

Curvaton and other spectators

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contents:

spectator dynamics

1. during

2. after

inflation

inflation

$$H_* \approx \text{const.}$$

dynamics unknown

slow rolling scalar(s)?

light **scalar spectators** exist

$$m \ll H_*$$

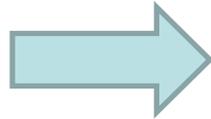
$$\rho_\sigma \ll \rho_{\text{inf}}$$

example: the **higgs**

others?

higgs as inflaton?

during inflation



spectators fluctuate

mean field

$$\sqrt{\langle \sigma^2 \rangle}$$

initial conditions for post-inflationary dynamics

field perturbations

$$\delta\sigma \approx H_*$$

isocurvature perturbation

spectators

can play a dynamical role after inflation

1. because of their field perturbations

-modulated (p)reheating $\Gamma_{\text{inf}} = \Gamma(\sigma)$

-modulated end of inflation $t_{\text{end}} = t(\sigma)$

-conversion of isocurvature into adiabatic (curvaton)

2. because of their classical evolution

-flat directions & Affleck-Dine BG

-moduli problems

DURING INFLATION

massless scalars in an expanding background



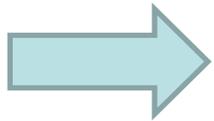
stochastic treatment

(cf. Starobinsky)

Langevin (simplified):

decompose field into UV and IR parts: $\Phi_{IR} \propto \int dk W(k, t) \phi_k(t)$

$$W(k, t) = \theta(k - xaH)$$


$$\dot{\Phi}_{IR} = -\frac{\partial}{3H\partial\Phi} V(\Phi_{IR}) + s(x, \eta) \quad k \ll aH$$


stochastic term, white noise correlators

$$\langle SS \rangle(dN) = (1 + x^3) \frac{H^2 dN}{4\pi^2}, \quad k = xa(N)H$$

inflationary fluctuations

massless field

$$\langle \phi^2 \rangle = \frac{1}{4\pi^2} H^2 N$$

$N = \#$ of e-folds

evolution of pdf: Fokker-Planck

$$\frac{\partial P}{\partial t} = \frac{1}{3H} \frac{\partial}{\partial \phi} [V'(\phi)P] + \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \phi^2} P$$

equilibrium pdf:

