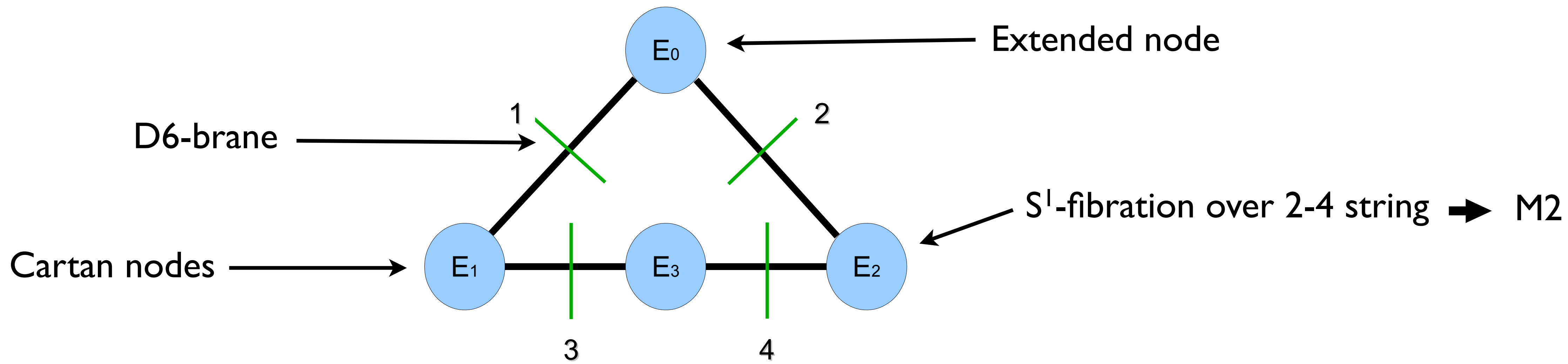

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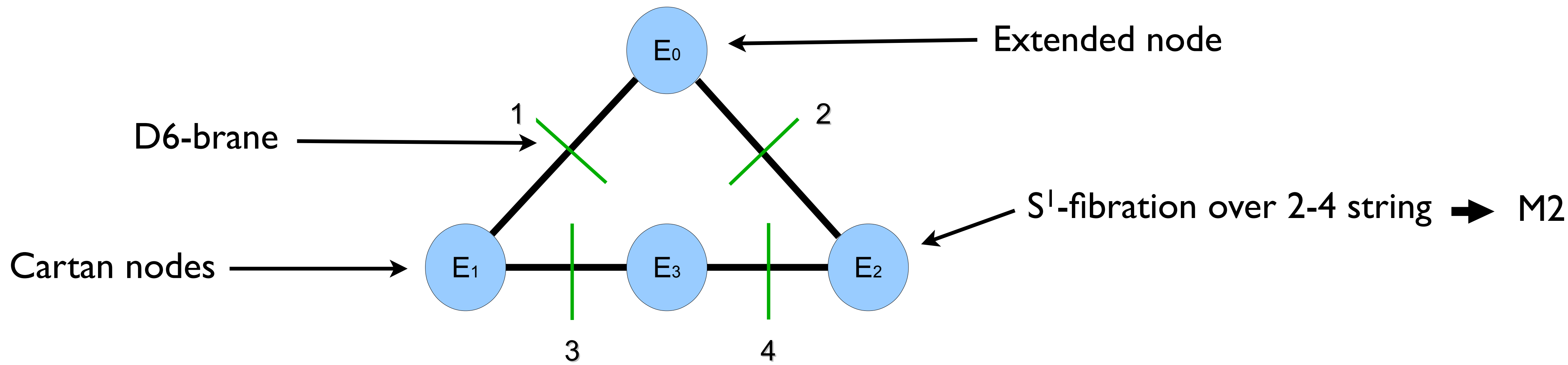
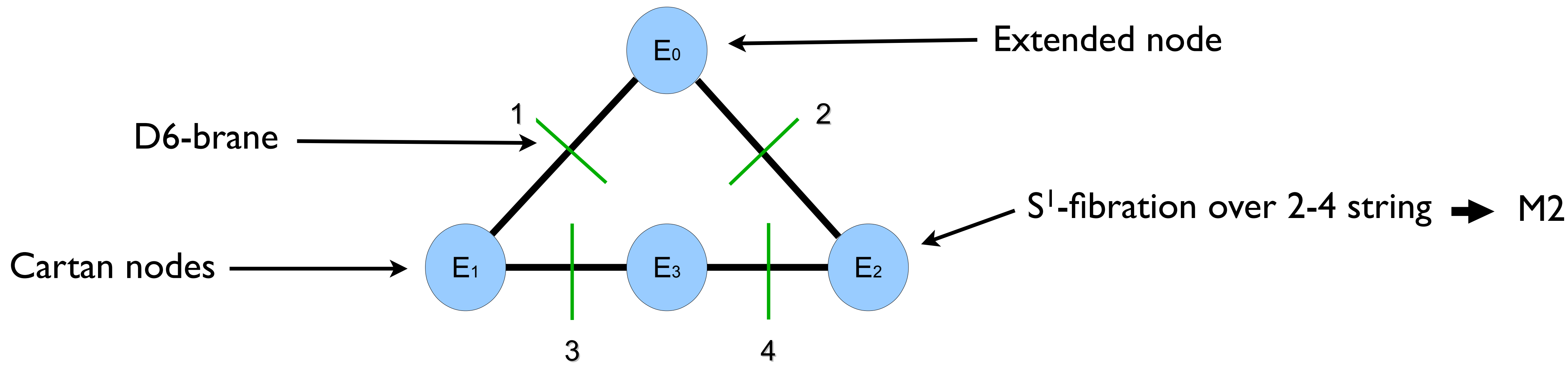


