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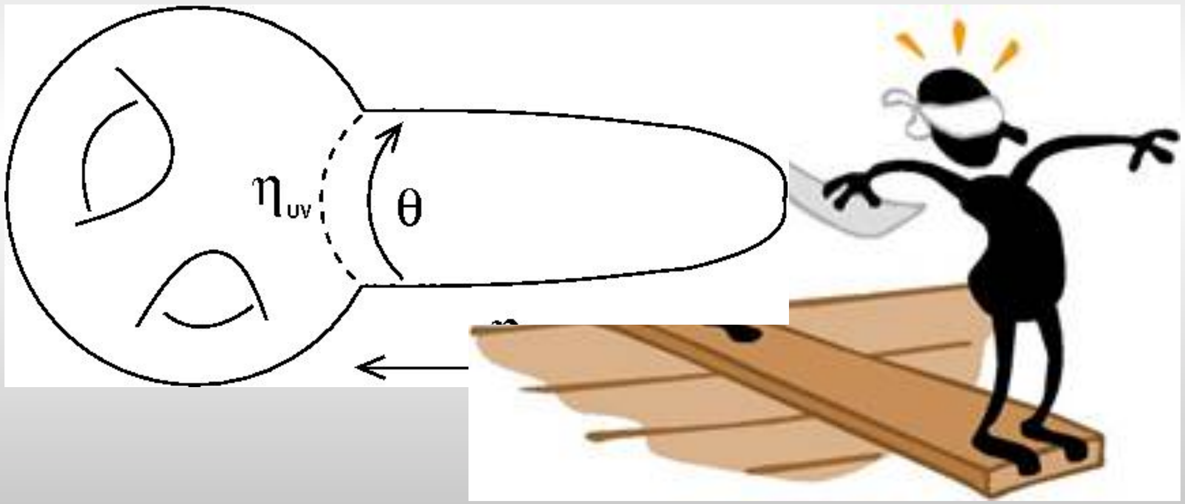
Brane Inflation – Walking the Planck



With Damien Easson, Dariush Kaviani, Gianmassimo Tasinato, Ivonne Zavala, David Mota

Outline

- Branes and extra dimensions – scalars for inflation.
- The brane inflation picture
- Looking into the infrared
- Multifield aspects.
- Walking the Planck! – Discussion.

WHY ?

Gravity and Extra Dimensions

- Einstein's GR is a successful theory, relating a fundamental observable (distance) to matter content of the Universe.
- But we could have modifications – extra dimensions – extra terms changing gravity in the UV and/or IR
- Can there be signals of those extra dimensions?

