RooStats Tutorials

Inversion of Hypothesis Tests

Inversion of Hypothesis Tests

- one-to-one mapping between hypothesis tests and confidence intervals

Table 20.1 Relationships between hypothesis testing and interval estimation

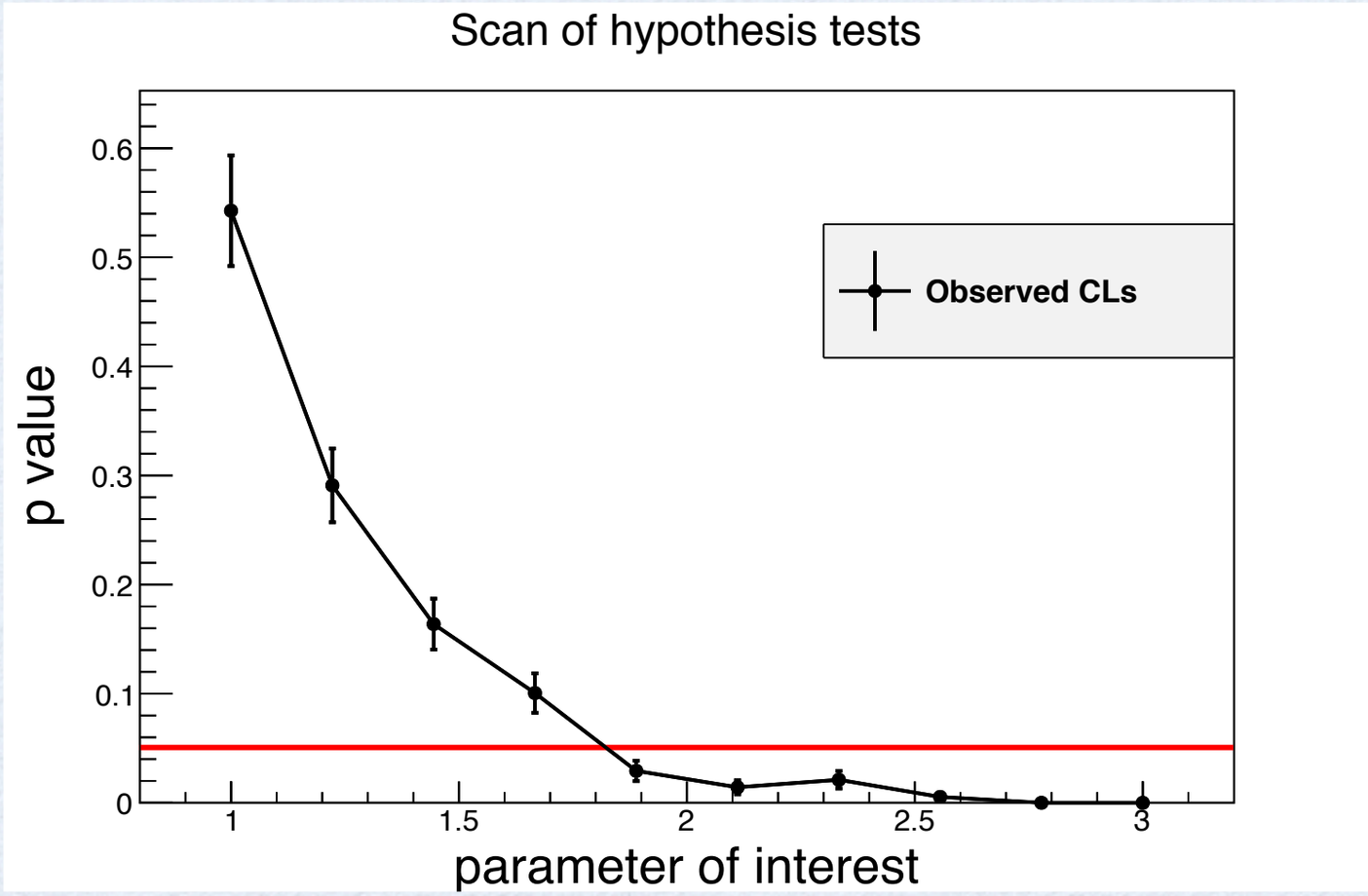
Property of test	Property of corresponding confidence interval
Size $= \alpha$	Confidence coefficient $= 1 - \alpha$
Power = probability of rejecting a false value of $\theta = 1 - \beta$	Probability of not covering a false value of $\theta = 1 - \beta$
Most powerful	Uniformly most accurate
$\longleftarrow$	$\left\{ \begin{array}{c} \textit{Unbiased} \\ 1 - \beta \geq \alpha \end{array} \right\}$ $\longrightarrow$
Equal-tails test $\alpha_1 = \alpha_2 = \frac{1}{2}\alpha$	Central interval

from G. Feldman visiting Harvard statistics department

They explained that in statistical theory there is a one-to-one correspondence between a hypothesis test and a confidence interval. (The confidence interval is a hypothesis test for each value in the interval.) The Neyman-Pearson Theorem states that the likelihood ratio gives the most powerful hypothesis test. Therefore, it must be the standard method of constructing a confidence interval.

