Hiding the cosmological constant

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1. Models of Lambda
2. Scalar field models
3. Modified gravity

Bonn - May 16th 2013

Some basic equations

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter.

$$\text{Energy density } \rho(t): \quad \text{Pressure } p(t)$$

$$\text{Related through}: p = w\rho$$

w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

Current bounds on H(z) -- Planck v WMAP

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

	Planck+WP+highL		*WMAP*7+SPT (S12)	
Parameter	Best fit	68% limit	Best fit	68% limits
$100\Omega_b h^2$	2.207	2.207 ± 0.027	2.223	2.229 ± 0.037
$\Omega_c h^2$	0.1203	0.1198 ± 0.0026	0.1097	0.1093 ± 0.0040
$10^9 A_s$	2.211	2.198 ± 0.056	2.143	2.142 ± 0.061
n_s	0.958	0.959 ± 0.007	0.963	0.962 ± 0.010
τ	0.093	0.091 ± 0.014	0.083	0.083 ± 0.014
$100\theta_*$	1.0414	1.0415 ± 0.0006	1.0425	1.0429 ± 0.0010
Ω_Λ	0.683	0.685 ± 0.017	0.747	0.750 ± 0.020
H_0	67.2	67.3 ± 1.2	72.3	72.5 ± 1.9

WMAP7

(curvature) -- $\Omega_k < 0.008$ (95%CL)

(de eqn of state) -- $1+w = 0.001 \pm 0.057$ -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then

$w_0 = -0.93 \pm 0.12$ and $w' = -0.38 \pm 0.65$ (68% CL)

Dark Energy

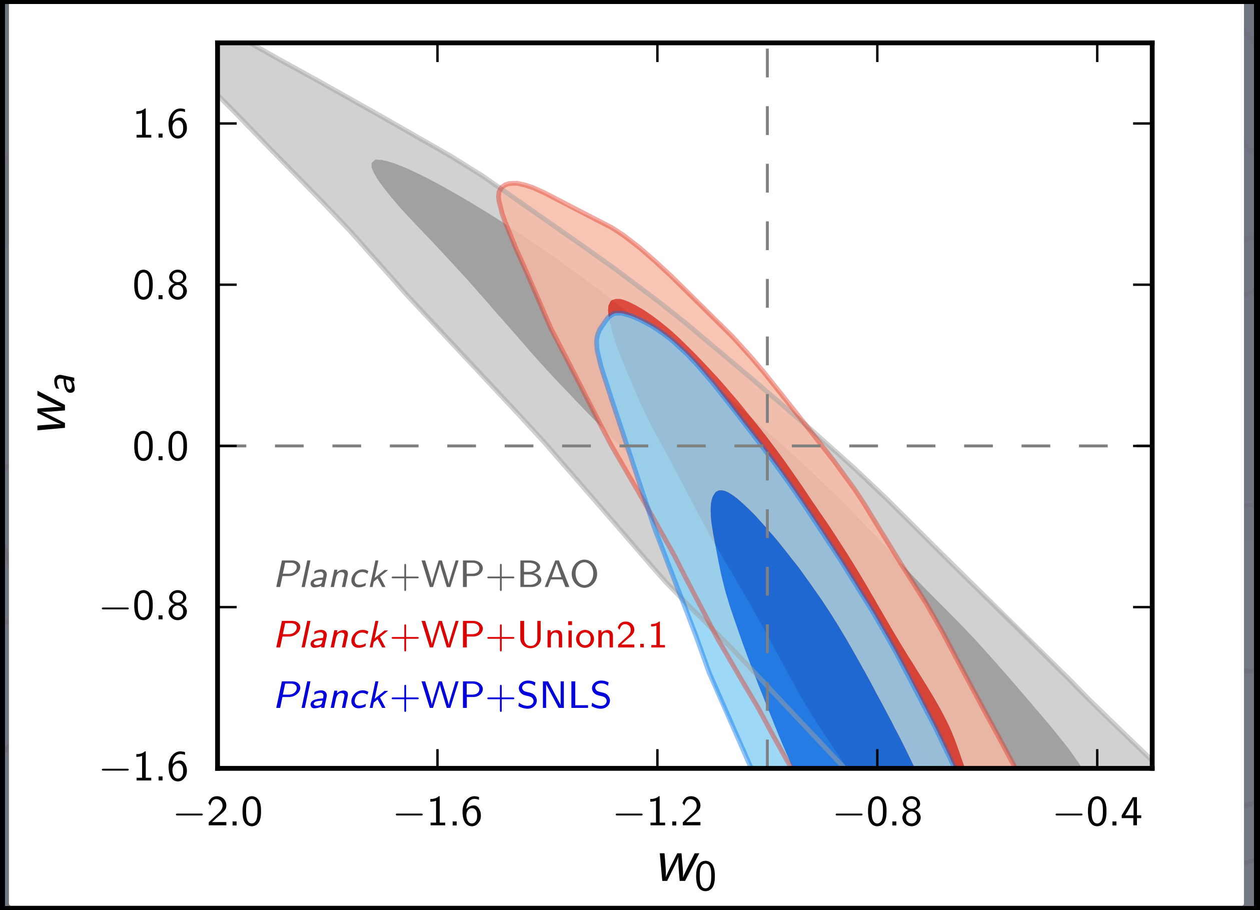
Parameterise eos: $w(a) \equiv \frac{p}{\rho} = w_0 + (1-a)w_a$

Planck alone weak constraints on DE because of degeneracy of w with H_0:

Break with other probes including lensing, SN, BAO ...

Example - if assume $w_a = 0$

$$w = -1.13 \pm 0.24 \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{BAO})$$
$$w = -1.09 \pm 0.17 \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{Union2.1})$$
$$w = -1.13^{+0.13}_{-0.14} \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{SNLS}),$$
$$w = -1.24^{+0.18}_{-0.19} \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{HST}).$$



For me -- a key question how should we parameterise w?

[Tarrant et el 2013]

WMAP7 and dark energy

(Komatsu et al, 2010)

Assume flat univ +
+BAO+ SNLS:

$$w = -0.980 \pm 0.053$$

Drop prior of flat univ:
WMAP + BAO +
SNLS:

$$w = -0.999^{+0.057}_{-0.056} \quad \Omega_k = -0.0057^{+0.0067}_{-0.0068}$$

Drop assumption of
const w but keep flat
univ: WMAP + BAO
+ SNLS:

$$\begin{aligned} w_0 &= -0.93 \pm 0.12 \\ w_a &= -0.38^{+0.66}_{-0.65} \end{aligned}$$

Coincidence problem – why now?

Recall: $$\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$$

If: $$\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$$

Universe dom by Quintessence at:

$$z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$$

$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \to z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

Univ accelerates at:

$$z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$$

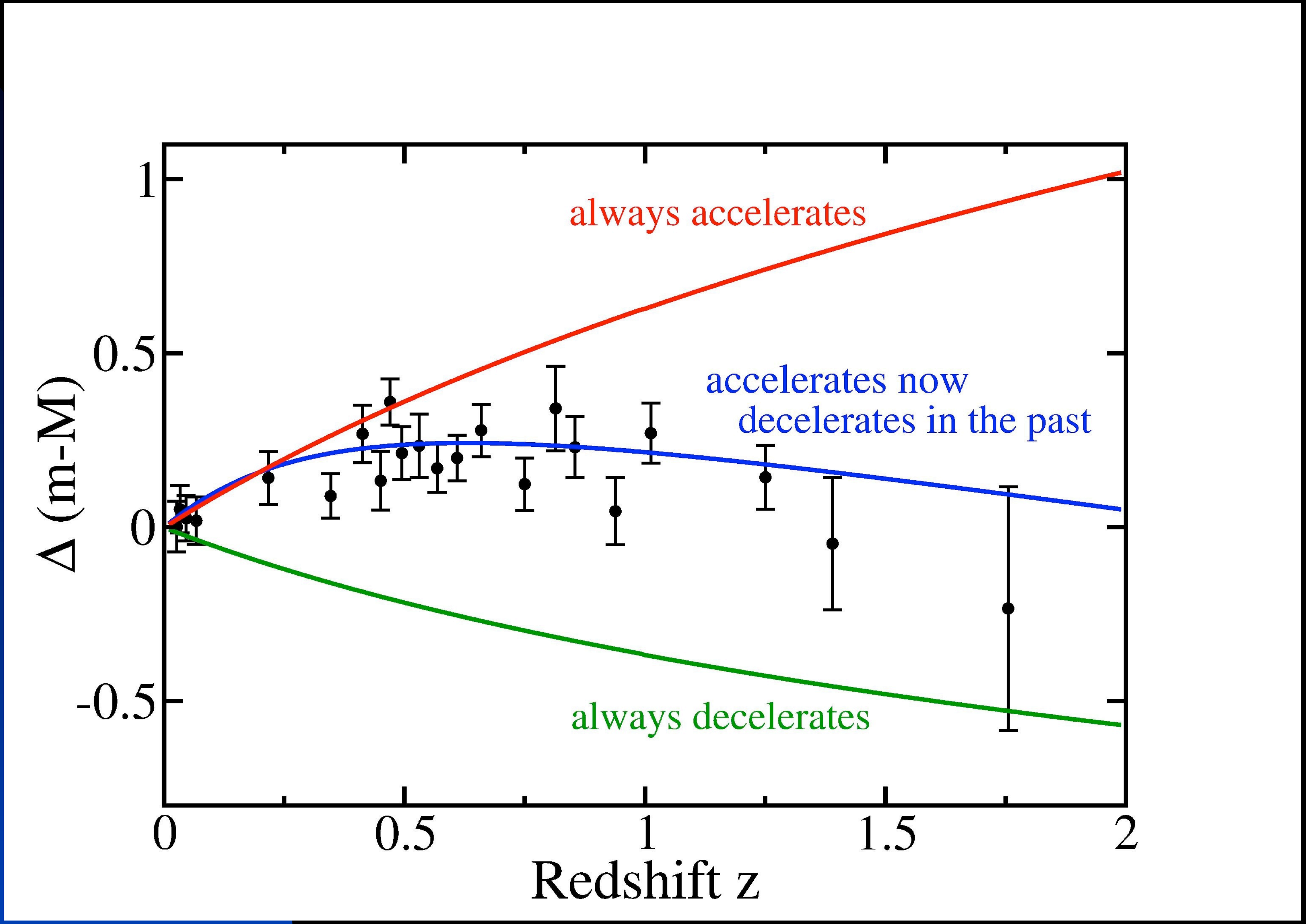
$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Constraint:

$$-0.11 < 1 + w < 0.14$$

Komatsu et al 2008 (WMAP5)

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



Potential are of impact for DES I think !