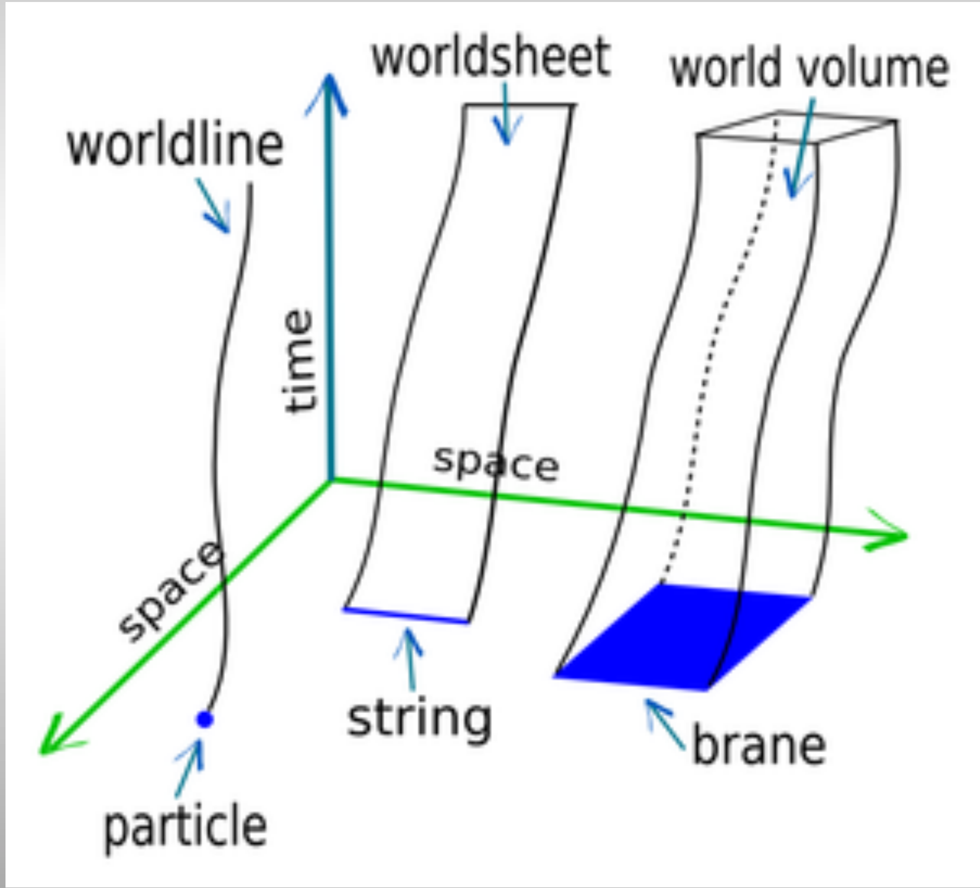
Braneworlds vs. Kaluza-Klein

- Extra dimensions large
- Localized in extra-D
- Geometry (typically) curves into bulk – a *warped compactification*
- 4D fields from fluctuations in brane

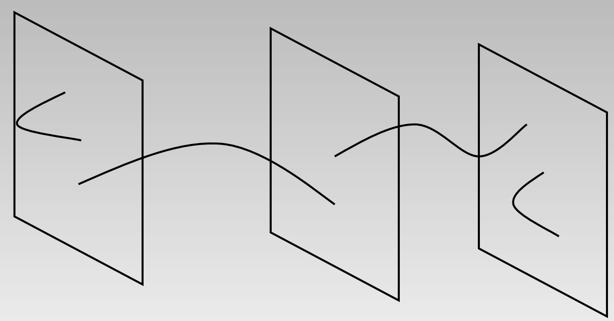
- Extra dimensions small
- Our world is “smeared” over internal dimensions
- Geometry independent of extra dimension
- 4D fields from symmetries of internal space

The Brane

Just like a particle is a pointlike object, a brane is a generic extended object in space – e.g. a string is a 1-brane.
The more spatial dimensions we have, the greater the number of possible branes.



String theory has many extra dimensions (6/7) and the possibility of brane-like solutions via D-branes.

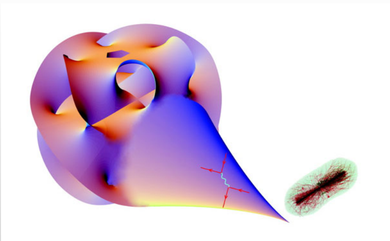


D-branes in string theory are surfaces on which open strings can end. The charges carried by the ends of the strings give gauge fields confined to the D-brane, which has a DBI effective action.

$$S \approx -g_s^{-1} \int d^{d+1}\sigma \sqrt{\gamma_{AB} + F_{AB}}$$

Branes and gravity

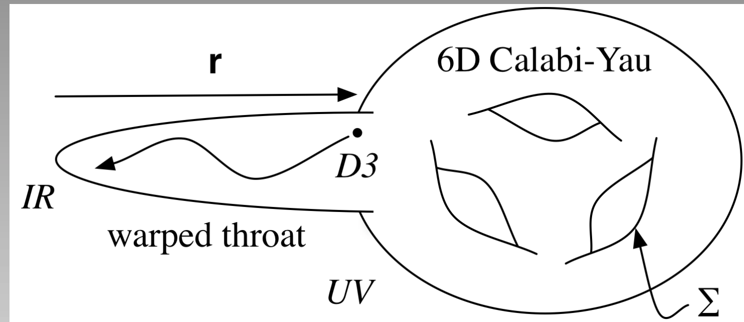
These branes have energy and tension (cosmological constant behaviour within the brane, no perpendicular components) :- *gravity is important*.