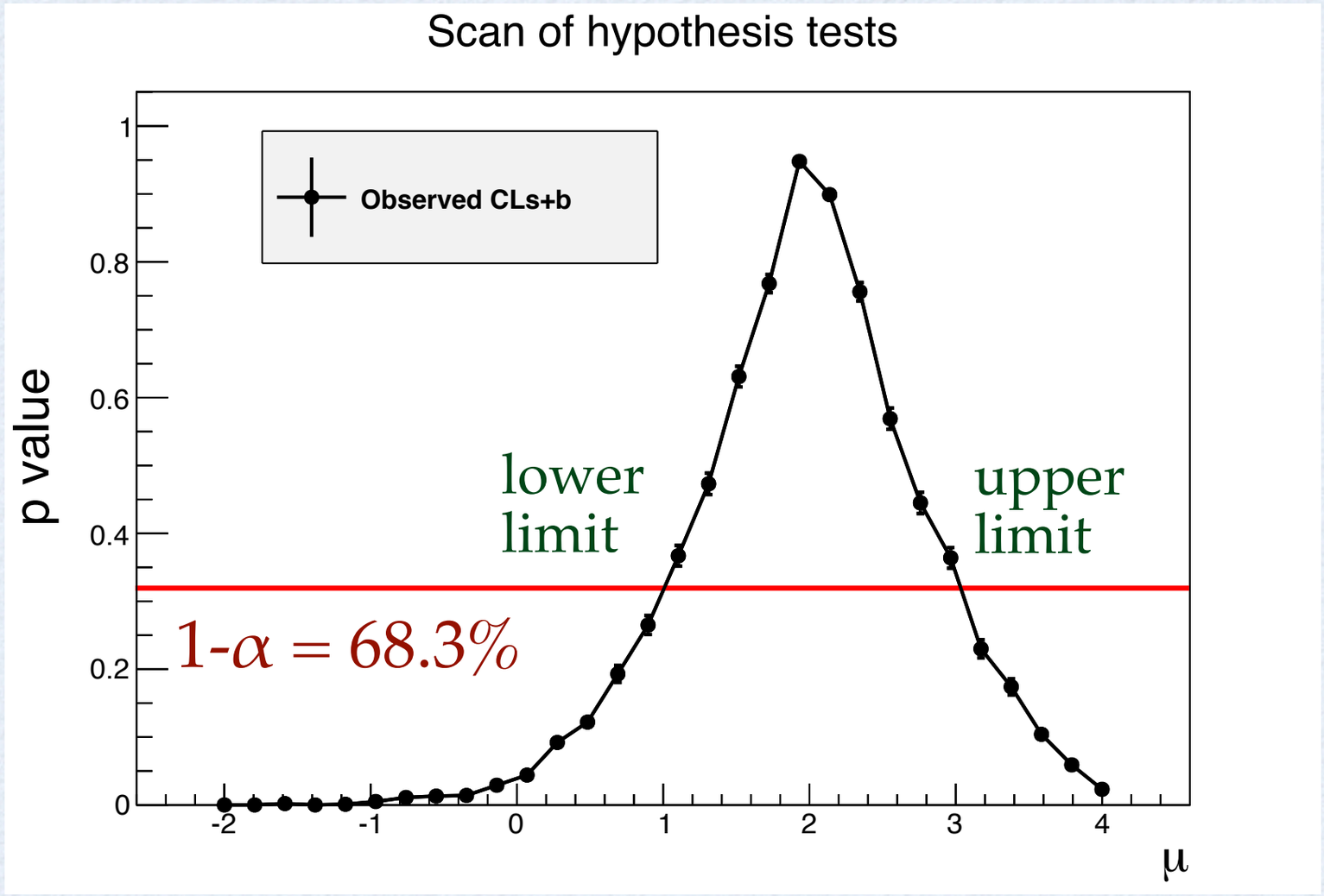
Hypothesis Test Inversion

- Performing an hypothesis test at each value of the parameter
- Interval van be derived by inverting the p-value curve, function of the parameter of interest (μ)
  - value of μ which has p-value α (e.g. 0.05), is the upper limit of 1-α confidence interval (e.g. 95%)



Hypothesis Test Inversion

- use one-sided test for upper limits (e.g. one-side profile likelihood test statistics)
- use two-sided test for a 2-sided interval



Example: 1-σ interval for a Gaussian measurement

HypoTestInverter class

- Input:
  - Hypothesis Test calculator (e.g. FrequentistCalculator)
    - possible to customize test statistic, number of toys, etc..
    - N.B: null model is S+B, alternate is B only model
- Interval calculator class
  - scan given interval of μ and perform hypothesis tests
  - compute upper/lower limit from scan result
    - can use $CL_s = CL_{s+b} \ / \ CL_b$ for the p-value
  - store in result (HypoTestInverterResult) also all the hypothesis test results for each scanned μ value
    - possible to merge later results
- Can compute expected limits and bands

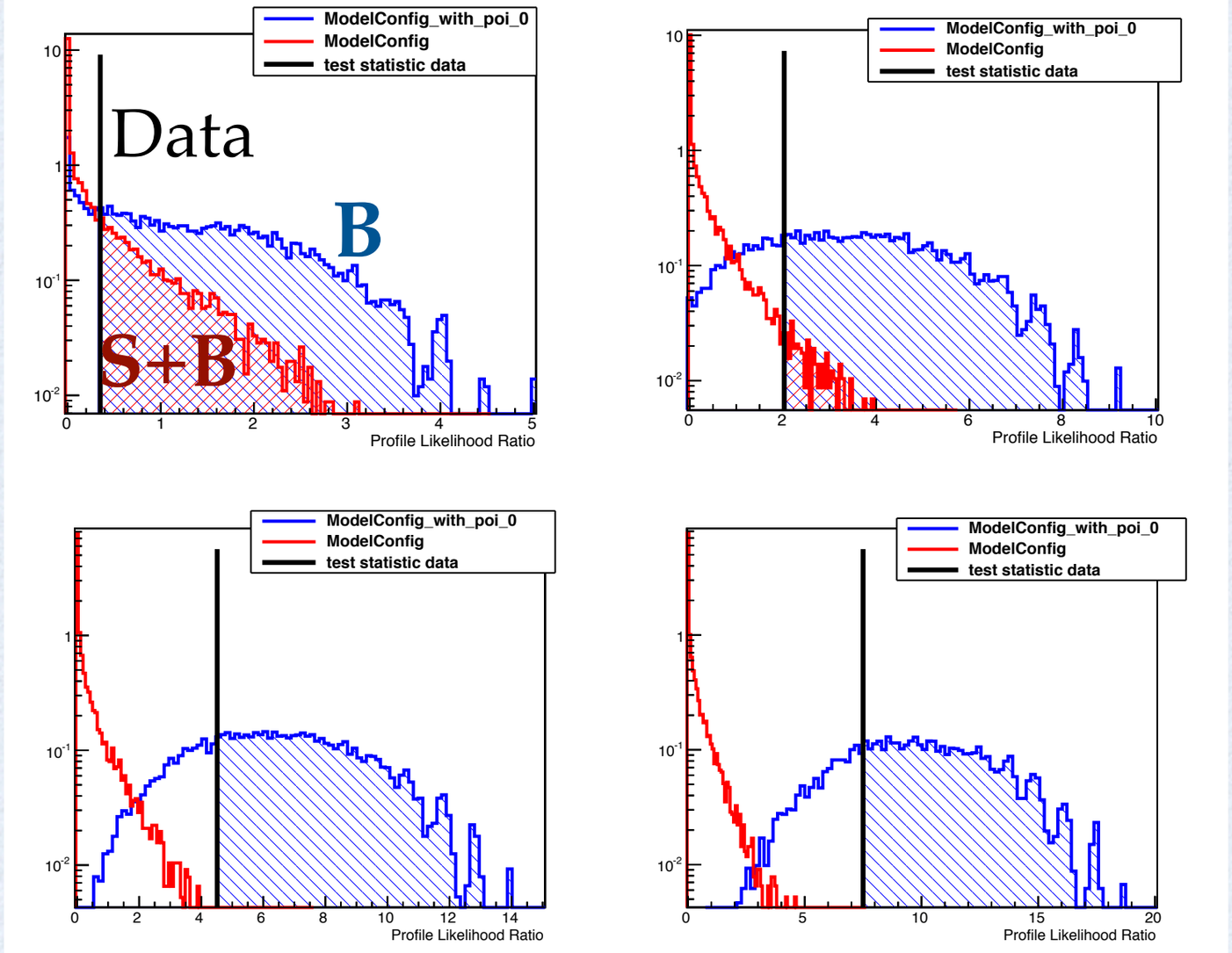
HypoTestInverter

- **`HypoTestInverter`** class in RooStats

```
// create first HypoTest calculator (N.B null is s+b model)
FrequentistCalculator fc(*data, *bModel, *sbModel);

HypoTestInverter calc(*fc);
calc.UseCLs(true);

// configure  ToyMCSampler and set the test statistics
ToyMCSampler *toymcs = (ToyMCSampler*)fc.GetTestStatSampler();

ProfileLikelihoodTestStat profll(*sbModel->GetPdf());
// for CLs (bounded intervals) use one-sided profile likelihood
profll.SetOneSided(true);
toymcs->SetTestStatistic(&profll);

// configure and run the scan
calc.SetFixedScan(npoints,poimin,poimax);
HypoTestInverterResult * r = calc.GetInterval();

// get result and plot it
double upperLimit = r->UpperLimit();
double expectedLimit = r->GetExpectedUpperLimit(0);

HypoTestInverterPlot *plot = new HypoTestInverterPlot("hi","",r);
plot->Draw();
```