Loops of i-j IIA open strings → Loops of boundaries on D6i and D6j ➡ $C^{(2)}$

The SU(2N) case

Resolved fiber over SU(4) locus ⟷ Affine Dynkin diagram of SU(4)



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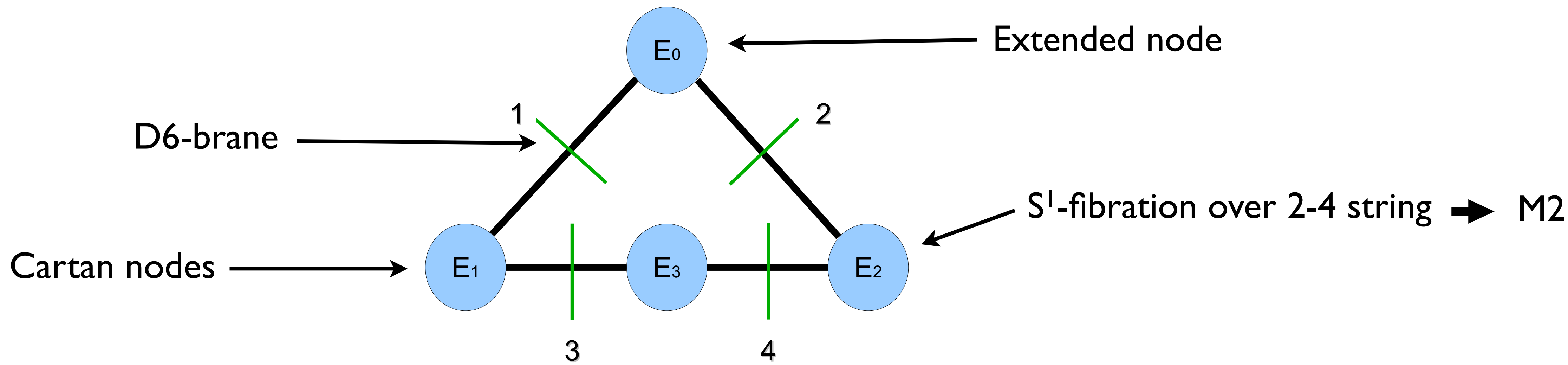

