Local and global aspects of supersymmetry breaking

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Generalities

Theory determined by K, W and f, or alternatively (away from W=0) by $G \equiv K + \ln |W|^2$

1. W (holomorphic) - no perturbative corrections
2. f (holomorphic) - No higher than one loop perturbative corrections
3. K (real analytic) - Has both perturbative and NP corrections.

Potential:

$$V = e^G(G_i G^i - 3) + V_D$$
$$= e^{K/M_P^2}(D_i W K^{i\bar{j}} D_{\bar{j}} \bar{W} - 3\frac{|W|^2}{M_P^2}) + V_D$$

Relevant decoupling limits:

a) $M_P \to \infty \qquad \frac{\phi}{M_P} \to 0 \qquad \frac{K}{M_P^2}, \frac{W}{M_P} \to 0$

a′) $M_P \to \infty \qquad \frac{\phi}{M_P} \to 0 \qquad \frac{K}{M_P^2} \to 0, \frac{W}{M_P} = m_{3/2} M_P \neq 0 \text{ and } < \infty$

b) $M_P \to \infty \quad \frac{\Phi}{M_P} \neq 0, \frac{Q}{M_P} \to 0 \quad \frac{W}{M_P^2} \neq 0 \text{ and } < \infty$

Note a’) is GMSB limit while b) allows for ‘moduli’ which have Planck scale vev’s – relevant for gravity mediated/string theory related phenomenology.

In naïve global limit a)

$$D_i W \to \partial_i W \text{ and } V \to \partial_i W K^{ij} \partial_{\bar{j}} \bar{W}$$

Derivatives:

$$\begin{aligned}
\partial_i V &= e^G (G_i + G^l \nabla_i G_l) + G_i V \\
V_{i\bar{j}} &= e^G (G_{i\bar{j}} + \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}m\bar{n}} G^m G^{\bar{n}}) + (G_{i\bar{j}} - G_i G_{\bar{j}}) V \\
V_{ij} &= e^G (2 \nabla_i G_j + G^k \nabla_i \nabla_j G_k) + (\nabla_i G_j - G_i G_j) V
\end{aligned}$$

Fermion mass matrix:

$$m_{ij} = e^{G/2} (G_{ij} + G_i G_j) = e^{K/2M_P^2} D_i D_j W$$

In naïve global limit a) as well as in a')

$$m_{ij} \to \partial_i \partial_j W$$

But fermions in SUGRA mix with gravitino. Unmixing gives

$$\begin{aligned}\tilde{m}_{ij} &= e^{G/2} \quad (\nabla_i G_j + \frac{1}{3} G_i G_j) \\ &= \quad e^{K/2} e^{-i\phi_W} \left(D_i D_j W - \frac{2}{3} \frac{D_i W D_j W}{W} \right) \\ &\to \quad \left(W_{ij} - \frac{2}{3} \frac{\partial_i W \partial_j W}{W} \right).\end{aligned}$$

Last line valid in limits a) and a')

Eg:

$$W = c + \sigma_i \phi^i + \mu_{ij} \phi^i \phi^j + y_{ijk} \phi^i \phi^j \phi^k, \; K = \sum_i |\phi|^2$$

$$m_{ij} = \mu_{ij} + O(\phi_0)$$

Even in limit a) $\tilde{m}_{ij} = \mu_{ij} - 2\sigma_i \sigma_j / 3c + O(\phi_0)$

To cancel CC $\sigma_i \sigma^i = 3|c|^2 / M_P^2$

Only in a') and b)!

Issues are relevant in GMSB context – SUSY breaking done in global context.

Mass matrix: $$\mathbf{M} = \begin{pmatrix} M^2_{k\bar{l}} & M^2_{kn} \\ M^2_{\bar{m}\bar{n}} & M^2_{\bar{m}n} \end{pmatrix}$$

$$\begin{aligned} M^2_{\bar{l}k} &= m_{\bar{l}}{}^{j} m_{jk} + \frac{1}{M_P^2}(K_{\bar{l}k}|F|^2 - F_{\bar{l}} F_k) - 2m^2_{3/2} - R_{\bar{l}k\bar{m}n} F^{\bar{m}} F^n \\ M^2_{kn} &= e^{K/2M_P^2} D_n D_k D_i W F^i - D_k D_l W \bar{m}_{3/2} \end{aligned}$$

$$m_{3/2} \equiv e^{K/2M_P^2} W / M_P^2$$

Even in global limits a), a')
$$\begin{aligned} M^2_{\bar{l}k} &= m_{\bar{l}}{}^{j} m_{jk} - R_{\bar{l}k\bar{m}n} F^{\bar{m}} F^n \\ M^2_{kn} &= \nabla_n \nabla_k \nabla_i W F^i \end{aligned}$$

Statement that zero mode of m is a zero mode of M not valid unless metric is flat. Many global thms only valid for flat case

- Non-canonical terms in K can arise from integrating out massive fields for example even at tree level.