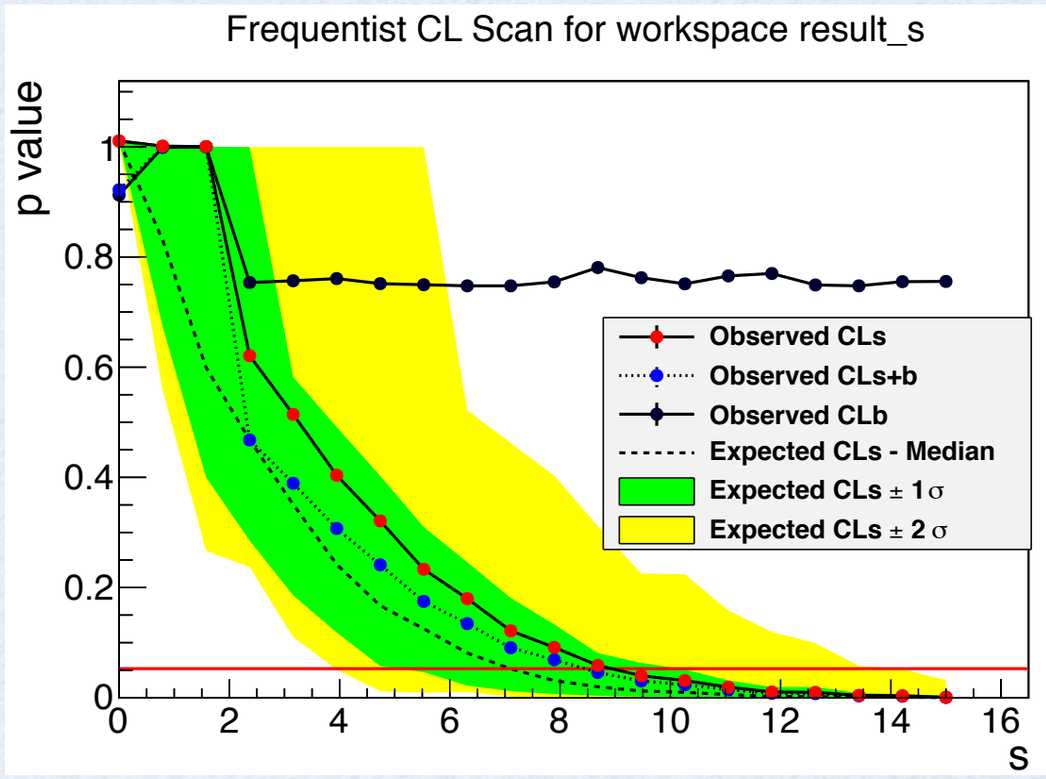
Running the HypoTestInverter

Hypothesis test results for each scanned point



p-value, CL_{s+b} (or CL_b) is integral of S+B (or B) test statistic distribution from data value

