Calculations of field distortions in a collider TPC

Thorsten Krautscheid

University of Bonn

Workshop on TPCs at high rate experiments
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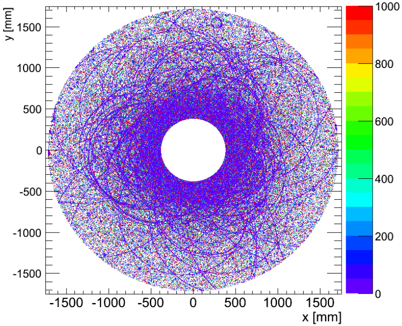
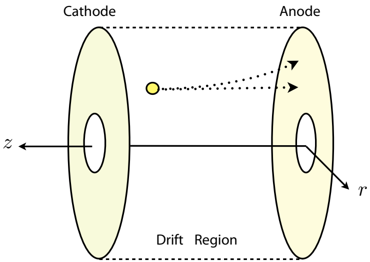
Outline

Motivation

Space charges

- (Primary) ions in active volume
- Charge induces electric field
  $\Rightarrow$ Distorted drift field





Distortions

- $\vec{E} \times \vec{B}$-Effects
- Spatial resolution
- Momentum resolution

Field calculations in a collider TPC

Electrostatic potential:

$$\Delta\Phi_0(\vec{x}) = 0$$

$$\Delta\Phi_Q(\vec{x}) = \frac{-4\pi}{\epsilon\epsilon_0}\rho(\vec{x})$$

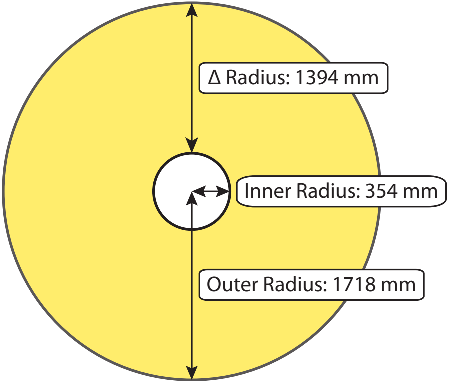
Electric field:

$$\begin{aligned}\vec{E}(\vec{x}) &= \vec{E}(\vec{x}) + \vec{E}_Q(\vec{x})\\ &= \vec{E}(\vec{x}) - \nabla\Phi_Q(\vec{x})\end{aligned}$$

Green's function:

$$\Delta G(\vec{x},\vec{x}') = -4\pi\delta(\vec{x}-\vec{x}')$$

$$\Phi_Q(\vec{x}) = \int_V G(\vec{x},\vec{x}')\,\rho(\vec{x}')\,\mathrm{d}\vec{x}'$$



$$\begin{aligned}\Phi_0(\vec{x}) &= 0, \qquad &\forall\vec{x}\in\partial V\\ G(\vec{x},\vec{x}') &= 0, \qquad &\forall\vec{x}\in\partial V\end{aligned}$$

Green's function

Modified Bessel functions:

$$G\left(\vec{x}, \vec{x'}\right) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{(2-\delta_{0m})}{\pi L}\cos\left(m(\varphi-\varphi')\right)\sin\left(\beta_n z'\right)\sin\left(\beta_n z'\right)$$

$$\cdot\frac{R_{mn1}(r_<)R_{mn2}(r_>)}{I_m(\beta_n a)K_m(\beta_n b)-I_m(\beta_n b)K_m(\beta_n a)}$$

Modified Bessel functions of real argument and imaginary order:

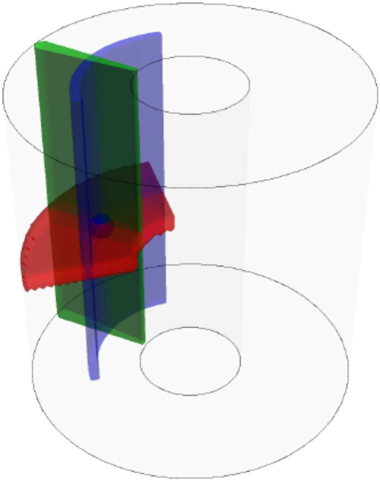
$$G\left(\vec{x}, \vec{x'}\right) = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cosh\left(\mu_{nk}(\pi-|\varphi-\varphi'|)\right)}{\mu_{nk}\sinh\left(\pi\mu_{nk}\right)}\sin\left(\beta_n z\right)$$

