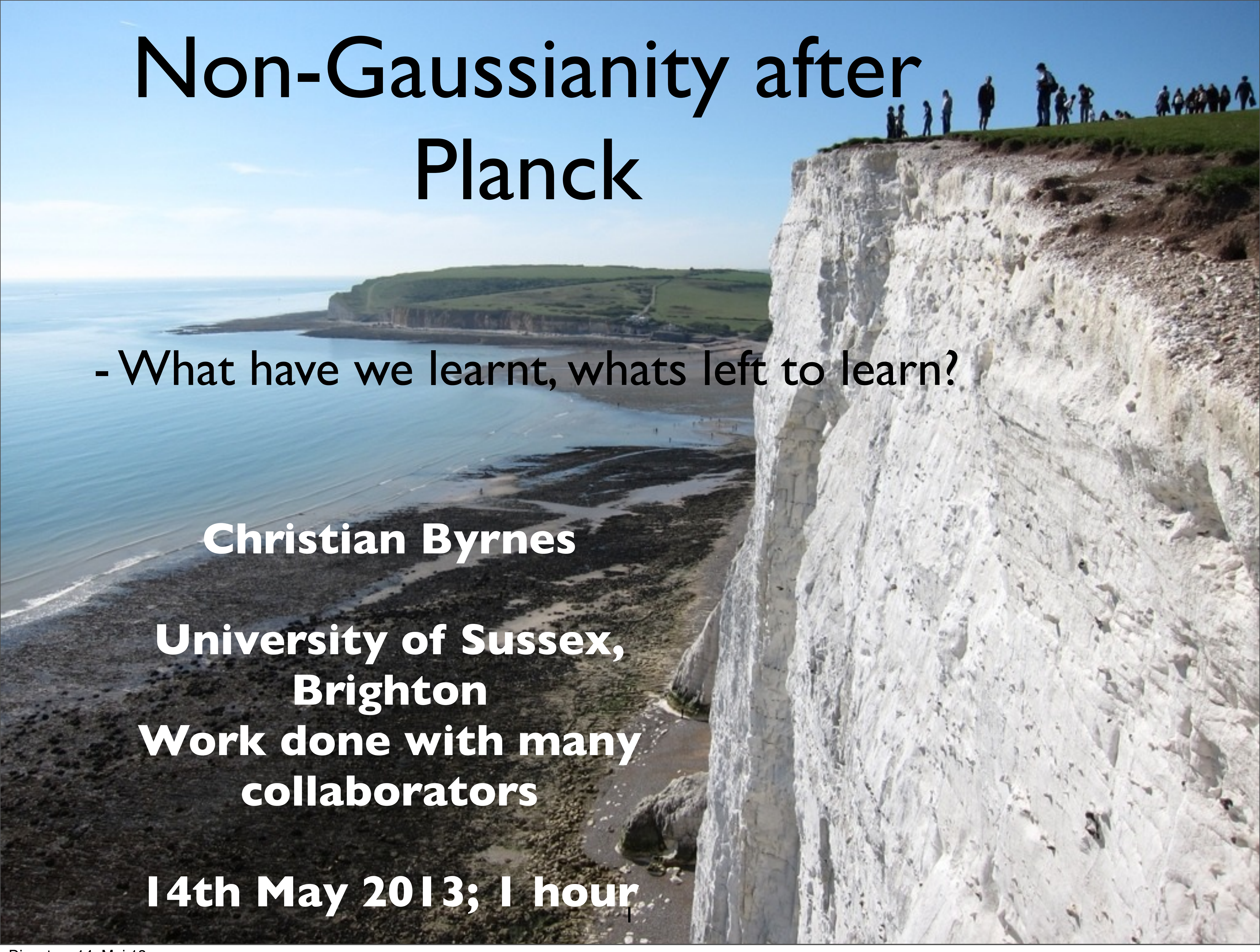


Non-Gaussianity after Planck

- What have we learnt, whats left to learn?

Christian Byrnes

University of Sussex, Brighton

Work done with many collaborators

14th May 2013; 1 hour

Why study primordial non-linearities?

Even in the “golden era” of cosmology, there is a lot we don't understand

The LCDM “standard model” of cosmology is phenomenologically simple but not motivated by theory

The inflationary paradigm is still successful after decades, but has hundreds of models, non are compelling

Success of the many new surveys, both CMB and LSS, must be utilised and interpreted in terms of realistic models

We need as many observables as possible

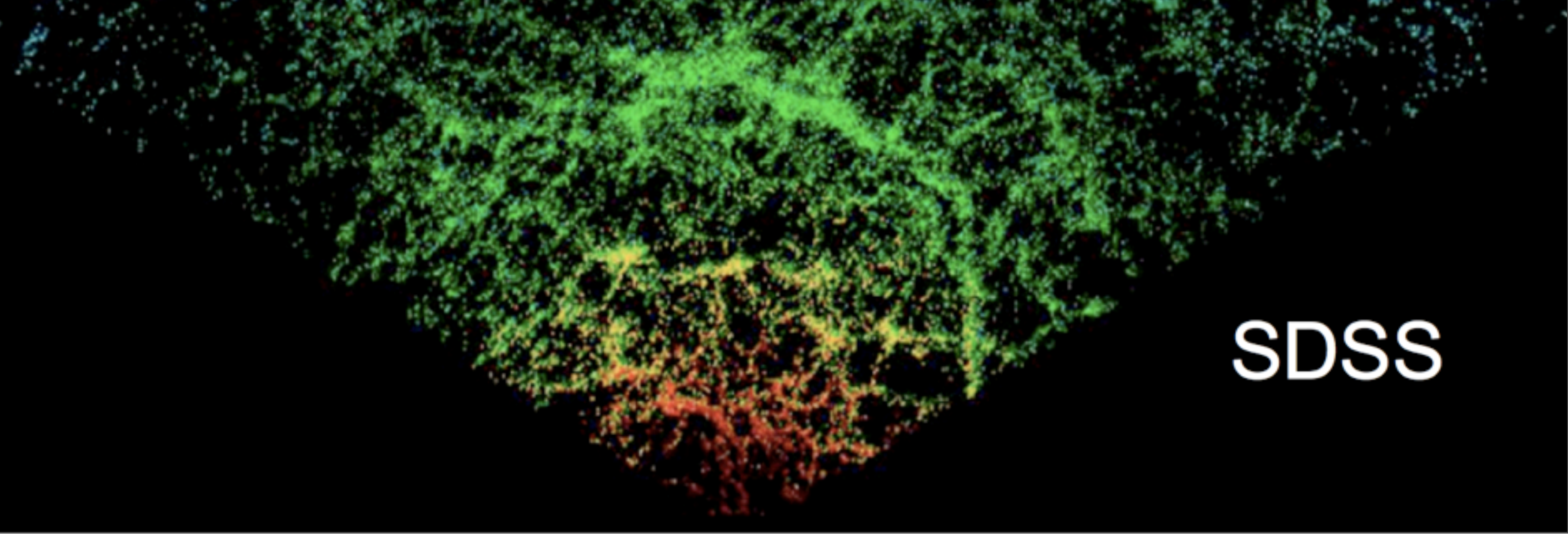
Non-linear perturbations may contain much more information

With large data sets, its neither practical nor desirable to search for every possible signal