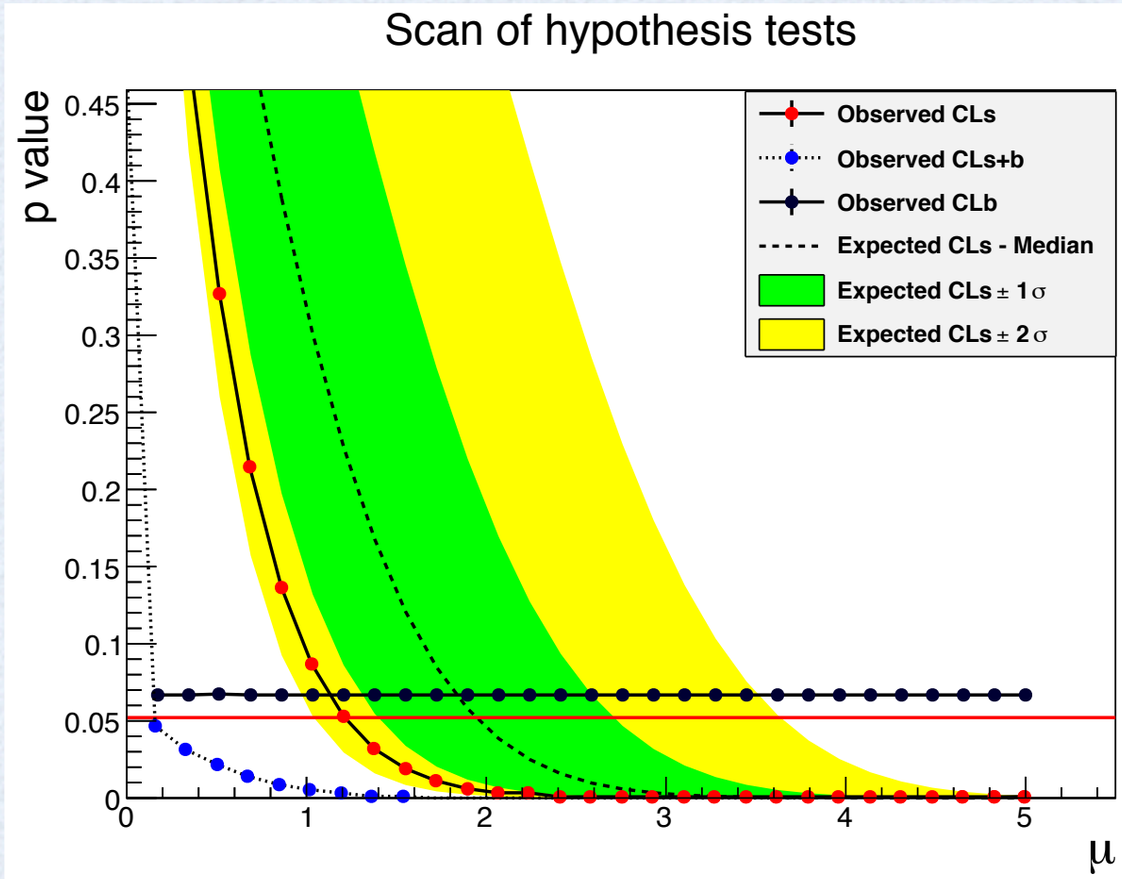
Scan result

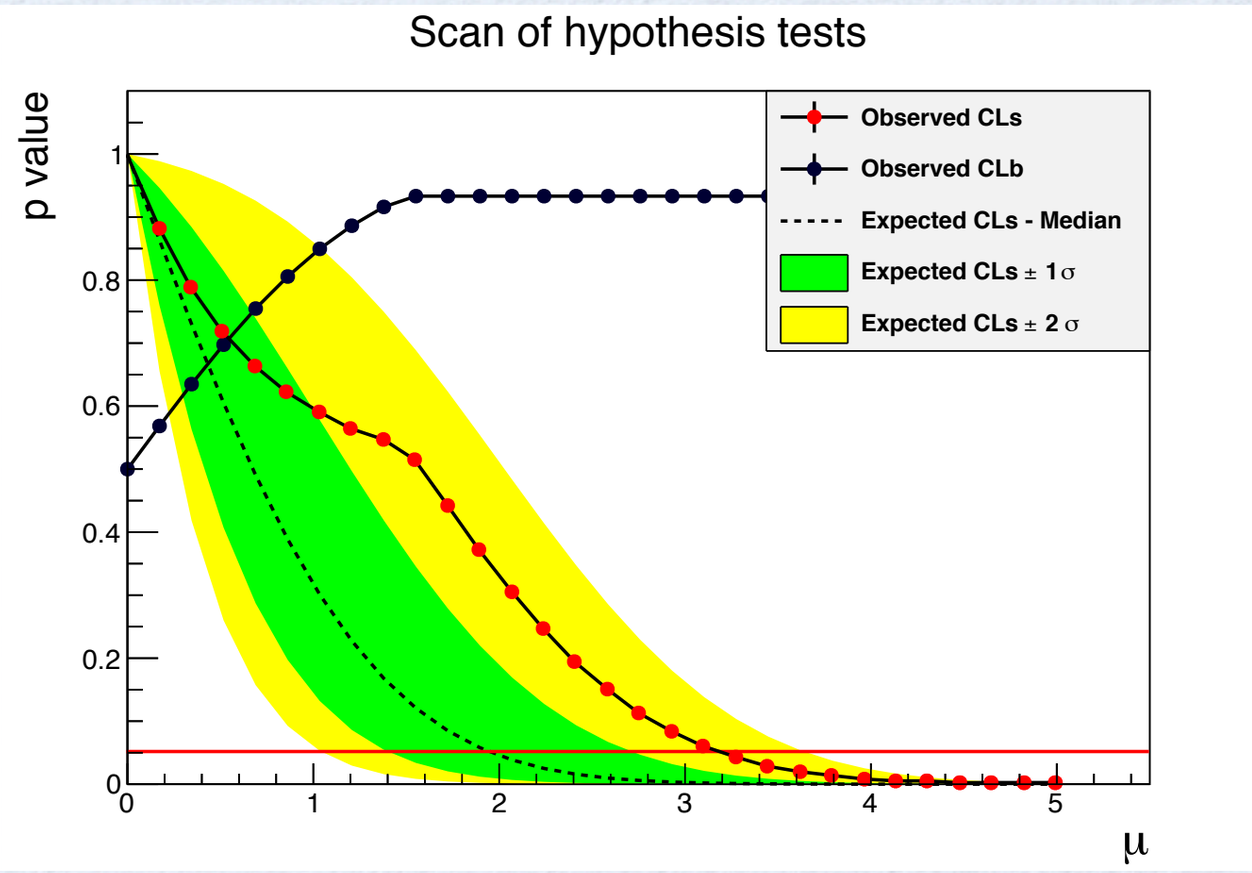


Expected limit and bands are obtained by replacing data test statistic value with quantiles of the B test stat. distribution

Example of Scan

- 95% CL limit on a Gaussian measurement:
  - Gauss(x,μ,1), with μ≥0





deficit, observation x = -1.5

excess, observation x = 1.5

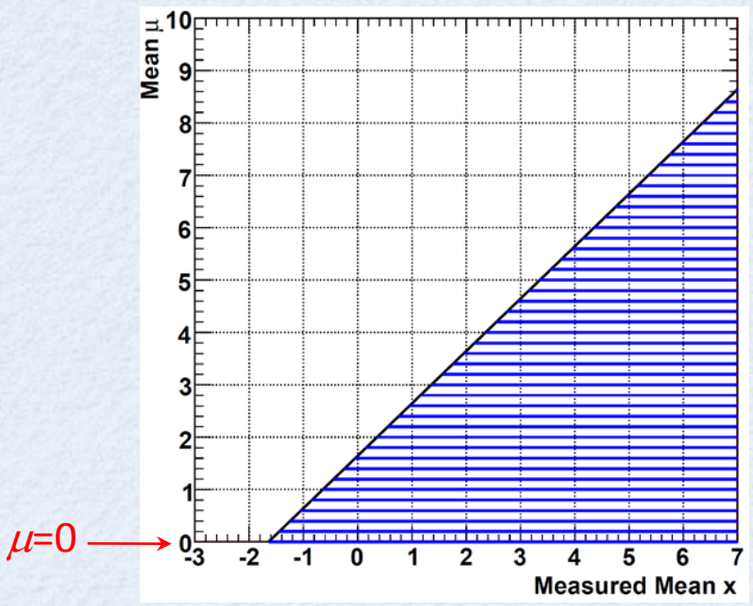
use CL_s as p-value to avoid setting limits which are too good

Limits on bounded measurements

from Bob Cousins:

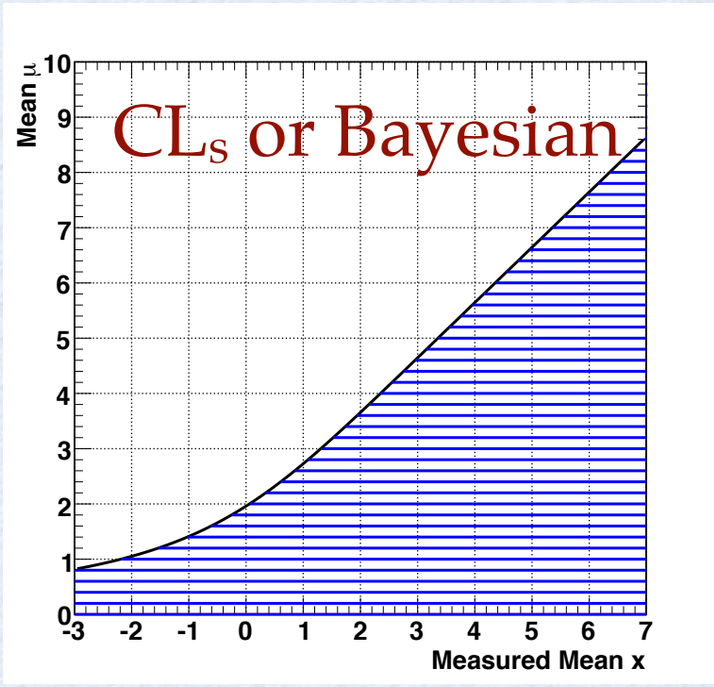
Downward fluctuations in searches for excesses