Many approaches to Dark Energy:

- A true cosmological constant -- but why this value?
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.
- Hiding the cosmological constant -- its there all the time but just doesn’t gravitate

String - theory -- where are the realistic models?

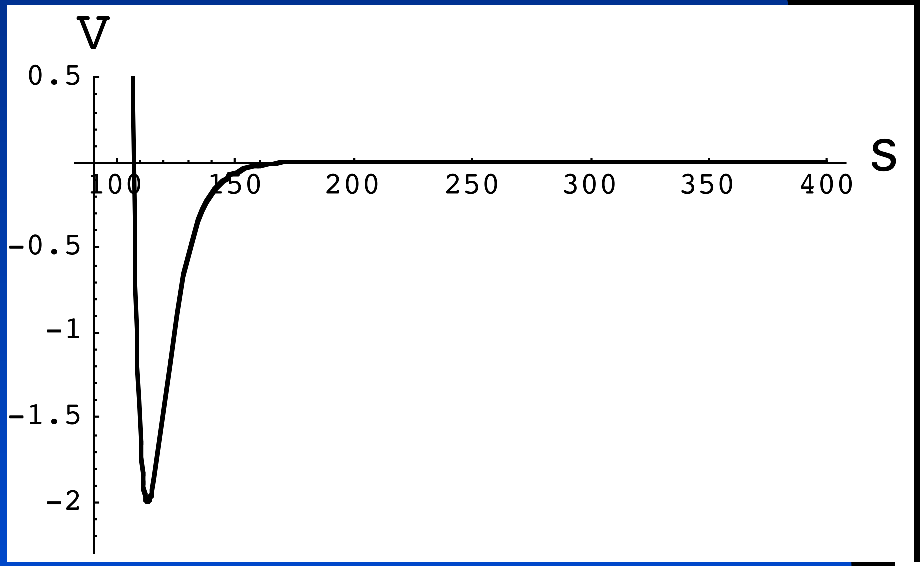
`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)
2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

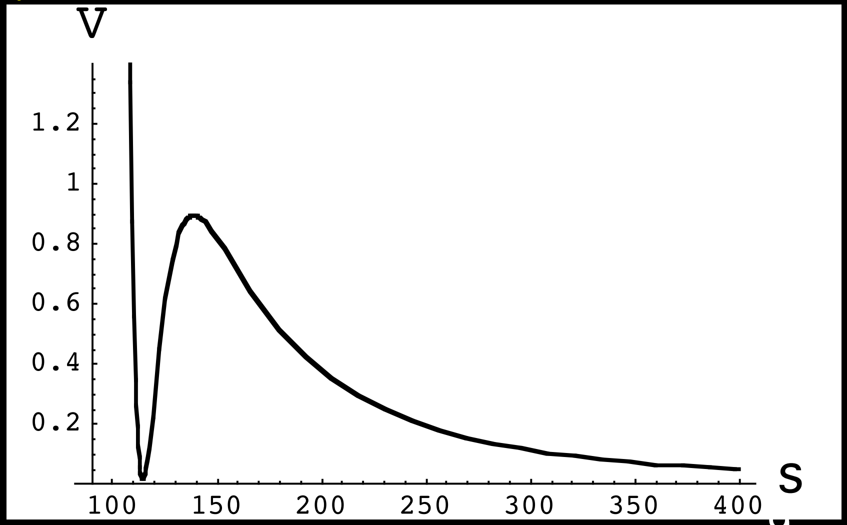
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\rm KKLT} = V_{\rm AdS} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

[Witten 2008]

Landscape gives a realisation of the multiverse picture.

There isn’t one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn’t a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

2. Λ from a self-tuning universe [Feng et al 2001].

Λ relaxes through nucleation of branes coupled to gauge potential, the particular branes depending on the compactification assumed.

3. Relaxation of Λ [Kachru et al 2000, Arkani Hamad et al 2000, Burgess et al].

Relies on presence of extra dimension to remove the gravitational effect of the vacuum energy.

3 brane solns in 5D eff theories leads to standard model vacuum energy warping the higher dimensional spacetime while preserving 4D flatness with no cosm constant. Problems with this!

4. Λ from the Cyclic Perspective [Steinhardt and Turok 2002, 2006].

Key feature, because many cycles and each cycle lasts a trillion years, universe today is much older than today’s Hubble time, so Λ has had long time to reduce to the observed value today.

5. Supersymmetric Large Extra Dims and Λ [Burgess et al, 2003-2013].

Solutions to 6D Supergravity

In more than 4D, the 4D vacuum energy can curve the extra dimensions instead of the observed 4 dimensions [Carroll and Guica; Aghababaie et al]

Proposal: Physics is 6D above 10^{-2} eV scale with supersymmetric bulk. We live in 4D brane with 2 extra dim.

Integrate out brane physics leads to large 4D vacuum energy, but it is localised in extra dimensions.

Integrate out classical contributions in bulk and find tensions cancel between bulk and brane.

Static and time dependent solutions exist, most of them runaway with rapid growing or shrinking dimensions.

Albrecht-Skiordis type quintessence evolution leads to late time acceleration and testable predictions.

Recent developments with stable solutions and particular new relations between particles - over to Cliff ...

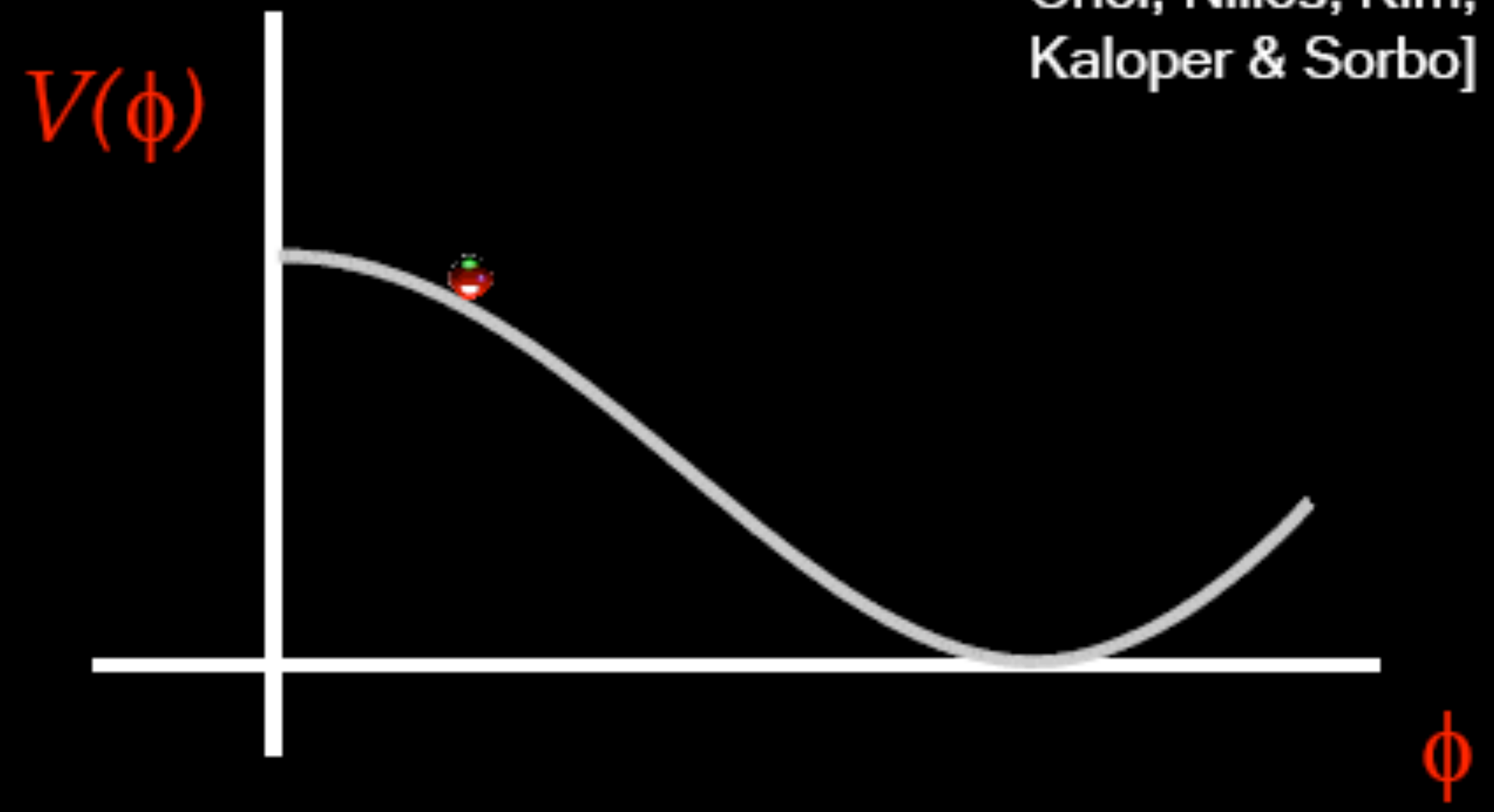
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym ϕ --> ϕ + const.

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al; Choi; Nilles; Kim; Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi / F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

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Strong CP problem intro axion : $m_a = \frac{\Lambda^2_{\mathrm{QCD}}}{F_a}$; F_a – decay constant

PQ axion ruled out but invisible axion still allowed:

$$10^9 \text{ GeV} \le F_a \le 10^{12} \text{ GeV}$$

Sun stability CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\text{vac}} = (10^{-3} \text{ eV})^4 \to m_{\text{axion}} \sim 10^{-33} \text{ eV}$$

Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints -- over to Hans Peter Nilles ...