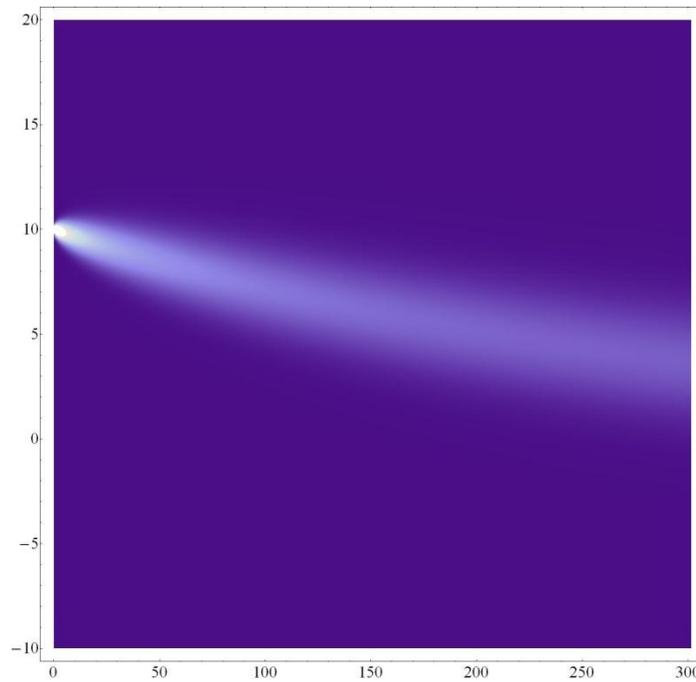
$$P \propto \exp(-8\pi^2 V / 3H^4)$$

example

$$V = \frac{1}{2} m^2 \sigma^2$$

$$m = 0.01H$$

σ / H



N

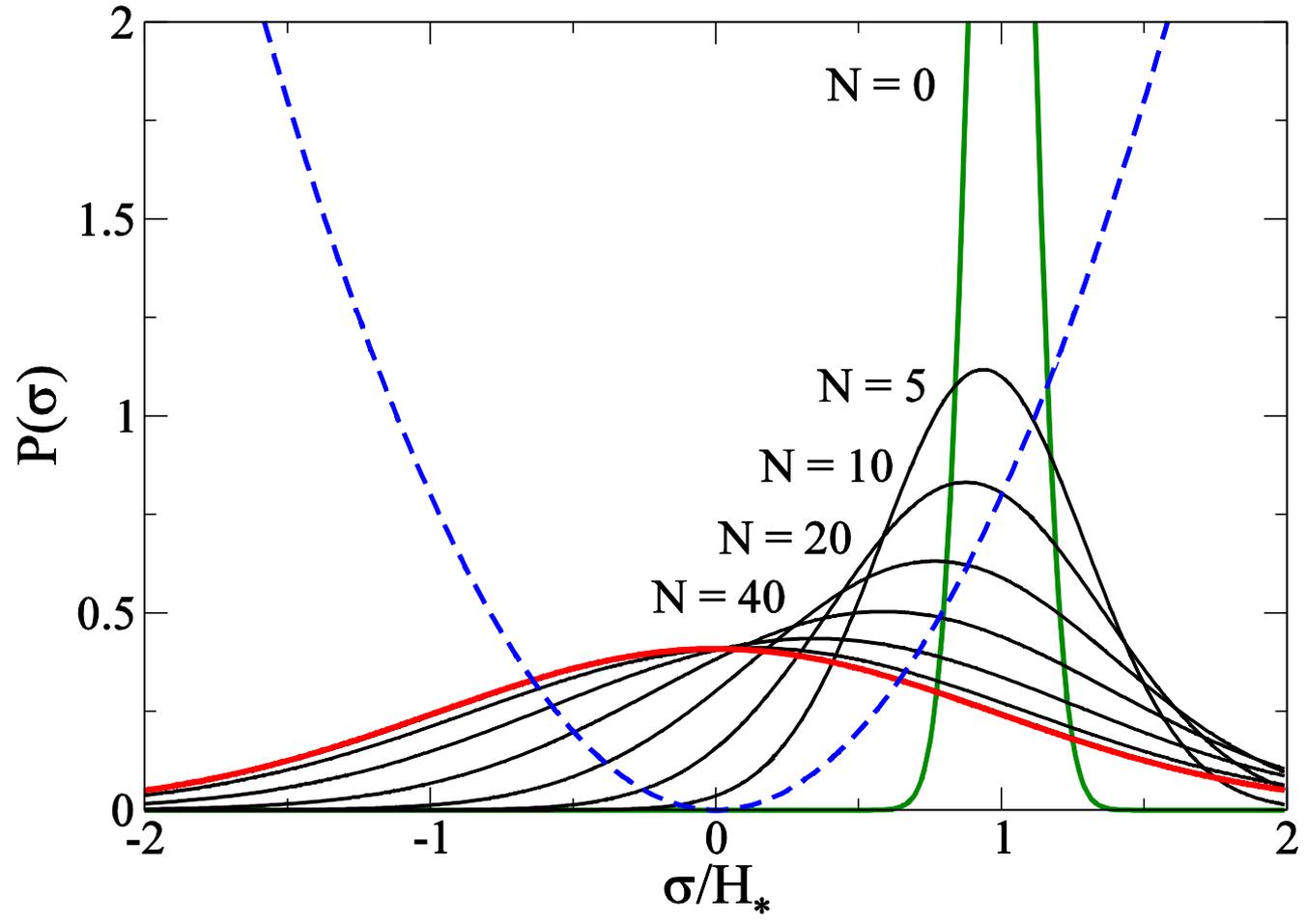
relaxation time

$$N_{rel} = \frac{3H_*^2}{m^2}$$

decoherence time

$$N_{dec} = \frac{3H_*^2}{2m^2}$$

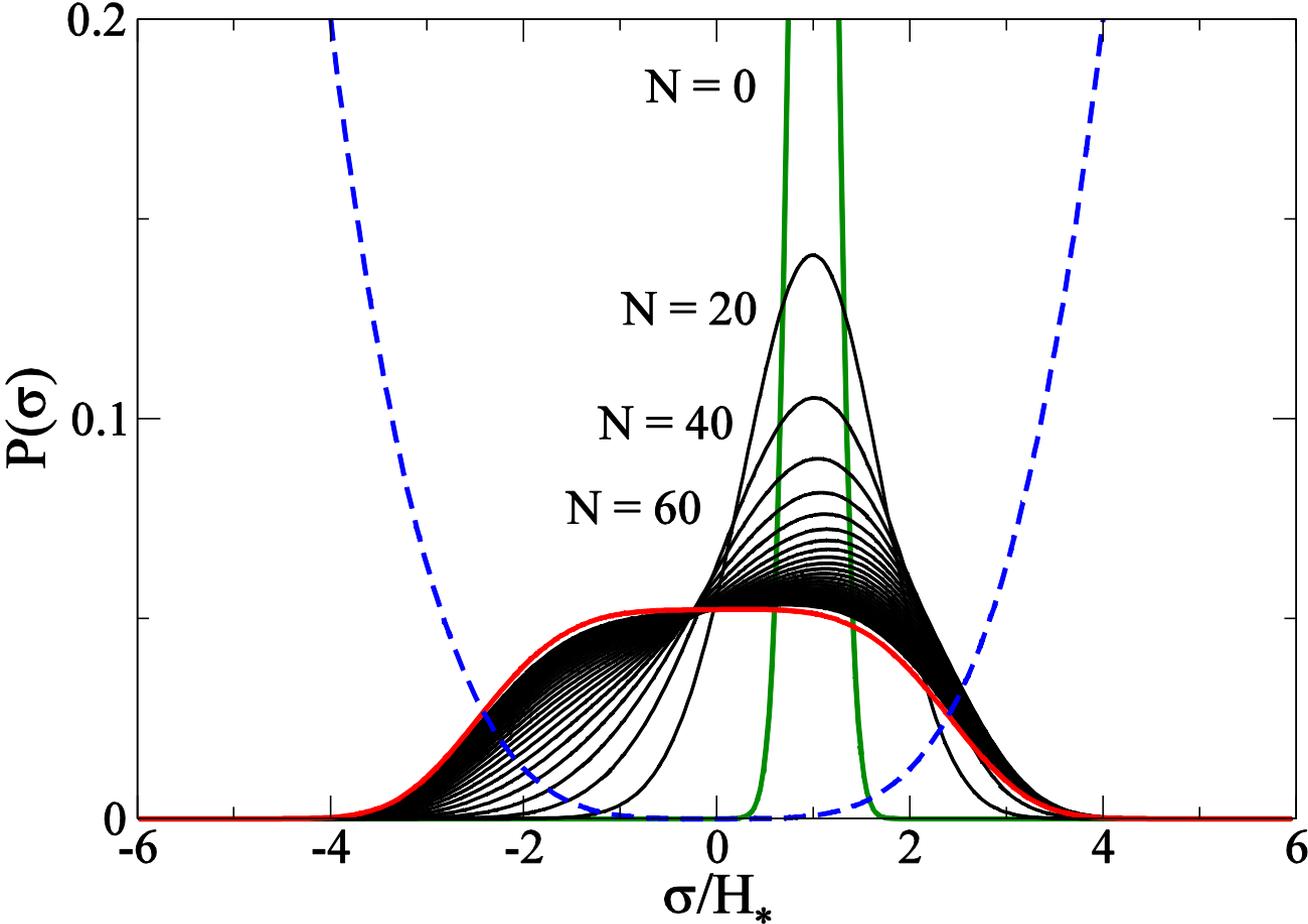
$m/H_* = 0.2$



quartic potential

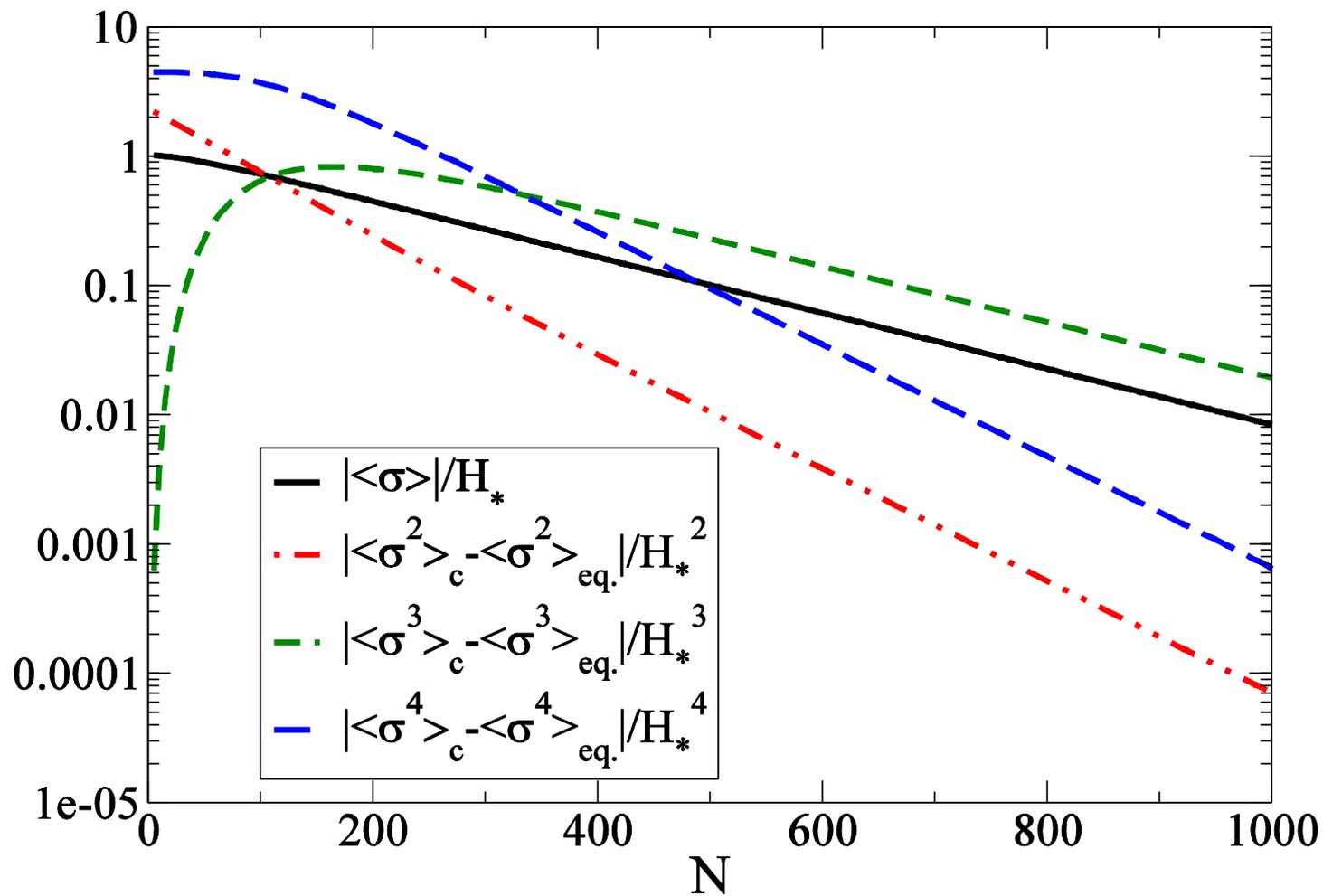
$$V = \frac{1}{4} \lambda \phi^4$$

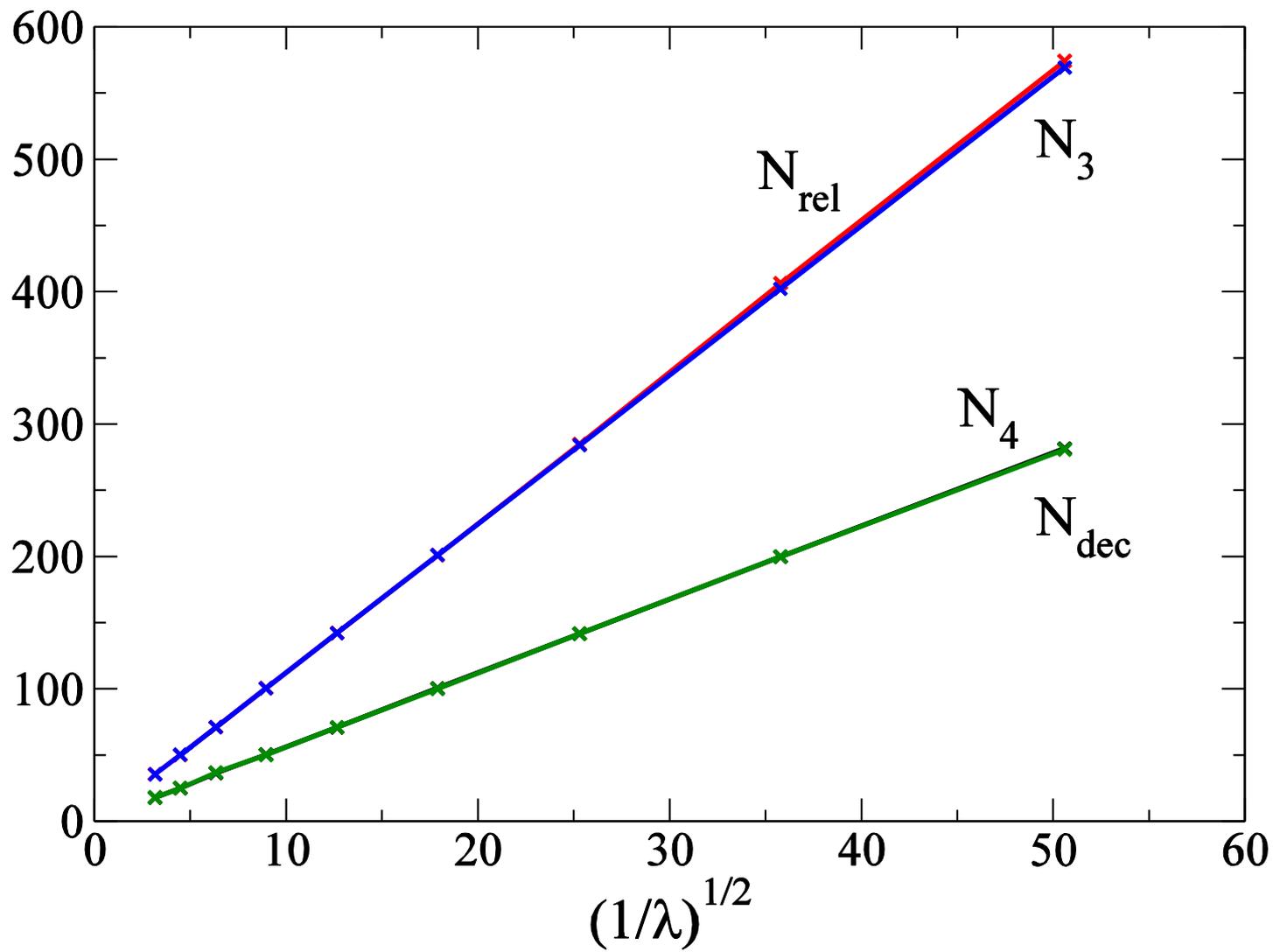
$$\lambda = 0.003125$$



quartic potential

$$V = \frac{1}{4} \lambda \phi^4$$





relaxation time

$$N_{rel} \approx \frac{11.3}{\sqrt{\lambda}}$$

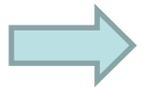
decoherence time

$$N_{dec} \approx \frac{5.65}{\sqrt{\lambda}}$$

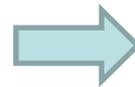
Example: the higgs

$$V \approx \frac{1}{4} \lambda h^4$$

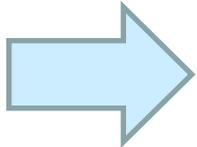
RGE $\rightarrow \lambda \approx 0.01$ at inflationary scales



