Fluxes and M5-instantons in F-theory

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Invitation

F-theory compactifications to 4 dimensions are so much fun:

- Realisation of local **GUT model building** ideas in global models

  $\rightsquigarrow$ brane world paradigm $\leftrightarrow$ exceptional gauge symmetry

  [Beasley,Heckman,Vafa][Donagi,Wijnholt] '08

- Better understanding of more formal aspects of **F-theory vacua**

  $\rightsquigarrow$ Exploitation of holomorphic geometry of elliptic 4-folds

  $\rightsquigarrow$ Extra M-theory ingredients: fluxes and instantons

Focus of this talk:

I) **Geometry (singularities)** (briefly)

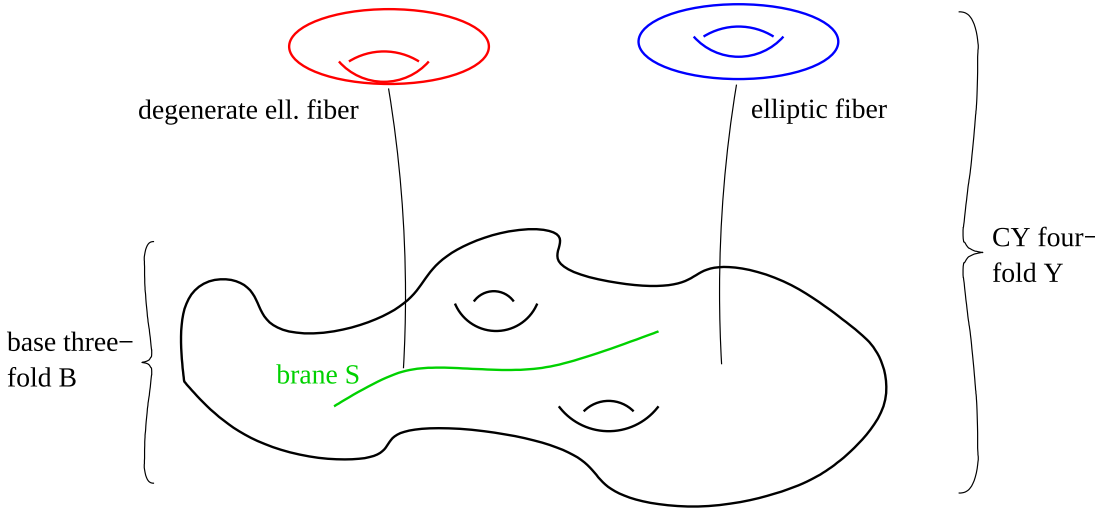
II) **Gauge fluxes** (briefly)

III) **M5-instantons**

I) The geometry

The framework

Elliptic fibration $Y_4 : T^2 \to B$



$$y^2 = x^3 + f(u_i)xz^4 + g(u_i)z^6$$

- fibre coordinates $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$
- B_6 coordinates: u_i
- singular over $\Delta \equiv 4f^3 + 27g^2 = 0$

Matter and **interactions** in 4D $\leftrightarrow$ **singularities of elliptic fiber**

Hierarchy of localisation of structure within base B:

- gauge fields G = SU(5) $\leftrightarrow$ GUT divisor (codim. 1)
- massless matter states $\mathbf{10}, \overline{\mathbf{5}}_m, \mathbf{5}_\mathbf{H}$ $\leftrightarrow$ curve (codim. 2)
- Yukawa couplings $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_\mathbf{H}, \mathbf{10}\,\overline{\mathbf{5}}_\mathbf{m}\,\overline{\mathbf{5}}_\mathbf{H}$ $\leftrightarrow$ point (codim. 3)

$\Longrightarrow$ Understanding **singularities** and their **resolution** is key

$SU(5) \times U(1)_X$ Tate models

- Example: **SU(5) singularity along $w = 0$ in Tate form**:

$$P_T : y^2 + a_1 xyz + a_3 yz^3 = x^3 + a_2 x^2 z^2 + a_4 xz^4 + a_6 z^6$$

$$a_1 = a_1, \quad a_2 = a_{2,1}\, w, \quad a_3 = a_{3,2}\, w^2, \quad a_4 = a_{4,3}\, w^3, \quad a_6 = a_{6,5}\, w^5$$

