<u>Heterotic non Calabi-Yau</u> <u>Compactifications</u>



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• Introduction

- Cosets as examples of half-flat mirror spaces
- Gauge fields on cosets
- Including α' corrections
- Moduli stabilization
- Calabi-Yau manifolds and NS flux
- Conclusion

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SU(3) structures classified by 5 torsions classes $\mathcal{W}_1,\ldots,\mathcal{W}_5$ with

$$\mathrm{d}J = -\frac{3}{2}\mathrm{Im}(\mathcal{W}_1\bar{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$

$$\mathrm{d}\Omega = -\mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \bar{\mathcal{W}}_5 \wedge \Omega$$

Interesting classes of SU(3) structure manifolds:

name	M_4	H	ϕ	\mathcal{W}_i	properties
CY	max. symm.	0	const	$\mathcal{W}_i = 0$	complex Kahler, CY
Strominger's	max. symm.	$\neq 0$	varies	$\mathcal{W}_1 = \mathcal{W}_2 = 0$ $\mathcal{W}_4 = \mathcal{W}_5/2 = d\phi$	complex
half-flat	domain wall	0	const	$\mathcal{W}_1^- = \mathcal{W}_2^- = 0$ $\mathcal{W}_4 = \mathcal{W}_5 = 0$	
nearly Kahler	domain wall	0	const	only $\mathcal{W}_1^+ eq 0$	
gen. half-flat	domain wall	$\neq 0$	varies	$\mathcal{W}_1^- = \mathcal{W}_2^- = 0$ $\mathcal{W}_4 = \mathcal{W}_5/2 = d\phi$	

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Why allow for a 4d domain wall?

- Domain wall still consistent with 4d covariant theory
- But 4d superpotential, e.g. $W = e_i T^i$, has runaway directions and domain wall is "simplest" solution.

Two main questions addressed in this talk:

• In the context of half-flat/nearly Kahler manifolds:

Can domain wall vacuum be lifted to a maximally-symmetric one, for example by α' corrections or non-perturbative effects?

• In the context of generalized half-flat manifolds:

Is there a new perspective on CY compactifications with H-flux?

(Gurrieri, Louis, Micu, Waldram, 2002)

half-flat mirror spaces: forms $(\omega_i, \tilde{\omega}^j)$ and (α_A, β^B) with

$$\mathrm{d}\omega_i = e_i\beta^0 \ , \quad \mathrm{d}\alpha_0 = e_i\tilde{\omega}^i$$

half-flat structure: $J = t^i \omega_i$, $\Omega = Z^A \alpha_A + G_A \beta^A$

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Monday, October 1, 2012

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Cosets as explicit examples:
$$\frac{G}{H} = \underbrace{\frac{SU(3)}{U(1)^2}}_{U(1)^2}, \underbrace{\frac{Sp(2)}{SU(2) \times U(1)}}_{\text{focus on this one}}, \underbrace{\frac{G_2}{SU(3)}}_{\text{focus on this one}}$$

(Chatzistavrakidis, Zoupanos, 2009)

"natural" vielbein e^1, \ldots, e^6 from left-inv. one-form on SU(3):

$$\begin{split} \omega_1 &= -\frac{1}{2\pi} \left(e^{12} + \frac{1}{2} e^{34} - \frac{1}{2} e^{56} \right) & \tilde{\omega}^1 &= \frac{4\pi}{3\mathcal{V}_0} \left(2e^{1234} + e^{1256} - e^{3456} \right) \\ \omega_2 &= -\frac{1}{4\pi} \left(e^{12} + e^{34} \right) & \tilde{\omega}^2 &= -\frac{4\pi}{\mathcal{V}_0} \left(e^{1234} + e^{1256} \right) \\ \omega_3 &= \frac{1}{3\pi} \left(e^{12} - e^{34} + e^{56} \right) & \tilde{\omega}^3 &= \frac{\pi}{\mathcal{V}_0} \left(e^{1234} - e^{1256} + e^{3456} \right) \\ \alpha_0 &= \frac{\pi}{2\mathcal{V}_0} \left(e^{136} - e^{145} + e^{235} + e^{246} \right) & \beta^0 &= \frac{1}{2\pi} \left(e^{135} + e^{146} - e^{236} + e^{245} \right) \end{split}$$