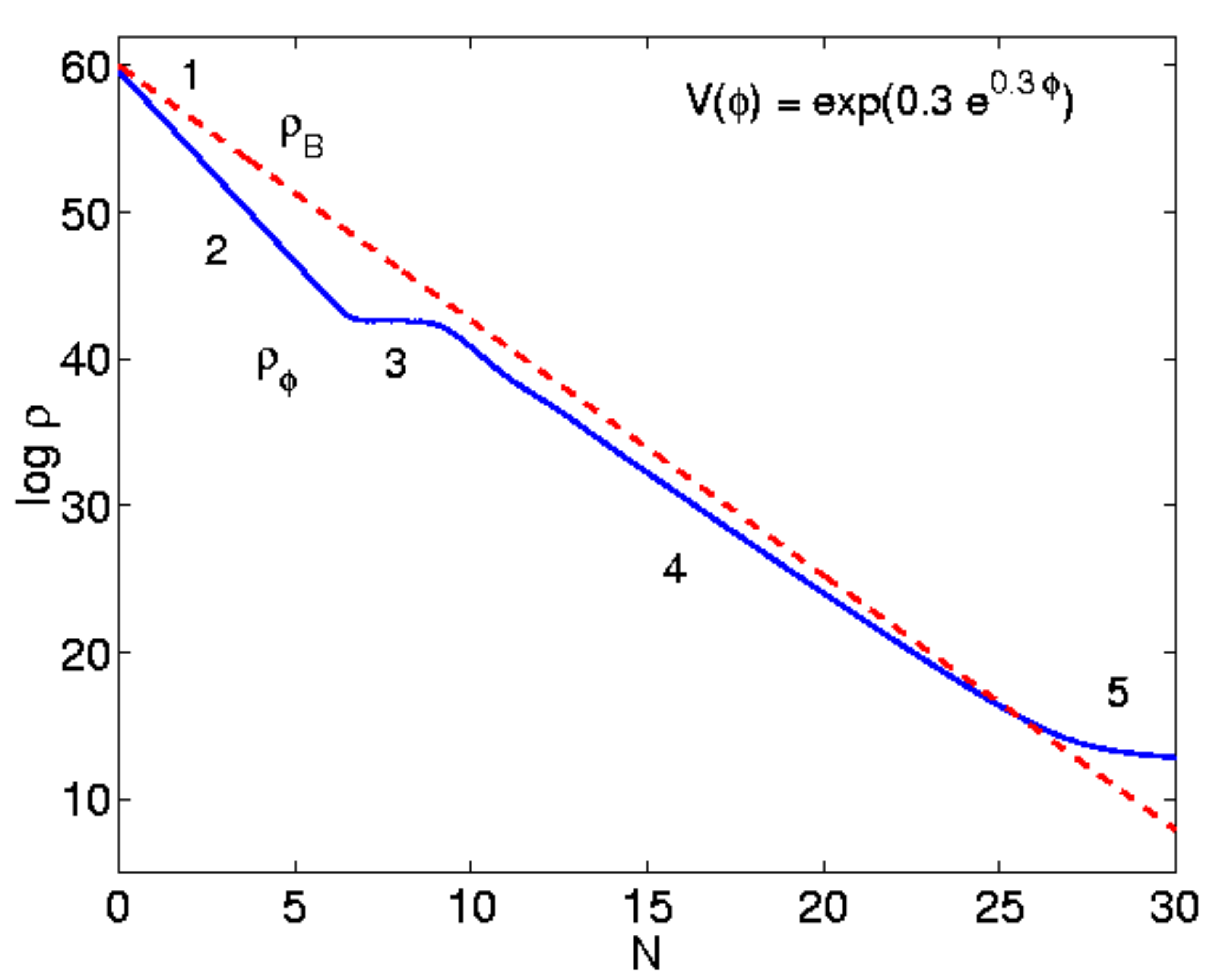
Slowly rolling scalar fields -- Quintessence

As of 14 Mar 2013, can really use this language !

Peebles and Ratra; Wetterich; Ferreira and Joyce

Zlatev, Wang and Steinhardt

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.

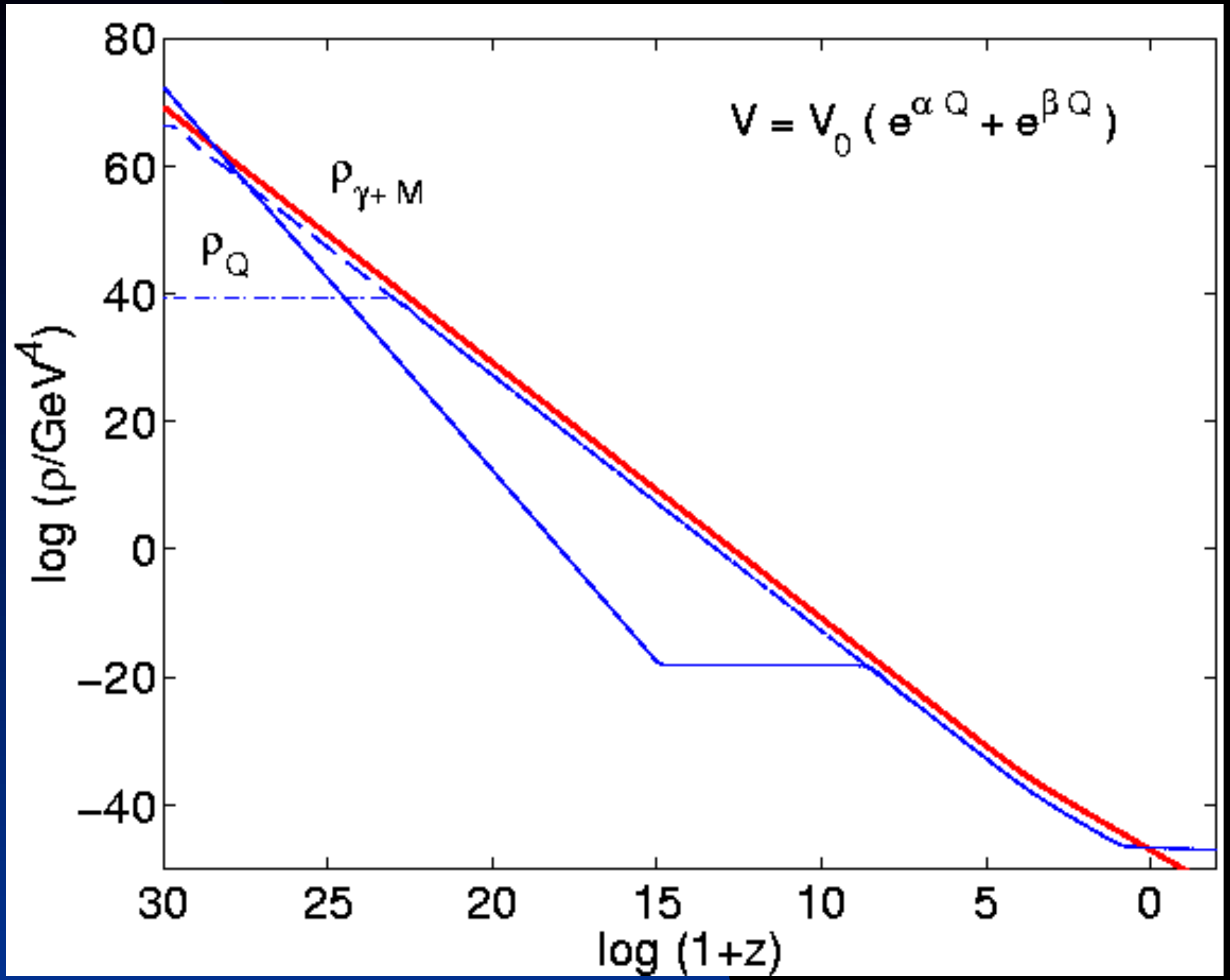


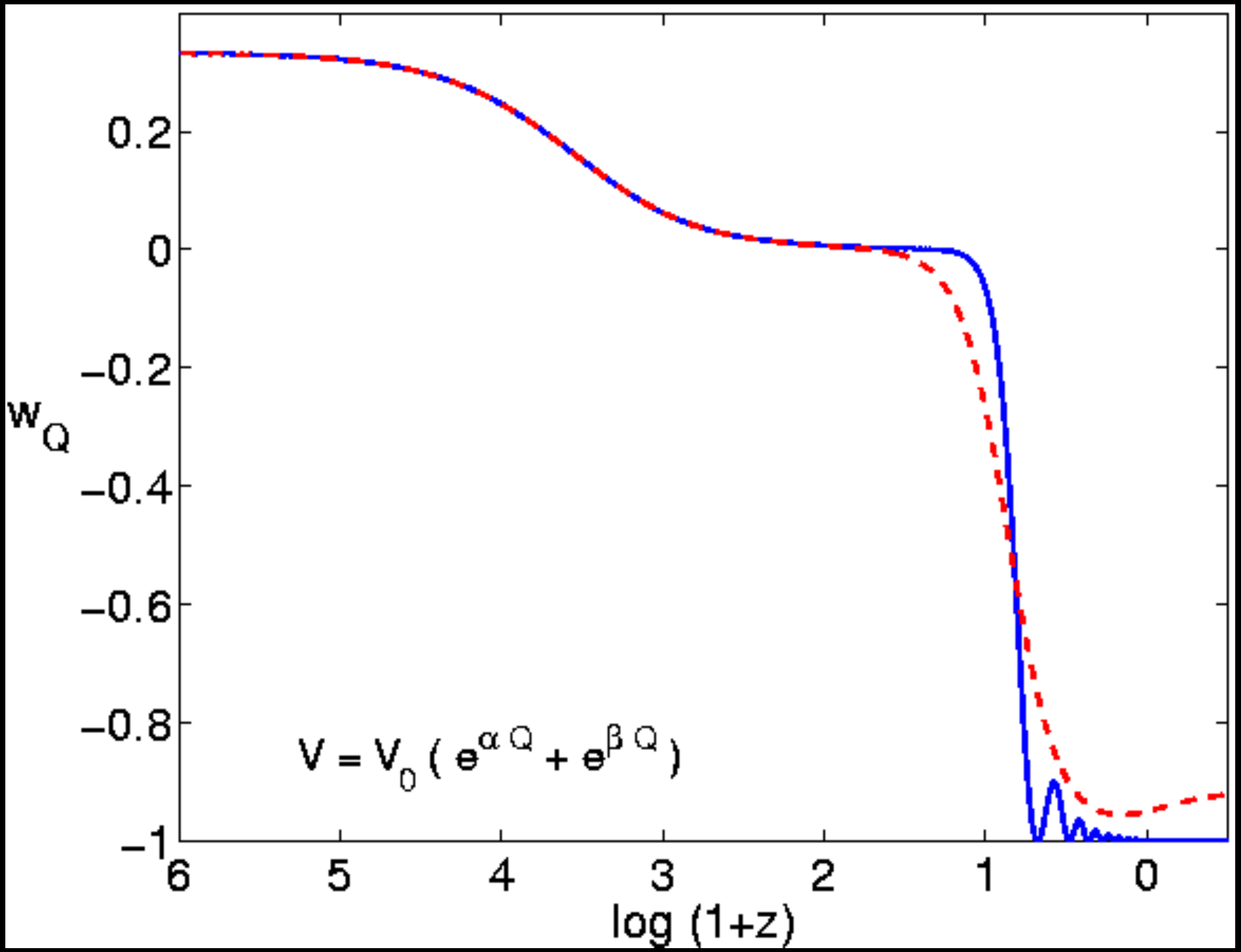
Nunes

Attractors make initial conditions less important

$$V(\phi) = V_1 + V_2 = V_{01}e^{-\kappa\lambda_1\phi} + V_{02}e^{-\kappa\lambda_2\phi}$$