- **Extra $U(1)$s** often desirable for phenomenology

  Simplest example: $a_6 \equiv 0 \leftrightarrow SU(5) \times U(1)_X$ [Grimm,TW '10]

Complete resolution of all singularities by sequence of blow-ups:

- $SU(5)$ first resolved torically in [Blumenhagen,Jurke,Grimm,TW]'09
- As 4 base-independent blow-ups [Krause,Mayrhofer,TW]'11

$$(x, y, w) \to (x e_1 e_4 e_2^2 e_3^2, y e_1 e_4^2 e_2^2 e_3^3, w e_1 e_2 e_3 e_4)$$

  ✓ applicable also to co-dim. 2 and 3 - see also [Grimm,Hayashi]'11

- Different, but equivalent approach in [Esole,Yau] [S.-Nameki,Marsano]'11
- $U(1)_X$: 1 extra blow-up $\quad y = \tilde{y} s, \quad x = \tilde{x} s$

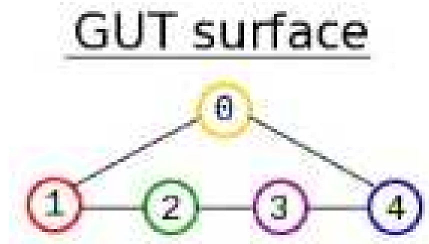
  [Grimm,TW'10],[Braun,Collinucci,Valandro][Krause,Mayrhofer,TW][Grimm,Hayashi]'11

Structure of resolved fiber

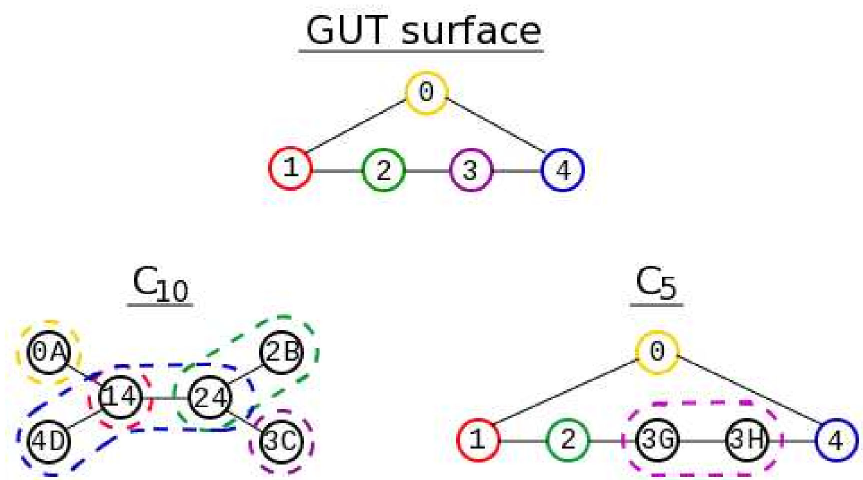
Over divisor: intersecting $\mathbb{P}^1_i$ - 1 for each resolution divisor $E_i : e_i = 0$

$\Rightarrow$ affine Dynkin diagram of SU(5)

(together with $\mathbb{P}^1_0$)