To avoid endless discussions about posterior detections and anomalies

Theorists are needed to motivate template searches

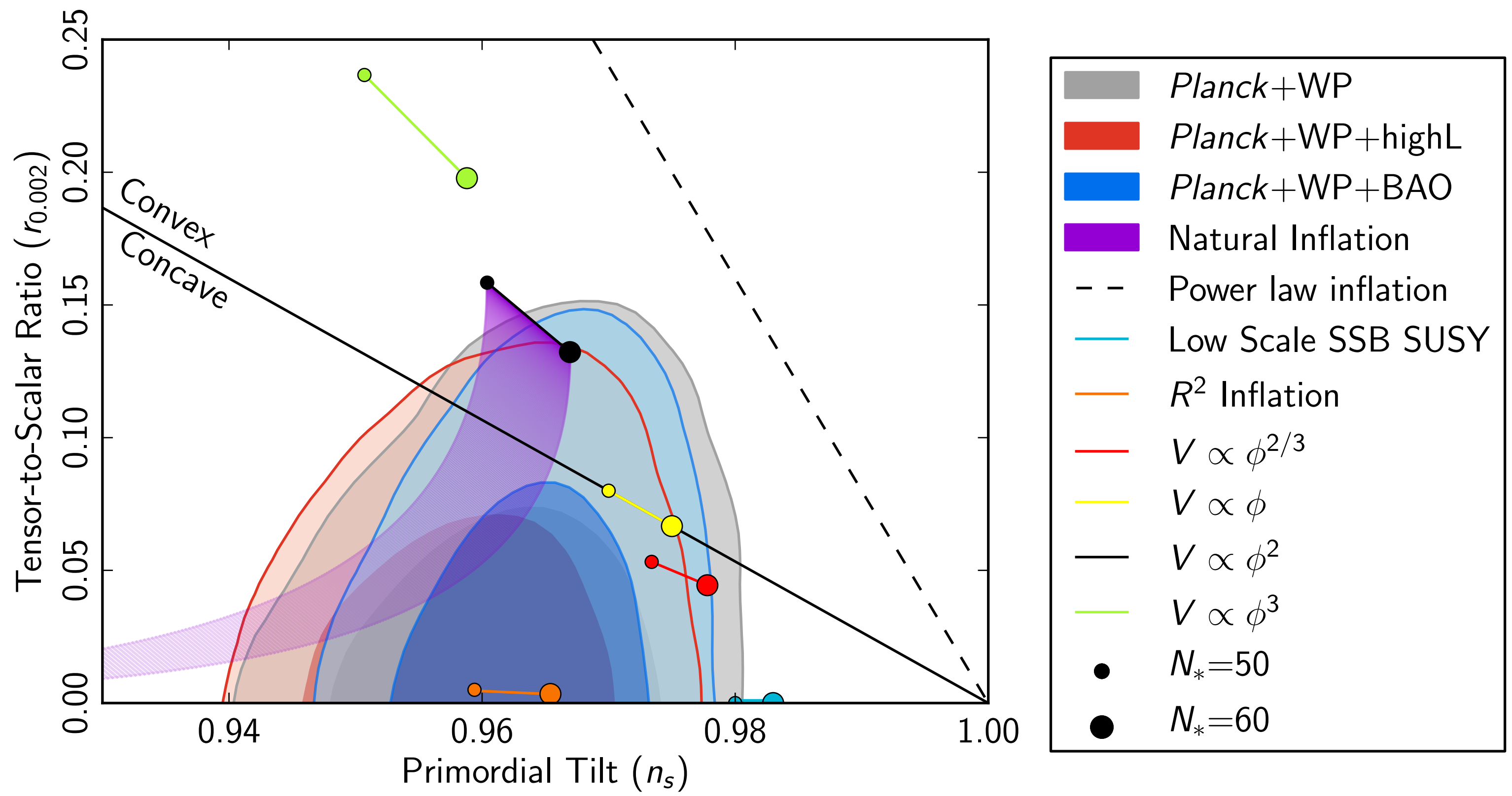
We all know theorists need observers, but observers also need theorists



Planck results: Vanilla rules!

- Except for anomalies, do they point to anything primordial?
- Is this a surprise?
- Gaussian statistics are not very informative, a window onto the early universe made of frosted glass
- A lot of discovery potential has gone for the foreseeable future
- Was all of the work done helpful?
- ISW-lensing bispectrum means we need non-G statistics
- Non-trivial results for single-field inflation

Single-field, slow-roll inflation



Planck collab XXII

Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Harrison-Zeldovich was ruled out with WMAP, now even scale invariance alone is convincingly ruled out

Leads to preference for a red spectral index

Why concave, and what does this imply?

$$n_s - 1 = -6\epsilon + 2\eta$$

$$r = \frac{P_T}{P_s} = 16\epsilon$$

$$r \ll 1 \;\Rightarrow\; \epsilon \ll 1 \;\Rightarrow\; \eta < 0$$

- Hence a negative mass squared at horizon crossing, but must have a positive mass squared at the minimum, if the potential gives a "graceful exit" from inflation

- Non-trivial evolution of the potential during inflation, monomial potentials (chaotic inflation) are disfavored

- Substantial progress, but there will always be many models which fit the data

Only one measured inflationary parameter

- The spectral index, all other parameters consistent with zero (tensors, isocurvature modes, non-Gaussianity, running of spectral index, cosmic string contribution, lots of additional parameters have been searched for)
- (Also the amplitude of perturbations since COBE, but for all models this is an overall scaling of the potential, its not predicted)
- However, notice that it was only in combination with the non-detection of gravitational waves that one finds evidence for a concave potential
- Shows that measuring a parameter to be close to zero is still a measurement, and may have important implications

Non-Gaussianity

- Constraints on the "headline" parameters are given, (WMAP9 in brackets)

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9),$$
$$f_{\rm NL}^{\rm equil} = -42 \pm 75 \qquad (51 \pm 136),$$
$$f_{\rm NL}^{\rm ortho} = -25 \pm 39 \qquad (-245 \pm 100).$$

- A factor of 2-4 improvement
- The central value always 1/2 sigma from zero
- Differences in the error bars are an artifact of the normalisation of fNL to an equilateral triangle, the local model is minimised for this shape, the others maximised
- Perhaps the biggest implication is that single-field DBI inflation was already ruled out, by the constraint on equilateral non-Gaussianity
- An extremely popular string motivated model of inflation (but ask Cliff/Gianmassimo/other experts...)

How Gaussian is the CMB?

- Depends heavily on the template for non-Gaussianity one chooses
- For the standard single-source local model

$$\zeta = \zeta_G + \frac{3}{5} f_{\rm NL}^{\rm local} \zeta_G^2 \qquad \sqrt{\mathcal{P}} = 2 \times 10^{-5}$$

$$|f_{\rm NL}^{\rm local} \zeta_G| \lesssim 10^{-4}$$

- However if the non-Gaussian term is uncorrelated with the Gaussian term then the bound greatly weakens

$$\zeta = \zeta_G + \alpha \sigma_G^2, \; f_{\rm NL}^{\rm local} \sim \alpha^3 P_\sigma^3 / P_\zeta^2,$$

$$\frac{\alpha \sigma_G^2}{\sqrt{\mathcal{P}_\zeta}} \lesssim 10^{-1}$$

- So the sky is over 99.99% Gaussian for the first "standard" template, but could be almost 10% non-Gaussian in the second case

Do the bounds on non-G rule out multifield inflation?