$$\cdot\sin\beta_n z'\frac{R_{nk}(r)R_{nk}(r'}{N_{nk}^2}$$

Ordinary Bessel functions:

$$G\left(\vec{x}, \vec{x'}\right) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{(2-\delta_{0m})}{2\pi}\cos\left(\varphi-\varphi'\right)$$

$$\cdot\frac{R_{mn}(r)R_{mn}(r')}{N_{mm}^2}\frac{\sinh\left(\beta_{mn}z_<\right)\sinh\left(\beta_{mn}(L-z_>)\right)}{\beta_{mn}sinh(\beta_{mn}L)}$$



see: Stefan Rossegger - *Static Green's function for a coaxial cavity including an innovative representation*

Electric field in radial (r) direction

Green's function represented by modified Bessel functions:

$$G\left(r,\varphi,z;r',\varphi',z'\right)=\frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left(2-\delta_{m0}\right)\cos\left(m\left(\varphi-\varphi'\right)\right)\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\frac{R_{mn1}\left(r_<\right)R_{mn2}\left(r_>\right)}{I_m\left(\beta_n a\right)K_m\left(\beta_n b\right)-I_m\left(\beta_n b\right)K_m\left(\beta_n a\right)}$$

Derivative with respect to r:

$$\frac{\partial}{\partial r}G\left(r,\varphi,z;r',\varphi',z'\right)=$$
$$\frac{1}{\pi L}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\cos\left(m\left(\varphi-\varphi'\right)\right)\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\frac{\partial}{\partial r}\left(\frac{R_{mn\,1}\left(r_<\right)R_{mn\,2}\left(r_>\right)}{I_m\left(\beta_n a\right)K_m\left(\beta_n b\right)-I_m\left(\beta_n b\right)K_m\left(\beta_n a\right)}\right)$$

with

$$\frac{\partial}{\partial r}\left(R_{mn\,1}\left(r_<\right)R_{mn\,2}\left(r_>\right)\right)=\begin{cases}R'_{mn}\left(a,r\right)R_{mn\,2}\left(r'\right), & \text{for } a\le r<r'\le b\\ R_{mn\,1}\left(r'\right)R'_{mn}\left(b,r\right), & \text{for } a\le r'<r\le b\end{cases}$$

wherein $R'_{mn}(s,t)$ is given by

$$R'_{mn}\left(s,t\right)=\frac{\beta_n}{2}\left(K_m\left(\beta_n s\right)\left(I_{m-1}\left(\beta_n t\right)+I_{m+1}\left(\beta_n t\right)\right)+I_m\left(\beta_n s\right)\left(K_{m-1}\left(\beta_n t\right)+K_{m+1}\left(\beta_n t\right)\right)\right)$$

Electric field in azimuthal (φ) direction

Green's function represented by modified Bessel functions of imaginary order and real argument:

$$G\left(r,\varphi,z;r',\varphi',z'\right) = \frac{1}{L}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\frac{\cosh\left(\mu_{nk}\left(\pi - \left|\varphi-\varphi'\right|\right)\right)}{\mu_{nk}\sinh\left(\pi\mu_{nk}\right)}\frac{R_{nk}(r)\,R_{nk}\left(r'\right)}{N_{nk}^2}$$

Derivative with respect to ϕ:

$$\frac{\partial}{\partial\varphi}G\left(r,\varphi,z;r',\varphi',z'\right)$$
$$= \frac{1}{L}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\frac{R_{nk}(r)\,R_{nk}\left(r'\right)}{N_{nk}^2}\cdot\frac{\partial}{\partial\varphi}\left(\frac{\cosh\left(\mu_{nk}\left(\pi - \left|\varphi-\varphi'\right|\right)\right)}{\mu_{nk}\sinh\left(\pi\mu_{nk}\right)}\right)$$

with

$$\frac{\partial}{\partial\varphi}\left(\cosh\left(\mu_{nk}\left(\pi-\left|\varphi-\varphi'\right|\right)\right)\right) = \begin{cases} -\mu_{nk}\sinh\left(\mu_{nk}\left(\pi-\left(\varphi-\varphi'\right)\right)\right), & \text{for } 0 \le \varphi' < \varphi \le 2\pi, \\ \mu_{nk}\sinh\left(\mu_{nk}\left(\pi-\left(\varphi'-\varphi\right)\right)\right), & \text{for } 0 \le \varphi < \varphi' \le 2\pi. \end{cases}$$