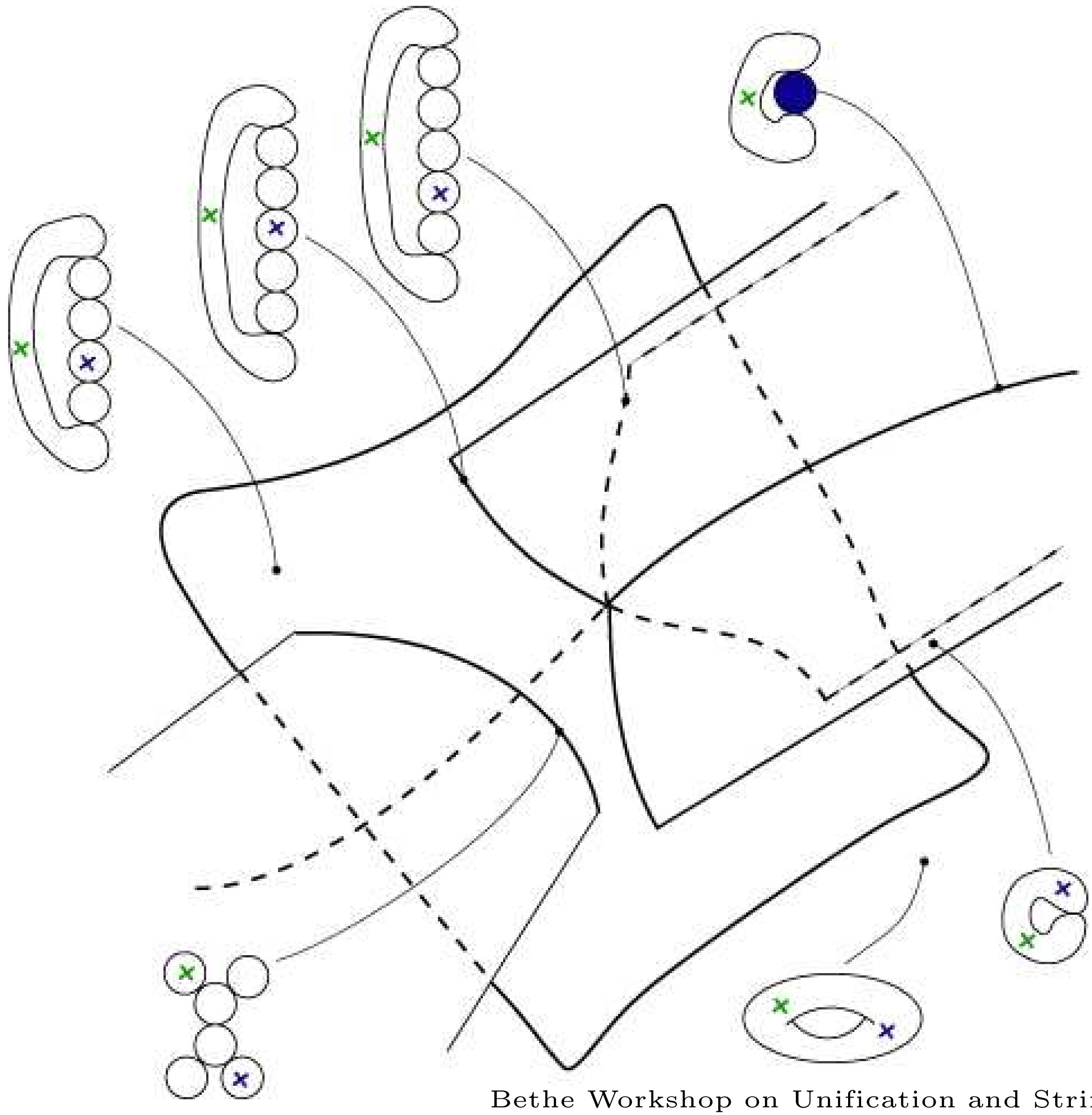
Over certain curves some $\mathbb{P}^1_i$ split $\rightarrow$ Dynkins of higher groups



Each weight β^k of repr. $R \leftrightarrow$ surface C^k_R fibered over curve $\mathcal{C}_R \subset B$

Structure of resolved fiber

[Krause,Mayrhofer,TW'11]



II) Gauge fluxes

Gauge-Fluxes - Overview

Chiral matter spectrum requires **gauge fluxes**

Gauge fluxes described by $\mathbf{G_4 \in H^{2,2}(Y_4)}$ with **'1 leg along fiber'**

a) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0$ b) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0 \quad \forall D_i \in H^2(B)$, Z: fibre

$$\mathbf{H^{2,2}(Y_4) = H^{2,2}_{\mathrm{vert}}(Y_4) + H^{2,2}_{\mathrm{hor}}(Y_4)}$$ [Greene,Plesser,Morrison'94]