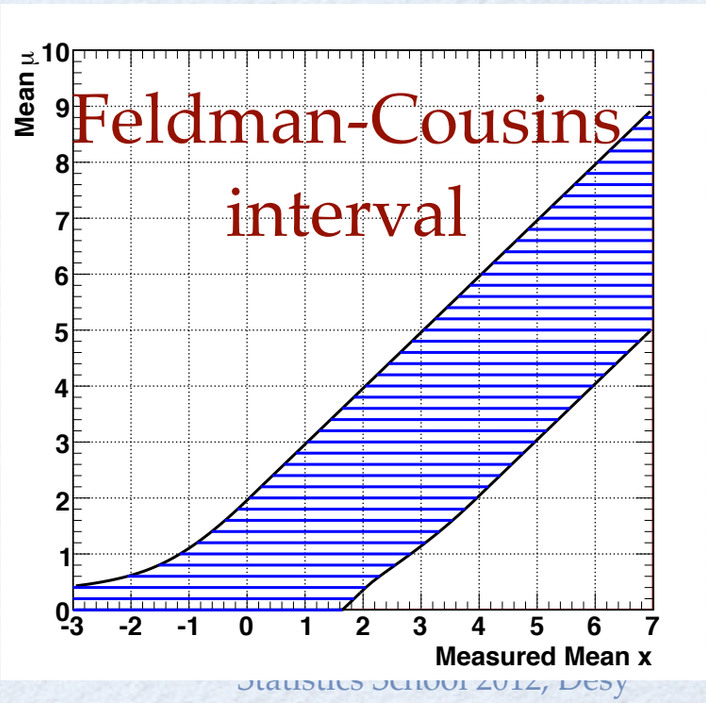
Classic example: Upper limit on mean μ of Gaussian based on measurement *x* (in units of σ).



If μ≥0 in model, as measured x becomes increasingly negative, standard classical upper limit becomes small and then null.

Issue acute 15-25 years ago in expts to measure ν_e mass in (tritium β decay): several measured $m_\nu^2 < 0$.





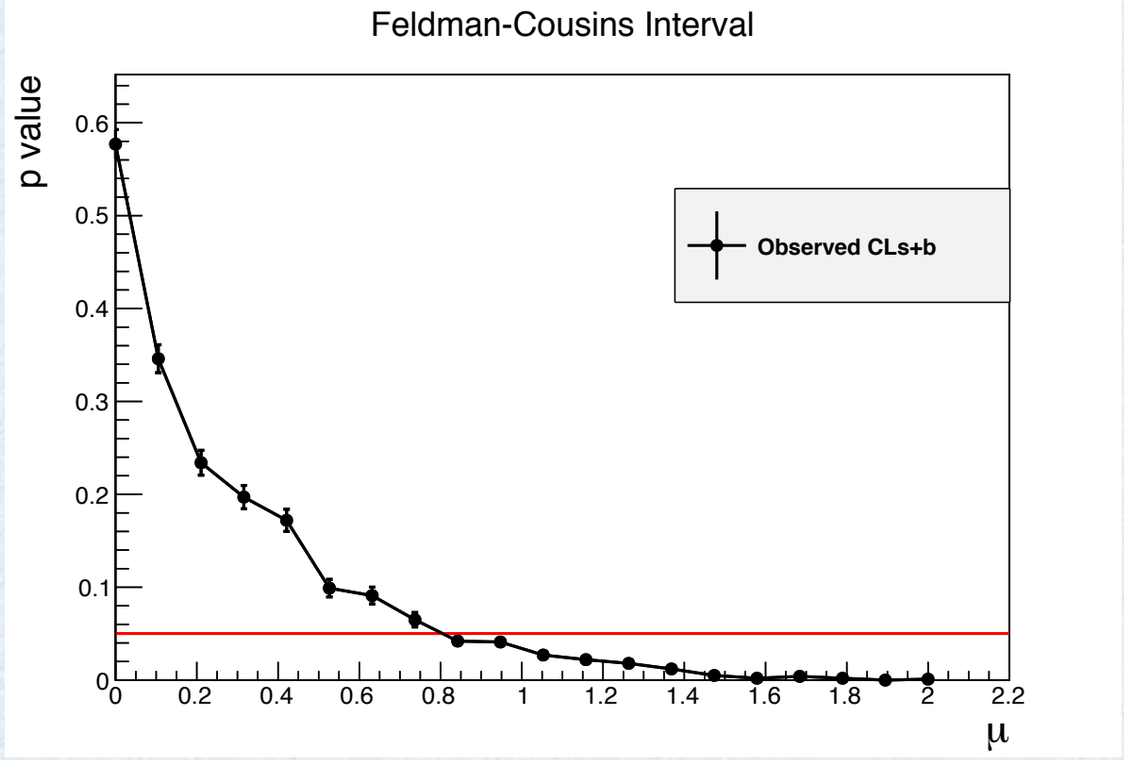
Frequentist 1-sided 95% C.L. Upper Limits, based on α = 1 – C.L. = 5% (called CL_{sb} at LEP).

For *x* < –1.64 σ the confidence interval is the *null* set!

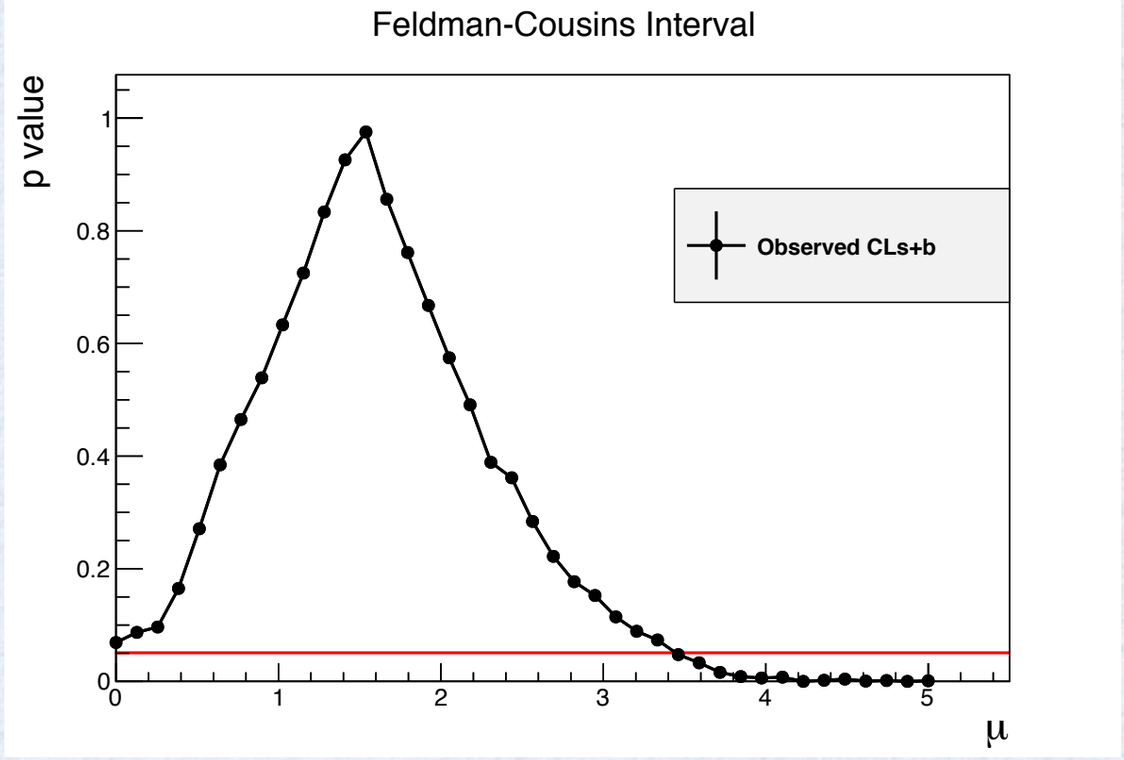
Bob Cousins, CMSDAS, 1/2012

Feldman-Cousins intervals

- HypoTestInverter class can compute also a Feldman-Cousins interval
  - need to use FrequentistCalculator and CL_{s+b} as p-value
  - use the 2-sided profile likelihood test statistic