- Thms proved in GMSB context (eg Komargodski+Shih 0902.0300) valid only if we ignore these. For instance existence of a flat direction in O'R models.

- In limit b) - relevant for gravity/moduli mediation all terms of expressions contribute and the connection between the zero modes is lost even for canonical K. ~~SUSY~~ direction $\mathbf{v} = [F^i/|F|^2]$ is not a zero mode of m. $v^i m_{ij} = 2\bar{m}_{3/2} v_j$ when $\partial_i V = 0$.

- But $F^i \tilde{m}_{ij} = -\frac{2}{3}\frac{D_i W}{W} V|_0$ which vanishes when CC=0.

- Results of K+S have important consequences for GMSB but valid only in limits a) or a') and only for canonical K.

sGoldstino mean squared mass: Gomez-Reino+Scrucca

$$
\begin{aligned}
M_{sg}^2 &= \frac{1}{3} V_{i\bar{j}} G^i G^{\bar{j}} \\
&= \frac{1}{3} e^G \left(2 G_{i\bar{j}} G^i G^{\bar{j}} - R_{i\bar{j}k\bar{l}} G^i G^{\bar{j}} G^k G^{\bar{l}}\right) \\
&= \frac{2}{3} K_{i\bar{j}} \frac{F^i F^{\bar{j}}}{M_P^2} - \frac{1}{|F|^2} R_{i\bar{j}k\bar{l}} F^i F^{\bar{j}} F^k F^{\bar{l}}). \\
&= 2 m_{3/2}^2 - \frac{1}{3 M_P^2 m_{3/2}^2} R_{i\bar{j}k\bar{l}} F^i F^{\bar{j}} F^k F^{\bar{l}}
\end{aligned}
$$

Unless $R >> 1/M_P{}^2$ as is possible when some scale $\Lambda \ll M_P$ is integrated out or $R \ll 1/M_P{}^2$ as in LVS, generically this is of the order of the squared gravitino mass

Even in former case there are limits on how large sGoldstino can be. Consider Kitano model:

$$K = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2} + q\bar{q} + \tilde{q}\bar{\tilde{q}} + \frac{\lambda^2}{(4\pi)^2} S\bar{S} \ln \frac{(S\bar{S})^2}{\Lambda'^2}$$

$$W = c + \mu^2 S + \lambda S q \tilde{q}$$

$$M_{sg}^2 = < M_S^2 > = 2m_{3/2}^2 (1 + 6\frac{M_P^2}{\Lambda^2}(1 + 6(\frac{\lambda}{4\pi})^4 \frac{M_P^2}{\Lambda^2}) \simeq 12 \left(\frac{M_P m_{3/2}}{\Lambda}\right)^2$$

Stability condition: $\lambda/4\pi < \Lambda/M_P$

$$S = \Lambda^2/\sqrt{12} M_P,\ q = \tilde{q} = 0$$

But this scale cannot be made arbitrarily small. Two conditions

$$\frac{12\sqrt{3}}{(4\pi)}\frac{m_{3/2}M_P^3}{\Lambda^4} < \frac{\lambda}{4\pi} < \frac{\Lambda}{M_P}$$

Gaugino mass

$$M \simeq \frac{\alpha}{4\pi}\frac{F^S}{S} \simeq \frac{\alpha}{4\pi}\frac{\sqrt{3}m_{3/2}M_P}{\lambda\Lambda^2/(2\sqrt{3}M_P)}$$

To have an open window and GMSB dominance

$$10^{-5}GeV \leq m_{3/2} \ll 100GeV$$

$$M_S = 2\sqrt{3}m_{3/2}\left(\frac{4\pi\lambda M}{6\alpha m_{3/2}}\right)^{1/2} \sim 10\sqrt{\lambda}\sqrt{m_{3/2}M}$$

So modulus is well below Gaugino mass scale.