- NO!
- In fact not even close, multifield models can easily mimic the predictions of single field models (at least within foreseeable experimental accuracy)
- Inflation suffers from a lack of predictivity
- In single-field models, the choice of minimum one rolls into and the model parameters, as well as the duration of reheating (which value of N to choose when calculating the spectral index)
- In multifield models, one (almost) always has to specify initial conditions as well, observables may heavily vary depending on these choices
- Don't know of any model which predicts $f_{NL}>>1$ for all initial conditions, however there are plenty which always predict $f_{NL}<<1$
- Latter less studied only because they are phenomenologically less interesting

If an isocurvature perturbation is converted into the adiabatic one after inflation

$$\zeta \sim (1-r)\zeta_\phi + r(\zeta_\chi + \zeta_\chi^2)$$

Gaussian inflaton field

subdominant non-G field

$$\zeta_\phi \sim \frac{\delta\phi_*}{\sqrt{\epsilon_*}}, \;\; \zeta_\chi \propto \frac{\delta\chi}{\chi}, \; V(\chi) \propto \chi^2 \Rightarrow \zeta_\chi^{(2)} \propto \left(\zeta_\chi^{(1)}\right)^2 = constant$$

$$\text{Curvaton scenario: } r\zeta_\chi \gg \zeta_\phi, \;\; r \simeq \Omega_\chi|_{\rm decay}, \;\; f_{\rm NL} \propto \tfrac{1}{r} \gtrsim 1, \;\; \tau_{\rm NL} = \left(\tfrac{6 f_{\rm NL}}{5}\right)^2$$

$$\text{Mixed scenario: } |f_{\rm NL}| \propto \tfrac{1}{r}\left(\tfrac{P_\chi}{P_\zeta}\right)^2 \propto k^{2(n_\chi - n_s)}, \;\; \tau_{\rm NL} = \tfrac{P_\zeta}{P_\chi}\left(\tfrac{6 f_{\rm NL}}{5}\right)^2 \geq \left(\tfrac{6 f_{\rm NL}}{5}\right)^2$$

$$\text{Dominant quadratic curvaton: } f_{\rm NL} = -\tfrac{5}{4}, \;\; g_{\rm NL} = \tfrac{9}{2}$$

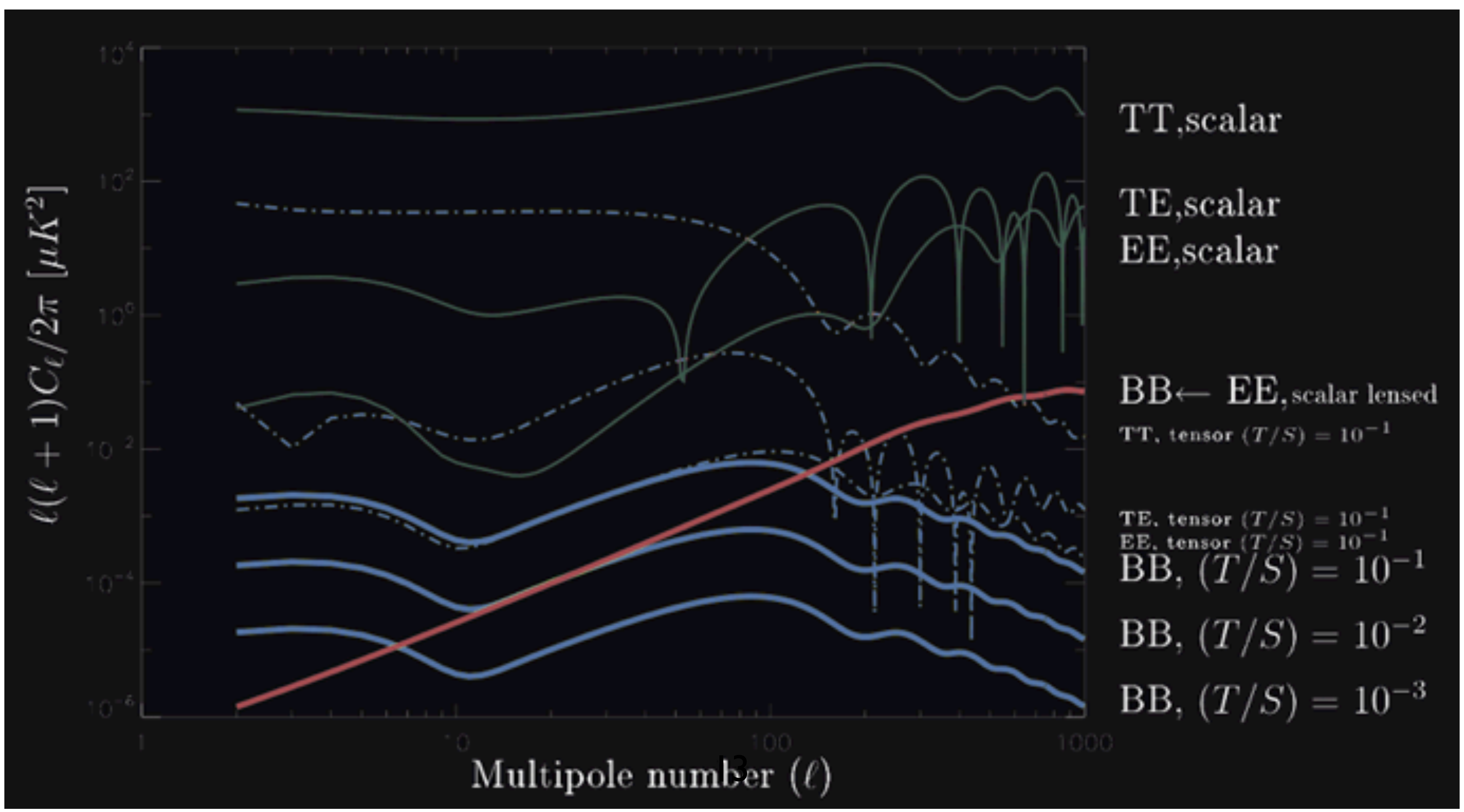
r measures the efficiency of the transfer from the initially subdominant field, which is isocurvature during inflation
The less efficient the transfer, the more non-G the perturbations, and tau$_{\rm NL}$ is relatively more important
However the Gaussian inflaton perturbations are more likely to dominate in this limit

- Previous slide made several assumptions:
  - 2 fields, one of which is Gaussian
  - Quadratic potential (implies negligble g_{NL})
  - Conversion takes place after the end of inflation (important, things work differently if during slow roll, and often get slow-roll value of f_{NL})
- Apart from the third assumption, prediction of (local) $|f_{NL}|>1$ is quite generic. Review: CB & Choi '10
- Can we observe $f_{NL}=1$, if so, when???

Gravity waves?

- Would be amazing, and large discovery potential (factor 10 increase in < decade)
- However only one number, short lever arm
- Single-field consistency relation so out of reach nobody even talks about it anymore (maybe with direct detection experiments it could be seen?)

$$r_T = 16\epsilon,\ n_T = -2\epsilon,\ r_T = -8n_T$$



B-Pol website

Testing single-source models?

- Poor prognosis for single-field models
- Looked more hopeful for single-source models (any one field generates the primordial curvature perturbation)

$$\tau_{NL} = \left(\frac{6f_{NL}}{5}\right)^2$$

- For f_{NL}=50 we would have measured both
- Now need an order of magnitude increase in sensitivity to tau_{NL}, is this ever possible?