- $\mathbf{H^{2,2}_{\mathrm{vert}}(Y_4)}$ generated by elements of $H^{1,1} \wedge H^{1,1}$: factorisable fluxes
  - fluxes associated with massless U(1)s [Grimm,TW '10], [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11],[Grimm,Hayashi'11]
  - extra special fluxes, e.g. 'spectral cover' fluxes [Marsano,Nameki'11][Küntzler,Nameki'12][Tatar,Walters'12]]
- $\mathbf{H^{2,2}_{\mathrm{hor}}(Y_4)}$: non-factorisable fluxes [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11]

Gauge flux from $U(1)_X$

Extra resolution S gives rise to **extra $U(1)_X$ not contained in SU(5)**

$$\mathsf{w}_X = S - Z - \bar{\mathcal{K}} + \sum t_i E_i \in H^{1,1}(\hat{Y}_4) \to C_3 = A_X \wedge \mathsf{w}_X$$

repr. R : $\quad \mathbf{10}_1, \quad \mathbf{5}_3, \quad \mathbf{5}_{-2}, \quad \mathbf{1}_5 \qquad \mathbf{q_R} = \int_{\mathbb{P}_\mathbf{R}} \mathsf{w}_\mathbf{X}$

Associated with $U(1)_X$: **Gauge flux G_4** [Grimm,TW' 10]

[Braun,Collinucci,Valandro '11] [Krause,Mayrhofer,TW '11] [Grimm,Hayashi'11]

$$C_3 = A_X \wedge \mathsf{w}_X \Longrightarrow \mathbf{G_4 = F_X} \wedge \mathsf{w}_\mathbf{X}, \qquad F_X \in H^{1,1}(\hat{Y}_4) \cap H^2(B_3)$$

Importance of G_4 flux: induces chirality for charged matter

$$\chi(R) = \int_{C_R} G_4 = \int_{C_R} F_X \wedge \mathsf{w}_X = q_R \int_{\mathcal{C}_R} F_X \qquad \mathbf{q_R} = \int_{\mathbb{P}_\mathbf{R}} \mathsf{w}_\mathbf{X}$$

✓ computation of charged singlets away from SU(5) GUT brane

✓ result accounts for corrections to semi-local split spectral cover due to global features

Flux consistency conditions

- Quantisation: $G_4 + \frac{1}{2}c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$ [Collinucci,Savelli'10&'12]
- D3/M2 tadpole: $N_{M_2} + \frac{1}{2}\int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{1}{24}\chi(\hat{Y}_4)$

$$\frac{1}{2}\int_{\hat{Y}_4} G_4 \wedge G_4 = \int_{B_3} F_X \wedge F_X \wedge (15\,[W] - 25\,c_1(B_3))$$

[Krause,Mayrhofer,TW'11]

- F-term condition: $G_4 \in H^{2,2}(\hat{Y}_4)$ ✓
- D-term condition: from detailed analysis of F/M- theory effective action [Grimm '10] [Grimm,Kerstan,Palti,TW '11]

$$D_X = -\frac{2}{\mathcal{V}_B}\int_{\hat{Y}_4} J \wedge G_4 \wedge \mathsf{w}_X = \int_{B_3} J \wedge F_X \wedge (15\,[W] - 25\,c_1(B_3))$$

✓ explicit example of 3-generation model on concrete 4-fold

[Krause,Mayrhofer,TW '11]

✓ classification of fluxes and explicit match with Type IIB fluxes

[Krause,Mayrhofer,TW '12] see Talk by C. Mayrhofer

III) M5-instantons

M5-instantons - Setup

Non-perturbative M5-brane superpotentials are important

- Kähler moduli stabilisation à la KKLT [Witten'96]
- generation of matter couplings

  neutrino masses, μ-terms, SUSY breaking F-terms, corrections to Yukawas, ...