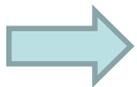
decoherence at ~ 60 efolds



mean field from equilibrium dist



$$h_* \approx 0.36 \lambda^{-1/4} H_* \approx 1.1 H_*$$



effective higgs mass

$$m_{h_*}^2 \approx V''(h_*) = 0.40 \lambda^{1/2} H_*^2 = 0.04 H_*^2$$

mean field can matter:

flat directions

$V=0$ along a ray in field space; e.g. MSSM

**fluctuations along flat directions → cosmological consequences
when decay**

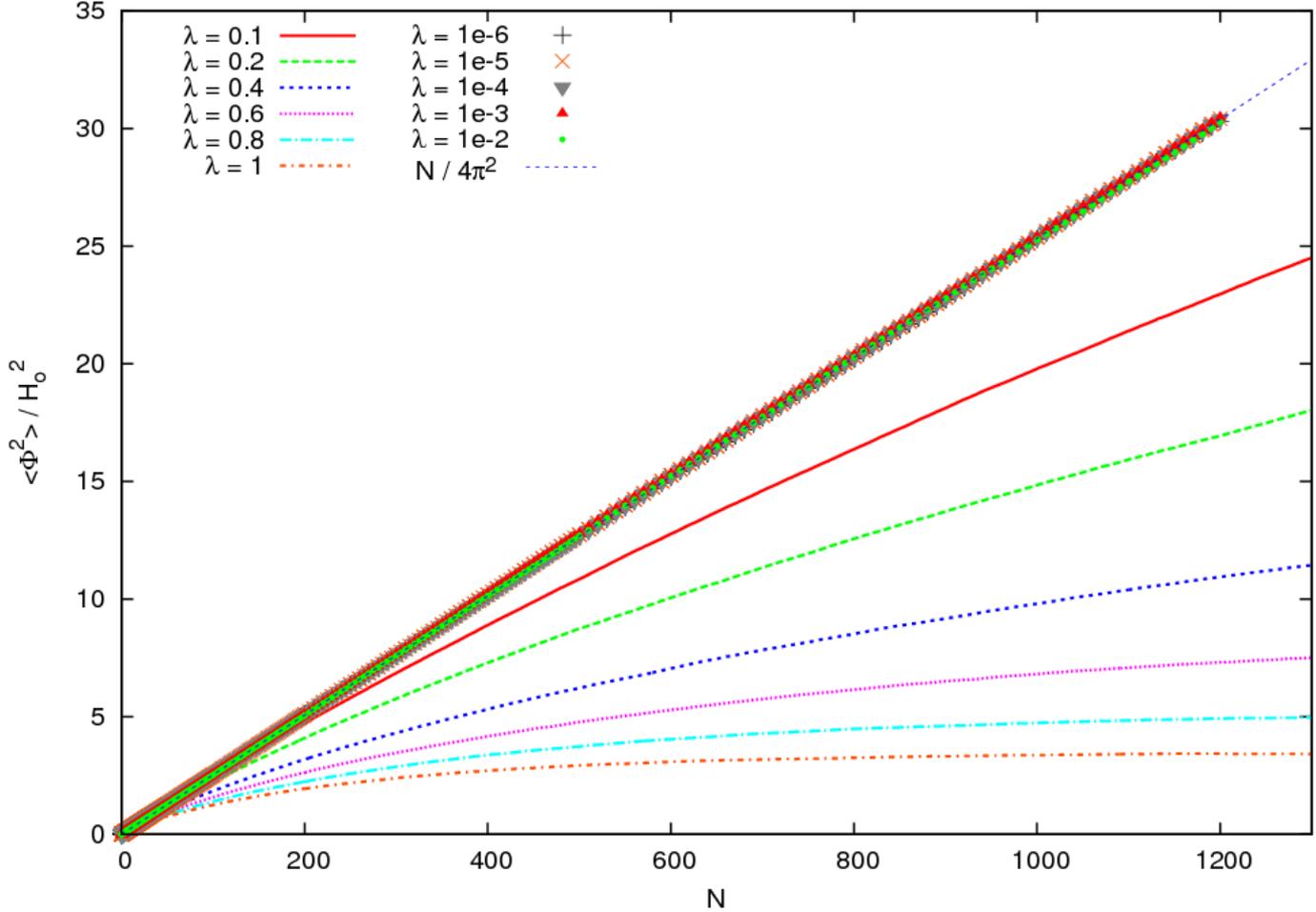
baryogenesis a la Affleck-Dine
etc

schematically

$$V(\phi, \chi) \approx \lambda^2 \phi^2 \chi^2 + g^2 \chi^4$$

Langevin eqs →

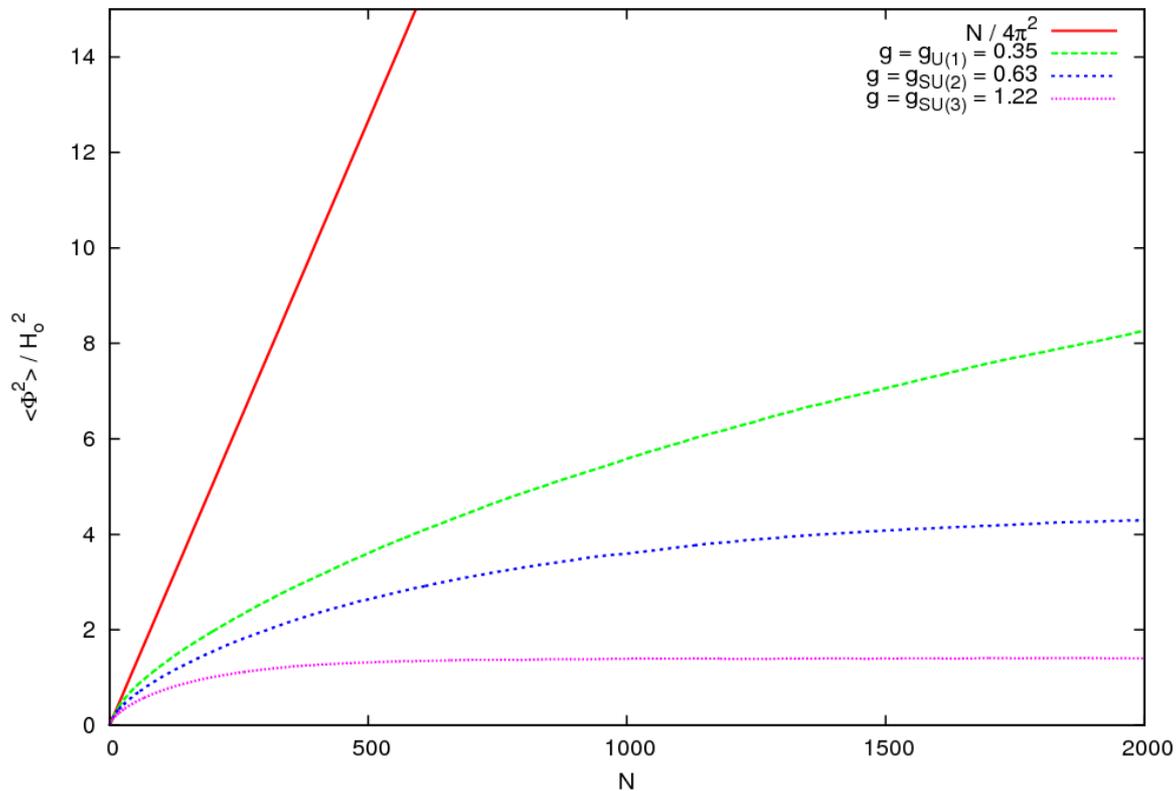
initial condition:
all fields at origin



consider MSSM D-term only with