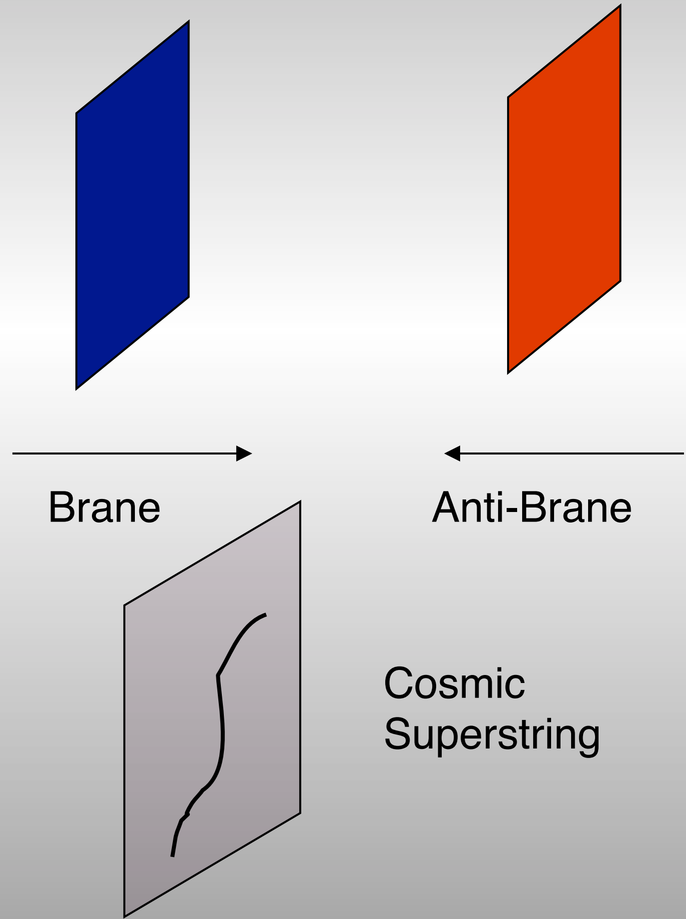


Wrapped branes can cause local throats to appear in the Calabi Yau - these can be modeled by exact geometries.

A 3-brane can extend along our non-compact directions and appear as a point moving in this background. The energy momentum of this brane can then affect our universe.

Brane Inflation





In brane inflation, we imagine an (anti-)brane moving on the internal extra dimensions and extended along the noncompact directions.

The higher dimensional information is encoded in a scalar describing the brane position: The Inflaton.

Inflation ends when the anti-brane meets the brane and annihilates.

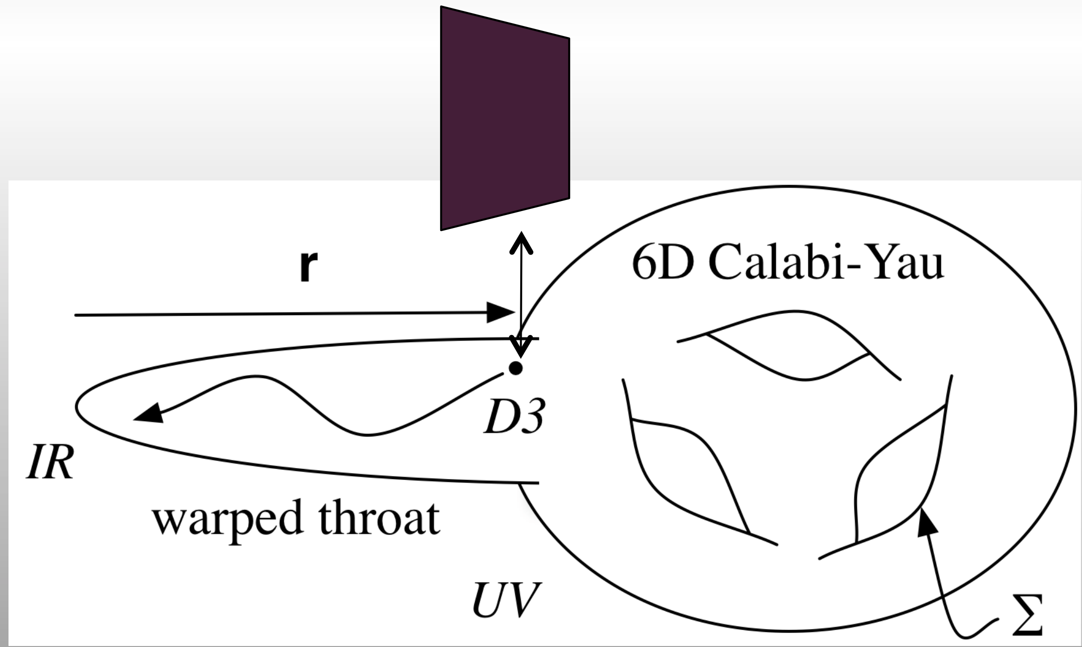
Dvali & Tye, 1999

The Big Picture

$$ds^2 = h^{-1/2} e^{-6u} d\mathbf{x}^2 + h^{1/2} e^{2u} d\mathbf{y}^2$$

FRW cosmology

Internal manifold



Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi, Baumann, Dymarsky, Klebanov….

Brane Inflation

$$ds^2 = h^{-1/2} e^{-6u} d\mathbf{x}^2 + h^{1/2} e^{2u} d\mathbf{y}^2$$

$$S_{grav} = \kappa_{10}^{-2} \left[\int d^6 y \, h \sqrt{\tilde{g}_6} \right] \int d^4 x \sqrt{g_4} \left(R_4 - 24 (\partial u)^2 \right) + \ldots$$

o Integrate over internal manifold (~KK)

o Write brane action in terms of position scalars

o Compute corrections dependent on brane position to desired order

$$-T_3 \int d^4 x \sqrt{g} \left[h^{-1} \sqrt{1 + h g_{mn} g^{\mu\nu} \partial_\mu y^m \partial_\nu y^n} - h^{-1} + V\left(y^m\right) \right]$$

Brane Energy-Momentum

o Brane action has 3 main pieces, its geometry, its charge, and corrections due to its presence.