EC and Nunes





$\alpha = 20; \beta = 0.5$

Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Fifth force !

1. Chameleon fields [Khoury and Weltman (2003) ...]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. Phantom fields [Caldwell (2002) ...]

The data does not rule out w<-1. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches).

3. K-essence [Armendariz-Picon et al ...]

Scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model?

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form. Similar fine tuning to Quintessence -- found in DBI models for instance, Gallileons, ...

4. Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} &: \quad \nabla_\mu G^\mu_\nu = 0 \to \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \to \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \quad \text{is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha\,(m)}_\alpha \nabla_\nu \phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha\,(m)}_\alpha \nabla_\nu \phi \end{aligned}$$

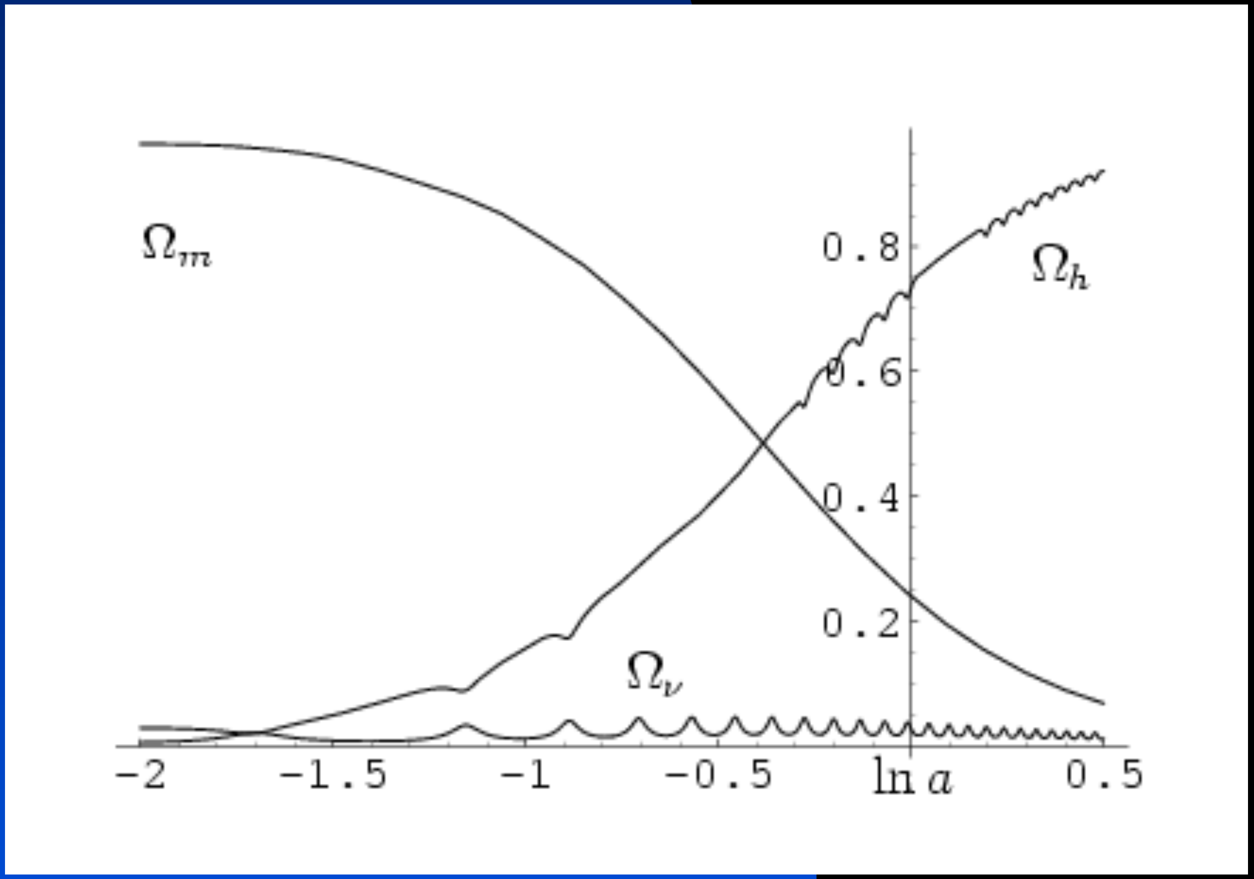
Ex: Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

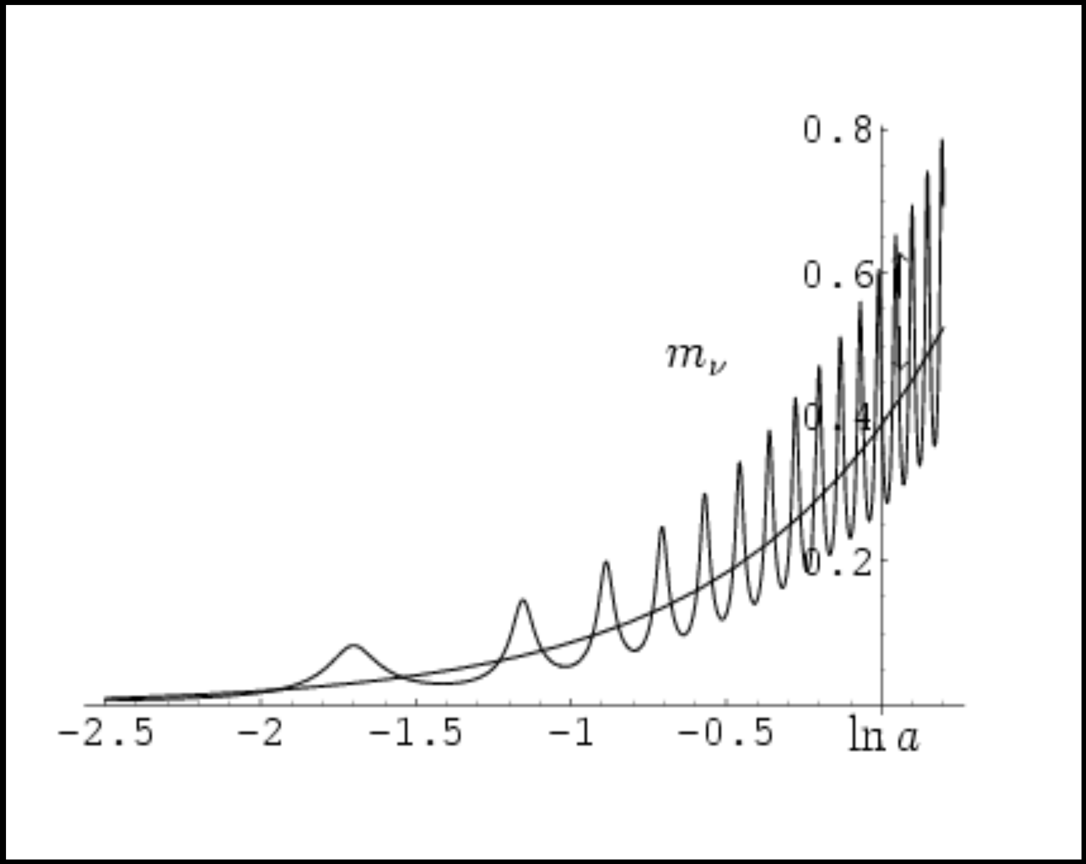
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by time when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$





m_ν

Perturbations in Interacting Dark Energy Models [Baldi et al (2008), Tarrant et al (2010)]

