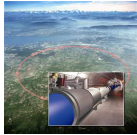


Towards a quantum treatment of leptogenesis

Mathias Garny (CERN)

September 25, 2014

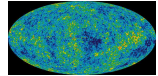
Physics beyond the Standard Model



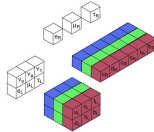
Collider exp.



Baryon
asymmetry

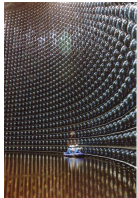


...



+ ?

...



Neutrino exp.



Dark matter



Leptogenesis

Standard Model (SM) extended by three heavy singlet neutrino fields $N_i = N_i^c$, $i = 1, 2, 3$ with Majorana masses $\hat{M} = \text{diag}(M_i)$ in the mass eigenbasis

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \bar{N} i \not{\partial} N - \frac{1}{2} \bar{N} \hat{M} N - \bar{\ell} \tilde{\phi} h P_R N - \bar{N} P_L h^\dagger \tilde{\phi}^\dagger \ell$$

Light neutrino masses via seesaw mechanism

$$m_\nu = -v_{EW}^2 h \hat{M}^{-1} h^T$$

$$0.06\text{eV} \lesssim \sum m_\nu \lesssim 0.23\text{eV} \quad \rightarrow \quad \text{TeV} \lesssim M_i \lesssim M_{GUT} \text{ for } m_e/v_{EW} < h_{ij} < 1$$

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Baryogenesis via leptogenesis

Fukugita, Yanagida 86

- ▶ B-violation via L-violating Majorana masses M_i
- ▶ CP-violation via Yukawa couplings $\text{Im}[(h^\dagger h)_{ij}] \neq 0$
- ▶ Out-of-equilibrium (inverse) decay $N_i \leftrightarrow \ell \phi^\dagger$ and $N_i \leftrightarrow \ell^c \phi$

$$(\Gamma_i/H)|_{T=M_i} \simeq \left. \frac{(h^\dagger h)_{ii} M_i / 8\pi}{1.66 g_* T^2 / M_{pl}} \right|_{T=M_i}$$

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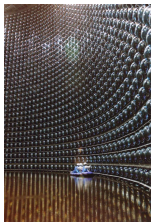
Baryogenesis via leptogenesis

Fukugita, Yanagida 86

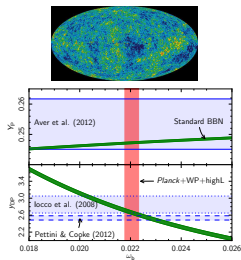
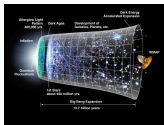
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Leptogenesis



$$m_\nu, \theta_{ij}, \delta_{CP}$$



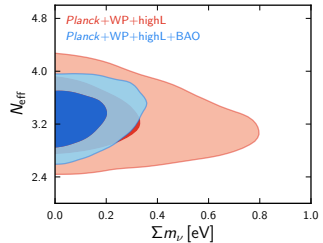
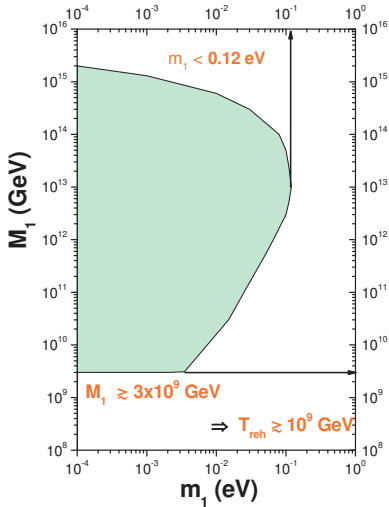
Planck XVI 1303.5076

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.2 \pm 0.15) \cdot 10^{-10}$$

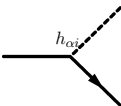
Relation between neutrino physics and baryon asymmetry depends on

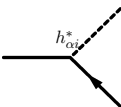
- ▶ Model building (seesaw, SO(10), ...)
- ▶ Microscopic theory for dynamics

Vanilla Leptogenesis



L-violating decay of heavy right-handed neutrino N_i

$$\mathcal{M}_{N_i \rightarrow \ell_\alpha \phi^\dagger} = \text{diagram} + \dots$$


$$\mathcal{M}_{N_i \rightarrow \ell_\alpha^c \phi} = \text{diagram} + \dots$$


L-violating decay of heavy right-handed neutrino N_i

$$\begin{aligned}\mathcal{M}_{N_i \rightarrow \ell_\alpha \phi^\dagger} &= \text{tree} + \text{loop} + \dots \\ \mathcal{M}_{N_i \rightarrow \ell_\alpha^c \phi} &= \text{tree} + \text{loop} + \dots\end{aligned}$$