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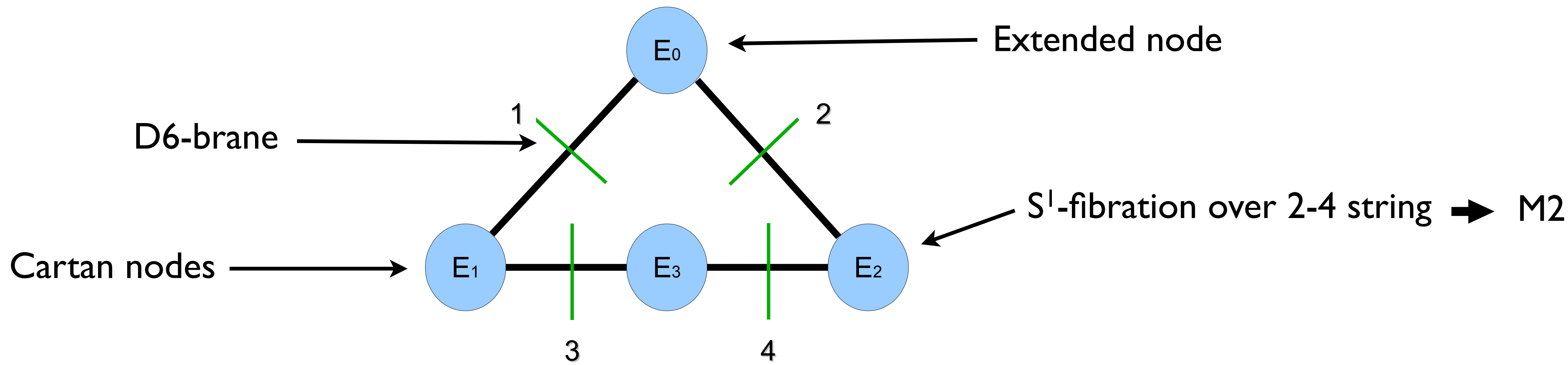

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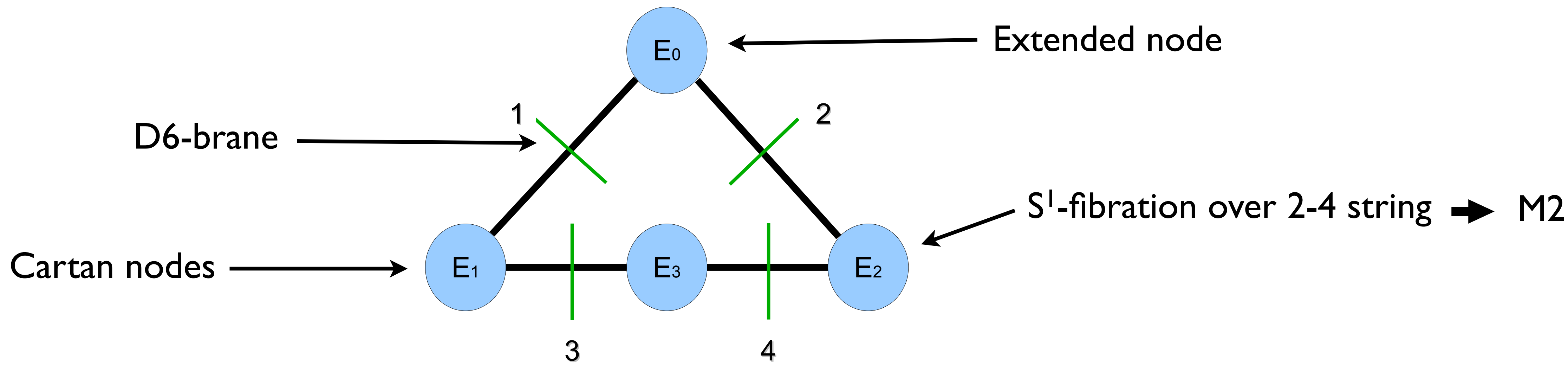