Perturb everything linearly : Matter fluid example

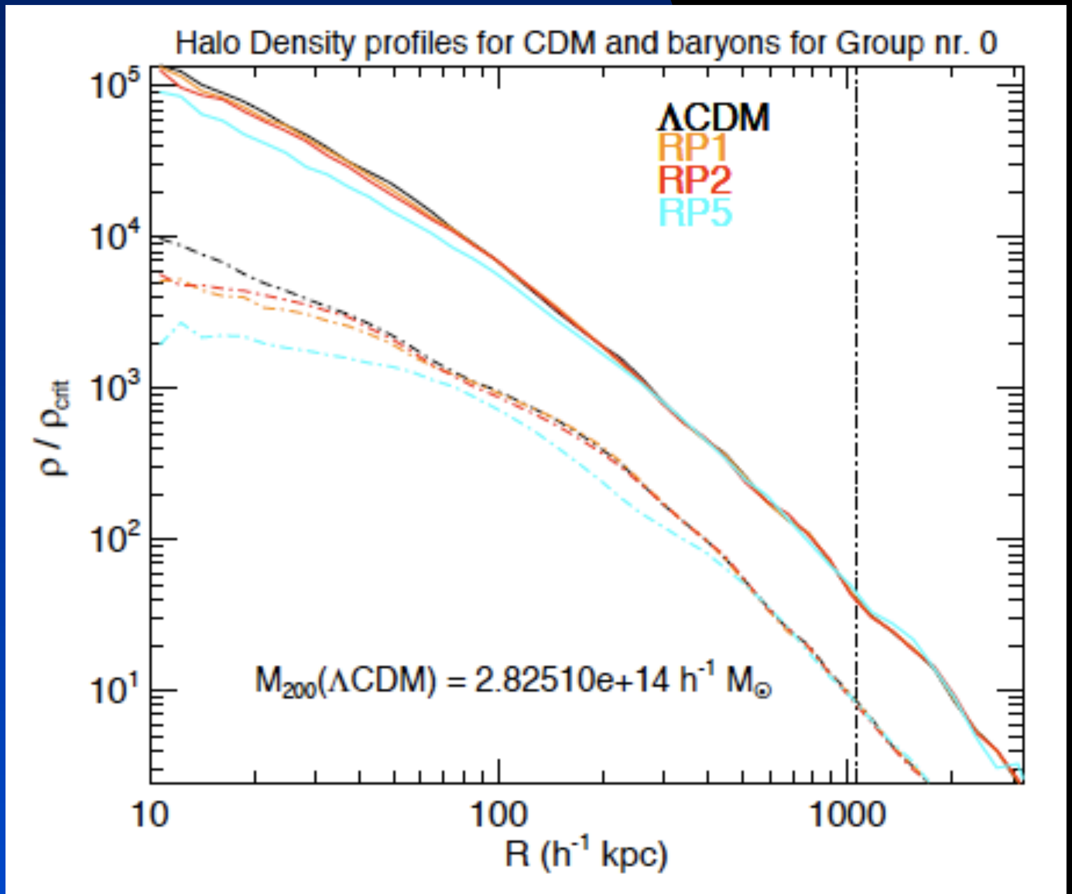
$$\ddot{\delta}_c + \left(2H - 2\beta\frac{\dot{\phi}}{M}\right)\dot{\delta}_c - \frac{3}{2}H^2[(1+2\beta^2)\Omega_c\delta_c + \Omega_b\delta_b] = 0$$

extra friction

modified grav interaction

vary DM particle mass

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

Still early days -- but this is where I think there should be a great deal of development -- see Puchwein et al yesterday (1305.2418)

Do we need Dark Energy ?

Buchert (2000), Kolb et al (2006), Wiltshire (2007), Hunt and Sarkar (2007), Garcia-Bellido and Haugbolle (2008), Moss et al (2010), Nadathur and Sarkar (2010) ... + many

Perhaps we dont need to fine tune a cosmological constant, what we see is a result of living in an inhomogeneous universe.

Live in a void arising from inhomo flucn in early universe -- Gpc scale.

We live close to centre of large void where Hubble flow is 30% faster than global rate. Void size 2.5 Gpc in otherwise EdS univ on large scales.

Apparent accn arises from curved photon paths in open patch of universe.

Fine tuning - must be within 100Mpc of centre of void so that induced dipole moment in CMB not too large.

CMB analysis of including higher multipoles suggest the standard LTB models used to describe the voids struggle to fit data, predicting low local Hubble rate, age problem and too little structure. [Moss et al (2010)]

But if allow for features in primordial power spectrum of density fluctuations, for example breaks in it arising from multiple periods of inflation -- then can address many of the issues with the CMB peaks.

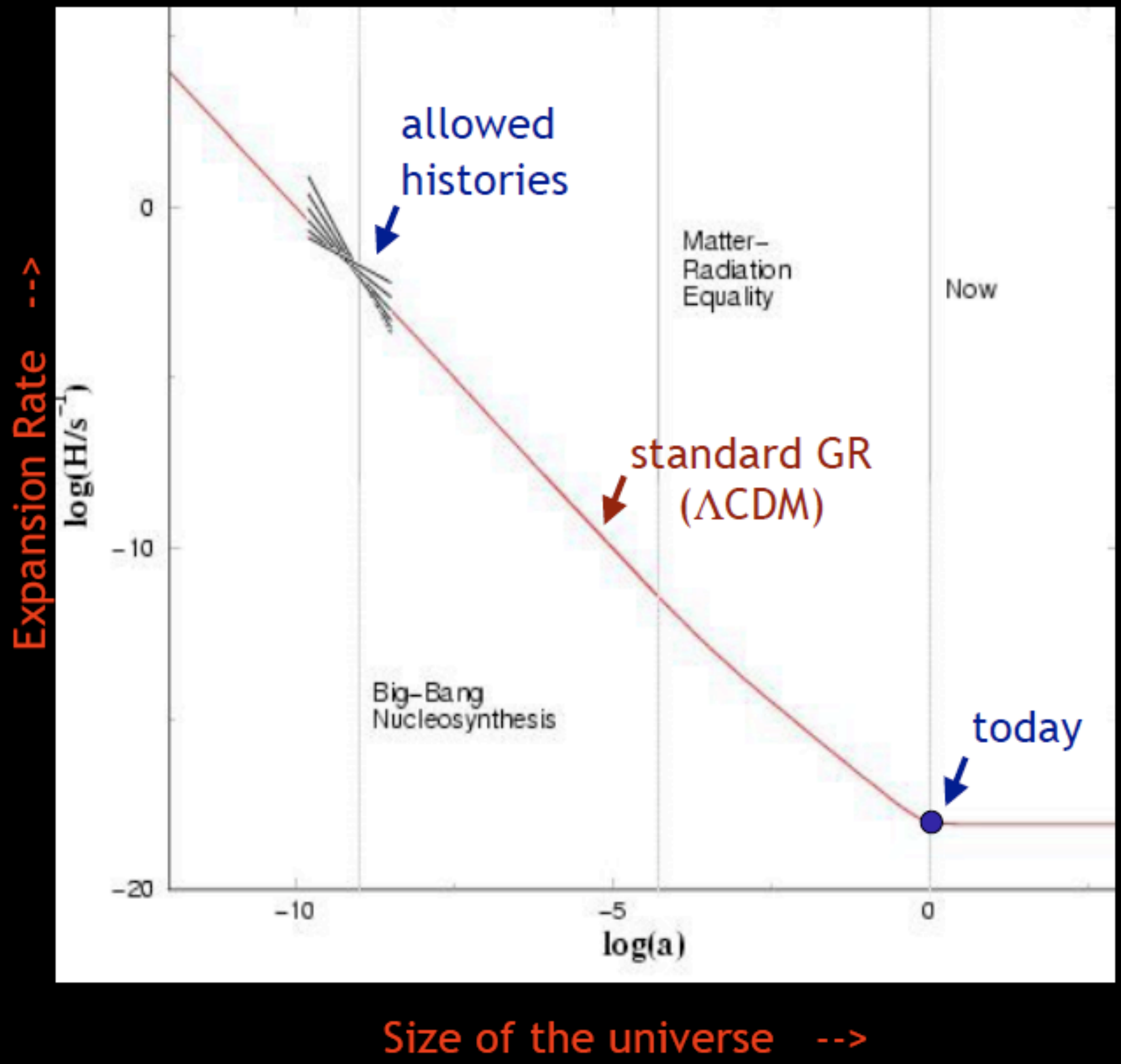
[Nadathur and Sarkar (2010)]

Should we be modifying gravity instead of looking for dark energy ?

Has become a big industry but it turns out to be hard to do too much to General Relativity without falling foul of data.

BBN occurred when the universe was about one minute old, about one billionth its current size. It fits well with GR and provides a test for it in the early universe.

Any alternative had better deliver the same successes not deviate too much at early times, but turn on at late times .



[Carroll & Kaplinghat 2001]

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

- f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical equations for the spacetime metric.

[Starobinski 1980, Carroll et al 2003, ...]

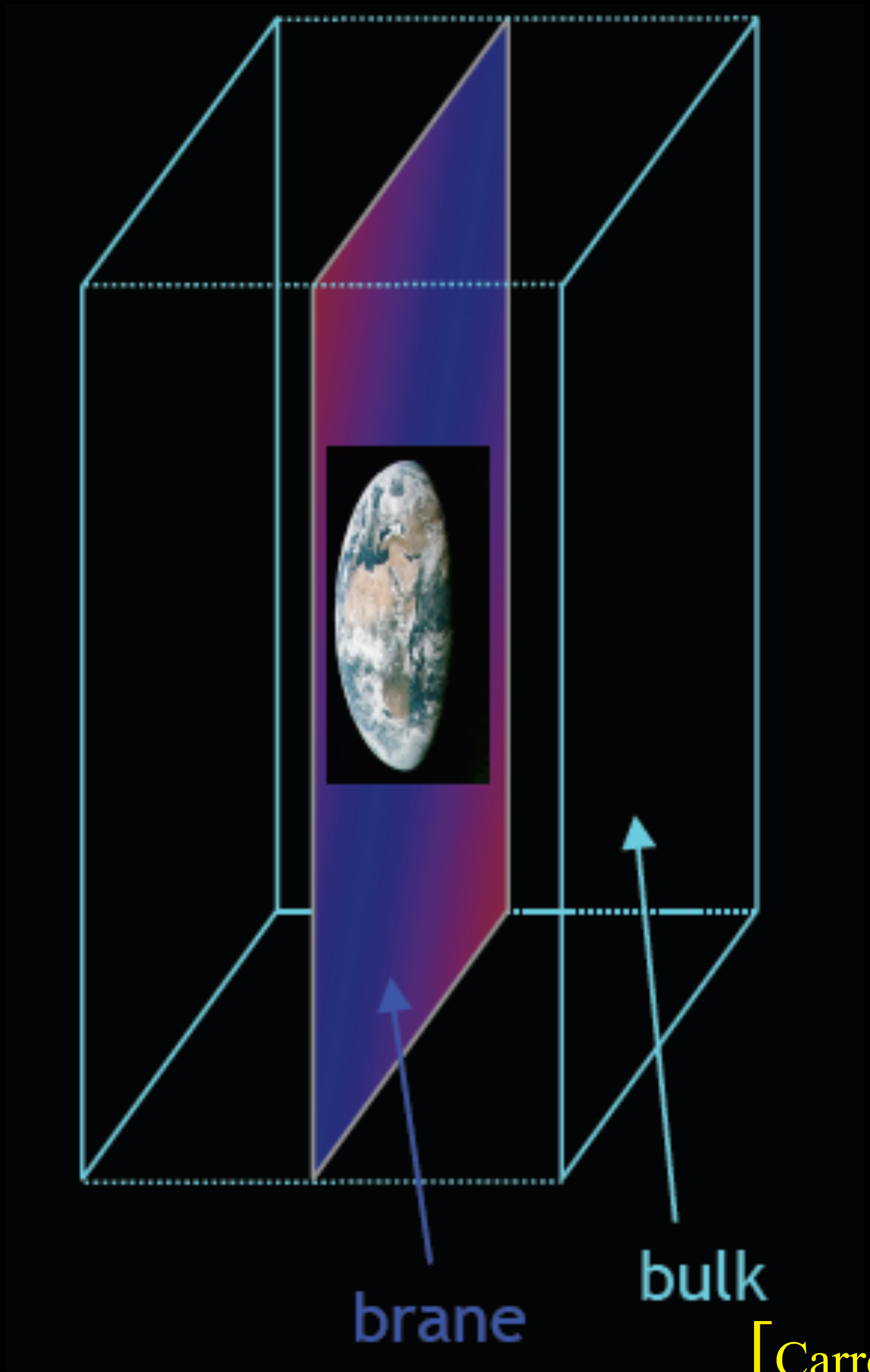
- Modified source gravity -- gravity depends on nonlinear function of the energy.

- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory [ghosts] and observations

- Scalar-tensor theories including higher order scalar-tensor lagrangians -- recent examples being Gallileon models



[Carroll]

f (R) models [Lots and lots of people...]

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_\mathrm{m} \right]$$

No Λ

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Has to look like a standard cosmological constant

Designer f (R) or f(G) models [Hu and Sawicki (2007), ...]

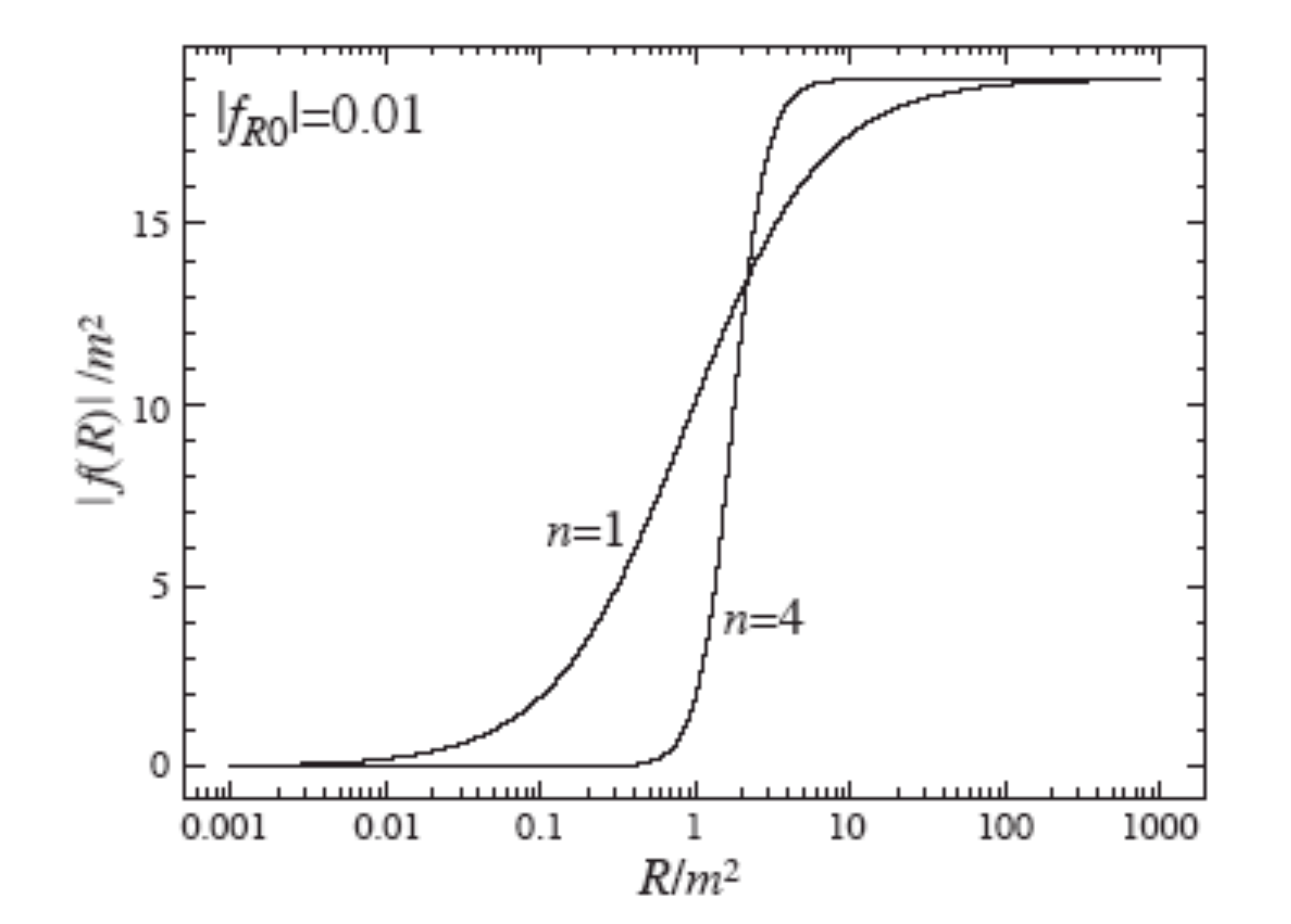
Construct a model to satisfy observational requirements:

1. Mimic LCDM at high z as suggested by CMB
2. Accelerate univ at low z
3. Include enough dof to allow for variety of low z phenomena
4. Include phenom of LCDM as limiting case.

$$\lim_{R\to\infty} f(R) = \text{const.}\,, \qquad \lim_{R\to 0} f(R) = 0\,,$$

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}\,,$$

$$f_{RR} \equiv \frac{d^2 f(R)}{dR^2} > 0$$



Bad news for chameleons [Barnaby, Burrage, Erickcek, Huang]

In f(R) and chameleon models dark energy is sourced by the trace of the energy momentum tensor of matter

The scalar gets 'kicked' every time a particle species decouples

This leads to the production of very high energy particles

During radiation domination

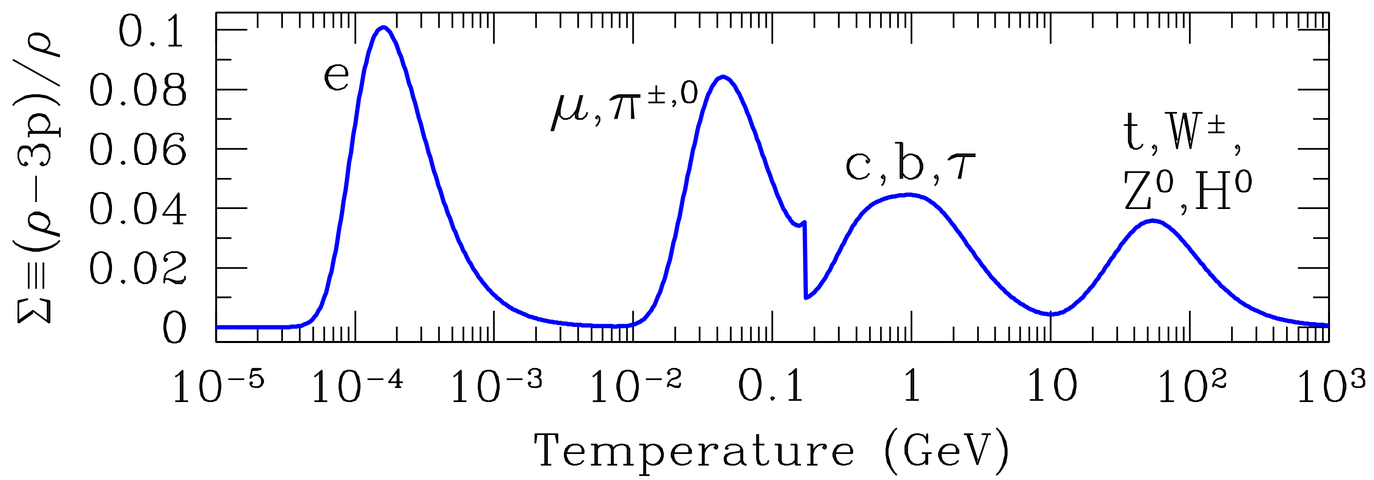
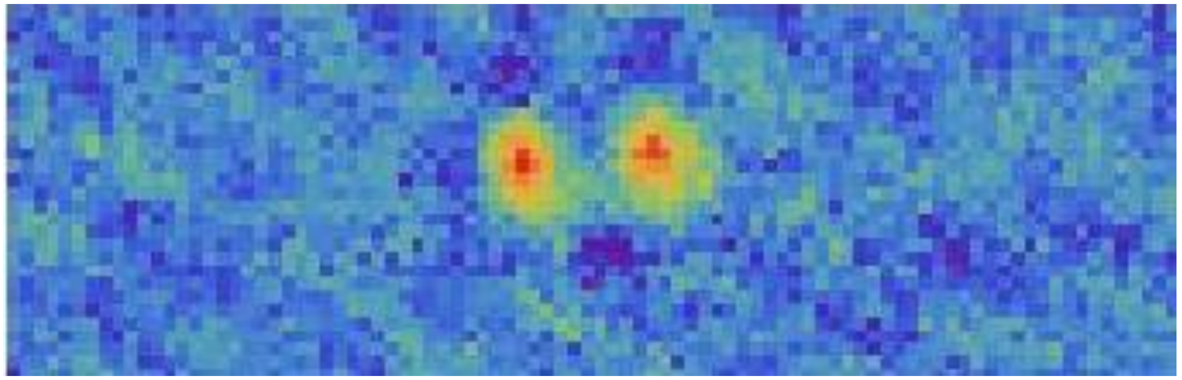