Multi-source generalisation

- In multi-source scenarios, this becomes an inequality

$$\tau_{NL} \geq \left(\frac{6f_{NL}}{5}\right)^2$$

- Lots of effort has gone into making as general a proof as possible - it is based on the definitions of the non-linearity parameters, not a model of inflation

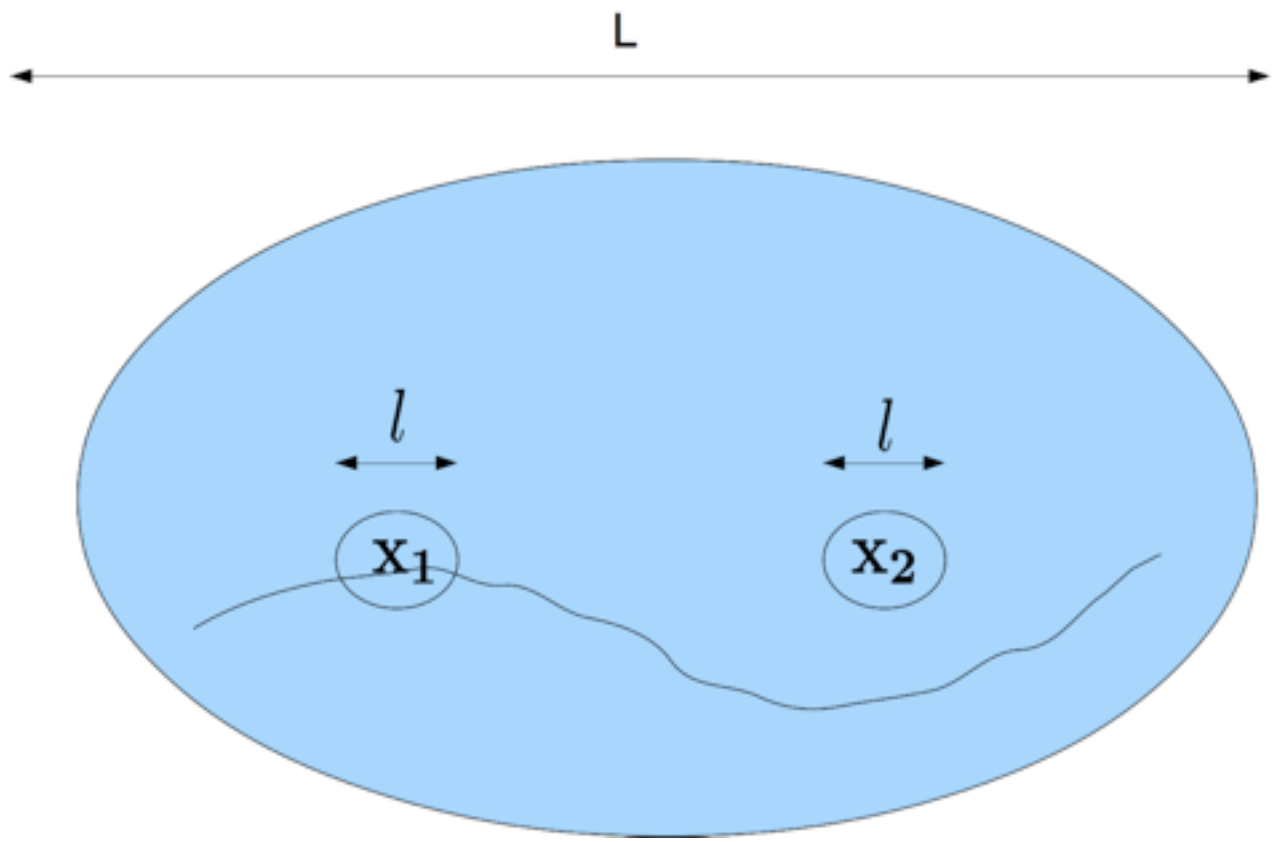
Suyama & Yamaguchi '08; Smith et al '11; Assassi et al '12, Kehagias & Riotto '12; Tasinato et al '12 + more

- Looks impossible to verify a breaking of the inequality, however a (very strong) breaking of the equality could still be observed, this would prove multiple fields generated the perturbations
- Currently tau_{NL}<2800 at 95% confidence

Is there anything which Planck did not do?

- Non-Gaussianity could be anything, so infinitely many things left to do!
- But of the "mainstream" targets, g_{NL} is the only obvious missing target
- In fact, τ_{NL} was the only trispectral shape to be constrained so far, huge range left to do (but difficult)
- τ_{NL} is large in the squeezed and collapsed limits, g_{NL} only in the squeezed limit
- WMAP and LSS constraints are weak, $|g_{NL}|<10^6$

Inhomogeneities



Long wavelength modes will shift the effective background in subpatches
In single-field inflation this shift has no observable consequences
In other cases it matters, especially under the presence of local non-Gaussianity, which correlates large and small scale modes
Shift depends on number of efoldings between the two scales l and L

$$N_{in} = \ln(L/l)$$

Our position in a larger Universe

- Think about the long mode L as corresponding to the "total" inflated region, denoted with f_{NL}^0
- And short mode "l" as our Hubble scale, denoted f_{NL}^{obs}
- The observables we measure will depend on the location of our observable universe

$$\zeta = \zeta_G + \frac{3}{5} f^0_{\rm NL} \left(\zeta_G^2 - \langle \zeta_G^2 \rangle \right) + \frac{9}{25} g^0_{\rm NL} \zeta_G^3$$

- The g_{NL} term may be larger than the f_{NL} term, does this have consequences?

Long-short wavelength split

$$\zeta = \zeta_G + \frac{3}{5} f_{\rm NL}^0 \left(\zeta_G^2 - \langle \zeta_G^2 \rangle\right) + \frac{9}{25} g_{\rm NL}^0 \zeta_G^3$$

$$\zeta_G = \zeta_{G,l} + \zeta_{G,s}$$

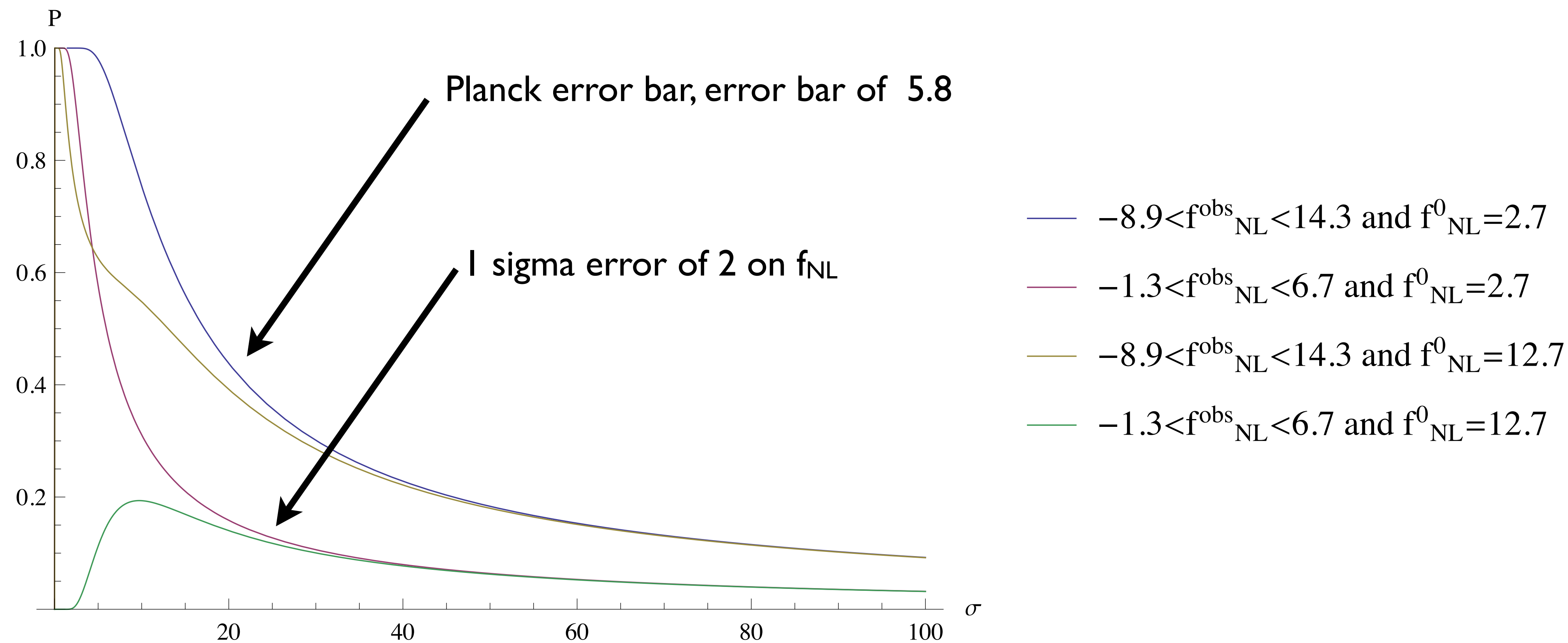
$$\zeta_s = \zeta_{G,s} + \frac{3}{5} f_{\rm NL}^{\rm obs} \left(\zeta_{G,s}^2 - \langle \zeta_{G,s}^2 \rangle\right) + \frac{9}{25} g_{\rm NL}^{\rm obs} \zeta_{G,s}^3$$

$$f_{\rm NL}^{\rm obs} = f_{\rm NL}^0 + \frac{9}{5} g_{\rm NL}^0 \zeta_{G,l}, \qquad g_{\rm NL}^{\rm obs} = g_{\rm NL}^0$$