$$
\begin{aligned}
S_{DBI} + S_{WZ} &= -T_3 \int d^4\xi \sqrt{-\det(\gamma_{AB} + \mathcal{F}_{AB})} + T_3 \int_{\mathcal{W}} C_4\,, \\
&= -T_3 \int d^4x \sqrt{-g} \left[h^{-1} \sqrt{\det(\delta^\mu_\nu - h y^{m,\mu} y^n_{,\nu} \tilde{g}_{mn})} - \alpha \right]
\end{aligned}
$$

$$
T_{\mu\nu} = T_3 \left\{ h^{-1} \sqrt{\det(\delta^\mu_\nu - h y^{m,\mu} y^n_{,\nu} \tilde{g}_{mn})} \left(g_{\mu\nu} - h y^m_{,\mu} y^n_{,\nu} \tilde{g}_{mn} \right) - \alpha g_{\mu\nu} \right\}
$$

$$
E = \frac{1}{h}\left[\gamma - q\right] + V \qquad P = \frac{1}{h}\left[q - \gamma^{-1}\right] - V
$$

Silverstein, Tong, Alishahiha

DBI v Slow Roll

Expanding the D-brane effective action in terms of the inflaton fields:

$$-T_3 \int d^4x \sqrt{g} \left[h^{-1} \sqrt{1 + h g_{mn} g^{\mu\nu} \partial_\mu y^m \partial_\nu y^n} - h^{-1} + V\left(y^m\right) \right]$$

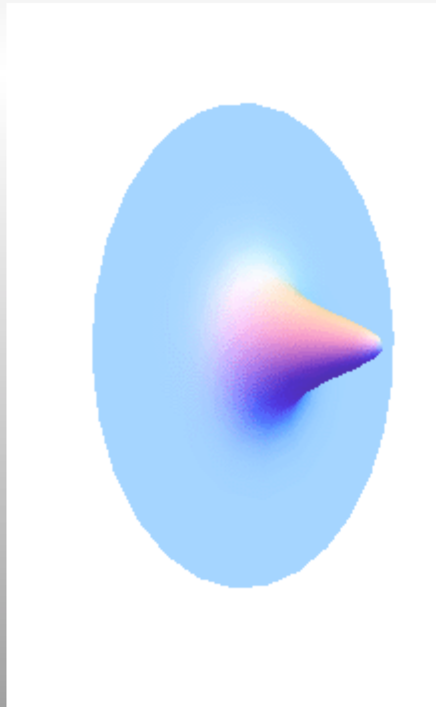
gives a non-canonical looking kinetic term. But, we can expand this at low velocity:

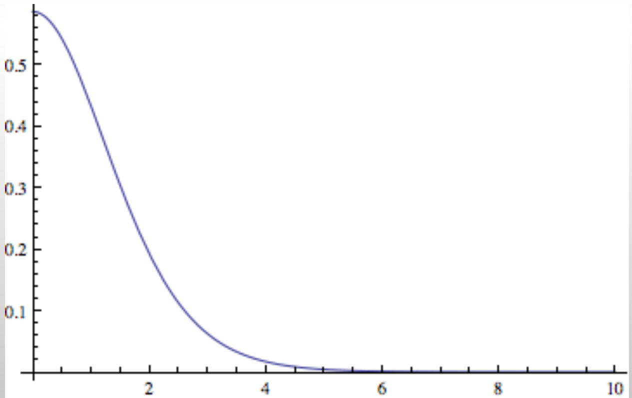
$$\approx -T_3 \int d^4x \sqrt{g} \left[\frac{1}{2} \left(\partial y^m\right)^2 + V\left(y^m\right) \right]$$

Slow roll

The IR Geometry

The Klebanov Strassler solution (based on the deformed conifold) is a candidate for a smooth throat. The warp factor has an (almost) analytic form:





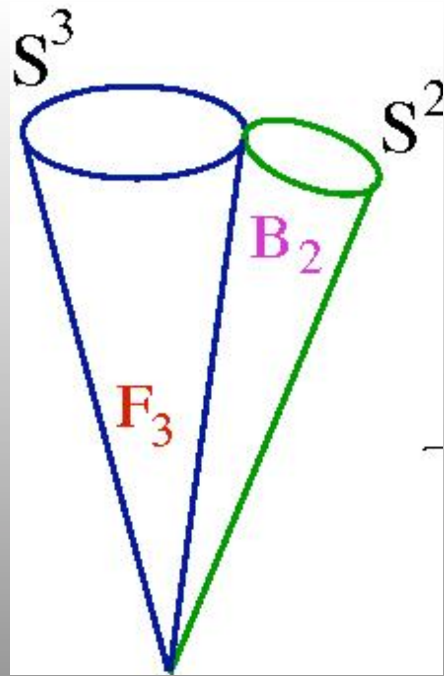
$$h = 2\left(g_s M \alpha'\right)^2 \varepsilon^{-8/3} \underbrace{\int_\eta^\infty \frac{x \coth x - 1}{\sinh^2 x}\left(\sinh x \cosh x - x\right)^{1/3}}_{I(\eta)}$$