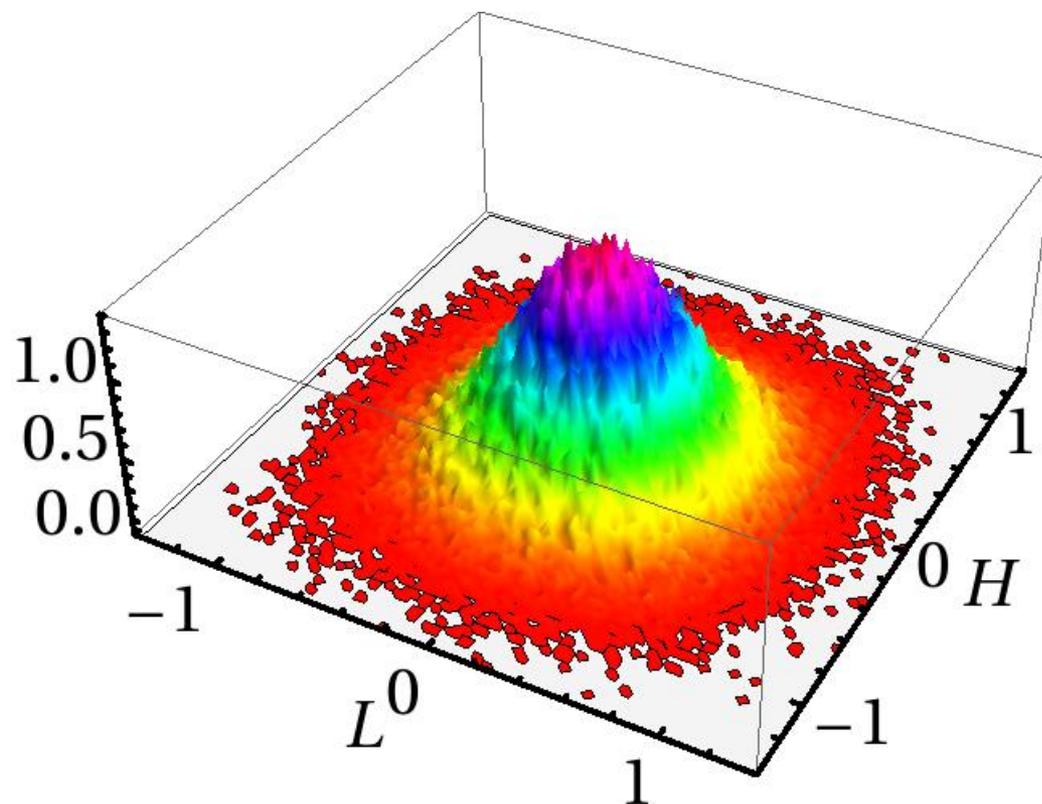
$$V(L, H) = \frac{g^2}{8} (L^2 - H^2)^2$$

all other = 0 but allow fluctuations to
displace fields from the flat direction
 $L = H$

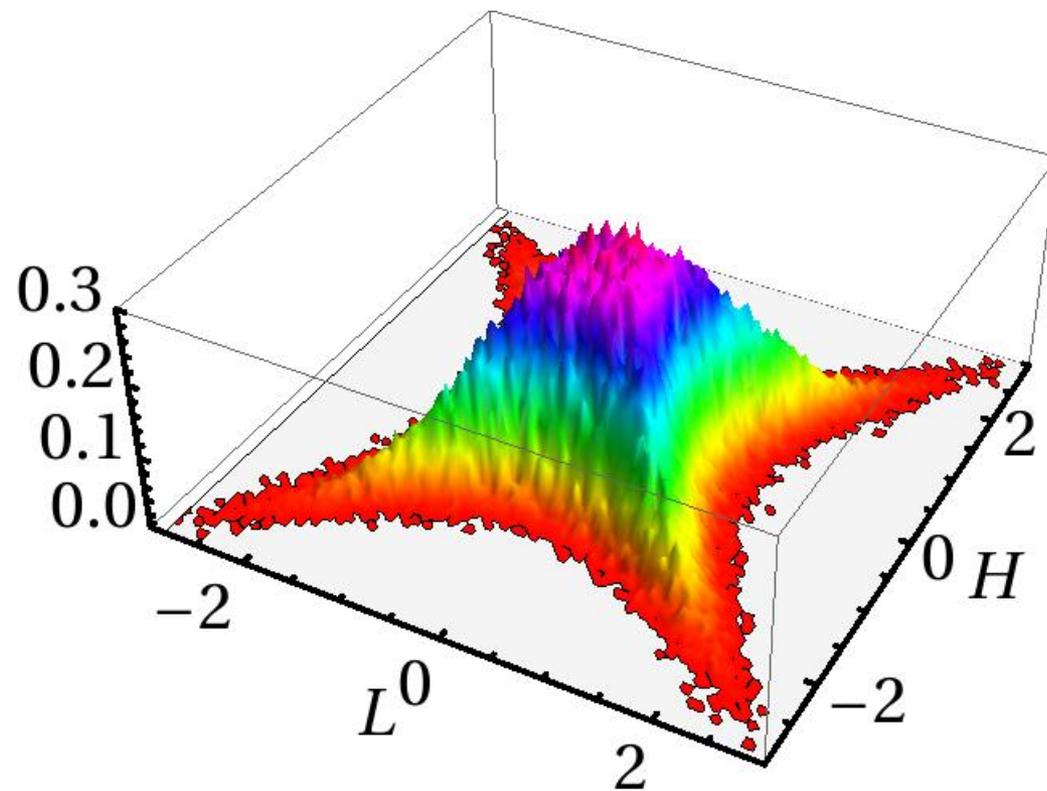


$N = 5$

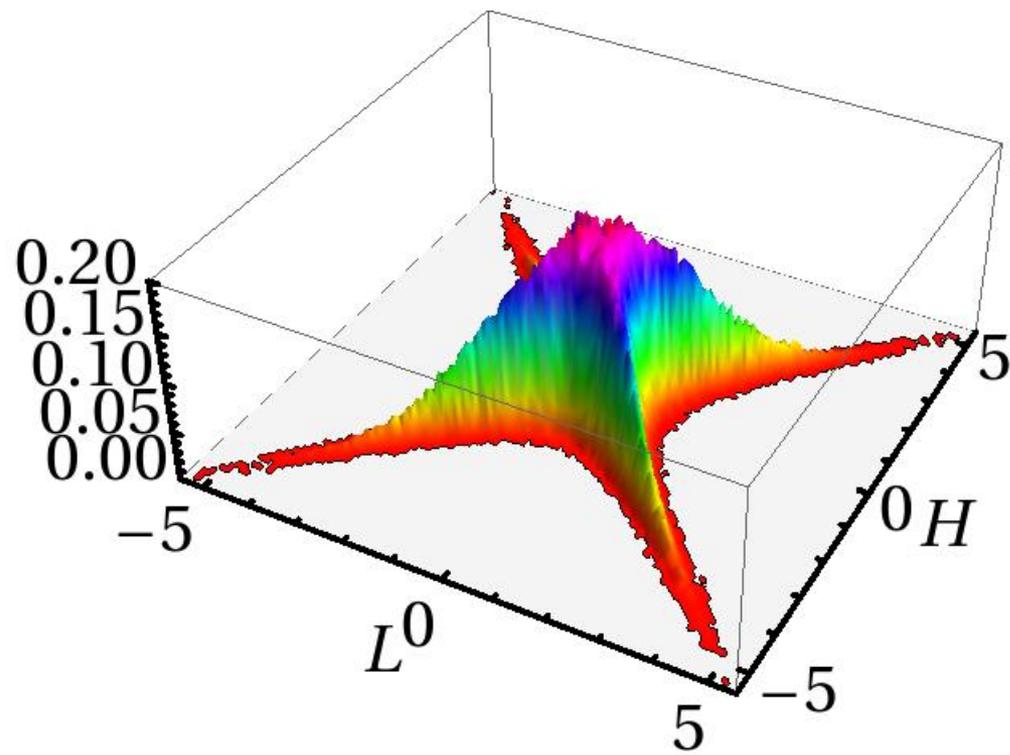
SU(2)



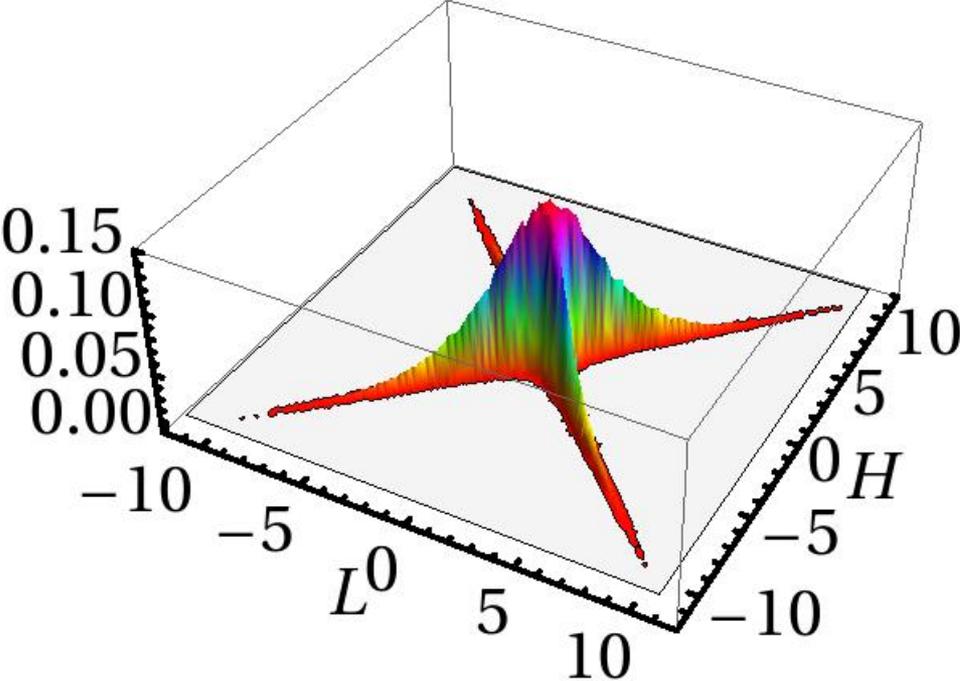
$N = 40$

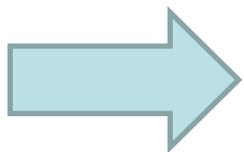


$N = 150$



$N = 1000$

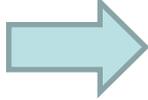




flat direction amplitude

blocked by fluctuations of non-flat directions

if fields initially at the origin

initially not at the origin?  technically complicated

KE, J. Bueno Sanchez

$$V(\phi, \chi) = \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m^2 \chi^2$$