- g_{NL} modulates the local value of f_{NL}
- If $|g_{NL}|>10^5$, which is what we need in order for it to be detectable, then it will "interfere" with the local value of f_{NL}

How big is the variation?

$$P(f_{\rm NL}^{\rm obs}|\sigma, f_{\rm NL}^0) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(f_{\rm NL}^{\rm obs} - f_{\rm NL}^0)^2}{2\sigma^2}\right) \qquad \sigma^2 \simeq g_{NL}^2 \mathcal{P}_\zeta N_{in}$$



Probability of f_{NL} matching the small observed value only around 10% for g_{NL}~10^6 and N_{in}~10, for the best choice of the global f_{NL}

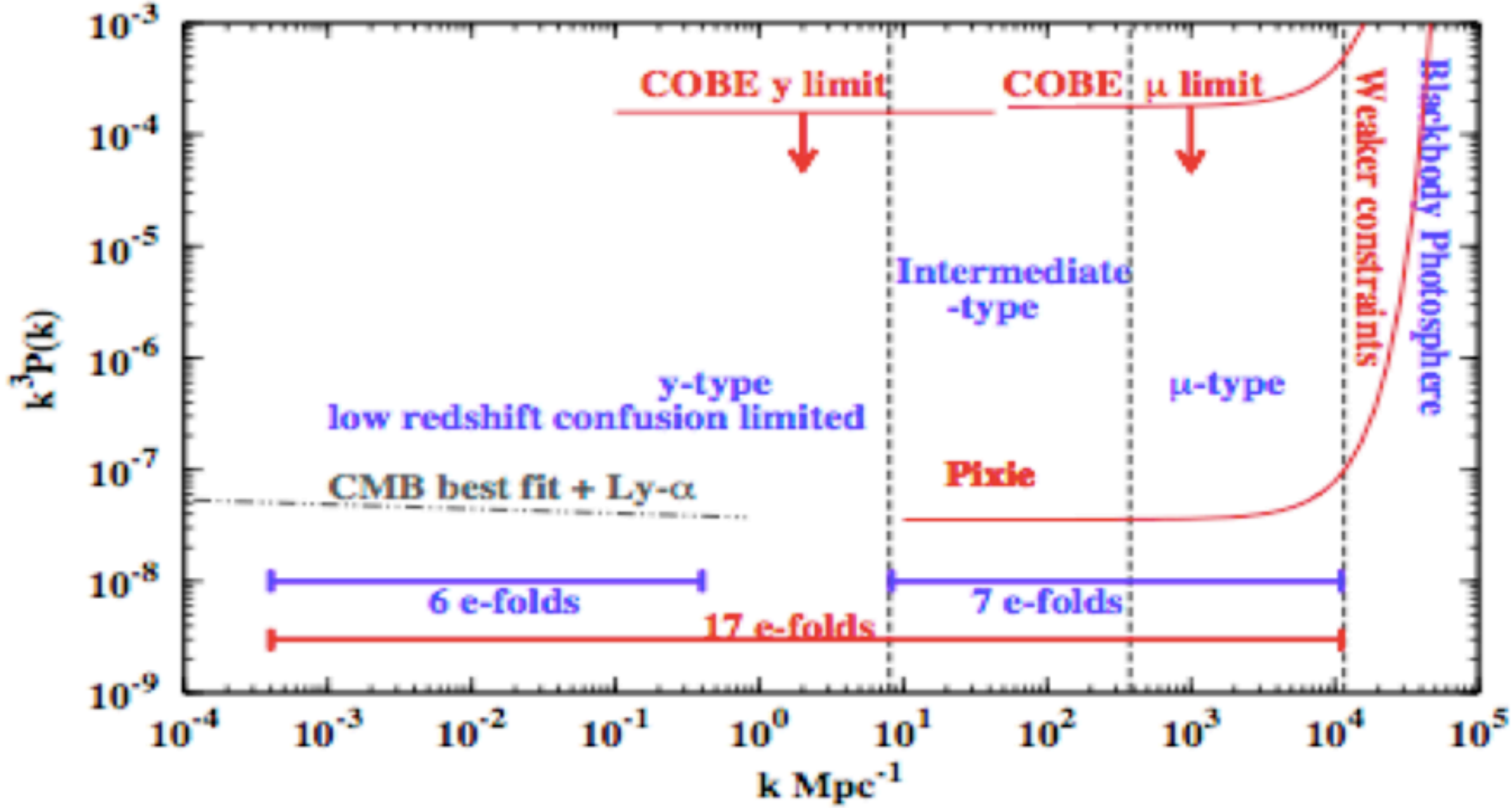
Model independent result, only assumption is the existence of super horizon perturbations

However note the long tail to positive sigma (the probability decreases as 1/sigma), this means we cannot make a constraint on g_{NL} from the tight constraint on f_{NL}

From very large to very small scales

- We have the “precision era” measurements on CMB and LSS scales
- These span approximately the largest 5-10 efoldings which are inside the Hubble scale today
- Lyman alpha, 21cm and spectral mu distortions in the CMB may add a similar range of scales in the (farish) future
- But inflation is believed to have lasted at least 50-60 efoldings
- So we only observe a small fraction of all scales
- Limits our ability to constrain the early universe

What we observe



Currently about 6 efolds
With Pixie, maybe 17 efolds
Still far short of 50-60 efolds

Khatri '13

What about the small scales?

- From a structure formation point of view, this is hopeless, we cant use the solar system to reconstruct the inflationary potential

The fact we don't see primordial black holes (PBHs) does give some constraints

Expect them to form on the horizon scale at the time of re-entry, if the over density is order unity

Gravitational waves could also give very small scale information (but wait for Lisa/DECIGO)

Where the constraints come from?

- The Hawking radiation from PBHs must not:
- stop the success of big bang nucleosythesis
- Mess up the CMB
- Be compatible with the observed extragalactic photon background
- PBHs must not have greater energy density than DM (but could be a DM candidate)
- Strongly scale/mass constraints in terms of beta, the fraction of the energy density of the universe in PBHs satisfies (over many scales):

$$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}\bigg|_{formation} \lesssim 10^{-20} - 10^{-5}$$

The Gaussian case

People usually assume this to be a good estimate

$$
\begin{aligned}
P(\zeta) &= \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{\zeta^2}{2\sigma^2}\right) \\
\beta &\simeq \int_{\zeta_c}^{\infty} P(\zeta) d\zeta \simeq exp\left(-\frac{\zeta_c^2}{2\sigma^2}\right) \\
\frac{\sigma}{\zeta_c} &= \sqrt{\frac{1}{2\ln(1/\beta)}} \qquad \zeta_c \simeq 1
\end{aligned}
$$

Green et al '04

Result is accurate to order of 10% (compared to more involved calculation using density perturbation with window functions)

$\mathcal{P}_\zeta \lesssim 10^{-2}$ on the relevant PBH scales

There is no theoretical prediction for the amplitude of perturbations on CMB scales, so no reason it should be so small on other scales, can we extrapolate over 50 efoldings?