Bound on the superpotential? Dine Festuccia Komargodski

Global SUSY argument! $k^i = iq^i\Phi^i$

$$k^i W_i = i2W$$

$$\mathbf{U} = \{U^i\}, \mathbf{V} = \{V^i\}$$

$$< \mathbf{U}, \mathbf{V} > = \overline{< \mathbf{V}, \mathbf{U} >} \equiv K_{i\bar{j}} V^i \bar{U}^{\bar{j}}$$

$$\bar{U}^{\bar{j}} = K^{\bar{j}l}\partial_l W \text{ and } V^i = k^i$$

$$< \mathbf{U}, \mathbf{V} > = i2W$$

Assume spontaneous breaking of R-symmetry

$$\phi^i = \phi_0^i e^{iq^i a(x)}$$

$$K_{i\bar{j}}\partial\Phi^i\partial\bar{\Phi}^{\bar{j}} \to K_{i\bar{j}} k_0^i k_0^{\bar{j}} (\partial a)^2$$

So the axion decay constant is given by

$$f_a^2 = K_{i\bar{j}} k_0^i k_0^{\bar{j}} = < \mathbf{V}_0, \mathbf{V}_0 >$$

Using Cauchy-Schwarz inequality:

$$4|W_0|^2 = | < \mathbf{U}_0, \mathbf{V}_0 > | \leq < \mathbf{U}_0, \mathbf{U}_0 > < \mathbf{V}_0, \mathbf{V}_0 > = |F|_0^2 f_a^2$$

$$|F|^2 = K^{i\bar{j}} \partial_i W \partial_{\bar{j}} \bar{W}$$

This bound is however not meanigful since in global SUSY the value of the superpotential at the minimum has no significance. Need to reconsider argument in SUGRA.

SUGRA is invariant under Kähler transformations

$$K \to K+\Lambda+\bar{\Lambda},\ W \to e^{-\Lambda}W$$

Bound is not invariant under this – so invalid in SUGRA

R-symmetric K implies

$$k^i\partial_i K + \bar{k}^{\bar{i}}\partial_{\bar{i}} K = iq^i\phi^i\partial_i K - iq^i\bar{\phi}^{\bar{i}}\partial_{\bar{i}} K = 0 \Rightarrow q^i\phi^i\partial_i K = \overline{q^i\phi^i\partial_i K}$$

$$F_i = e^{K/2}D_iW \to e^{(\bar{\Lambda}-\Lambda)/2}F_i \Rightarrow |F_i| \to |F_i|$$

$$k^iF_i = (2i + k^iK_i)e^{K/2}W = i(2+\sum_i q^i\phi^iK_i)e^{K/2}W$$

$$|<\mathbf{k},\mathbf{F}>_0| = |2+\sum_i q^i\phi_0^i\partial_iK_0|m_{3/2}$$

From Cauchy-Schwarz inequality

$$f_a^2 < \mathbf{F}, \mathbf{F} > |_0 \geq |2 + \sum_i q^i \phi_0^i \partial_i K_0|^2 m_{3/2}^2$$

This is still not Kähler invariant. But for models in which sum is +ve get a Kähler invariant bound.

$$f_a^2 < \mathbf{F}, \mathbf{F} >_0 \geq 4 m_{3/2}^2 M_P^4$$

This is the natural Kähler invariant version of global result. But need to tune CC to zero!

$$< \mathbf{F}, \mathbf{F} >_0 \equiv F_i K^{i\bar{j}} \bar{F}_{\bar{j}}|_0 = 3 m_{3/2}^2 M_P^2$$

$$f_a^2 \geq \frac{4}{3} M_P^2$$

So the bound on W/gravitino mass disappears!

Generically get: $f_a^2 \gtrsim M_P^2$

No FI terms in SUGRA? Komargoski + Seiberg; Dienes+Thomas

A SUGRA example: Note no 'new' global symmeries!

$$G \equiv K + \hat{\xi} V + \ln \frac{|W|^2}{M_P^6}$$

Similar model by Catino, Villadoro, Zwirner

$$K = \bar{S} e^V S + \sum \bar{\Phi}\Phi, \; W = S^{\hat{\xi}/M_P^2} W_I(\Phi)$$

$$\begin{aligned} V &\to V + i(\Lambda - \bar{\Lambda}) \\ S &\to e^{-i\Lambda} S \end{aligned}$$

Full superspace integral in action can be taken to be (for $S \neq 0$) :

$$-3M_P^2 \int d^4\theta \mathbf{E} e^{-[K + \hat{\xi} V + \hat{\xi}(\ln(S/M_P) + h.c.)]/3M_P^2}$$

Note that this is gauge invariant by itself – so is the chiral integral in this Kähler frame

Now take global limit:

$$M_P^2 \rightarrow \infty, \quad \mathrm{E} \rightarrow 1,$$
$$\xi = \frac{\hat{\xi}}{M_P^2} \rightarrow O(\frac{1}{M_P^2})$$

$$\int d^4\theta[-3M_P^2 + (K + \hat{\xi}V + \hat{\xi}(\ln S/M_P + h.c.) + O(\frac{1}{M_P^2})] \rightarrow \int d^4\theta(K + \hat{\xi}V)$$

if $\hat{\xi}$ is quantized in Planck units (i.e. $\xi = n \in \mathcal{Z}$)

$$\int d^4\theta K + nM_P^2(V + \ldots)$$

limit does not exist!