The Conifold throat

Conifolds correspond to special points in the moduli space of Calabi-Yau manifolds, and are regular manifolds apart from isolated “conical” singularities. The standard example is a simple cone based on a complex hypersurface in $\mathbf{C}^4$.

$$\sum_{A=1}^{4} (z_A)^2 = 0 \qquad (z = x + i\, y)$$

$$\mathbf{x}^2 = \mathbf{y}^2 = r^2/2$$
$$\mathbf{x} \,.\, \mathbf{y} = 0$$



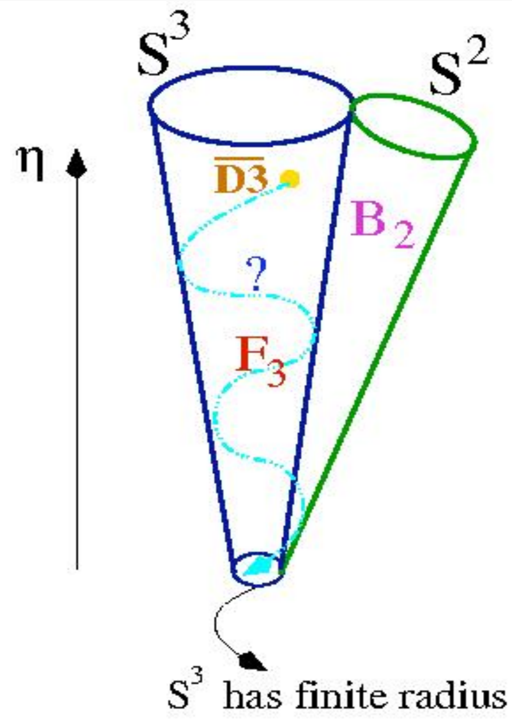
Deformed Conifold

The Klebanov-Strassler solution uses the deformed conifold, supported by fluxes.

$$\sum (z_A)^2 = \varepsilon^2$$

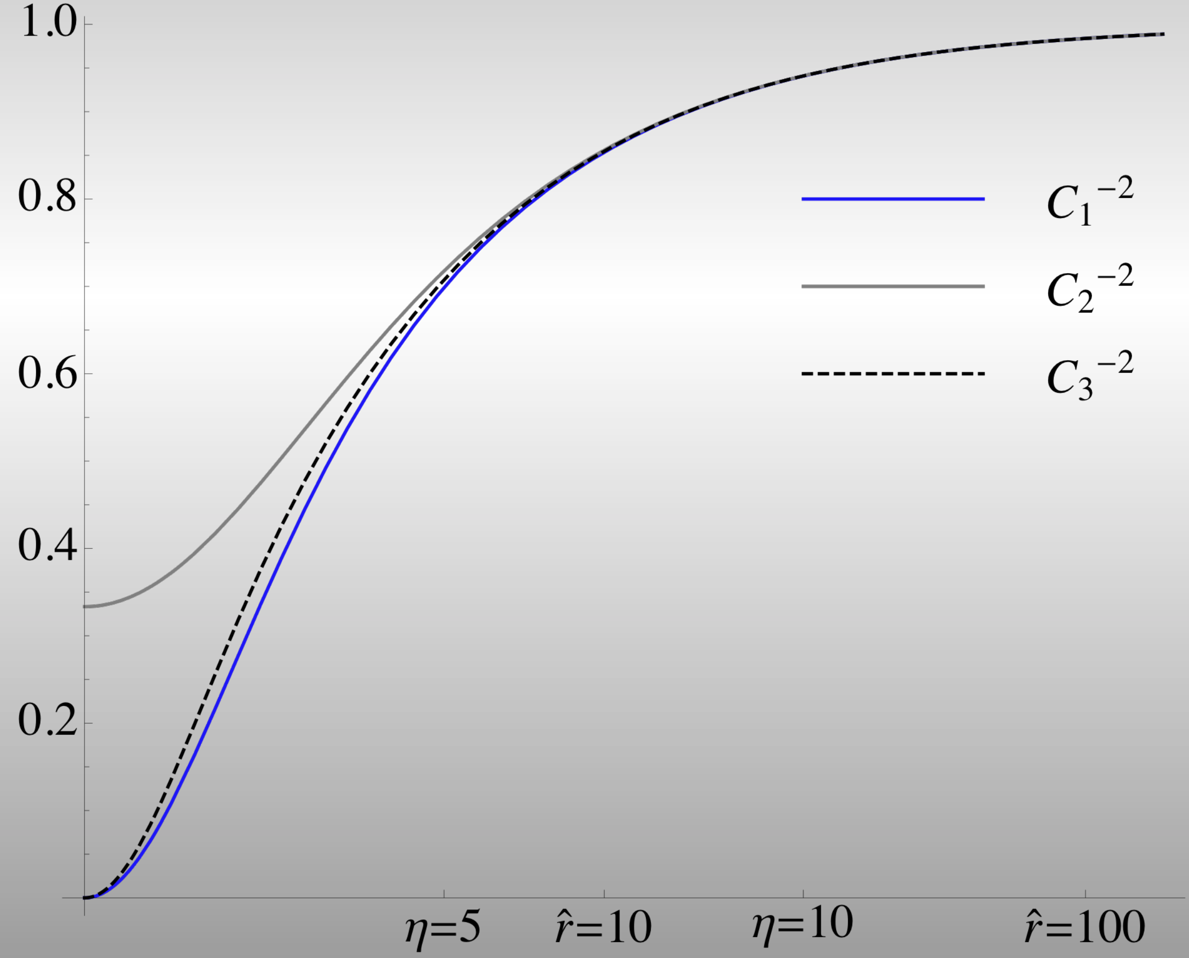
$$ds^2 = \frac{K}{2}\varepsilon^{4/3}\left[\left(\frac{d\eta^2 + g_5^2}{3K^3}\right) + \cosh^2\frac{\eta}{2}\left(g_3^2 + g_4^2\right) + \sinh^2\frac{\eta}{2}\left(g_1^2 + g_2^2\right)\right]$$