Bottom line: Only sensitive to log of the observational constraints

So small changes in amplitude of perturbations changes PBH formation rate exponentially

We will see that even small non-Gaussianity is very important

PBH formation is very rare, so we are measuring the tails of the pdf's, typically larger than 5 sigma deviations

So skewness/kurtosis really matters!

Lets take it into account, and see how the normal constraints on the power spectrum change

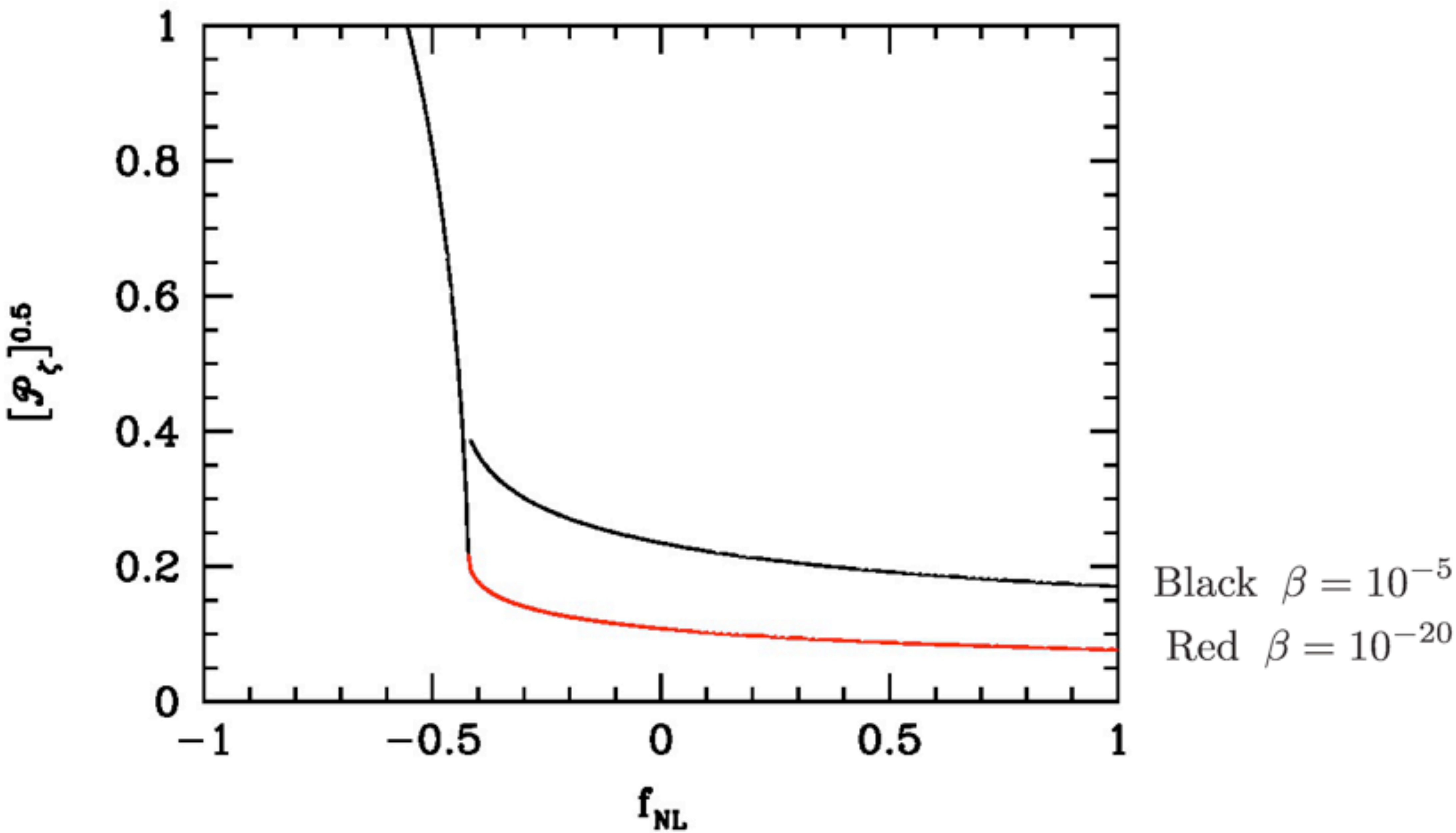
Quadratic non-Gaussianity

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \left(\zeta_G^2 - \langle \zeta_G^2 \rangle \right) + \frac{9}{25} \cancelto{0}{g_{NL}} \zeta_G^3$$

Results will depend on the sign of the non-Gaussianity, if positive its easier to form overdensities because the linear and quadratic terms act in the same direction (similarly to the speculated “too big, too early clusters” which could be explained by large and positive f_{NL})

Otherwise the two terms tend to cancel each other, and zeta is bounded from above