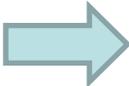
$$m_\phi^2 = \frac{1}{2} g^2 \chi^2 \quad \text{"light field"}$$

$$m_\chi^2 = \frac{1}{2} m^2 \chi^2 + \frac{1}{2} g^2 \phi^2 \quad \text{"heavy field"}$$

initially $\langle \phi \rangle = \phi_0 \Rightarrow m_\chi^2(0) = g^2 \phi_0^2 \gg H^2$

dynamics depends on coarse graining scale

there is a cross-over scale below which both fields become light
as ϕ random walks $\rightarrow 0$

 probability for trapping of fields around the origin

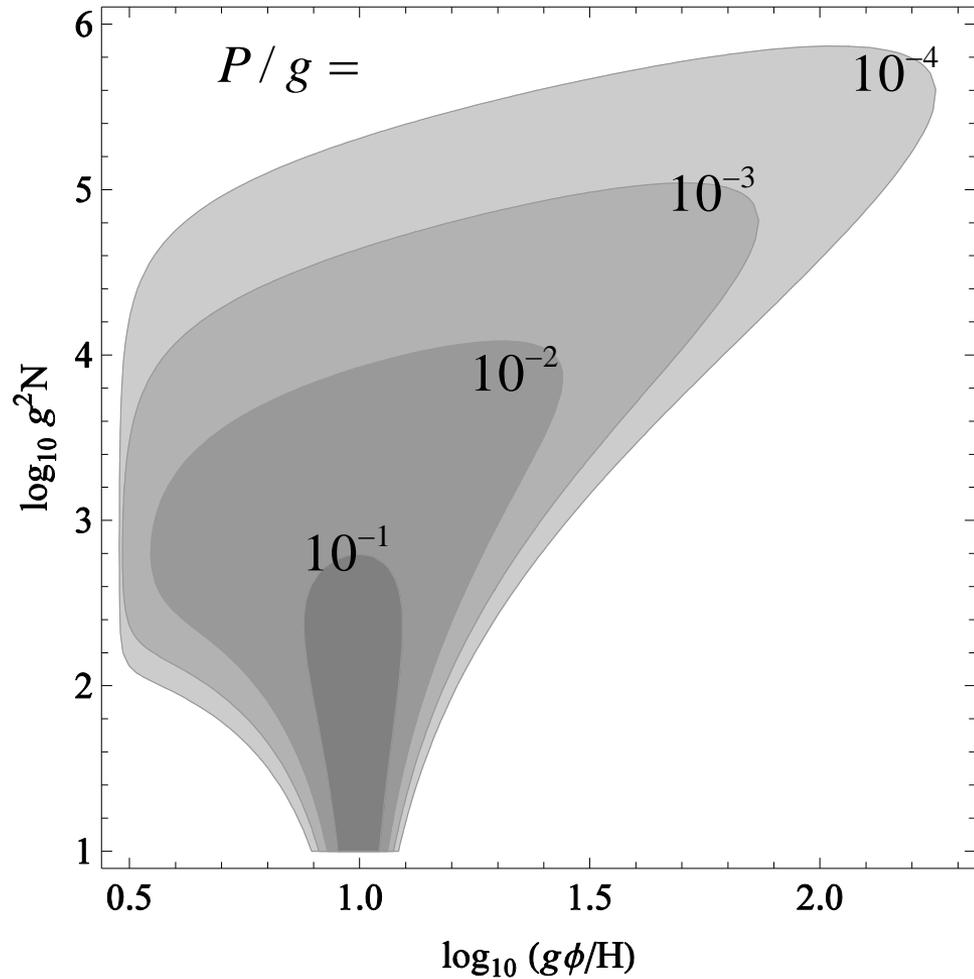
cross-over scale at $\phi = \phi_c \approx \text{few} \times H / g$

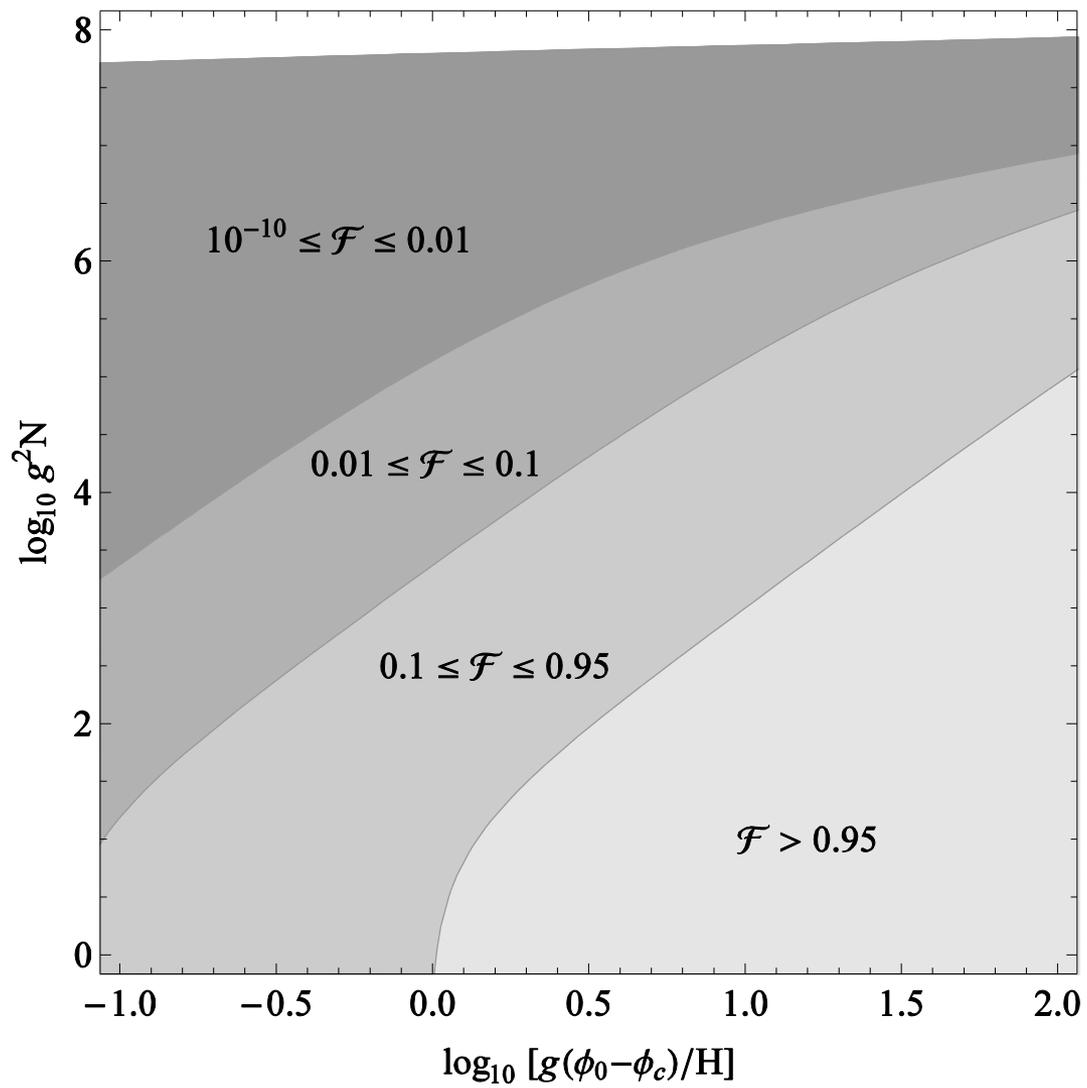
difficult to compute

”absorbing barrier”

$$\phi_c = 3H / g$$

$$\phi_0 = 10H / g$$





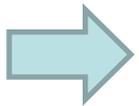
if a spectator remains around for some time after inflaton decay, it can generate the observed curvature perturbation