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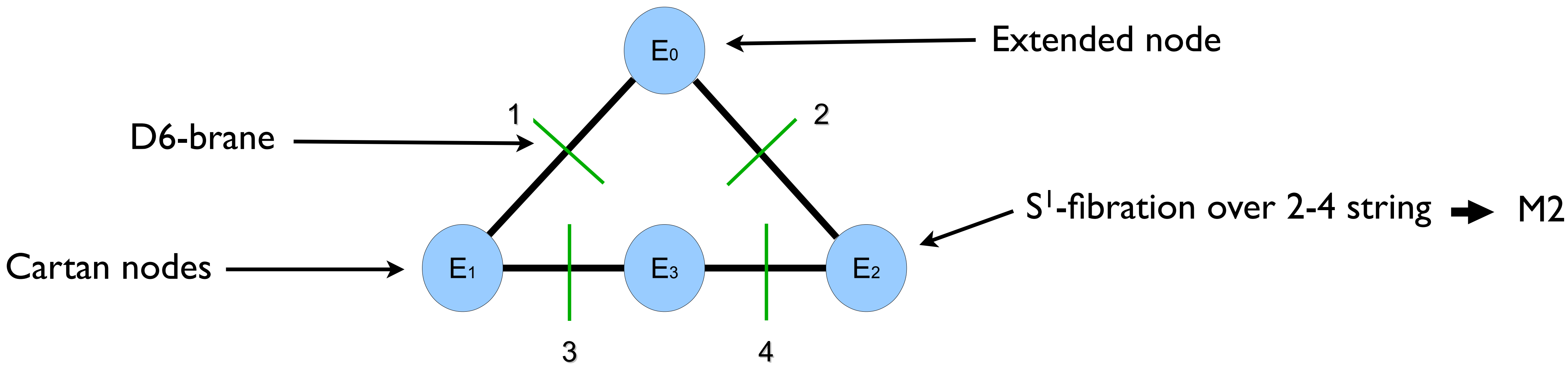

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(gauge flux)

$C^{(4)}$ complete-intersection ➡ $\int_{C^{(4)}} c_2$ is even

Krause, Mayrhofer, Weigand `12

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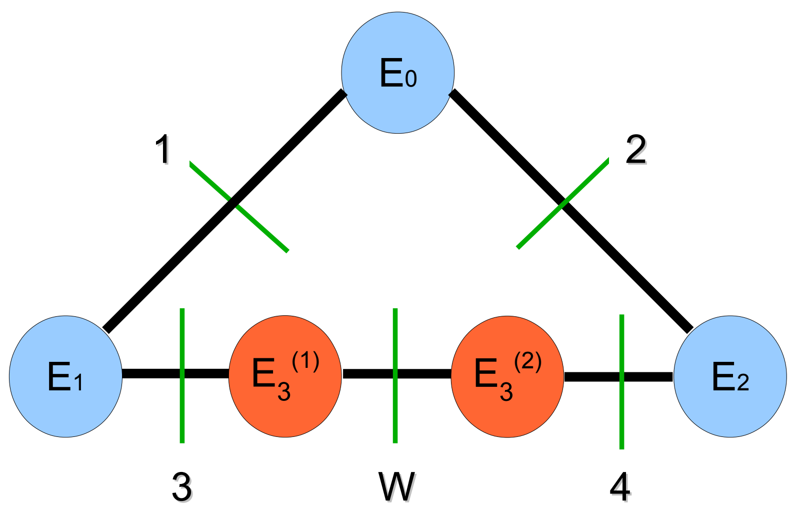

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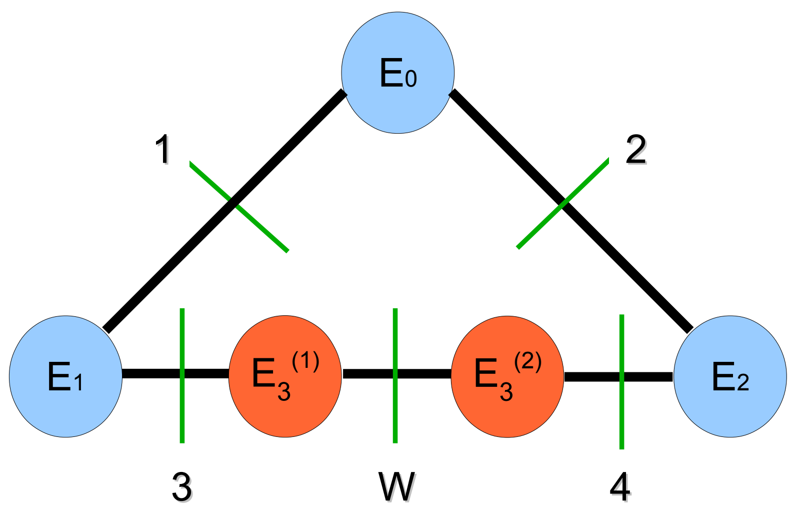