Normalization

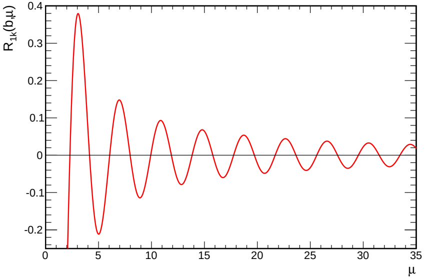
$$\int_a^b R_{nk}(r)\,R_{ns}(r)\,\frac{\mathrm{d}r}{r} = \delta_{ks}N_{nk}^2$$

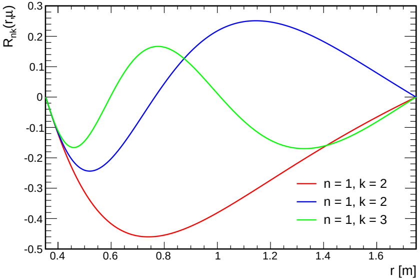
Electric field in azimuthal (ϕ) direction

$$R_{nk}(r,\mu) = L_{i\mu}(\beta_n a)\, K_{i\mu}(\beta_n r) - K_{i\mu}(\beta_n a)\, L_{i\mu}(\beta_n r)$$

with $L_{i\mu}(\beta_n r) := \frac{1}{2}\left(I_{-i\mu}(\beta_n r) + I_{i\mu}(\beta_n r)\right)$ and

μ_{nk} being the $k-$th zero of $R_{nk}(b,\mu)$:





Electric field in longitudinal (z) direction

Green's function represented by ordinary Bessel functions:

$$G\left(r,\varphi,z;r',\varphi',z'\right)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left(2-\delta_{m0}\right)\frac{\cos\left(m\left(\varphi-\varphi'\right)\right)}{2\pi}\frac{R_{mn}\left(r\right)R_{mn}\left(r'\right)}{\bar{N}_{mn}^{2}}\frac{\sinh\left(\beta_{mn}z_{<}\right)\sinh\left(\beta_{mn}\left(L-z_{>}\right)\right)}{\beta_{mn}\sinh\left(\beta_{mn}L\right)}$$

Derivative with respect to z:

$$\frac{\partial}{\partial z}G\left(r,\varphi,r',\varphi',z'\right)=$$

$$\frac{1}{2\pi}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left(2-\delta_{m0}\right)\cos\left(m\left(\varphi-\varphi'\right)\right)\frac{R_{mn}\left(r\right)R_{mn}\left(r'\right)}{\bar{N}_{mn}^{2}}\frac{\partial}{\partial z}\left(\frac{\sinh\left(\beta_{mn}z_{<}\right)\sinh\left(\beta_{mn}\left(L-z_{>}\right)\right)}{\beta_{mn}\sinh\left(\beta_{mn}L\right)}\right)$$

with

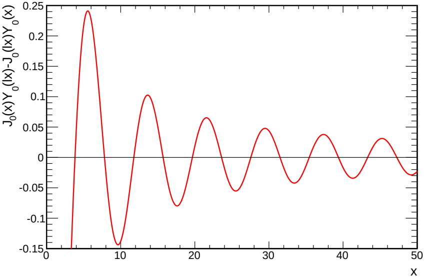
$$\frac{\partial}{\partial z}\left(\sinh\left(\beta_{mn}z_{<}\right)\sinh\left(\beta_{mn}\left(L-z_{>}\right)\right)\right)=\begin{cases}\beta_{mn}\cosh\left(\beta_{mn}z\right)\sinh\left(\beta_{mn}\left(L-z'\right)\right), & \text{for}\quad 0\leq z<z'\leq L,\\ -\beta_{mn}\cosh\left(\beta_{mn}\left(L-z\right)\right)\sinh\left(\beta_{mn}z'\right), & \text{for}\quad 0\leq z'<z\leq L.\end{cases}$$