  [Blumenhagen,Cvetič,TW][Ibanez,Uranga][Florea,Kachru,McGreevy,Saulina]'06
- interesting from formal perspective e.g. relation to U(1)s and G_4-fluxes

Superpotential requires M5-instanton on vertical divisor [Witten'96]

$$D_M = \pi^{-1} D_M^{\mathrm{b}} \subset Y_4$$

- generic fiber of D_M smooth except at intersections with 7-branes
- extra effect not encoded in singular geometry
- M5-instantons are among the most 'globally sensitive' objects
- intrinsically M/F-theoretic description desirable

Instantons in F-theory

Lots of recent work on M5/D3-instantons including

[Blumenhagen,Collinucci,Jurke'10],[Donagi,Wijnholt'10]

[Marsano,Schäfer-Nameki,Saulina'08/'11]

[Cvetič,Garcia-Etxebarria,Richter'09],[Cvetič,Garcia-Etxebarria,Halverson'10/'11]

[Marchesano,Martucci'10]

[Grimm,Savelli'11]

[Grimm,Kerstan,Palti,TW'11] [Bianchi,Collinucci,Martucci'12]

[Cvetič,Donagi,Halverson,Marsano'12]

Focus of [Kerstan,TW'12]: Better understanding of

1) Structure of M5-partition function

2) Interplay with 7-branes/ gauge flux

Partition function - Overview

Technical complication: M5 hosts 2-form $\mathcal{B}$ with self-dual 3-form field strength $*\mathcal{H} = i\mathcal{H}$ see [Pasti,Sorokin,Tonin '96&'97] for first explicit action

We work with **Witten's auxiliary action** [Witten'96]

- $S_{M5} = 2\pi(\mathrm{Vol}_{D_M} + i\int C_6) + S_{\mathcal{B}}, \quad S_{\mathcal{B}} = -2\pi \int \left[\mathcal{H}\wedge *\mathcal{H} - 2i\,\mathcal{H}\wedge C_3^- + \frac{1}{4} C_3 \wedge * C_3\right]$
- $*\mathcal{H} = i\mathcal{H}$ only imposed later by extracting holomorphic piece in W (watch out for C_3^-!)

For superpotential need holomorphic piece of the partition function

$$Z_{M5} = e^{-2\pi(\mathrm{Vol}_{D_M} + i\int C_6)} \int \mathcal{D}\mathcal{B}\; e^{-S_{\mathcal{B}}}$$

- This is merely part of the partition function corresponding in IIB language to fluctuations of gauge potential along instanton
- to be supplemented by fermionic partners and deformation moduli(ni) [Kallosh,Kashani-Poor,Tomasiello][Martucci et al.]'05, [Tsimpis]'07
- assume first absence of bulk fluxes (isolated rigid instanton)

Partition function - Overview

Partition function includes **classical** + **quantum piece (Pfaffian)**:

$$Z_{M5} = \sum_{\mathcal{H}_0 \in H^3(D_M,\mathbb{Z})} e^{-S_{M5}[\mathcal{H}_0]} \int \mathcal{D}\delta\mathcal{B}\; e^{-S'_{M5}[d\delta\mathcal{B},\mathcal{H}_0]}$$

$$S'_{M5} \simeq -\int d(\delta\mathcal{B}) \wedge *d(\delta\mathcal{B}) + 2\mathcal{H}_0 \wedge *d(\delta\mathcal{B}) + \ldots.$$

Quantum piece (Pfaffian) $\leftrightarrow$ non-zero modes $\delta\mathcal{B}$

- non-trivial info on bulk moduli via Hodge $* = *_0 + \hat{*}$
- zeroes expected when instanton hits other branes [Ganor'97]; IIB: [Baumann et al'06],...
- hard to compute explicitly in SUGRA/closed string channel
  (cf. Type II: open string channel very helpful!)
- recent computation in spectral cover approach by het. duality
  [Cvetič,Donagi,Halverson,Marsano '12]

Ignoring moduli dependence: Classical and quantum piece independent

$\Longrightarrow$ focus on $\sum_{\mathcal{H}_0 \in H^3} e^{-S_{M5}[\mathcal{H}_0]}$

Classical partition function

Formal evaluation of classical sum over $\mathcal{H}_0$ possible

- Expansion of $\mathcal{H}_0, C_3$ along integer symplectic basis (E_M, F^M) of $H^3(D_M)$ and summation yields [Henningson,Nilsson,Salomonson'99]

$$Z_{M5} = \sum_{\alpha,\beta=0,\frac{1}{2}} e^{-2\pi(\mathrm{Vol}_{M5}+i\int C_6)} \mathcal{Z}_-\left[{\alpha \atop \beta}\right] \mathcal{Z}_+\left[{\alpha \atop \beta}\right]$$