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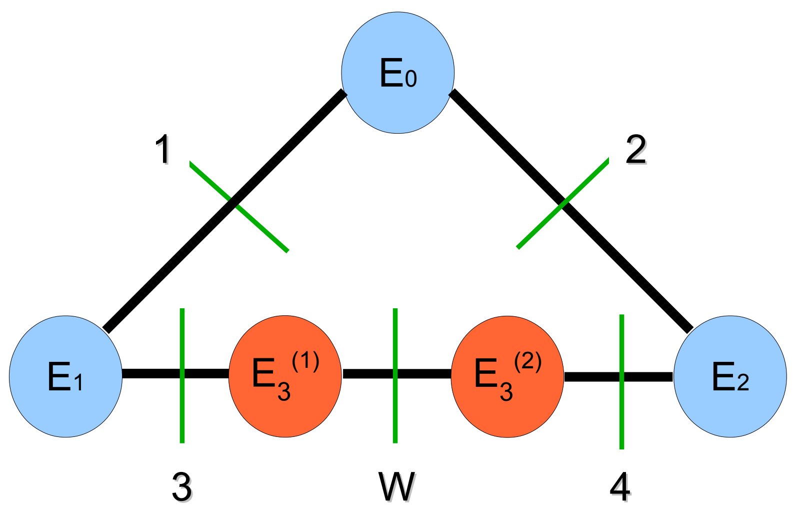

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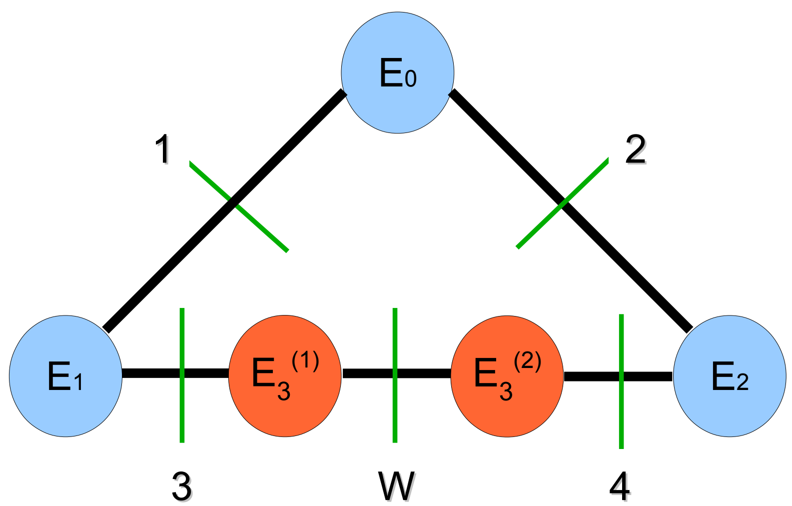

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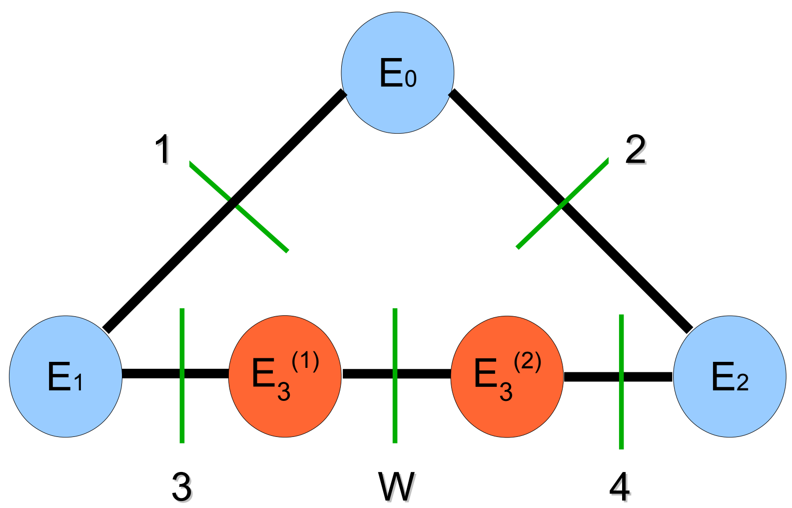