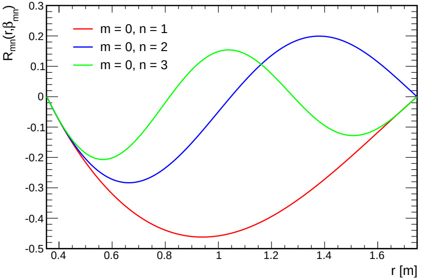
Normalization

$$\bar{N}_{nm}^{2}=\frac{2}{\pi^{2}\beta_{mn}^{2}}\left(\frac{J_{m}^{2}\left(\beta_{mn}a\right)}{J_{m}^{2}\left(\beta_{mn}b\right)}-1\right)$$

$$R_{mn}\left(r,\beta_{mn}\right) = Y_m\left(\beta_{mn}a\right) J_m\left(\beta_{mn}r\right) - J_m\left(\beta_{mn}a\right) Y_m\left(\beta_{mn}r\right)$$

x_{mn} being the n-th zero of $R_{mn}\left(r,l\right)$ with $x_{mn} = \beta_{mn}b$ and $l = \frac{a}{b}$





φ-Symmetry

Field in radial (r) direction:

$$E_r\left(r, z; r', z'\right) = \frac{1}{\pi L}\sum_{n=1}^{\infty}\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\cdot\begin{cases} R'_{0n}\left(a, r\right) R_{0n\,2}\left(r'\right), & \text{for } a \leq r < r' \leq b \\ R_{0n\,1}\left(r'\right) R'_{0n}\left(b, r\right), & \text{for } a \leq r' < r \leq b \end{cases}$$

because of $I_{-n}(x) = I_n(x)$ and $K_{-n}(x) = K_n(x)$, $R'_{mn}(s,t)$ simplifies to

$$R'_{mn}\left(s, t\right) = \frac{\beta_n}{2}\left(K_m\left(\beta_n s\right)\left(I_{m-1}\left(\beta_n t\right) + I_{m+1}\left(\beta_n t\right)\right) + I_m\left(\beta_n s\right)\left(K_{m-1}\left(\beta_n t\right) + K_{m+1}\left(\beta_n t\right)\right)\right)$$

Field in longitudinal (z) direction:

$$E_z\left(r, z, r', z'\right) = \frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{R_{0n}\left(r\right) R_{0n}\left(r'\right)}{\bar{N}_{0n}^2}\cdot\begin{cases} \beta_{mn}\cosh\left(\beta_{0n} z\right)\sinh\left(\beta_{0n}\left(L - z'\right)\right), & \text{for } 0 \leq z < z' \leq L \\ -\beta_{mn}\cosh\left(\beta_{0n}\left(L - z\right)\right)\sinh\left(\beta_{0n} z'\right), & \text{for } 0 \leq z' < z \leq L \end{cases}$$

φ-Symmetry

Field in radial (r) direction:

$$E_r\left(r, z; r', z'\right) = \frac{1}{\pi L}\sum_{n=1}^{\infty}\sin\left(\beta_n z\right)\sin\left(\beta_n z'\right)\cdot\begin{cases} R'_{0n}\left(a, r\right) R_{0n\,2}\left(r'\right), & \text{for } a \leq r < r' \leq b \\ R_{0n\,1}\left(r'\right) R'_{0n}\left(b, r\right), & \text{for } a \leq r' < r \leq b \end{cases}$$

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$$R'_{0n}\left(s, t\right) = \beta_n\left(K_0\left(\beta_n s\right) I_1\left(\beta_n t\right) + I_0\left(\beta_n s\right) K_1\left(\beta_n t\right)\right)$$

Field in longitudinal (z) direction:

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Event generation and full detector simulation

Guinea-Pig

- Simulation of beam-beam interaction
- Suitable for any beam parameters of ILC and CLICK
- Output: ASCII

Mokka

- Full detector simulation
- Based on Geant4
- Simulation of the interaction of particles with matter
- Includes all ILC detector models
- Takes input from Guinea-Pig, STDHEP, ...
- Produces output for individual detectors
- LCIO output

Marlin

Marlin

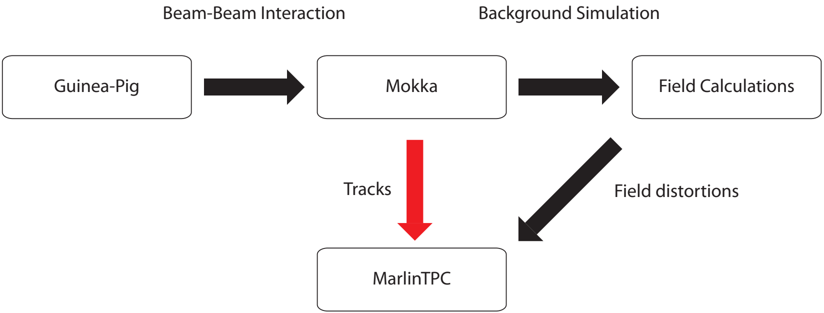
Modular Analysis & Reconstruction Framework for the LINear Collider