- $\mathcal{Z}_-\left[{\alpha \atop \beta}\right] = e^{-\frac{\pi}{2} C_-^M Z_{MN}(C_-^N - C_+^N)} \times$

$$\sum_{k_M} e^{i\pi\left((k+\alpha)_M \overline{Z}^{MN}(k+\alpha)_N + 2(k+\alpha)_M(\beta^M - C_-^M)\right)}$$

- $\mathcal{Z}_- \leftrightarrow C_- \leftrightarrow$ **self-dual piece of $\mathcal{H}$**
- $\int_{D_M} E_M \wedge F^N = \delta_M^N, \quad F^N = X^{MN} E_N + Y^{MN}(*E_N), \quad Z = X + iY$

Witten ('96): $W_{\mathrm{cl.}} = e^{-2\pi(\mathrm{Vol}_{M5}+i\int C_6)} \mathcal{Z}_-\left[{\alpha_c \atop \beta_c}\right]$

- Requires choice of correct spin structure α_c, β_c
- Finding α_c, β_c in M-theory is in general hard!

Fluxed E3-brane instantons

Recall from in Type IIB orientifolds:

unfluxed $E3$-brane instantons contribute to superpotential if

- instanton divisor D_E is rigid
- $D_E = D_{E'}$, but not pointwise: $O(1)$ instantons

More structure possible: [Grimm,Kerstan,Palti,TW'11], [Donagi,Wijnholt '10]

Orientifold odd instanton flux compatible with superpotential

$$\mathcal{F}_E^- \in H_-^{(1,1)}(D_E, \mathbb{Z})$$

- $(D_E, \mathcal{F}_E^-)$ is invariant as a whole $\longrightarrow$ same projection of zero modes
- Instanton must be BPS with no lines of marginal stability
  D-term $\quad 0 = \int_{D_E} J \wedge \mathcal{F}_E = \int_{D_E^+} J \wedge \mathcal{F}_E^+ + \int_{D_E^-} J \wedge \mathcal{F}_E^- \equiv 0$ ✓
- Freed-Witten quantisation condition:

$$\mathcal{F}_E^- + \underbrace{\iota^* B_+ + \frac{1}{2} c_1(K_{D_E})}_{\text{must cancel mod } \mathbb{Z}} \in H^2(D_E, \mathbb{Z})$$

Comparison with Type IIB

[Kerstan,TW'12]

✓Quantitative match with rigid fluxed $O(1)$ E3-instantons in Type IIB

✓sum over $\mathcal{H}$-flux $\leftrightarrow$ sum over E_3-flux $\mathcal{F}_E$

$$W^{cl.}_{E3} = \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]},$$

✓$H^{1,1}_{-}(D_E) \to H^{2,1}(D_M) + H^{1,2}(D_M)$ [Blumenhagen,Collinucci,Jurke'10]

$$W^{cl.}_{E3} = \exp\left[-\pi\left(\frac{1}{2}C^{\alpha}_{E}\mathcal{K}_{\alpha\beta\gamma}v^{\beta}v^{\gamma} + iC^{\alpha}_{E}(c_{\alpha} - \frac{1}{2}\mathcal{K}_{\alpha ab}c^{a}b^{b})\right)\right]$$
$$\times \exp\left[-\frac{i\pi}{\tau-\overline{\tau}}\delta_{MN}G^{M}(G^{N}-\overline{G}^{N})\right]\sum_{\mathcal{F}^{M}\in\,\mathbb{Z}} e^{-i\pi\left(2\delta_{MN}G^{M}\mathcal{F}^{N}+\tau\delta_{MN}\mathcal{F}^{M}\mathcal{F}^{N}\right)}$$

✓allows us to fix spin structure of M5-instanton: $\alpha_M = 0 = \beta^M$

$$\mathcal{F}^{M} \leftrightarrow \delta^{MN}k_N \;; \quad G^{M} \leftrightarrow C^{M}_{-} \;; \quad \overline{G}^{M} \leftrightarrow C^{M}_{+} \;; \quad \overline{Z}^{MN} = -\tau\delta^{MN}.$$