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4d "moduli" fields: 3 T-moduli $T^i = t^i + i\tau^i$ dilaton $S = s + i\sigma$ no "complex structure" moduli Four-dimensional theory:

(Gurrieri, Lukas, Micu, 2004)

$$K = -\ln(S + \bar{S}) - \ln\kappa , \quad \kappa = d_{ijk}t^i t^j t^k$$

$$W \sim \int_X \Omega \wedge (H + idJ) \sim e_i T^i = T^3$$

runaway directions -> domain wall

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So far this generalizes to all half-flat mirror manifolds, but gauge fields are difficult to deal with in general....

Gauge fields on cosets

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-> line bundles $L = \mathcal{O}_X(\mathbf{p})$, with $c_1(L) = p \omega_1 + q \omega_2$

$$V = \bigoplus_{a=1}^{n} \mathcal{O}_X(\mathbf{p}_a)$$
 where $c_1(V) \sim \sum_{a=1}^{n} \mathbf{p}_a = 0$

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-> X nearly Kahler

Bianchi identity:
$$dH = \frac{\alpha'}{4} \left(\operatorname{tr} F \wedge F + \operatorname{tr} \tilde{F} \wedge \tilde{F} - \operatorname{tr} R^{-} \wedge R^{-} \right)$$

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Can be solved: $H \sim \mu(\mathbf{p}_a, \tilde{\mathbf{p}}_a) \alpha_0$ -> Killing spinor equations

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$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T})$$

$$W \sim \int_X \Omega \wedge (H + idJ) \sim T + \mu$$

Can now fix remaining T-modulus but dilaton still runaway...

Monday, October 1, 2012

Moduli stabilization

Add gaugino condensate: $W = T + \mu + ke^{-cS}$

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F-flat conditions $F_S=F_T=0$ lead to au=0 , $y=0,\pi$ and

$$(1-x)e^{-x} = \frac{\mu}{k}$$
 $t = \frac{3x}{1-x}\mu$

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Two main differences to CY gaugino condensation and H-flux:

- Additional T-dependent contribution from torsion
- H-flux is not harmonic but bundle-induced

What about consistent field values, t > 1 (α' expansion valid) and s > 1 (weak coupling)?

Tension: large t needs large flux μ and large s requires small μ

"Compromise" values for μ which lead to marginally consistent field values can be obtained for suitable bundle choices.

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$$J \wedge dJ = J \wedge J \wedge d\phi$$

$$J \wedge H = *d\phi$$

$$dJ = 2\phi'\Omega_{-} - \Omega'_{-} - 2d\phi \wedge J + *H$$

$$d\Omega_{+} = J \wedge J' - \phi'J \wedge J + 2d\phi \wedge \Omega_{+}$$

$$\Omega_{-} \wedge H = 2\phi' * \mathbf{1}$$

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flow eqs. $2\phi' * \mathbf{1} = \Omega_- \wedge H$ $J \wedge J' = \phi' J \wedge J$ $\Omega'_- = 2\phi' \Omega_- + *H$

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A domain wall solution to these eqs. exists. It can also be obtained by solving 4d theory on CY with superpotential

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- Previously: H-flux requires non-Kahler spaces (Strominger system)
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- Now: Keep space CY even in the presence of H-flux
 - + : CY methods available : need to lift domain wall

Conclusion

- Non-Calabi-Yau compactifications of string theory are interesting but progress is hampered by the lack of examples. (Larfors, Lust, Tsimpis, 2010) (Gray, Larfors, Lust, 2012)
- Half-flat and gen. half-flat spaces provide solutions of the het. string if combined with a 4d domain wall.
- Coset provide explicit examples of half-flat spaces and α' corrections can be worked out explicitly.
- For cosets, a combination of α' and non-pert. effects can lift the domain wall to AdS. There are consistency issues....
- H-flux is consistent with keeping the internal space CY if the 4d space-time is a domain wall.

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Thanks!