- Software package for analysis, reconstruction and simulation
- Modular: Applications are split in individual units $\Rightarrow$ Processor
- Uniform data model: LCIO (Linear Collider I/O)
- Event based data flow, new data can be added to each event in each processor
- Conditions data available at runtime
- Order and parameters controlled by steering file (XML)

MarlinTPC

- Collection of processors for TPC related data

Simulation flow



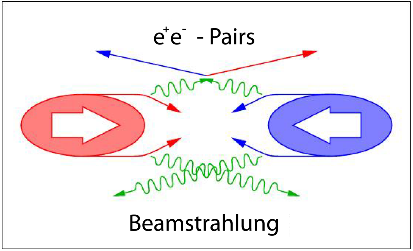
1) TPC Simulation
2) Reconstruction
3) Analysis

Beamstrahlung at the ILC

Beamstrahlung

- High luminosity (500 GeV): $\mathcal{L} = 1.47 \times 10^{34}$ cm^{-2} s^{-1}
- Beams have to be focussed on a tiny spot:
  - $\sigma_x = 474$ nm
  - $\sigma_y = 5.9$ nm

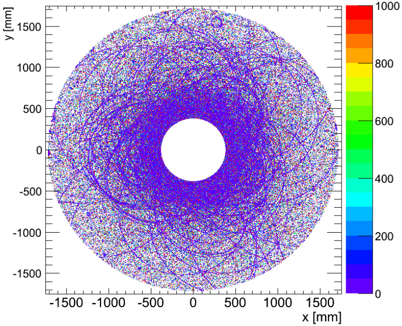
  $\Rightarrow (\frac{1}{1000})^2$ LEP
- Bunches have a very high space charge
- Particles get redirected and emit photons
- Photons can produce e^+e^--pairs by scattering

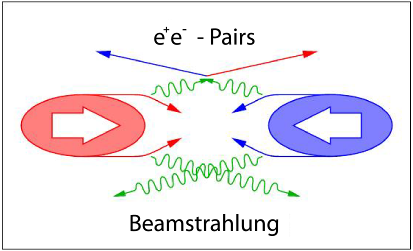


Pairs

- 10^5 pairs per BX
- Focussed in forward direction (small θ)
- Backscattering into TPC $\approx 217 \times 10^3 e^-$ per BX ≈ 34.76 fC

Beamstrahlung at the ILC





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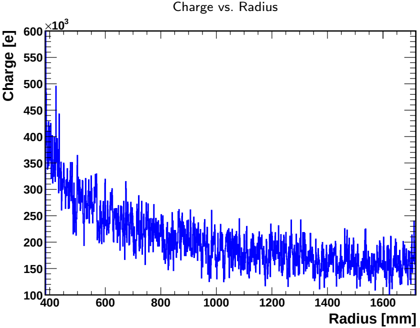
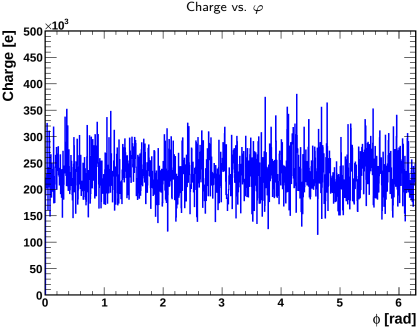
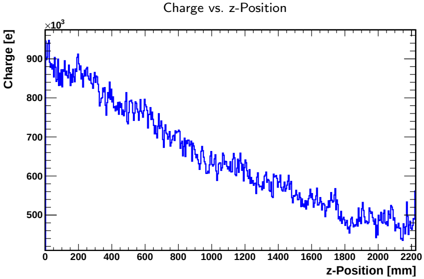
Pair background





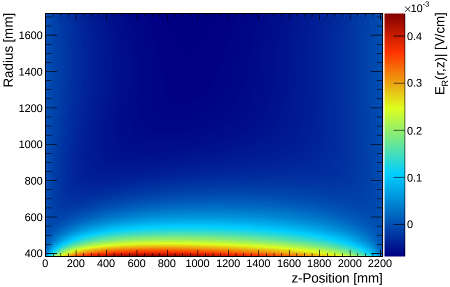


Background distribution

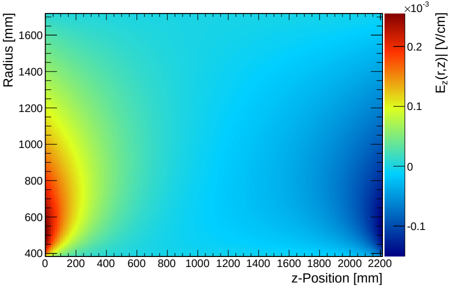
- Strong r-dependency
  Decrease from inner to outer field cage
- No φ-dependency
- Strong z-dependency
  Decrease from cathode to anode