the curvaton

curvature perturbation generated after inflation

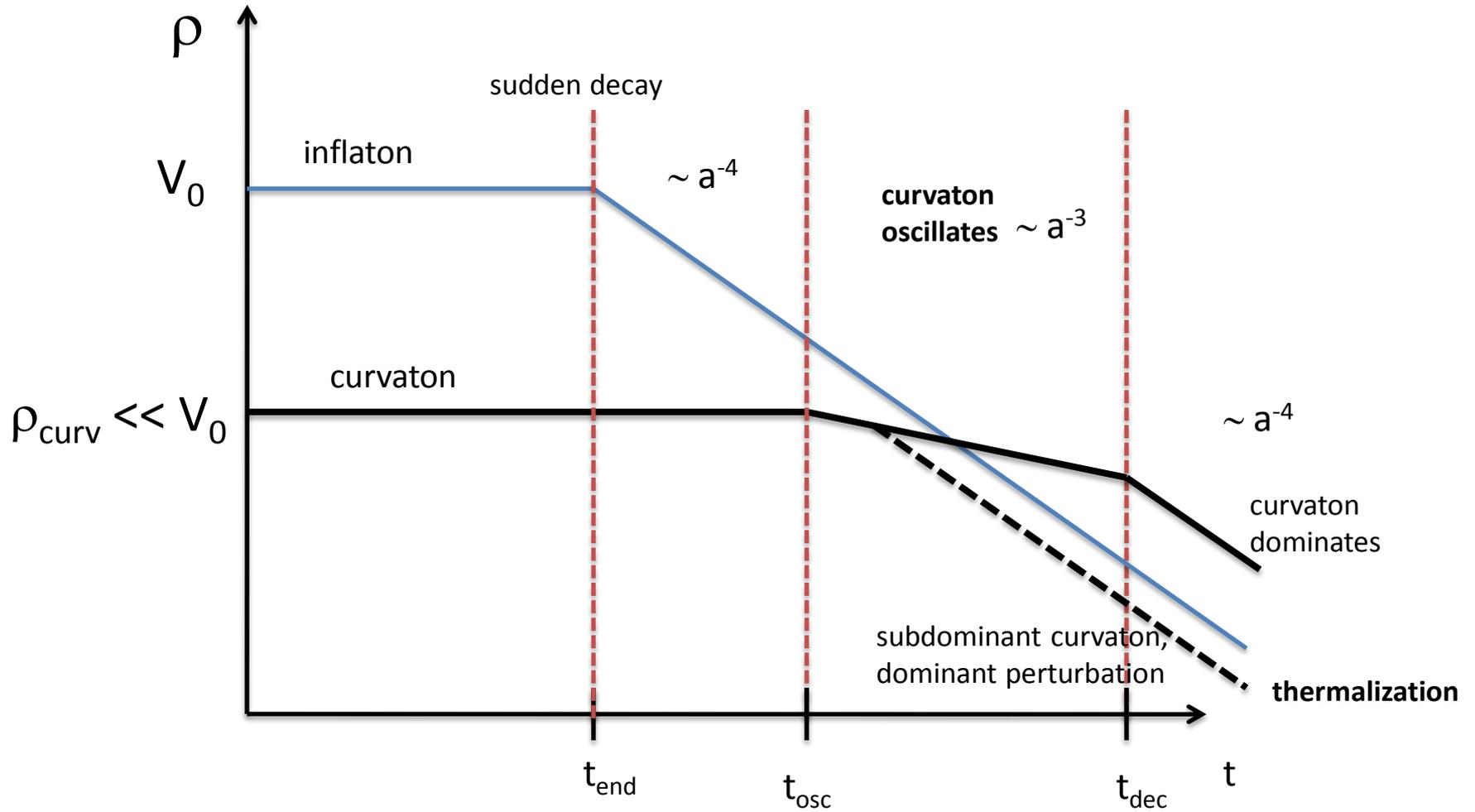
require

- curvaton decay products thermalize with radiation



initial curvaton isocurvature perturbation is transformed to an adiabatic perturbation

- $\zeta_{\text{inf}} \ll \zeta_{\sigma} \approx 10^{-5}$



curvaton perturbation remains \sim constant (depends on potential)

perturbation grows

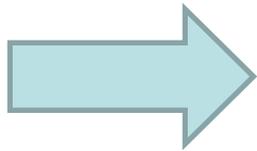
curvature
perturbation

$$\zeta = \frac{H_*}{3\pi\sigma_*} r_{\text{eff}} \approx 10^{-5}$$

$$r_{\text{eff}} \approx r_{\text{dec}} = \frac{3\rho_\sigma}{3\rho_r + 4\rho_\sigma}$$

simplest potential

$$V = \frac{1}{2} m^2 \sigma^2$$



$$f_{NL} \approx \frac{3}{8r}$$

large non-gaussianity = subdominant curvaton

Initial condition for the curvaton field?

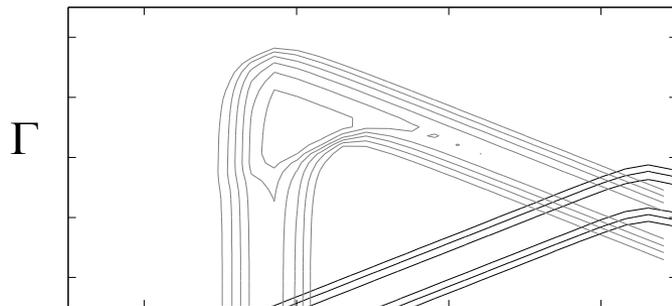
$$\zeta = \frac{H_*}{3\pi\sigma_*} r_{\text{eff}} \Rightarrow r_{\text{eff}} = \frac{\sigma_*^2}{\sigma_*^2 + 4M_P^2(\Gamma/m)^{1/2}}$$

stochastic treatment



$$P(10^{-6} \leq \zeta \leq 10^{-5}, N)$$

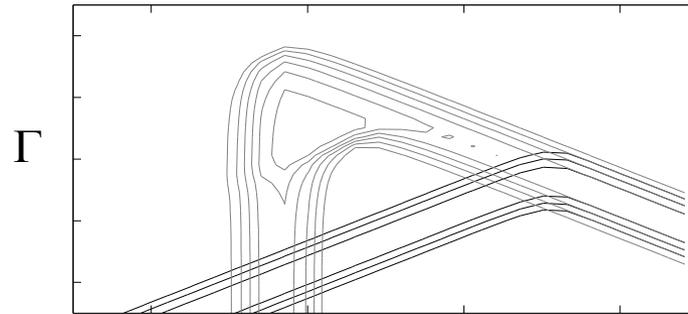
Lerner, Melville



m contours $P = 0.1, \dots, 0.9$

$N=100$

$$\sigma(0)=0$$



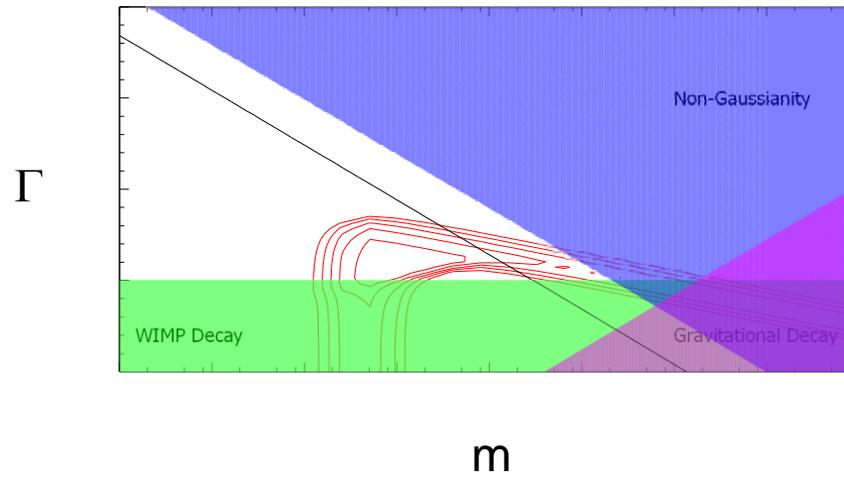
m

$N=10^4$



need a very large number of e-folds

$$H = 10^{10} \text{ GeV}$$



constraints on probable models

N.B.: interactions are important

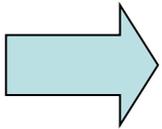
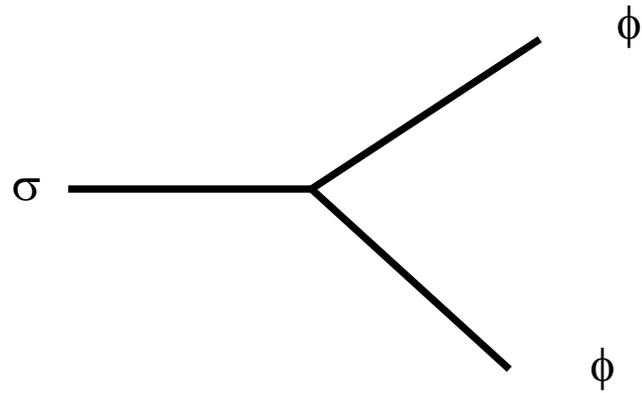
- large field values → probe interaction terms**
- interactions → non-linearities**