$$K(\eta) = \frac{\left(\sinh\eta\cosh\eta - \eta\right)^{1/3}}{\sinh\eta}$$



$$d\tilde{s}_6^2 = dr^2 + r^2\left[\frac{C_3^2(r)}{9}(g^5)^2 + \frac{C_1^2(r)}{6}\{(g^3)^2 + (g^4)^2\} + \frac{C_2^2(r)}{6}\{(g^1)^2 + (g^2)^2\}\right]$$

The throat logarithmically approaches a cone, but smooths off in the IR



The Potential

The potential typically includes an $m^2\phi^2$ piece, but what is ϕ? Conventionally, inflaton is canonically normalized – here we take the radial distance up the throat.

$$\phi = \sqrt{T_3}\,\frac{\varepsilon^{2/3}}{\sqrt{6}}\int_0^{\eta}\frac{dx}{K(x)}$$

There are also corrections to the background from moduli stabilization. To leading order these appear via perturbations in the warp factor and 5-form flux:

$$\Delta V = T_3\left(h^{-1} - \alpha\right)$$

This combination appears in the Einstein equations, and satisfies (to leading order) the Laplace equation.

$$\nabla^2 \Phi_- = 0$$

On KS background, lowest angular correction is:

$$\Phi_- = \left(\sinh\eta\cosh\eta - \eta\right)^{1/3} \cos\vartheta$$

So we take the potential:

$$V = T_3\left[m^2 r^2(\eta) + c\left(\sinh\eta\cosh\eta - \eta\right)^{1/3} \cos\vartheta\right]$$

V_{LIN}

$$V = T_3\left[m^2 r^2(\eta) + c\left(\sinh\eta\cosh\eta - \eta\right)^{1/3}\cos\vartheta\right]$$

$\varepsilon^{4/3} e^{2\eta/3}$

NOT a simple power of r.

General linear potential will be a sum of angular harmonic functions – we took the simplest. Also one for which we found an analytic radial eigenfunction. Away from the tip, radial eigenfunctions are known powers of r.

Parameter constraints

There are constraints on the choice of parameters. The Planck mass is given by integration over the internal space, and leads to a bound

$$\bigstar \qquad M_p^2 \geq \frac{\varepsilon^{4/3} M^2}{6(2\pi)^4 \alpha'^2} \int_0^{\eta_{uv}} I(\eta) \sinh^2 \eta$$

The integral grows very rapidly, so the UV cutoff is important in determining the flux and deformation parameter.