Large influence of small f_{NL}

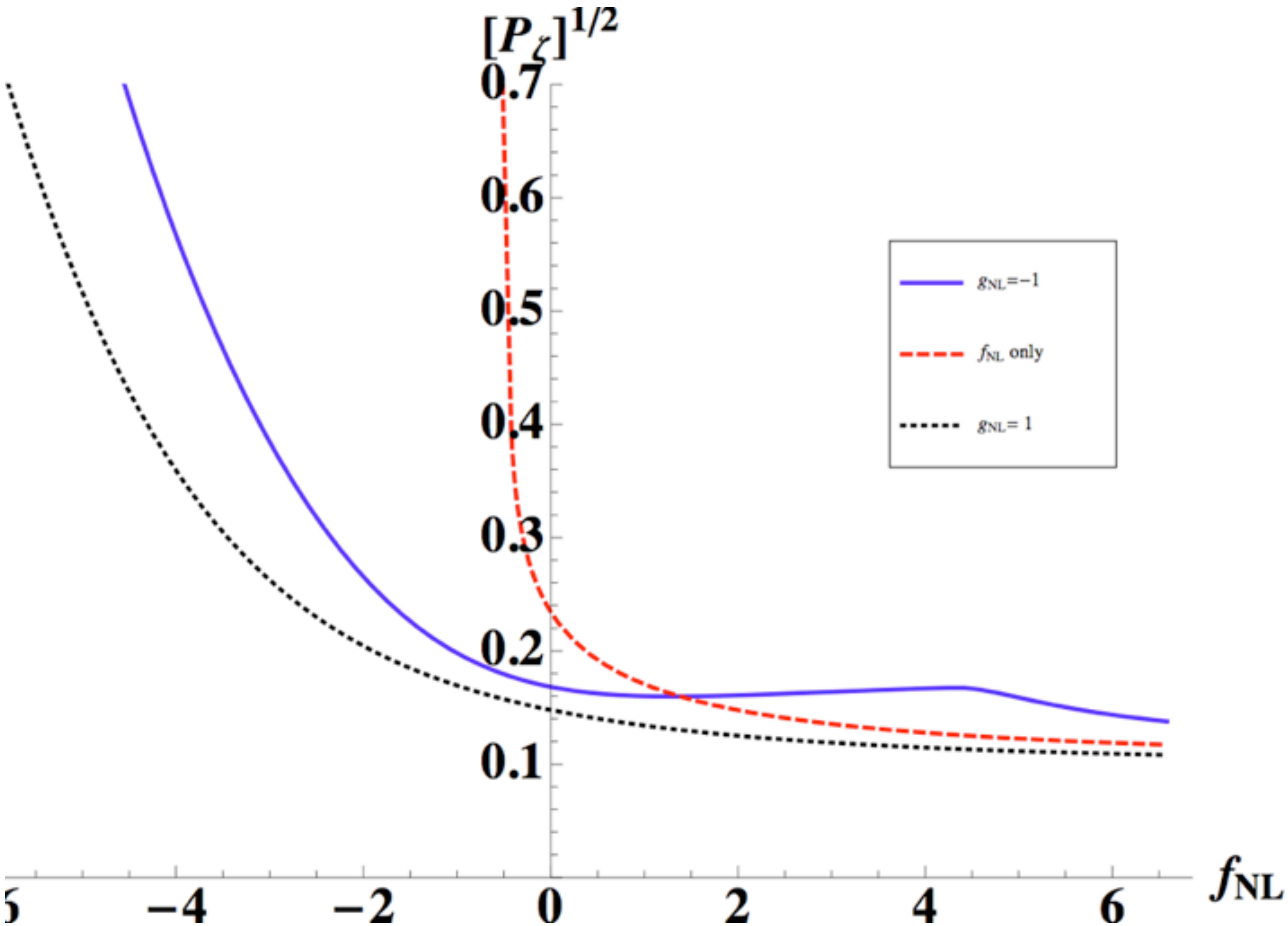


Results especially dramatic for negative f_{NL}

If PBHs are detected in the future, a negative f_{NL} (and all higher order parameters zero) on the relevant scales is ruled out, unless it has a tiny amplitude

Generalisation in progress with Sam Young

What if g_{NL} was not zero?



Credit: Sam Young

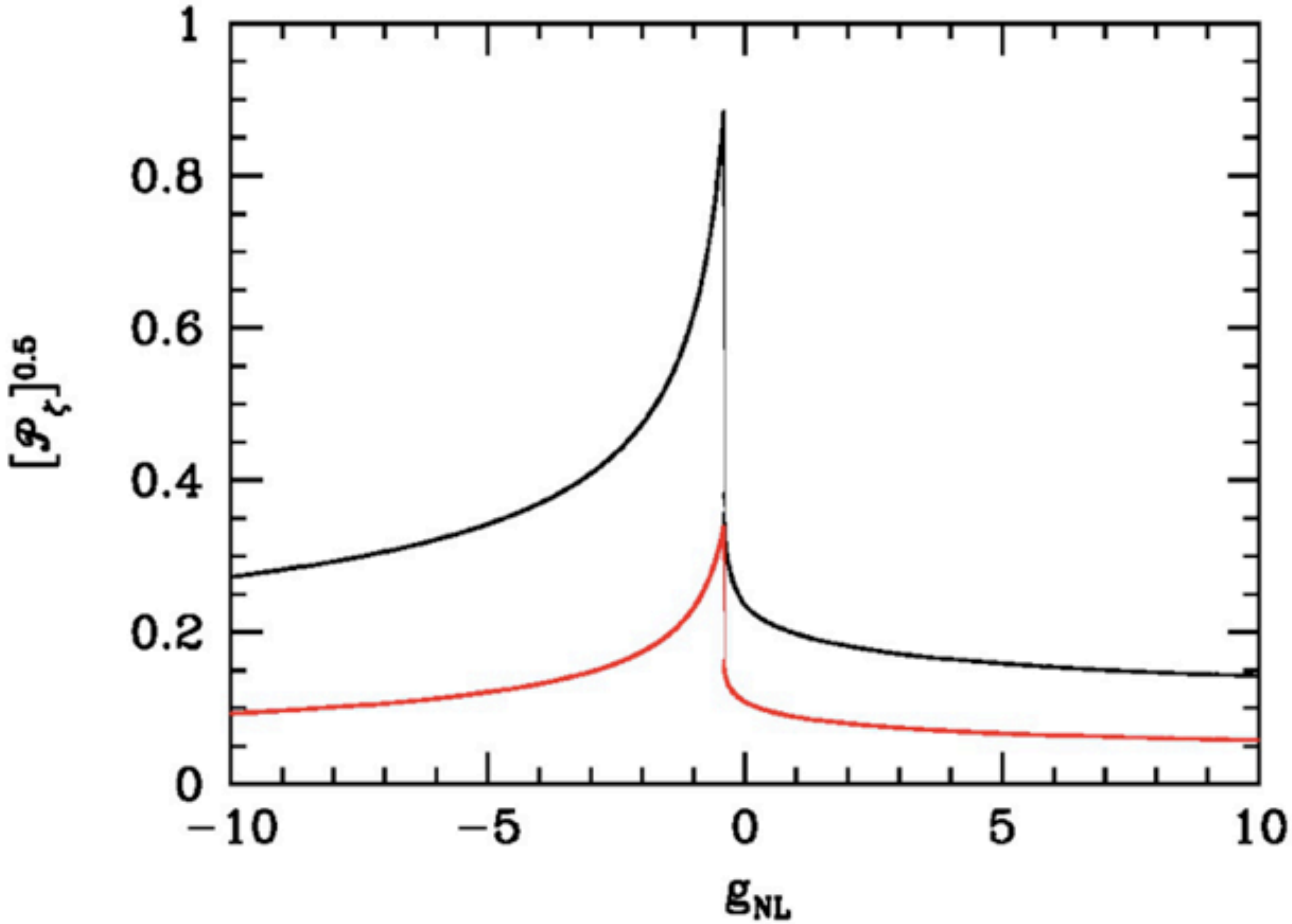
Very large and positive f_{NL}

- Results are about the square of the Gaussian case, hence much more stringent

$$\beta = 10^{-20}, \;\; \underbrace{\mathcal{P}_\zeta \simeq 10^{-2}}_{\text{Gaussian}} \rightarrow \underbrace{\mathcal{P}_\zeta \simeq 10^{-4}}_{\text{Chi-squared}}$$

- Limit of very small and very large non-Gaussianity was previously known, we recover those results and interpolate between them
- Very small: Seery & Hidalgo '06
- Chi squared non-Gaussianity: Avelino '05 and Lyth '12