Note that in the case $\xi < 1$ S=0 is not a minimum of potential

For simplicity consider Φ neutral. In global limit get:

$$V_{global} = \mid \frac{\partial W_I}{\partial \Phi} \mid^2 + \frac{g^2}{8}(S\bar{S} + \hat{\xi})^2$$

Gauge invariant min at $S = 0$

$$V_{global,0} = \frac{g^2}{8}\hat{\xi}^2$$

The full SUGRA potential however is:

$$e^{(S\bar{S}+\Phi\bar{\Phi})/M_P^2} \left(\frac{S\bar{S}}{M_P^2}\right)^{\hat{\xi}/M_P^2} [\frac{\mid W_I \mid^2}{S\bar{S}}(\frac{\hat{\xi}}{M_P^2} + \frac{S\bar{S}}{M_P^2})^2 + \mid \partial_\Phi W_I + \frac{\bar{\Phi}}{M_P^2} W_I \mid^2 - 3\frac{\mid W_I \mid^2}{M_P^2}]$$
$$+ \frac{g^2}{8}(S\bar{S} + \hat{\xi})^2$$

$M_P^2 \to \infty$ with $W_I, \hat{\xi}, S, \Phi$ fixed

gives global potential above.

But in limit a') i.e.

$W_I/M_P \equiv \mu^2$ fixed but without assuming that $s_0 \to O(1/M_P^2)$

$$V \to e^{S\bar{S}/M_P^2}[\mu^4 \frac{S\bar{S}}{M_P^2} + |\partial_\phi W_I|^2 - \mu^4] + \frac{g^2}{8}(S\bar{S} + \hat{\xi})^2$$

Still the min. is at S=0.

On the other hand with $\xi = \hat{\xi}/M_P^2 < 1$ and fixed,
there is no gauge invariant minimum.

$$|s_0|^2 = \frac{S\bar{S}_0}{M_P^2} \neq 0$$

Gauge invariant minimum is an artifact of the decoupling limit

Above discussion part of paper to be published with K. Dienes and B. Thomas

Quantum Effects:

It has been argued that $\hat{\xi} = nM_P{}^2$ Seiberg
Even if this is the case one can still get an effective low energy theory by taking the scale of the superpotential W_I well below the Planck scale.

Also U(1) and mixed anomalies can be cancelled by adding charged fields as in Catino et al

Quantization of FI term related to quantization of U(1) charge for fermion – but this is not required for instance in Kähler frame where theory is defined by G.

For $\xi < 1$ min. not at W=0. In any case in functional integral such points in field space are generically a set of measure zero. It's not clear that that is a problem!

Green-Schwarz anomaly cancellation

Add new field $$T \to T - iM\Lambda$$

$$\Delta K = -3\ln(T + \bar{T} + MV)$$

$$f = \frac{1}{g^2} + b_{UU}T$$

$$\begin{aligned} V &= V_F + V_D \\ &= \frac{|c|^2}{(T+\bar{T})^3} e^{S\bar{S}} (S\bar{S})^{\xi-1} (S\bar{S}+\xi)^2 \\ &\quad + \frac{g^2}{8(1+\frac{g^2}{2} b_{UU}(T+\bar{T}))} (S\bar{S} + \xi - \frac{3M}{T+\bar{T}})^2 \end{aligned}$$

For ξ >1 SUSY minimum at

$$S\bar{S} = 0, \ \Re T = 3M/2\xi.$$

Note FI term prevents runway to infinity

Note again that there are no ‘new’ global symmetries but theory has no strict global limit!

$\xi < 1$ SUSY and gauge invariance are broken

$\Re T$ runs away to infinity at the global minimum

A local minimum with finite T may exist if M,ξ are such that a quartic eqn for ReT has real roots.

Conclusions

- Main point of this work is that a generic SUGRA can have different global limits. The action of global SUSY in Minkowski space is just one possibility.
- If one starts from the latter only a limited class of SUGRA's can be constructed.
- Arguments made in the global context often need to be revisited in the light of SUGRA. In particular CC can be tuned to zero (after SUSY breaking) only in SUGRA.
- Two issues relevant to GMSB. One mass of sGoldstino and other a bound on the superpotential need to be revisted – the latter in fact goes away!
- Arguments relating to FI terms in SUGRA when latter is constructed starting from global SUSY need to be revisted. There are consistent SUGRA's with FI terms and no 'new' global symmetries – naïve global limit will not exist though.