non-linearities: sensitivity to the initial condition

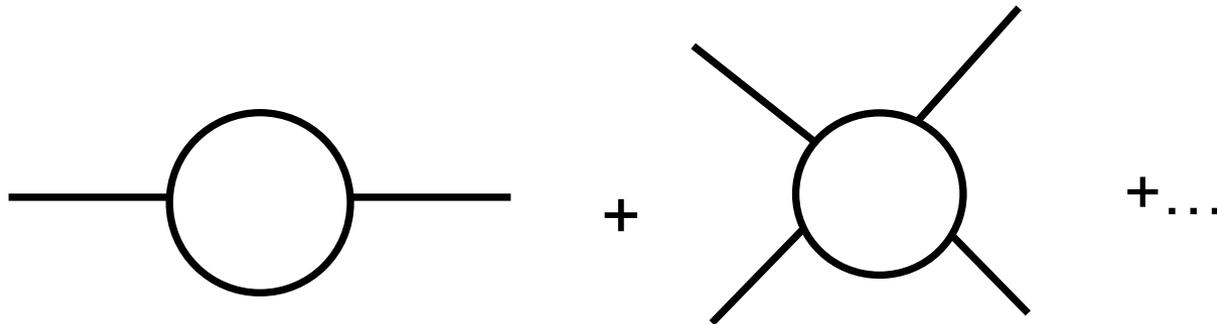
- in particular: non-gaussianity

$$f_{NL} \approx \frac{3}{8r} \rightarrow \frac{3}{8r} - g(n, \lambda)$$

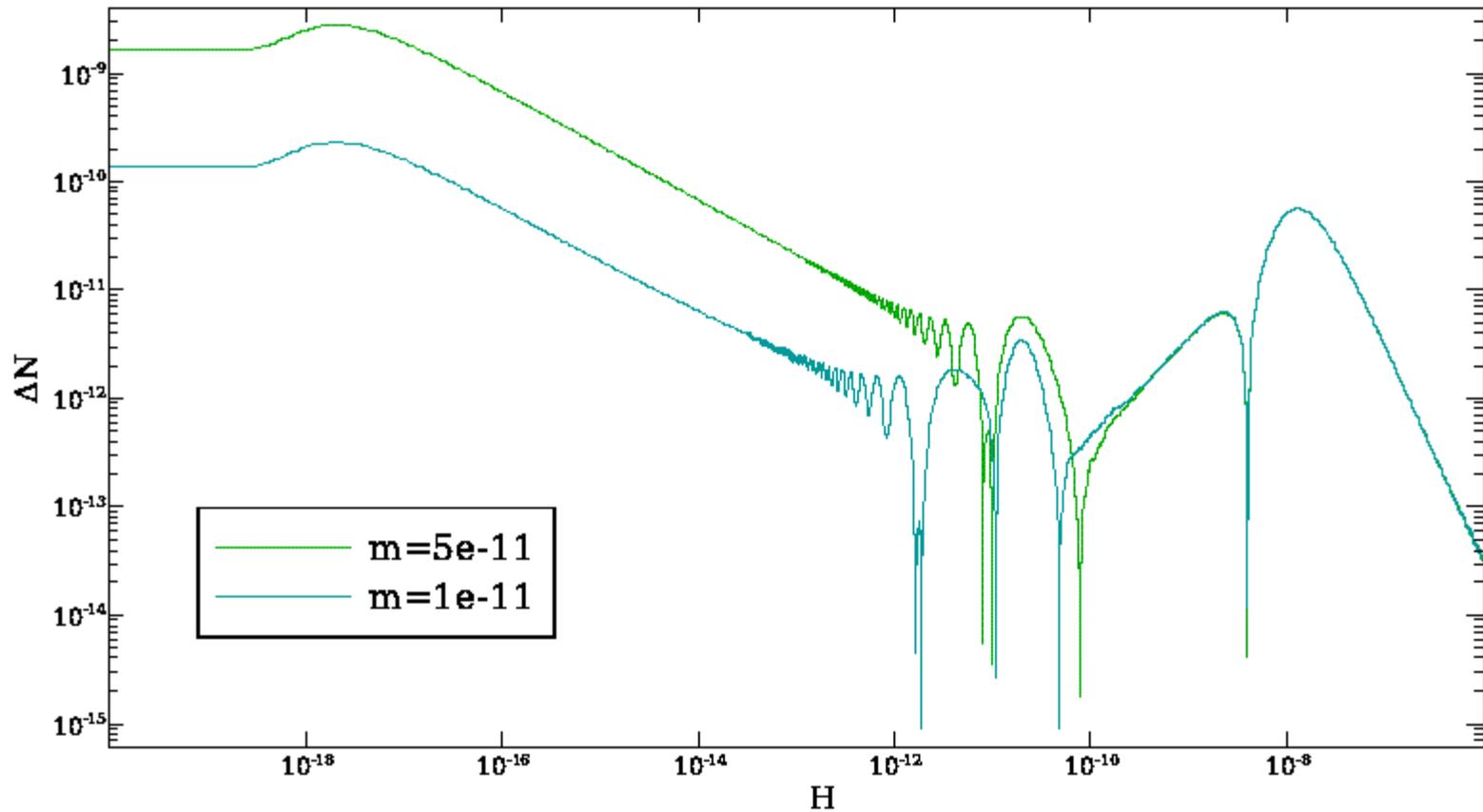
curvaton must decay:



higher order operators must exist:



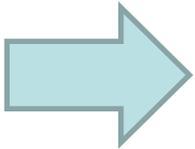
perturbation sensitive to non-linearities:



$$V = \frac{1}{2} m^2 \sigma^2 + \lambda \sigma^6$$

CURVATON DECAY

the amplitude of the curvature perturbation depends on the time of decay of the curvaton



must account for the decay mechanism

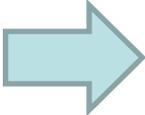
OPTIONS

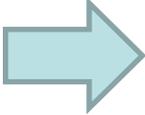
1. throw in a Γ

2. couple the curvaton to SM and compute

Higgs as the curvaton?

Choi & Huang
de Simone, Perrier, Riotto

NO: $V \approx \frac{1}{4} \lambda h^4$  higgs oscillations behave as radiation

 relative density does not grow

**but could be a field modulating either a) end of inflation
or b) inflaton decay rate**

N:B:: need precise calculations – 2-loop RGE

$$\lambda(H_*) \approx 0.01, g(H_*) \approx 0.5$$

HIGGS MODULATED (P)REHEATING

KE, Meriniemi, Nurmi

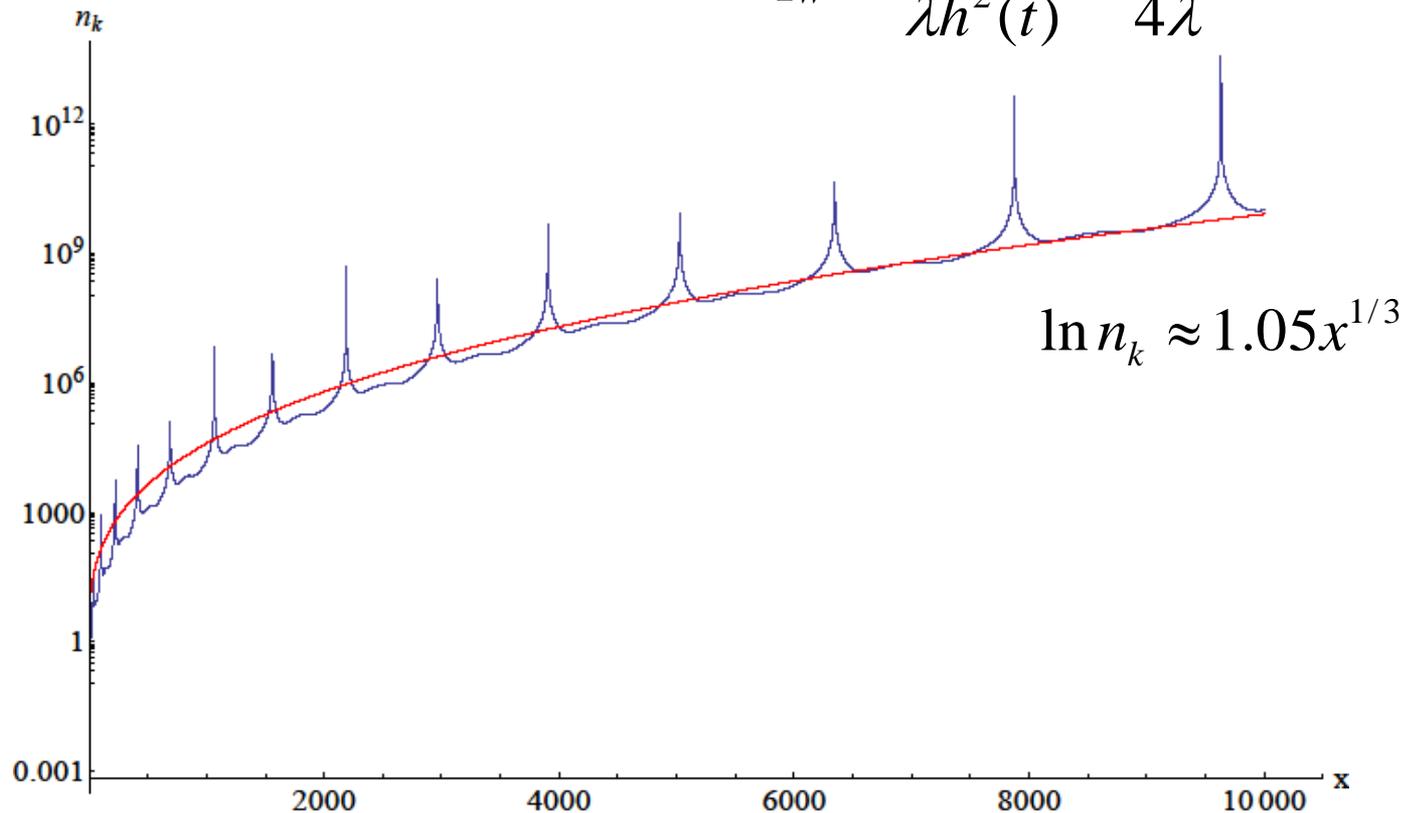
must require: higgs does not decay before the inflaton

- perturbative top- and W, Z channels blocked
- fastest perturbative channel is bb – very slow
- decay by resonant production of gauge bosons
- higgs self-decay at the edge of instability band - weak

non-perturbative decay of the higgs into gauge bosons

instability bands for $k = 0$ conformal model: $q = 1...3; 6...10; ...$

$$q_W = \frac{m_W^2(t)}{\lambda h^2(t)} = \frac{g^2}{4\lambda}$$

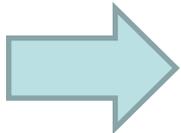


$$x = \sqrt{\lambda h_{osc}^2} (t - t_{osc})$$

Table 1: Numerical values for μ_k and approximations for n_ϕ^{res} with different H_* .

H_*/GeV	λ	(q_W, μ_k)	(q_Z, μ_k)	n_ϕ^{res}
10^4	0.09	(1.1, 0.14)	(1.5, 0.23)	170
10^7	0.04	(2.3, 0.23)	(3.2, 0.00)	300
10^{10}	0.01	(8.1, 0.24)	(12.2, 0.00)	380

estimate the time $\rho_W \approx \rho_h = \frac{\lambda}{4} \left(\frac{h}{a} \right)^4$

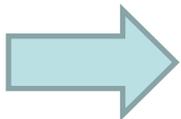


of inflaton oscillations before higgs decay

170 ... 380

$H_* = 10^4 \dots 10^{10}$

assuming $m_\phi \approx H_*$



**curvature perturbation from higgs modulated (p)reheating
implies rapid inflaton decay**

Curvaton coupled to SM higgs

Only renormalisable coupling to standard model:

$$V(\sigma, \Phi) = \frac{1}{2} m_\sigma^2 \sigma^2 + g^2 \sigma^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

curvaton
(real scalar)