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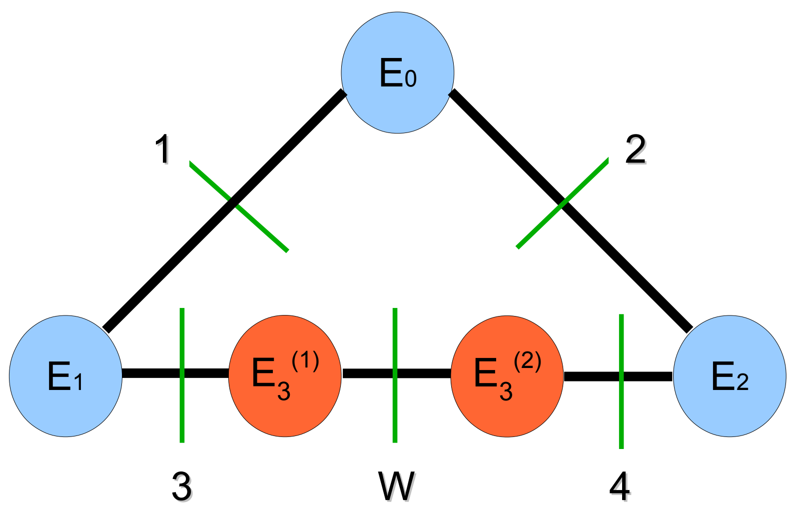

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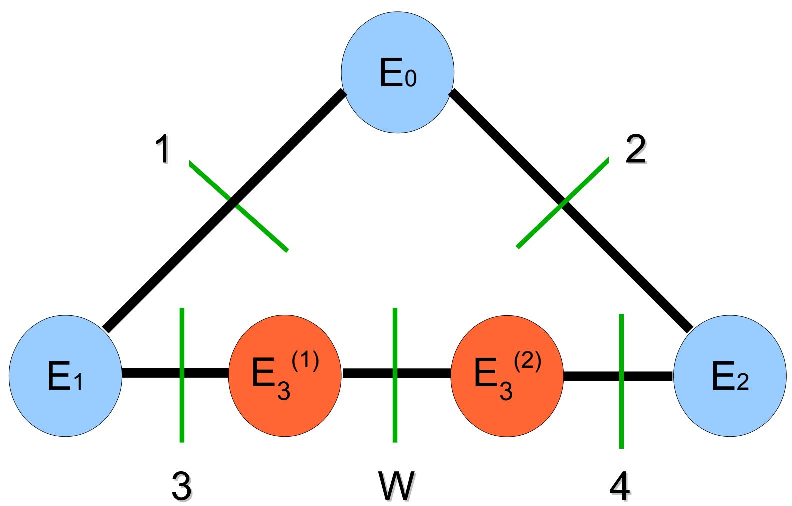

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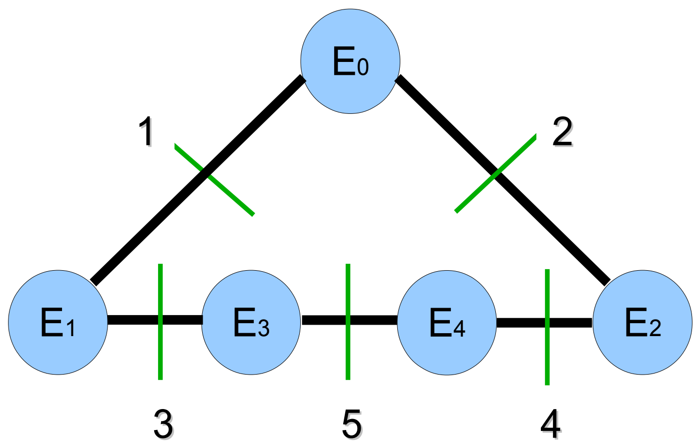
Same procedure applies for the Sp(N) series

The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

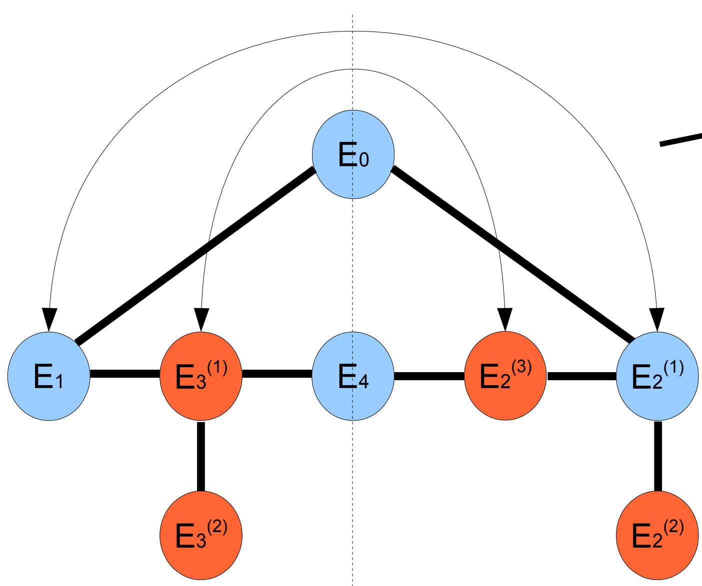


→ same strategy, but different fluxless object needed!