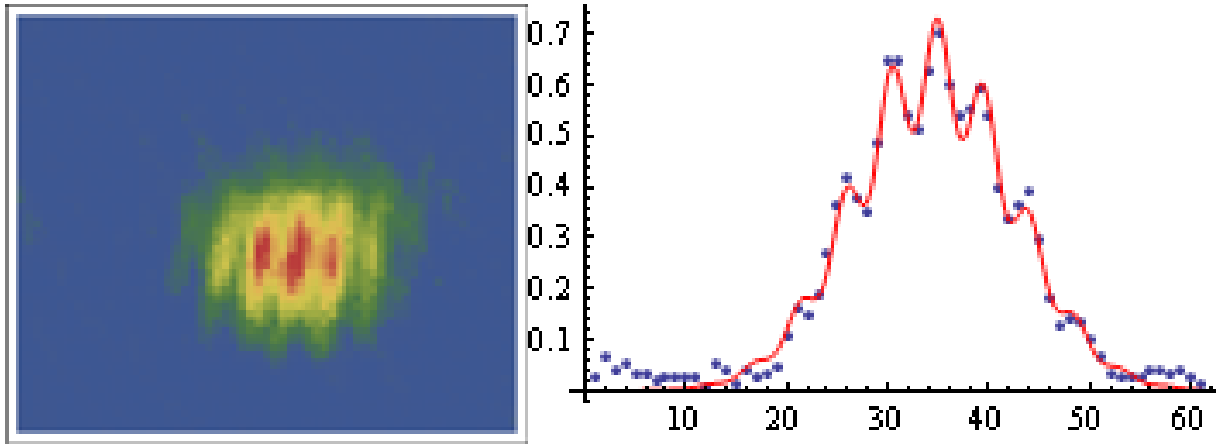
FIG. 1: The kick function $\Sigma = (\rho - 3P)/\rho$ vs Jordan-frame temperature. We account for all SM particles. The discontinuity at $T_J = 170\,\mathrm{MeV}$ corresponds to the QCD phase transition.

Net effect - breakdown in calculability prior to BBN due to same matter couplings required to avoid fifth force constraints

Dark Energy Direct Detection Experiment [Burrage, EC, Hinds]

Coherent waves in Bose-Einstein condensates can be used for interferometry





Credit: Centre for Cold Matter, Imperial

Interference of waves in condensates at different heights has already detected gravitational effects

(Dimopoulos, Geraci 2003. Baumgärtner et al. 2010)

Interference of waves in condensates held in different environments can be used to directly detect screening mechanisms

What should we do to help determine the nature of DE ?

1. We need to define properly theoretically predicted observables, or determine optimum ways to parameterise consistency tests (i.e. how should we parameterise w(z)?)

2. Need to start including dynamical dark energy, interacting dark matter-dark energy and modified gravity models in large scale simulations -[see Puchwein, Baldi and Springel].

3. Include the gastrophysics + star formation especially when considering baryonic effects in the non-linear regimes.

4. On the theoretical side, develop models that go beyond illustrative toy models. Extend Quintessential Axion models. Are there examples of actual Landscape predictions? De Sitter vaccua in string theory is non trivial -[see Burgess et al].

5. Recently massive gravity and galileon models have been developed which have been shown to be free of ghosts. What are their self-acceleration properties?

6. Will we be able to reconstruct the underlying Quintessence potential from observation?

7. Never mind evidence of evolution, will we ever be able to determine whether $w \neq -1$?

8. Look for alternatives, perhaps we can shield the cosmological constant from affecting the dynamics through self tuning-- The Fab Four

9. Given the complexity (baroque nature ?) of some of the models compared to that of say Λ, we should be using Bayesian model selection criterion to help determine the relevance of any one model.

Many more things to be done on a phenomenological and theoretical side.

Things are getting very exciting with the Dark Energy Survey beginning to take data and proposed longer term Euclid mission.

Self tuning - with the Fab Four

"The Beatles [have made] negligible contributions to cosmological theory"
PRL, June 2011

In GR the vacuum energy gravitates, and the theoretical estimate suggests that it gravitates too much.

Basic idea is to use self tuning to prevent the vacuum energy gravitating at all.

The cosmological constant is there all the time but is being dealt with by the evolving scalar field.

with Charmousis, Padilla and Saffin

PRL 108 (2012) 051101; PRD 85 (2012) 104040

with Padilla and Saffin

JCAP 1212 (2012) 026

Horndeski's theory: [G.W. Horndeski, Int. Jour. Theor. Phys. 10 (1974) 363-384

Most general scalar-tensor theory with second order field equations:

$$\begin{aligned}\mathcal{L}_H \;=\; & \delta^{ijk}_{\mu\nu\sigma}\Big[\kappa_1 \nabla^\mu \nabla_i \phi R_{jk}{}^{\nu\sigma} - \frac{4}{3}\kappa_{1,\rho}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi \\ & +\kappa_3\nabla_i\phi\nabla^\mu\phi R_{jk}{}^{\nu\sigma} - 4\kappa_{3,\rho}\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\nabla^\sigma\nabla_k\phi\Big] \\ & +\delta^{ij}_{\mu\nu}\left[(F+2W)R_{ij}{}^{\mu\nu} - 4F_{,\rho}\nabla^\mu\nabla_i\phi\nabla^\nu\nabla_j\phi + 2\kappa_8\nabla_i\phi\nabla^\mu\phi\nabla^\nu\nabla_j\phi\right] \\ & -3[2(F+2W)_{,\phi} + \rho\kappa_8]\nabla_\mu\nabla^\mu\phi + \kappa_9(\phi,\rho),\end{aligned}$$

$$\rho = \nabla_\mu\phi\nabla^\mu\phi$$

$\kappa_1,\ \kappa_3,\ \kappa_8,\ \kappa_9$ — —Four indep func of ϕ and ρ

W can be set to zero and F can be derived from κ's.

Equivalent to Deffayet et al, PRD80 (2009) 064015

(see also Kobayashi et al 1105.5723 [hep-th])

The action which leads to self tuning solutions can be rewritten in a more natural way in which we see how the scalar fields couple directly to various curvature invariants:

$$
\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}
$$

where $\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

and $P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$

In other words it can be seen to reside in terms of the four arbitrary potential functions of ϕ coupled to the curvature terms.

Covers most scalar field related modified gravity models studied to date.

Assume no derivative couplings to matter to avoid violation of Equivalence Principle.

Can assume matter only couples to metric.

Begin the Cosmology

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\, d\Omega_{(2)}\right]$$

$$L_H^{\rm eff}(a,\dot a,\phi,\dot\phi) = a^3 \sum_{n=0}^{3}\left(X_n - Y_n\frac{\kappa}{a^2}\right)H^n$$

$$\begin{aligned}
X_0 &= -\tilde{Q}_{7,\phi}\dot\phi + \kappa_9 \\
Y_0 &= \tilde{Q}_{1,\phi}\dot\phi + 12\kappa_3\dot\phi^2 - 12F \\
X_1 &= -12F_{,\phi}\dot\phi + 3(Q_7\dot\phi - \tilde{Q}_7) + 6\kappa_8\dot\phi^3 \\
Y_1 &= -Q_1\dot\phi + \tilde{Q}_1 \\
X_2 &= 12\kappa_{1,\phi}\dot\phi^2 - 12\kappa_3\dot\phi^2 + 24\kappa_{3,\rho}\dot\phi^4 \\
&\quad -12F - 24F_{,\rho}\dot\phi^2 \\
X_3 &= 8\kappa_{1,\rho}\dot\phi^3
\end{aligned}$$

$$\begin{aligned}
Q_1 &= \frac{\partial\tilde{Q}_1}{\partial\dot\phi} = -12\kappa_1 \\
Q_7 &= \frac{\partial\tilde{Q}_7}{\partial\dot\phi} = 6F_{,\phi} - 3\dot\phi^2\kappa_8
\end{aligned}$$

Friedmann equation: $$\mathcal{H}(a,\dot{a},\phi,\dot{\phi}) = \frac{1}{a^3}\left[\dot{a}\frac{\partial L_H^{\rm eff}}{\partial \dot{a}} + \dot{\phi}\frac{\partial L_H^{\rm eff}}{\partial \dot{\phi}} - L_H^{\rm eff}\right] = -\rho_m$$

At most cubic in Hubble parameter H

$$\mu_3 H^3 + \mu_2 H^2 + \mu_1 H + \mu_0 = \rho_m$$

Scalar eom: $$\mathcal{E}(a,\dot{a},\ddot{a},\phi,\dot{\phi},\ddot{\phi}) = \frac{\partial L_H^{\rm eff}}{\partial \phi} - \frac{d}{dt}\left[\frac{\partial L_H^{\rm eff}}{\partial \dot{\phi}}\right] = 0$$

Linear in both $\ddot{\phi}$ and $\ddot{a}$.

Self tuning in Horndeski.

1. Vacuum solution is always Minkowski whatever the vacuum energy

2. Solution remains Minkowski even after a phase transition where the vacuum energy changes instantaneously.

In other words the vacuum energy does not gravitate at all because of the influence of the evolving scalar field and curvature.

$$< \rho_m >_{\text{vac}} = \rho_\Lambda, \quad H^2 = -\frac{\kappa}{a^2}, \quad \phi = \phi_\Lambda(t)$$

piecewise constant but discontinuous at transition

``On shell in a'' always satisfied

continuous everywhere and not constant

Scalar field eqn of motion should be trivial ``on-shell-in-a”

The scalar is completely determined by the vacuum Friedmann equation.