Primary ionization

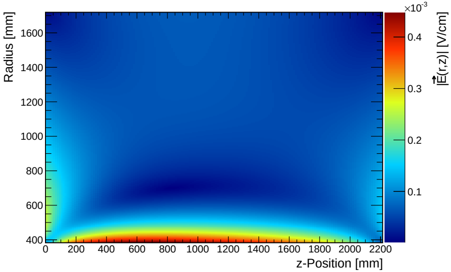
Field in radial (r) direction



Field in longitudinal (z) direction



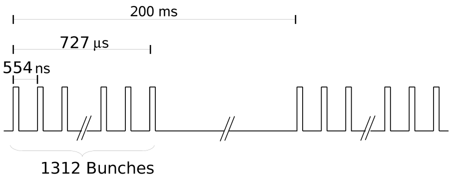
Total field distortions

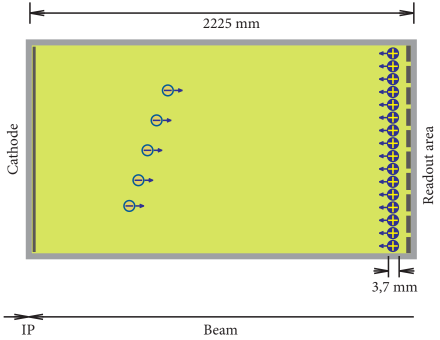


Electric field

- Primary ionization only
- Calculated for one BT
- Biggest distortions near inner field cage

Ion discs



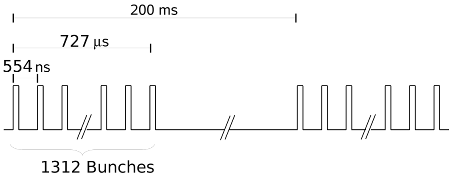


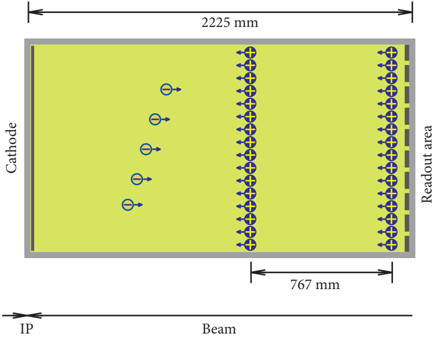
Secondary ions

- Gating only possible between BT
- Pair background gets amplified at the andode
- Ion disc builds up during BT
- Amount of charge determined by ion back drift ratio
- Without gate:
  - Discs drift towards cathode
  - Number of discs determined by drift velocity in the gas

$\Rightarrow$ Work in progress

Ion discs





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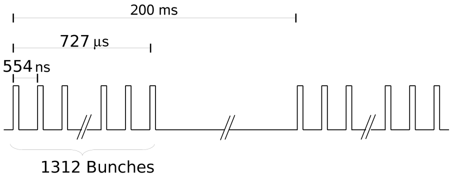
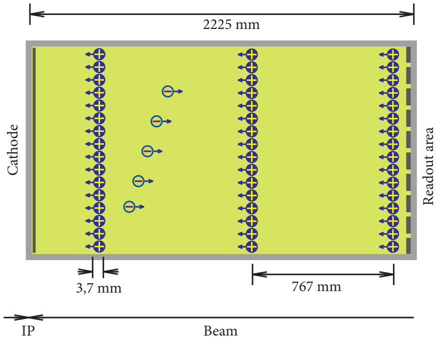
⇒ Work in progress

Ion discs





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$\Rightarrow$ Work in progress

Summary

- Space charge effects are calculable for all spatial directions
- Different representations of Green function
  see: Stefan Rossegger - *Static Green's function for a coaxial cavity including an innovative representation*
- Different representations for different E-Field directions
- Plenty of simulation tools for ILC
  - Guinea-Pig
  - Mokka
  - Marlin
- Field calculations at ILC ongoing