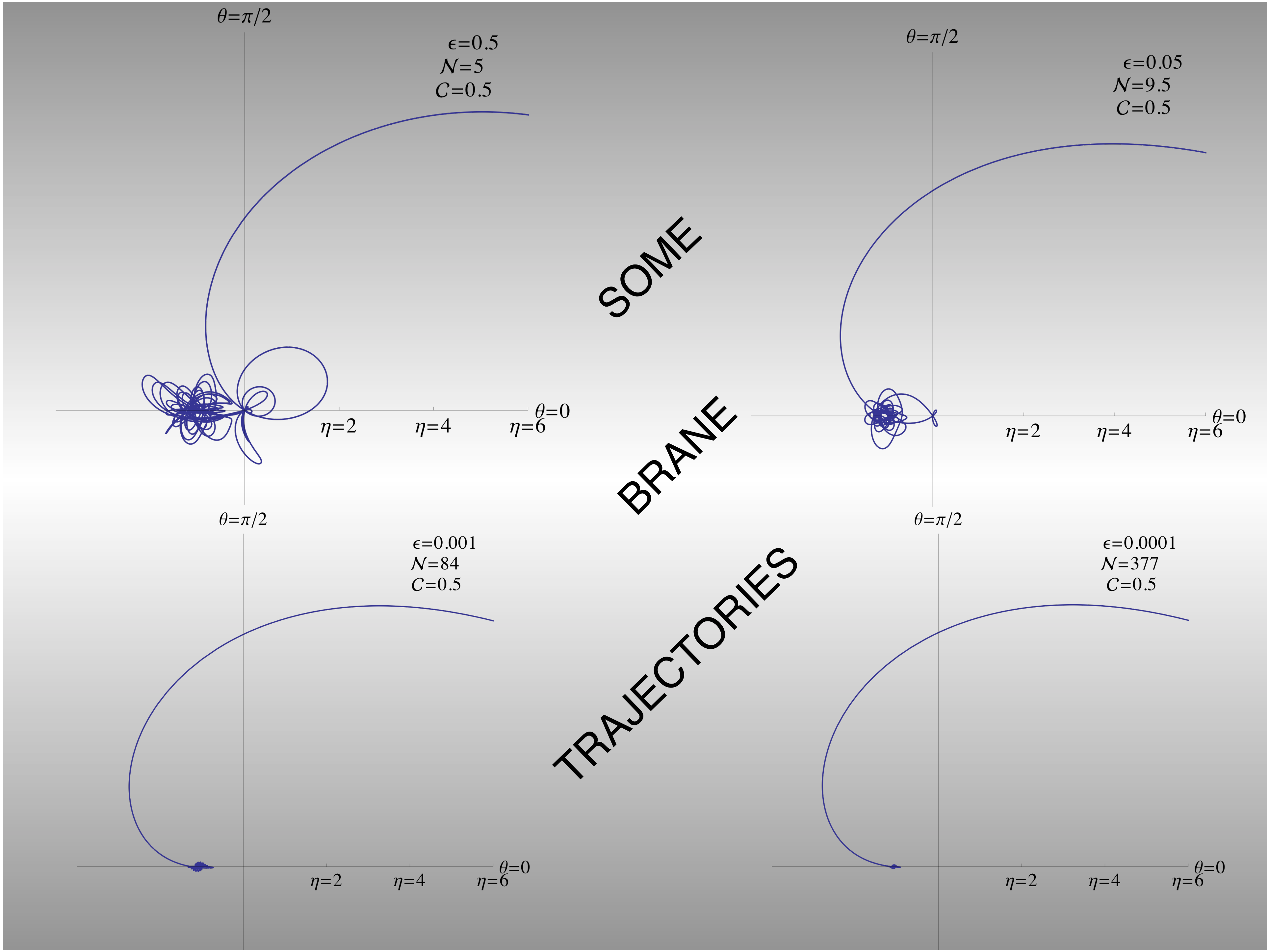
Cosmological Eqns

$$\hat{H}^2 = \frac{T_3}{3M_p^2}\left[(g_sM)^2\hat{V} + \frac{\epsilon^{4/3}}{\hat{h}}(\gamma-1)\right] \longrightarrow \frac{2\pi g_s}{J_{UV}}\left[\epsilon^{-4/3}\hat{V} + \frac{\gamma-1}{(g_sM)^2\hat{h}}\right]$$

$$\dot{\hat{H}} = -\frac{\epsilon^{4/3}T_3}{2M_p^2\hat{h}}(\gamma-\gamma^{-1}) \longrightarrow -\frac{3\pi g_s(\gamma-\gamma^{-1})}{(g_sM)^2J_{UV}\hat{h}}$$

$$\begin{aligned}\ddot{\eta} &= -\frac{3\hat{H}}{\gamma^2}\dot{\eta} + \frac{\hat{h}'}{\hat{h}}(\gamma^{-1}-1)\dot{\eta}^2 + 3K^2\frac{\hat{h}'}{\hat{h}^2}(\gamma^{-1}-1)^2 \\ &\quad + \frac{K'}{K}\dot{\eta}^2 + 3K^2B'\dot{\vartheta}^2 + \frac{(g_sM)^2}{\epsilon^{4/3}\gamma}\left[\hat{h}\dot{\vartheta}\dot{\eta}\hat{V}_\vartheta - (6K^2-\hat{h}\dot{\eta}^2)\hat{V}_\eta\right]\end{aligned}$$

$$\ddot{\vartheta} = -\frac{3\hat{H}}{\gamma^2}\dot{\vartheta} + \left[\frac{\hat{h}'}{\hat{h}}(\gamma^{-1}-1) - \frac{B'}{B}\right]\dot{\eta}\dot{\vartheta} + \frac{(g_sM)^2}{\epsilon^{4/3}\gamma}\left[\hat{h}\dot{\eta}\dot{\vartheta}\hat{V}_\eta - (1-\hat{h}B\dot{\vartheta}^2)\frac{\hat{V}_\vartheta}{B}\right]$$