M5-instantons and gauge flux

Now: intersection of M5 with 7-branes in presence of flux

Flux dependence via $S_{\mathcal{B},G_4} = S_{\mathcal{B}} - 2\pi i \int \mathcal{B} \wedge \iota^* G_4$

Superpotential without 'charged matter operators' $\Rightarrow \iota^* G_4 \stackrel{!}{=} 0$

- G_4 breaks large gauge trafos on $\mathcal{B}$ to $H^2(D_M, \mathbb{R}) \to H^2(D_M, \mathbb{Z})$
  extra integration over $H^2(D_M, \mathbb{R})/H^2(D_M, \mathbb{Z})$ gives zero:

  $$\int d\mathcal{B}\, e^{2\pi i \int \mathcal{B} \wedge \iota^* G_4} \simeq \delta(G_4)$$ [Donagi,Wijnholt'10]

  $\Longleftrightarrow$ integration over charged zero modes in IIB language [Kerstan,TW'12]

  $$\int \mathcal{D}\lambda_a \mathcal{D}\tilde{\lambda}_b \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]} = 0$$

- equivalently: $\iota^* G_4 \neq 0$ implies Freed-Witten anomaly
  e.g.[Marsano,Saulina,Schäfer-Nameki'11; Grimm,Savelli'11]

Quantitative criterion: Compute $\int_{C_{(4)}} G_4$ **with** $C_{(4)} \in H_4(D_M)$

[Kerstan,TW'12]

M5-instantons and gauge flux

[Kerstan,TW'12]

Task:
Test for surface $C_{(4)} \in H_4(D_M)$ with $C_{(4)}$ neither inside base nor wrapping fiber

Result: There are two types of such surfaces

1) First type of $C_{(4)} \leftrightarrow U(1)_X$

- $U(1)_X$ from $C_3 = A_X \wedge \mathsf{w}_X \qquad \mathsf{w}_X \in H^{1,1}(\hat{Y}_4)$
- 4-cycle $C_4^X \subset D_M$ by intersecting divisor in class w_X with D_M

$$[C_{(4)}^X] = -D_M \wedge \mathsf{w}_X \in H^{2,2}_{\text{vert.}}(\hat{Y}_4)$$

- $q_X = \int_{C_{(4)}^X} \iota^* G_4 = -\int_{\hat{Y}_4} D_M \wedge \mathsf{w}_X \wedge G_4 \leftrightarrow U(1)_X$ shift of C_6
- In Type IIB picture: This is **sensitive to linear combination of zero modes** with two brane stacks! `cf. Talk by C. Mayrhofer`

M5-instantons and gauge flux

2) Proposal

- If $D_M \cap \mathcal{W} \neq 0$ **extra surfaces** exist which are **not in** $H^{2,2}_{\text{vert}}(\hat{Y}_4)$
- These are algebraic only for special complex structure.
  in different contexts: [Braun,Collinucci,Valandro'11;Collinucci,Savelli'12]

Evidence:

- Suppose $D_M \cap \mathcal{W} \subset C_{10}$ (10-matter surface) $\Longrightarrow$ take surface $C_{10}|_{D_M}$
  ✓ $\int_{C_{10}|_{D_M}} \iota^* G_4 \neq 0$ yields correct $U(1)_a$ charge in IIB limit!
  ✓ measures effect of intersection with $SU(5)$ stack individually
- If $D_M \cap \mathcal{W}$ not in C_{10}, one can deform intersection locus, possibly on auxiliary space defined in [Kerstan,TW'12]

Conclusion:

Consistent with intuition about zero modes even in absence of IIB dual

Conclusions

✓ **F-theory is an attractive corner for model building.**

✓ **Our understanding of the underlying geometry has improved a lot.**

✓ **Explicit resolution of singularities gives a handle on gauge fluxes.**

✓ **Structure of partition function and of selection rules for M5-instantons agrees with Type IIB intuition**

Next steps:

- Better understanding of non-generic fluxes
- Vertex operators/microscopic picture for M5-instantons
- M5-instantons with chirality inducing $\mathcal{H}$ flux?