observation x = -1.5



observation x = 1.5

Feldman-Cousins Interval

from Kyle Cranmer:

A different way to picture Feldman-Cousins

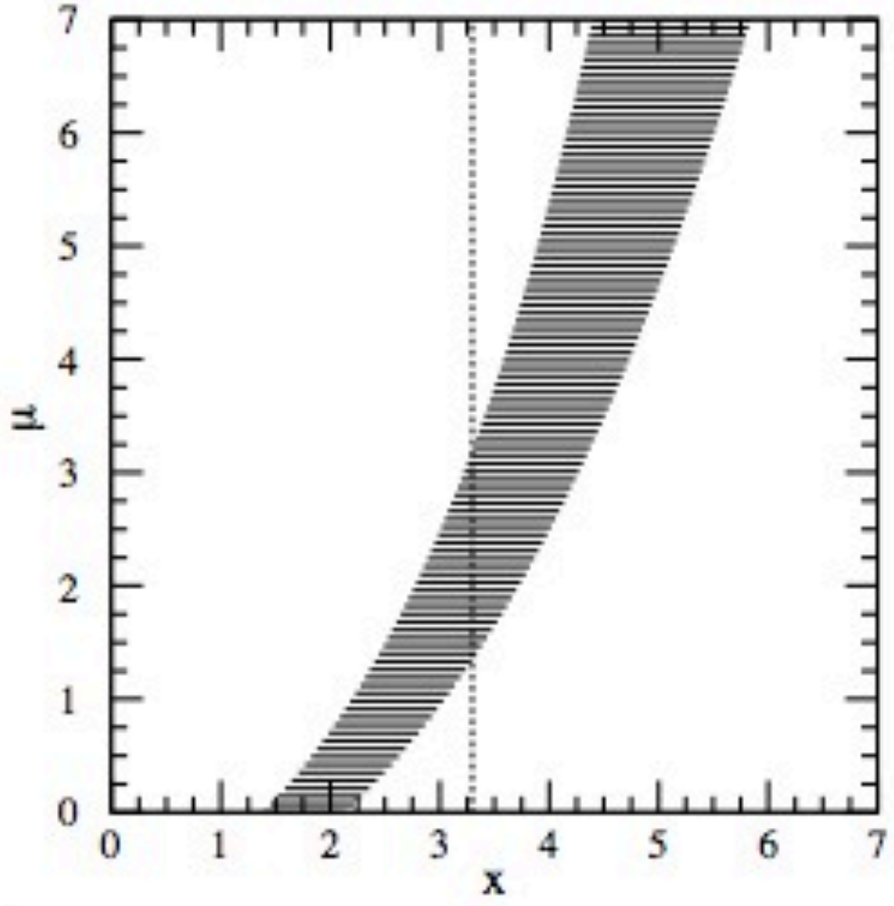
CENTER FOR COSMOLOGY AND PARTICLE PHYSICS

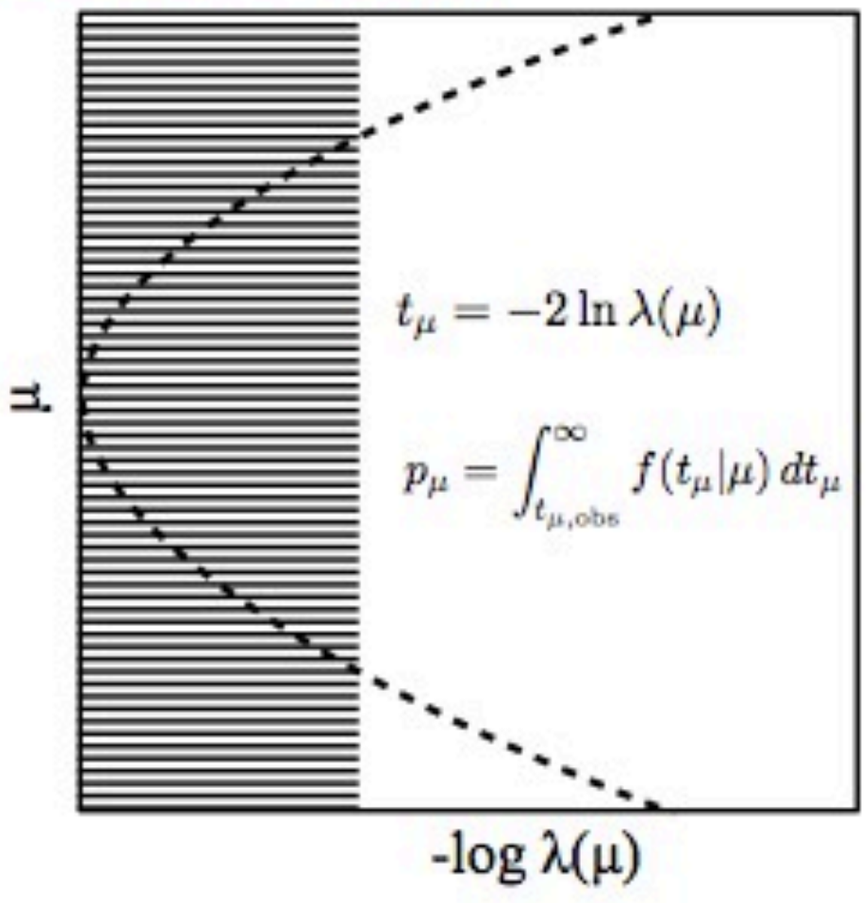
Most people think of plot on left when thinking of Feldman-Cousins

- bars are regions "ordered by" $R = P(n|\mu)/P(n|\mu_{\text{best}})$, with $\int_{x_1}^{x_2} P(x|\mu)dx = \alpha.$

But this picture doesn't generalize well to many measured quantities.