Small g_{NL}, big changes again

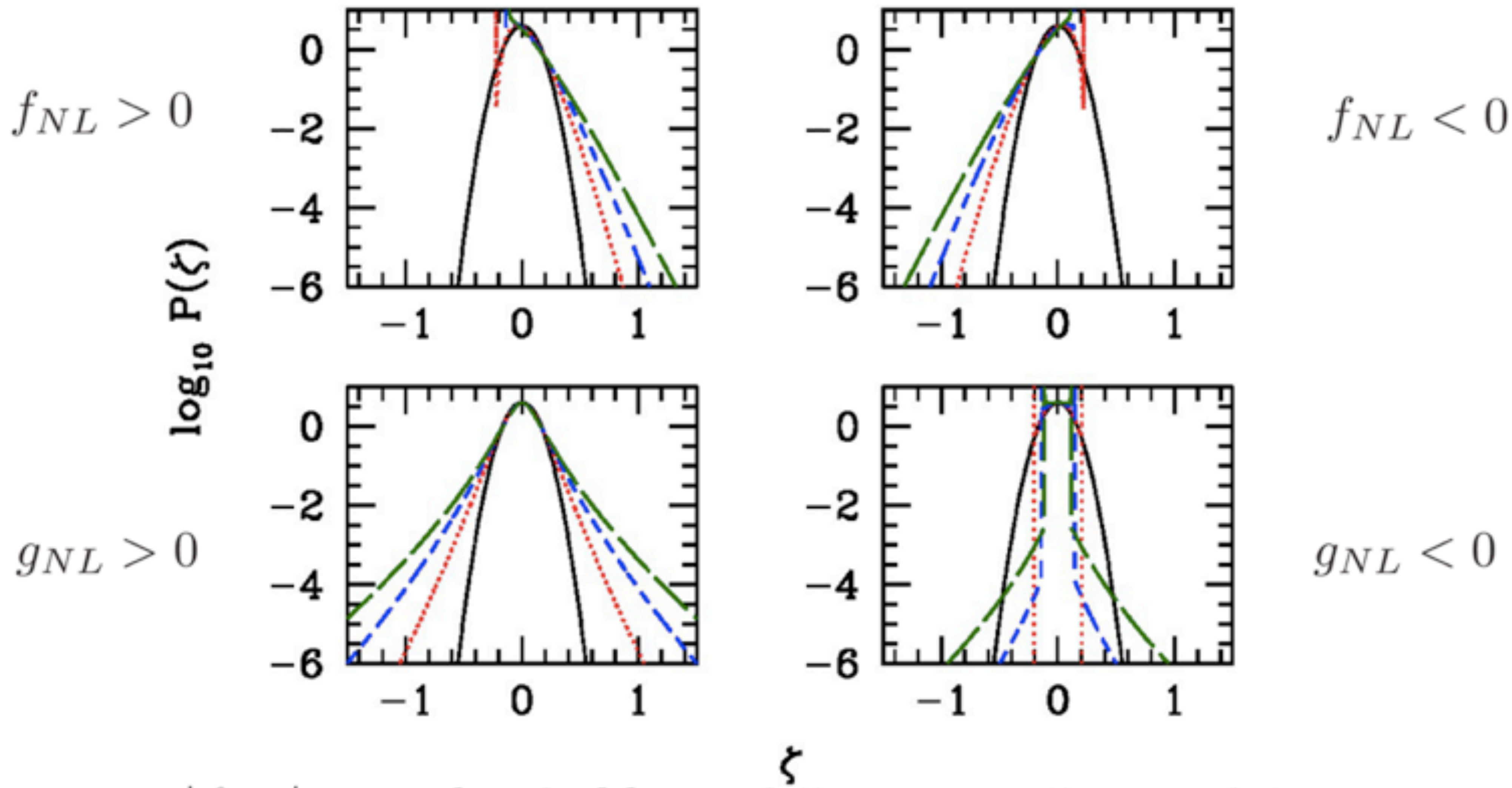


Black $\beta = 10^{-5}$

Red $\beta = 10^{-20}$

There is a symmetry as $g_{NL} \to \pm$ infinity, because the Gaussian is an even function

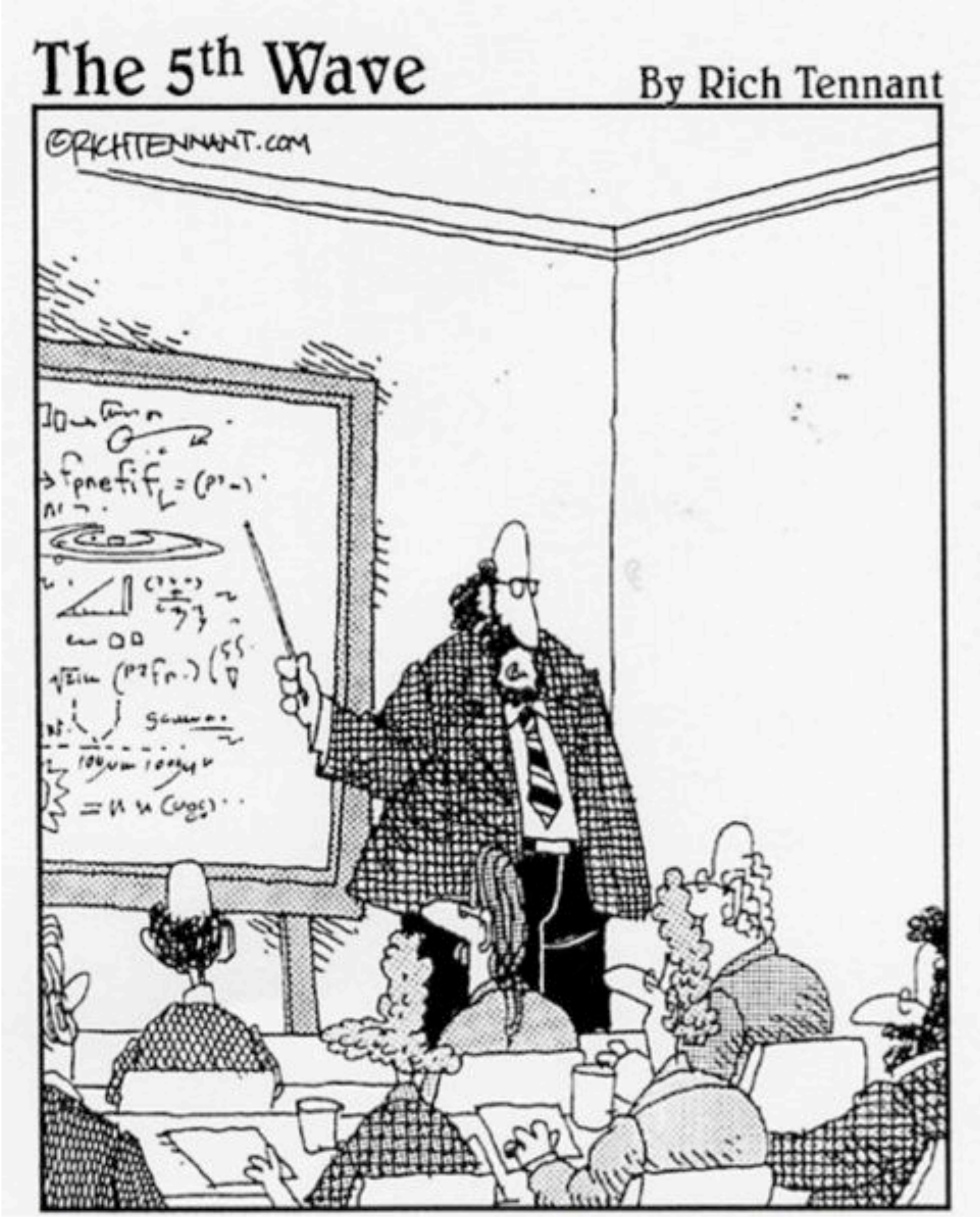
What happens to the pdf's?



$|f_{NL}|$: red = 2, blue = 3.5, green = 5, $\sigma = 0.1$

$|g_{NL}|$: red = 10, blue = 20, green = 30, $\sigma = 0.1$

Even things which probably dont exist can matter!



"After the discovery of 'antimatter' and 'dark matter', we have just confirmed the existence of 'doesn't matter', which does not have any influence on the Universe whatsoever."

Future prospects

- More shapes to be searched for with Planck, lots to do especially with the trispectrum
- For shapes already constrained, the local model has the best prospects (scale dependent bias)
- The galaxy bispectrum is quite poorly explored
- Don't expect any significant improvements until Euclid at best
- Higgs field is likely to be a second light degree of freedom during inflation (unless itself the inflaton, requires huge non-minimal coupling to gravity)
- Anomalies such as power spectrum modulation may be non-Gaussian signatures (wait for polarization)
- Large scale magnetic fields definitely exist and are non-Gaussian

Conclusions

- Even today, non-Gaussianity arguably remains the best window onto the early universe (and provides the tightest constraints)
- Forecasts are important to tell theorists what remains interesting
- PBHs, CMB distortions, 21cm, how much can we hope to see?
- Progress is needed on top-down theories, reheating, conversion to standard model, initial conditions
- A way to theoretically discriminate between the plethora of surviving models will be essential