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Logic of SUSY breaking and AMSB

- A SUSY Wilsonian effective action is determined by (quantum corrected) K, W and f at the two derivative level
- An anomaly can only change the gauge coupling function f at this level:
- Once this quantum correction is $f \to f + \frac{3c}{4\pi^2}\tau$ incorporated the gaugino mass given by the Appendix G of WB
- So $\Delta m = \frac{3c}{16\pi^2} g^2 F^\tau \partial_\tau \tau = \frac{c}{32\pi^2} g^2 F^A K_A$. In particular this vanishes when $F^A = 0$ consistent with the vanishing of gaugino mass in SUSY

Logic of SUSY breaking and AMSB II

- In AMSB on the other hand and extra term proportional to W exists. i.e. gaugino mass does not have the form expected i.e. $F^A \partial_A f$ for a supersymmetric action
- In particular gaugino mass term in the Wilsonian action does not vanish in the supersymmetric limit.

Logic of SUSY breaking and AMSB III

- Can there be infra-red effects in AdS as claimed by Griapios et al.?
- Such effects background dependent have no place in Wilsonian action. 1PI action not meaningful in this context – soft terms are calculated at some high scale and RG evolved down to TeV scale within a Wilsoninan context!
- Related issues about breakdown of non-renomalization thms in global SUSY

In WZ model it has been claimed that coupling is renormalized at two loops in m=0 limit.

Based on a (chiral) two loop contribution

$$\int d^8z \frac{D^2}{\Box} G(\Phi) = -\frac{1}{4}\int d^6z \frac{\bar{D}^2 D^2}{\Box} G(\Phi) = -4\int d^6z G(\Phi)$$

But the actual propagator at a generic point in field space is

$$\begin{aligned} G_{--}(z,z') &= D^2 D'^2 \frac{1}{\Box - \mathcal{M}^2}(1+\Delta)\delta^8(z-z') \\ &= D^2 D'^2 \frac{1}{\Box - \Psi\bar{\Psi}}(\Psi\frac{\bar{D}^2}{4}\Box^{-1} + O(\bar{\Psi}, D\Psi, \bar{D}\bar{\Psi}))\delta^8(z-z') \end{aligned}$$

If this is expanded $\Psi = m + \lambda\Phi$

$$G_{--}(z,z') = D^2 D'^2 \frac{1}{\Box}(\Psi\frac{\bar{D}^2}{4}\Box^{-1} + O(\bar{\Psi}, D\Psi, \bar{D}\bar{\Psi}))\delta^8(z-z')$$

get infra-red issue in m=0 theory only at origin of field space!

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• The effects observed by Conlon et al, in string theory calculations related to supposed violations of non-renormalization theorems.

• But 1PI action should be calculated at an arbitrary point in field space –propagator has a natural IR cutoff at a generic point in field space $1/(k^2 + \lambda\phi)$

•It should NOT be expanded in $\lambda\phi$! No violation of NR thms at a generic point – the claim on "no non-renormalization" thm in the massless WZ model is an artifact of working at a infra-red singular point in field space.

•Infra-red issues in gaugino mass calculations have the same problem. In any case we should be looking at the Wilsonian action and its parameters at some high scale. These are then RG evolved down to the TeV scale. This is a perfectly well defined procedure – has no infra red issues!

Conclusions

- A two derivative SUSY theory is necessarily restricted to small SUSY breaking i.e. F/M^2 ; M is "lowest integrated out scale".
- In such a situation the general KL formulae are valid – at all scales from M down to gravitino/soft mass scale. They are RG invariant. In GMSB case only needed modification is to add contribution of messenger threshold.
- The textbook AMSB formulae for scalar masses etc which are RG invariant can be obtained without any insertion of Weyl compensator factors. They follow if one makes a certain factorization assumption for the (moduli dependent) matter metric. The AMSB term for gaugino masses would represent an explicit breaking of SUSY. It should not be there!.
- GMSB formulae can also be obtained from the general KL formulae when messenger threshold is included - additional (loop suppresed) term compared to usual formula for gaugino mass is obtained.
- The simplest mediation mechanism is inoAMSB where in a sequestered situation the gaugino mass is generated by the Weyl anomaly in accordance with KL and scalar masses and A, Bμ terms come from RG running. Explicit string theory realization in IIB LVS

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