- Instead, just use R as the test statistic... and R is $\lambda(\mu)$

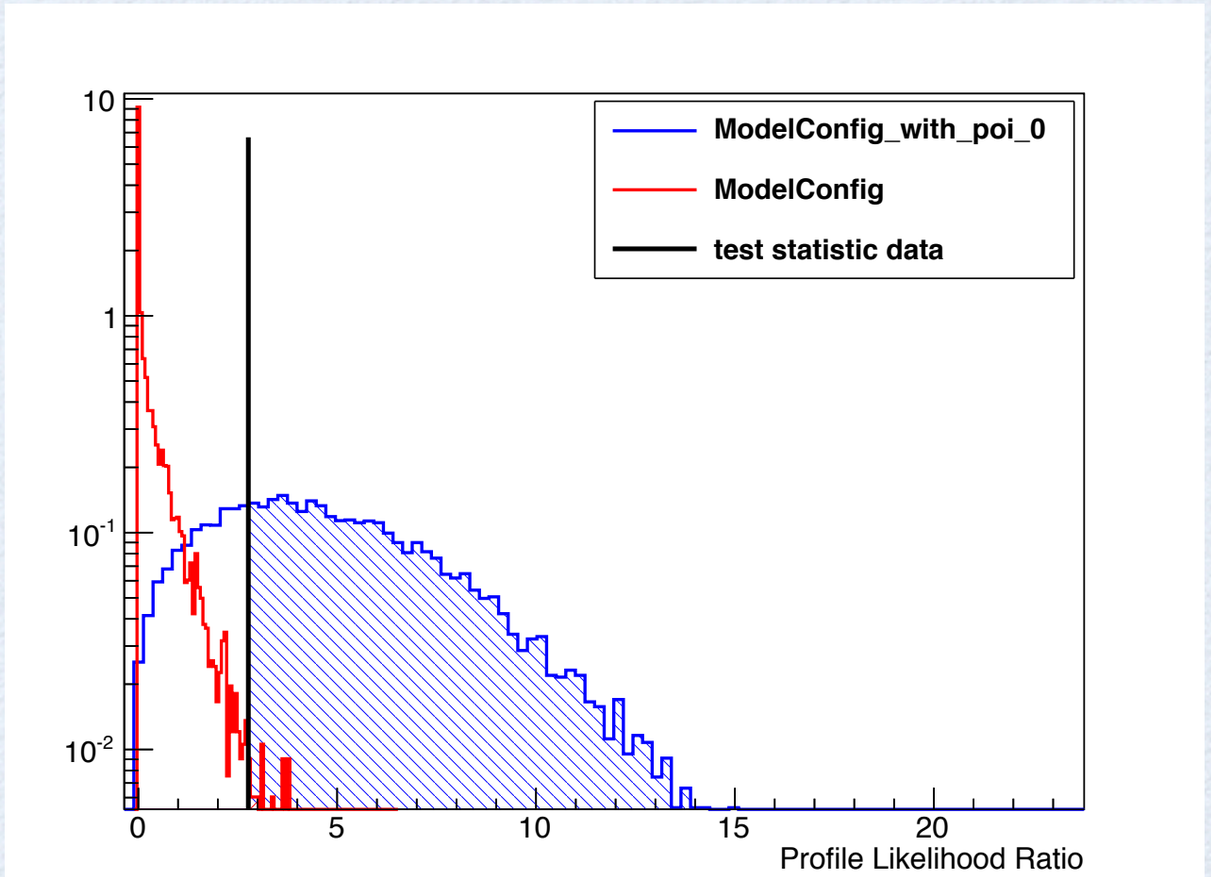




Kyle Cranmer (NYU) CERN School HEP, Romania, Sept. 2011 134

Asymptotic Formulae

- Use the asymptotic formula for the test statistic distributions
- For one-sided profile likelihood test statistic:
  - null model ($\mu = \mu_{TEST}$)
    - half χ^2 distribution
  - alt model ($\mu \neq \mu_{TEST}$)
    - non-central χ^2
    - use Asimov data to get the non centrality parameter λ
- p-values for null (CL_{s+b}) and alt (CL_b) can be obtained without generating toys
- expected limits can be also obtained using the alt distribution

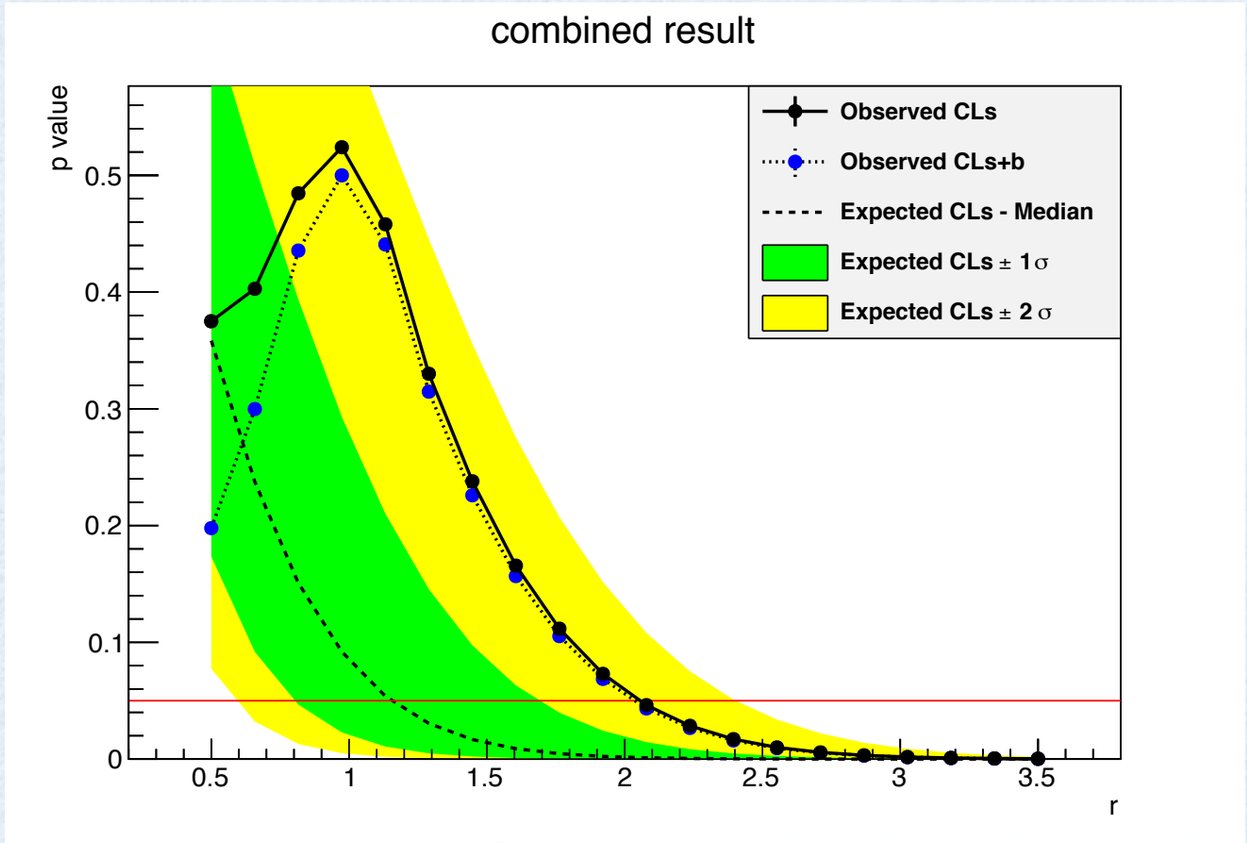


➡ see Cowan, Cranmer, Gross, Vitells, arXiv:1007.1727,EPJC 71 (2011) 1-1

HypoTestInverter

- **`AsymptoticCalculator`** class in RooStats
  - HypoTestCalculator class implementing the asymptotic formulae

```
// create first HypoTest calculator (N.B null is s+b model)
AsymptoticCalculator ac(*data, *bModel, *sbModel);

HypoTestInverter calc(*ac);
// run inverter same as using other calculators
........
```