In this self tuning vacuum:

1. the matter tells the scalar how to move - this requires that the ``on-shell-in-a” gravity equation be dependent on $\dot{\phi}$

2. the scalar tells the spacetime not to curve, but crucially only in the vacuum - the scalar equation should not be independent of $\ddot{a}$

Some equations for Fab Four Cosmology:

$$\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
\mathcal{L}_{ringo} &= \sqrt{-g}V_{ringo}(\phi)\hat{G}
\end{aligned}$$

$$\hat{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$P^{\mu\nu\alpha\beta} = -\frac{1}{4}\varepsilon^{\mu\nu\lambda\sigma}\, R_{\lambda\sigma\gamma\delta}\, \varepsilon^{\alpha\beta\gamma\delta}$$

$$\mathcal{H}_{\text{john}} + \mathcal{H}_{\text{paul}} + \mathcal{H}_{\text{george}} + \mathcal{H}_{\text{ringo}} = -[\rho_\lambda + \rho_{\text{matter}}]$$

$$\mathcal{E}_{john} + \mathcal{E}_{paul} + \mathcal{E}_{george} + \mathcal{E}_{ringo} = 0$$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2\, d\Omega_{(2)}\right]$$

$$\mathcal{H}_{john} = 3V_{john}(\phi)\dot{\phi}^2\left(3H^2 + \frac{\kappa}{a^2}\right))$$

$$\mathcal{H}_{paul} = -3V_{paul}(\phi)\dot{\phi}^3 H\left(5H^2 + 3\frac{\kappa}{a^2}\right)$$

$$\mathcal{H}_{george} = -6V_{george}(\phi)\left[\left(H^2 + \frac{\kappa}{a^2}\right) + H\dot{\phi}\frac{V'_{george}}{V_{george}}\right]$$

$$\mathcal{H}_{ringo} = -24V'_{ringo}(\phi)\dot{\phi}H\left(H^2 + \frac{\kappa}{a^2}\right)$$

$$\mathcal{E}_{john} = 6\frac{d}{dt}\left[a^3V_{john}(\phi)\dot{\phi}\Delta_2\right] - 3a^3V'_{john}(\phi)\dot{\phi}^2\Delta_2$$

$$\mathcal{E}_{paul} = -9\frac{d}{dt}\left[a^3V_{paul}(\phi)\dot{\phi}^2 H\Delta_2\right] + 3a^3V'_{paul}(\phi)\dot{\phi}^3 H\Delta_2$$

$$\mathcal{E}_{george} = -6\frac{d}{dt}\left[a^3V'_{george}(\phi)\Delta_1\right] + 6a^3V''_{george}(\phi)\dot{\phi}\Delta_1 + 6a^3V'_{george}(\phi)\Delta_1^2$$

$$\mathcal{E}_{ringo} = -24V'_{ringo}(\phi)\frac{d}{dt}\left[a^3\left(\frac{\kappa}{a^2}\Delta_1 + \frac{1}{3}\Delta_3\right)\right]$$

Note each term vanishes identically when $\Delta = 0$

where:

$$\Delta_n = H^n - \left(\frac{\sqrt{-\kappa}}{a}\right)^n$$

fab four cosmology

TABLE I: Examples of interesting cosmological behaviour for various fixed points with $\sigma = 0$.

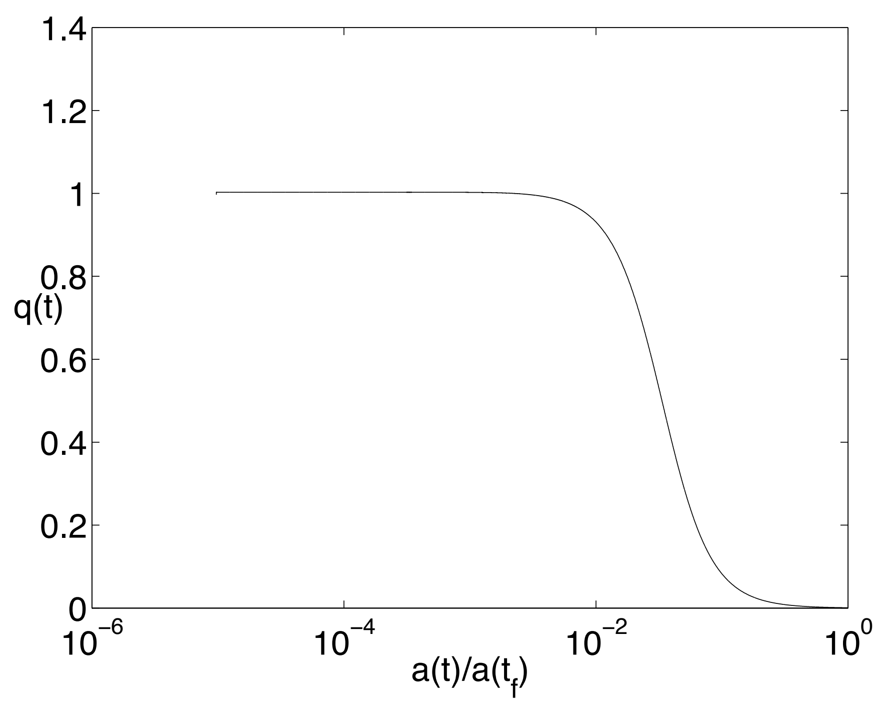
Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Stiff fluid	$H^2 \propto 1/a^6$	$c_1\phi^{\frac{4}{\alpha}-2}$	$c_2\phi^{\frac{6}{\alpha}-3}$	0	0
Radiation	$H^2 \propto 1/a^4$	$c_1\phi^{\frac{4}{\alpha}-2}$	0	$c_2\phi^{\frac{2}{\alpha}}$	$-\frac{\alpha^2}{8}c_1\phi^{\frac{4}{\alpha}}$
Curvature	$H^2 \propto 1/a^2$	0	0	0	$c_1\phi^{\frac{4}{\alpha}}$
Arbitrary	$H^2 \propto a^{2h}, \quad h \neq 0$	$c_1(1+h)\phi^{\frac{4}{\alpha}-2}$	0	0	$-\frac{\alpha^2}{16}h(3+h)c_1\phi^{\frac{4}{\alpha}}$

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

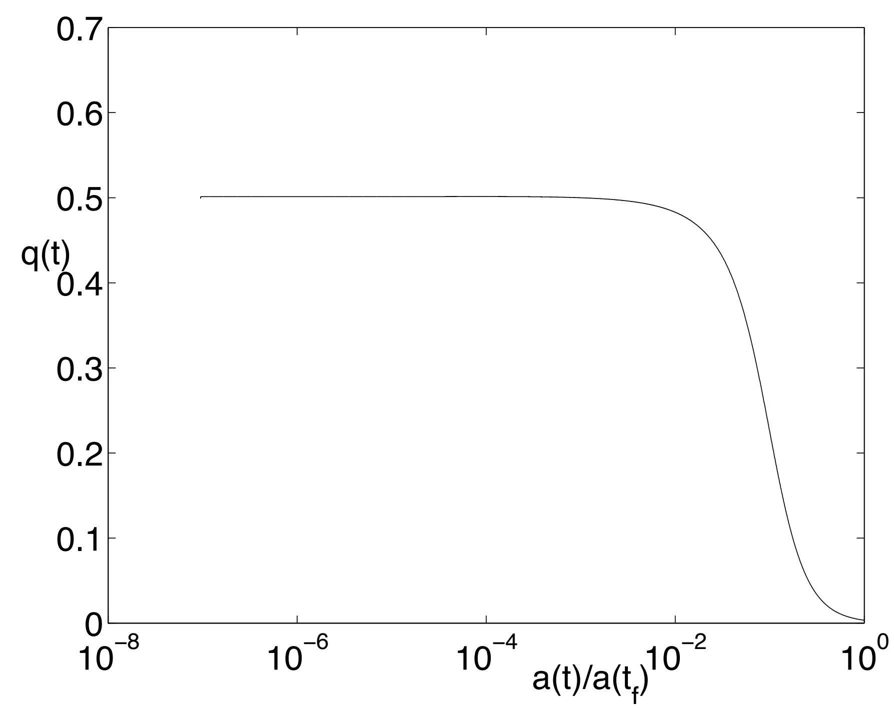
$$a \sim t^p \sim t^{-1/h}$$

$$q = -\frac{p(p-1)}{p^2} = -(1+h)$$

"radiation"



"matter"



Stability? see Kobayashi et al: 1105.5723; De Felice et al: 1108.4242

Tensor pertns:
$$\frac{1}{8}\int dt d^3x a^3\left[\mathcal{G}_T\dot{h}_{ij}^2-\frac{\mathcal{F}_T}{a^2}(\nabla h_{ij})^2\right]$$

Scalar pertns:
$$\int dt d^3x a^3\left[\mathcal{G}_S\dot{\xi}^2-\frac{\mathcal{F}_S}{a^2}(\nabla \xi)^2\right]$$

Find stable $F_T>0$, $G_T>0$, $F_S>0$, $G_S>0$ for say:

Case	cosmological behaviour	$V_j(\phi)$	$V_p(\phi)$	$V_g(\phi)$	$V_r(\phi)$
Matter I	$H^2\propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	$c_2\phi^{\hat{n}+6}$	0	$\frac{2\hat{n}-3}{16(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$
Matter II	$H^2\propto 1/a^3$	$c_1\phi^{\hat{n}+4}$	0	$c_2\phi^{\hat{n}+3}$	$-\frac{(\hat{n}+3)(2\hat{n}+5)}{8(2\hat{n}+7)(\hat{n}+6)}c_1\phi^{\hat{n}+6}$

Also true for radiation and inflation ...

But can we put them together somehow ?

Possible to have a self tuning `classical' solution in which the system adjusts itself to the Minkowski vacuum irrespective of the magnitude of the cosmological constant and whether it changes. It relies on breaking the assumption of Poincare invariance demanded by Weinberg in his original no-go theorem. In particular we have to have the scalar field evolving in time.

Remains to be seen whether we can satisfy solar system tests and obtain realistic cosmological solutions.

The role of quantum corrections remains to be evaluated (although initial calculaations suggest they can be controlled). They could spoil the party, although we note the crucial role played in the geometrical structure of the model.

There is always the question of stability of the solutions

Gregory Hormdeski left physics in 1981 having obtained a faculty position at Waterloo, Canada. He was on leave in Amsterdam and went to a Van Gogh exhibition.

His love of art was too strong and the inspiration he took from Van Gogh overpowering. He now works from his studio in Santa Fe.

Summary

•Data currently consistent with a pure cosmological constant -- but why that value?

•Why is the universe inflating today?

•Is w = - 1, the cosmological constant ? If not, then what value has it?

•Is w(z) -- dynamical. How should this be parameterized when considering surveys like DES and Euclid?

•New Gravitational Physics -- perhaps modifying Einstein equations on large scales? Key differences arising in perturbations.

•Perhaps we will only be able to determine it from anthropic arguments and not from fundamental theory.

•or -- we can avoid the need for a lambda term all together?