SOME
BRANE
TRAJECTORIES
$\theta=\pi/2$
$\epsilon=0.5$
$\mathcal{N}=5$
$C=0.5$
$\theta=0$
$\eta=2$
$\eta=4$
$\eta=6$
$\theta=\pi/2$
$\epsilon=0.05$
$\mathcal{N}=9.5$
$C=0.5$
$\theta=0$
$\eta=2$
$\eta=4$
$\eta=6$
$\theta=\pi/2$
$\epsilon=0.001$
$\mathcal{N}=84$
$C=0.5$
$\theta=0$
$\eta=2$
$\eta=4$
$\eta=6$
$\theta=\pi/2$
$\epsilon=0.0001$
$\mathcal{N}=377$
$C=0.5$
$\theta=0$
$\eta=2$
$\eta=4$
$\eta=6$

2^{nd} Order Corrections

These come from corrections to the superpotential, dependent on the D3 position and interactions with other wrapped branes. The total potential also includes an uplift term:

$$V_F = \frac{C_1}{U^2} g^{2/n} e^{-2a\sigma} \left[\frac{U}{6} + \frac{1}{a} \left(1 - C_2 \frac{e^{a\sigma}}{g^{1/n}} \right) + F_5 \right] , \qquad V_D = \frac{C_3}{U^2}$$

σ is the Kahler modulus and instantaneously shifts to minimize V

$$\begin{aligned} C_1 &= 2\kappa^2 a^2 |A_0|^2 , \quad C_2 = \left| \frac{W_0}{A_0} \right| \\ g(\eta,\theta) &= 1 + \frac{\epsilon}{\mu} \cosh\frac{\eta}{2} \cos\frac{\eta}{2} \\ U &= 2\sigma - \gamma k(\eta) \\ k(\eta) &= \epsilon^{4/3} \int_0^\eta d\eta [\cosh\eta \sinh\eta - \eta]^{1/3} \\ \gamma &= \frac{\sigma_0 T_3}{3M_p^2} \end{aligned}$$

$$\begin{aligned} F_5 =& \frac{\epsilon^{2/3}\sin^2(\theta/2)}{6n^2a^2\mu^2\gamma g^2 K} + \\ & \gamma\epsilon^{2/3} K^2 \sinh^2\frac{\eta}{2} \left(K\epsilon^{1/3}\cosh\frac{\eta}{2} - \frac{\cos(\theta/2)}{2na\mu\gamma g} \right)^2 \end{aligned}$$

D. Kaviani 1212.5831

Parameter constraints

But also it seems that the geometric part of the F-term potential has a hidden constraining feature once angular freedom is switched on:

$$F_5 = \frac{\epsilon^{2/3}\sin^2(\theta/2)}{6n^2a^2\mu^2\gamma g^2 K} + \gamma\epsilon^{2/3}K^2\sinh^2\frac{\eta}{2}\left(K\epsilon^{1/3}\cosh\frac{\eta}{2} - \frac{\cos(\theta/2)}{2na\mu\gamma g}\right)^2$$

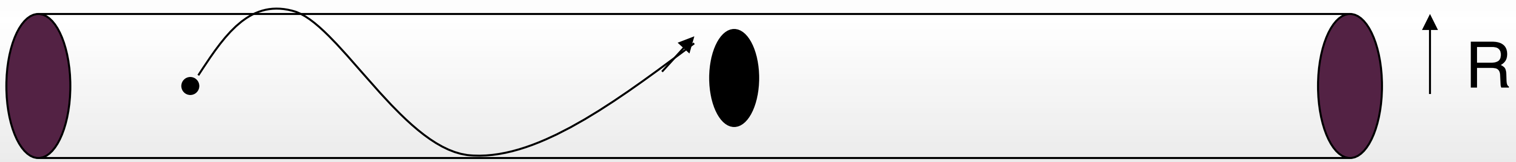
Second part is always small, but the first is maximized at π and prevents μ from being too small.

DISCUSSION POINTS

- DBI is dead – right?
- Have we been honest about parameter constraints?
- Is it consistent to ignore UV/IR corrections to the mid-throat region?
- Should we worry about the IR?
- Should we use perturbative SUGRA for highly anisotropic/ time dependent situations?
- Have we got backreaction right?

Back-reaction:

To check the probe brane does not significantly warp the internal geometry, use a toy exact solution – D3 on $\mathbf{R}^9$ x $\mathbf{S}^1$



As the D-brane moves on the S^1 a horizon forms, and becomes larger as the brane moves faster. The proper energy of the D3 with respect to a Poincare observer also increases.