RooStats Exercises (Part 2)

Getting Started

- all RooStats classes are in a namespace
- recommended to add at beginning of macro:
  - using namespace RooStats
- this will also load automatically the RooStats library
- note that RooStats methods start with upper case letter while RooFit start with lower case
- RooStats calculator are quite verbose, useful to suppress many info messages"

```
RooMsgService::instance().setGlobalKillBelow(RooFit::WARNING) ;
```

Roostats reference guide:

http://root.cern.ch/root/htmldoc/ROOSTATS_Index.html

RooStats tutorial macros:

http://root.cern.ch/root/html/tutorials/roostats

RooStats Exercise (7)

- Exercise 7:
  - Compute limit on the exponential background model (same as Ex. 6)
  - run SPlusBExpoModel.C
    - will create a file SPlusBExpoModel.root
  - run the HypoTestInverter using the frequentist calculator (use macro HypoTestInverterDemo.C)
    - look and try to understand the code of the macro
    - use the CL_s option (especially for nobs $\lesssim$ b)
      - use one-sided profile likelihood test statistics and CL_s
    - will take some time
      - use npoints = 5; number of points to scan
      - fc->SetToys(200,100);) number of toys
  - N.B. comment line toymcs->SetNEventsPerToy(1);
    - model is from an extended pdf

RooStats Exercise (7b)

- Exercise 7b:
  - run the asymptotic calculator
    - use same macro, HypoTestInverterDemo.C but create the AsymptoticCalculator instead of the FrequentistCalculator
      - uncomment relevant code in the macro
    - pass to the AsHypoTestInverter class to the HypoTestInverter constructor
  - look at how the result is plotted (HypoTestInverterPlot)
    - plot->Draw("obs"); plot only observed p-value
    - plot->Draw(); plot obs+expected limits with bands
    - plot->Draw("CLb 2CL"); plot CL_b, CL_{s+b}, CL_s

RooStats Exercises (8)

- Exercise 8
  - Use Poisson model (with background)
    - run macro PoissonModelWithBackg.C (generate model)
    - use possibly same values (nobs, b) used for Bayesian limit (exercise 5)
    - compute 95% upper limits using the HypoTestInverter class

(→ slide 6)

  - use macro HypoTestInverterDemo.C
    - look and try to understand the code of the macro
    - use the CL_s option (especially for nobs ≲ b)
      - use one-sided profile likelihood test statistics and CL_s
    - look at how the result is plotted (HypoTestInverterPlot)
      - plot->Draw("obs"); plot only observed p-value
      - plot->Draw(); plot obs+expected limits with bands
      - plot->Draw("CLb 2CL"); plot CL_b, CL_{s+b}, CL_s

RooStats Exercises (8)

- Option:
  - compute Feldman-Cousins 95% upper limit
    - use two-sided profile likelihood test statistics and CL_{s+b} instead of CL_s for p-value to scan
- Option:
  - use instead of Poisson simple Gauss model (e.g. with N=100 or N=1)
    - generate it with GaussianModel.C
    - when using FC and GaussianModel with N=1 you can check the result with FC paper
      http://arxiv.org/pdf/physics/9711021v2.pdf
      or google Feldman-Cousins

Solution

- use model created with previous exercises
- use macro HypoTestInverterDemo.C passing workspace name
  - comment/uncomment code depending on exercise
- can also use the tutorials/roostats/ StandardHypoTestInvDemo.C

StandardHypoTestInvDemo.C

- How does it work:
  - input workspace file, workspace name
  - name of S+B model (null) and for B model (alt)
    - if no B model is given, use S+B model with poi = 0
  - data set name
  - options:
    - calculator type (frequentist, hybrid, or asymptotic)
    - test statistics
    - use CL_s or CL_{s+b} for computing limit
    - number of points to scan and min, max of interval
- Example:

load the macro after having create the workspace using given macro (e.g. SPlusBExpoModel.root)

```
root[] .L StandardHypoTestInvDemo.C
```

run for CLs (with frequentist calculator (type = 0) and one-side PL test statistics (type = 3) scan 10 points in [0,100]

```
root[] StandardHypoTestInvDemo("SPlusBExpoModel.root","w","ModelConfig","","data",0,3, true, 10, 0, 100)
```

run for Asymptotic CLs (scan 20 points in [0,100])

```
root[] StandardHypoTestInvDemo(SPlusBExpoModel.root","w","ModelConfig","","data",2,3, true, 20, 0, 100)
```

run for Feldman-Cousins (scan 10 points in [0,100])

```
root[] StandardHypoTestInvDemo(SPlusBExpoModel.root","w","ModelConfig","","data",0,2, false, 10, 0, 15)
```

Documentation and user support

- RooStats TWiki: https://twiki.cern.ch/twiki/bin/view/RooStats/WebHome
- RooStats users guide (under development, to be completed)
  - http://root.cern.ch/viewcvs/branches/dev/roostats/roofit/roostats/doc/usersguide/RooStats_UsersGuide.pdf
- Paper: ACAT 2010 proceedings: http://arxiv.org/abs/1009.1003
- ROOT reference guide: http://root.cern.ch/root/htmldoc/ROOSTATS_Index.html
- RooFit and RooStats tutorial macros: http://root.cern.ch/root/html/tutorials
- RooFit's users guide: http://root.cern.ch/drupal/content/users-guide
- RooStats November tutorials:
  - Lecture of L. Lista on statistics: http://indico.cern.ch/conferenceDisplay.py?confId=73545
  - Tutorial contents: http://indico.cern.ch/conferenceDisplay.py?confId=72320
- RooStats user support:
  - Request support via ROOT talk forum: http://root.cern.ch/phpBB2/viewforum.php?f=15 (questions on statistical concepts accepted)
  - Submit bugs to ROOT Savannah: https://savannah.cern.ch/bugs/?func=additem&group=savroot
- Contacts for statistical questions:
  - ATLAS statistics forum: hn-atlas-physics-Statistics@cern.ch (Cowan, Gross et al)
    - TWiki: https://twiki.cern.ch/twiki/bin/view/AtlasProtected/StatisticsTools
  - CMS statistics committee: (Cousins, Demortier et al)
    - via hypernews: hn-cms-statistics@cern.ch or directly: cms-statistics-committee@cern.ch