Orientifold plane

The new nodes pop up along the "antisymmetric-matter" curve $S \cap O7 \subset CY_3$

Again: Constrain CY_4 complex structure such that $S \cap O7$ is reducible and choose $C^{(2)}$ to be one component



→ Affine Dynkin diagram of SO(10)

The new integral, holomorphic 4-cycles are the orange nodes fibered over $C^{(2)}$

Result for SU(2N+1) N≥2

$$\int_{C^{(4)}} c_2(\mathrm{CY}_4) = \int_{C^{(2)}} S$$

Interpretation: $C^{(4)}$ lifts loops of closed, non-orientable strings intersecting S in $C^{(2)}$

This procedure works also for the SU(2N) series and lends better itself to treating the "U(1)-restricted" cases. Grimm, Weigand `10

Weak coupling limit

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For any SU(N) singularity: $a_2 = s\, a_{2,1}$ ➡ Type IIB CY$_3$: $(\xi + a_1)\,(\xi - a_1) = 4\, s\, a_{2,1}$

Conifold singularity

Donagi, Wijnholt '09

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The points $\{s = a_1 = a_{2,1} = 0\} \subset B_3$ typically accommodate "special" Yukawas in GUT models.

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Alternatively, we improve the implementation of Sen's limit

Idea: Obtain a "better-behaved" singularity of CY$_3$ M.Esole, R.S. `12

$$\begin{cases} a_{2,1} & \to \quad \epsilon\, a_{2,1} + s\, a_{2,2} \\ a_{3,m_3} & \to \quad \epsilon\, a_{3,m_3} \\ a_{4,m_4} & \to \quad \epsilon\, a_{4,m_4} \\ a_{6,m_6} & \to \quad \epsilon^2\, a_{6,m_6} \end{cases}$$

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➡ Type IIB CY3 : $\boxed{(\xi + a_1)\,(\xi - a_1) = 4\, s^2\, a_{2,2}}$

"Suspended pinch point" (spp) singularity in cod. 2: $\{a_1 = s = 0\} \subset B_3$

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Now there is an O-invariant and crepant resolution, which commutes with the double cover:

$$\widetilde{\mathrm{CY}}_3 : \begin{cases} (\xi + a)(\xi - a) = 4\sigma^2 a_{2,2} \\ a\, v = a_1 \\ \sigma\, v = s \end{cases}$$

$(a, \sigma) \neq (0, 0)$ proper transforms

$v = 0$ exceptional divisor

7-branes and tadpoles

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Focus on SU(5) : $a_2 = s a_{2,1},\ a_3 = s^2 a_{3,2},\ a_4 = s^3 a_{4,3},\ a_6 = s^5 a_{6,5}$

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$4s a_{2,1} \to 4s^2 a_{2,2}$

$a_{2,1} a_{3,2}^2 \to 0$

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This condition amounts to constrain its deformation moduli:

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Collinucci, Denef, Esole '08
Braun, Collinucci, Valandro `11

7-branes and tadpoles

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  - Blow-up + Charge conservation ➡ smooth IIB vacua lifting to F-theory with G-flux

Outlook

✦ When B_3 is non-spin we find "unexpected" patterns of quantization, for which a closer understanding is desirable.

➡ We give evidence that the G-flux contains the discrete information of a half-integral B-field.

✦ The SU(3) case behaves misteriously... G_4 always integral!

What is responsible to cancel 7-brane FW anomalies? Kapustin's mechanism?

Kapustin `99

✦ The outlined picture of the lift may be useful for several consistency checks.

➡ Prove that the G-flux quantization is designed to lead to well-defined chiral indices.

✦ The application of spp-singularity to GUT model building needs further study.

- Not all expected matter spectrum is realized in a standard way.
- Effective IIB realization of Yukawa couplings: Suitable D-instanton effects?