SM higgs

KE, Figueroa, Lerner

Free parameters: $g, m_\sigma, \sigma^*, H^*$

- no perturbative decay (no three-point coupling)
- but expect non-perturbative decay, just like preheating
- there is a thermal background from inflaton decay
- higgs has a thermal mass $m^2(H) = g_T^2 T^2, g_T^2 \approx 0.1$

resonant production of higgs particles

oscillating curvaton with zero crossings

production takes place at resonant bands

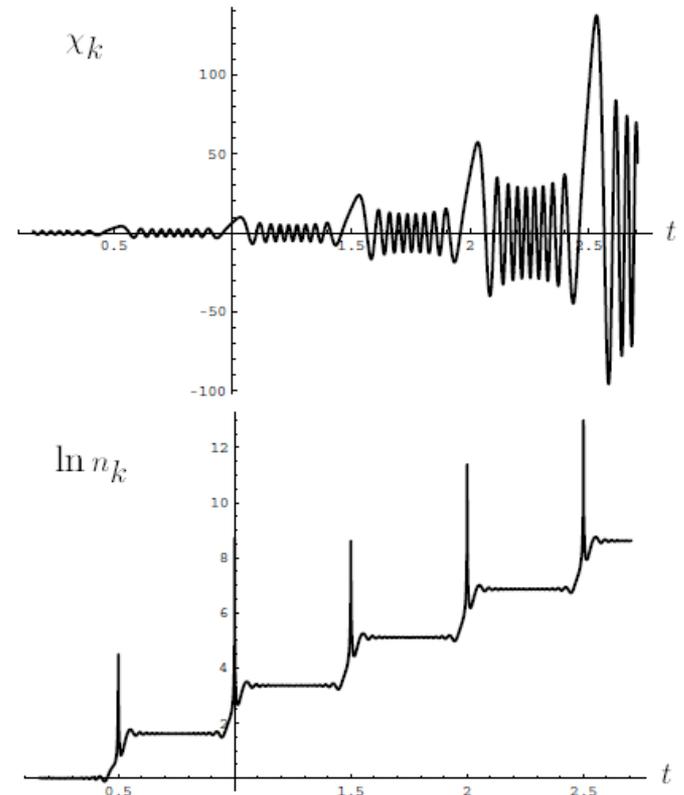
resonant
parameter

$$q \equiv \frac{g^2 \sigma^2}{4m_\sigma^2}$$

- broad resonance:
 $q \gg 1$
- narrow resonance:
 $q \ll 1$

determined by the effective frequency of the higgs

broad resonance



- curvaton is oscillating
- higgs has mass $g\sigma$
- resonant production of higgs with momentum k
 - depends on the dispersion relation
 - requires non-adiabaticity at zero crossing

(some differences between broad and narrow resonance)

corrected by thermal mass

(thermal background also induces mass for curvaton)

IR modes with $k < k_{\text{cut}}$

$$K_{\text{cut}}(j) = \frac{k_{\text{cut}}(j)}{a} \approx j^{-3/8} \sqrt{gm\sigma_*}$$

jth zero crossing

need to consider

- as the curvaton is oscillating, the resonance parameter q also evolves
 - unblocking: broad or narrow resonance?
- as the curvaton is oscillating, its relative energy density is increasing
 - unblocking: radiation or matter (=curvaton oscillation) dominated

dispersion relation

- Higgs equation of motion: j = time = # zero crossings

$$\frac{d^2 \chi_\alpha}{dx^2} + \left(\kappa^2(j) + g_{\text{T}}^2 a^2(j) \frac{T^2(j)}{k_{\text{cut}}^2(j)} + x^2 \right) \chi_\alpha = 0$$

- effective frequency:

$$\omega_k^2(j) = \kappa^2(j) + \frac{m_\sigma}{H_*} g_{\text{T}}^2 \frac{8}{14\pi} \left(\frac{T_*}{k_{\text{cut}}(j)} \right)^2 + x^2$$

$$\kappa^2(j) \approx \left(\frac{K}{K_{\text{cut}}(j)} \right)^2 \quad x \equiv K_{\text{cut}}(j)t$$

Adiabaticity violated if...

$$0 \leq k^2 \leq \underbrace{k_{cut}^2(j)}_{> 0} - \underbrace{\frac{8m_\sigma g_T^2}{14\pi H_*} T_*^2}_{< 0}$$

- RHS should be > 0
- Thermal mass of Higgs blocks resonance!
- Unblocked after **many** oscillations:

$$j \gtrsim j_{NP}|_{RD} \equiv \frac{g_T^8}{g^4 g_*^2} \left(\frac{M_P}{\sigma_*} \right)^4$$

it is not enough that the resonance becomes unblocked
 – energy must also be transferred to higgs particles

- if decay products do not thermalise:

$$\rho_H(j) \approx 0.028 f(q) q(j)^{1/4} \frac{(1 + \frac{2}{e})^{\Delta j - 1}}{\left(\frac{1}{3} + \frac{(j_{\text{NP}} + \Delta j)}{j_{\text{EQ}}}\right)^2} \left(\frac{\sigma_*}{M_P}\right)^6 \frac{1}{\left(1 + \frac{\Delta j - 1}{(e/2 + 1)}\right)^{\frac{3}{2}}} \times (g m_\sigma \sigma_*)^2$$

where

$$f(q) \equiv 1 + \frac{2 + e}{\exp(g_T q^{1/4} - 1)}$$

- if decay products thermalise ($m_\sigma \ll T(j_{\text{NP}})$):

$$\rho_H(j_{\text{NP}} + \Delta j) \approx \rho_H(j_{\text{NP}}) \left[1 + \frac{1}{g_*} 0.01357 \Delta j \right]$$

but: for a range in parameters, thermal blocking persists until electroweak symmetry breaking

don't know what happens after that – assume that the curvaton decays

note: EWSB is not a phase transition but a smooth cross-over

Possible timescales

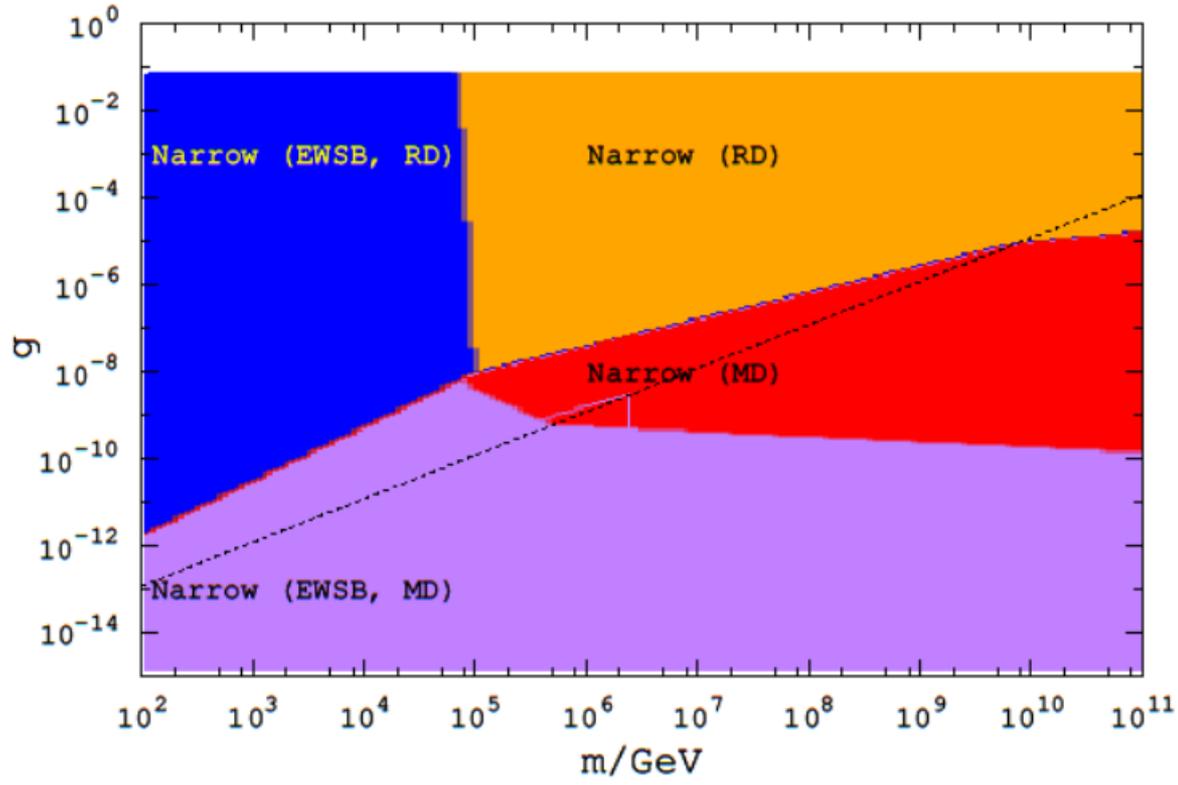
- e.g. narrow resonance in matter-domination



- depends on resonance parameter $q \equiv \left(\frac{g\sigma(t)}{2m} \right)^2$
- q decreases with time
- narrow resonance: $T_{\text{NP}} = \frac{m_\sigma(1 + \mathcal{O}(q))}{g_T}$
- narrow resonance energy transfer:

$$\Delta j \simeq -\frac{\log(g^2 q^{1/2}(j_{\text{NP}}))}{\pi q(j_{\text{NP}})}$$

higgs-curvaton coupling



curvaton mass

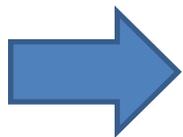
MORE DETAILS ...

KE, Lerner, Rusak

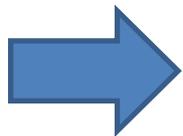
What actually happens at the onset of inflaton oscillation?

Inflaton decay not instantaneous – radiation background builds up

$$\rho_{\text{inf}} = 3M_P^2 H_*^2 \left(\frac{a}{a_0} \right)^{-3} e^{-\Gamma t}$$



$$\rho_{SM} \approx \frac{6}{5} M_P^2 H_* \Gamma a^{-4} \left[a^{5/4} e^{-\Gamma t} - 1 \right]$$



$$T_{\text{max}, SM} \approx 0.330 \left(M_P^2 H_* \Gamma \right)^{1/4}$$