$\Leftrightarrow$ interference of tree and **loop** processes

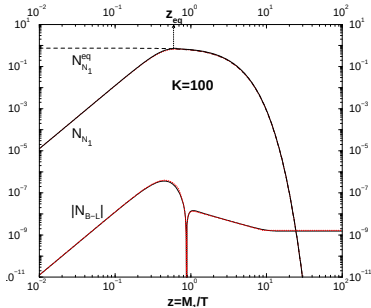
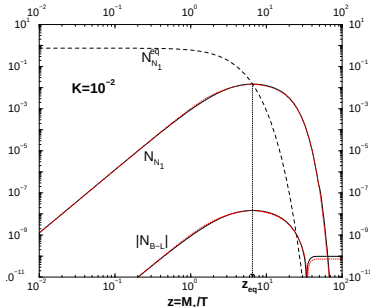
$$\begin{aligned}\epsilon_i &= \frac{\Gamma_{N_i \rightarrow \ell \phi^\dagger} - \Gamma_{N_i \rightarrow \ell^c \phi}}{\Gamma_{N_i \rightarrow \ell \phi^\dagger} + \Gamma_{N_i \rightarrow \ell^c \phi}} \\ &= \sum_{j \neq i} \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{8\pi(h^\dagger h)_{ii}} \left\{ \frac{M_j}{M_i} \left[1 - \left(1 + \frac{M_j^2}{M_i^2} \right) \ln \left(1 + \frac{M_i^2}{M_j^2} \right) \right] + \frac{M_i M_j}{M_i^2 - M_j^2} \right\} \\ \epsilon_1 &\lesssim 10^{-6} \frac{M_1}{10^{10} \text{GeV}} \frac{m_{\text{atm}}}{m_{\nu_1} + m_{\nu_3}} \quad \text{unless} \quad \Delta M_N \ll M_N \quad \text{Davidson, Ibarra 02}\end{aligned}$$

Rate equations

$$\frac{dN_{N_i}}{dt} = - \underbrace{\Gamma_{N_i}}_{\text{equilibration rate}} (N_{N_i} - N_{N_i}^{\text{eq}})$$

equilibration rate

$$\frac{dN_{B-L}}{dt} = \underbrace{\sum_i \Gamma_{\text{QP},i} (N_{N_i} - N_{N_i}^{\text{eq}})}_{\text{source term}} - \underbrace{\Gamma_W N_{B-L}}_{\text{washout term}}$$



$$K = (\Gamma/H)_{T=M_N} = \tilde{m}_\nu / \text{meV}$$

Standard Boltzmann approach

$$N_L(t) = \int d^3x \sqrt{|g|} \int \frac{d^3p}{(2\pi)^3} \sum_{\ell} [f_{\ell}(t, \mathbf{x}, \mathbf{p}) - f_{\bar{\ell}}(t, \mathbf{x}, \mathbf{p})]$$

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$$p^{\mu} \mathcal{D}_{\mu} f_{\ell}(t, \mathbf{x}, \mathbf{p}) = \sum_i \int d\Pi_{N_i} d\Pi_h$$

$$\times (2\pi)^4 \delta(p_{\ell} + p_h - p_{N_i})$$

$$\times \left[|\mathcal{M}|_{N_i \rightarrow \ell \phi^{\dagger}}^2 f_{N_i} (1 - f_{\ell}) (1 + f_{\phi}) + \dots \right. \\ \left. - |\mathcal{M}|_{\ell \phi^{\dagger} \rightarrow N_i}^2 f_{\ell} f_{\phi} (1 - f_{N_i}) + \dots \right]$$



(Naive) LO rates

- Equilibration $\mathcal{O}(h^2)$ $(N_i \leftrightarrow \ell\phi^\dagger)_{tree}$

$$\Gamma_{N_i} = \Gamma_{N_i}^0 \left\langle \frac{M_i}{E_i} \right\rangle_T \quad \text{where } \Gamma_{N_i}^0 = \frac{(h^\dagger h)_{ii} M_i}{8\pi}$$

- Source term $\mathcal{O}(h^4)$ $(N_i \leftrightarrow \ell\phi^\dagger)_{1-loop}, \quad (\ell\phi^\dagger \leftrightarrow \ell^c\phi)_{1-loop}|_{RIS}$

$$\Gamma_{CP,i} = \epsilon_i \Gamma_{N_i} \quad \epsilon_i = \frac{\Gamma_{N_i \rightarrow \ell\phi^\dagger} - \Gamma_{N_i \rightarrow \ell^c\phi}}{\Gamma_{N_i \rightarrow \ell\phi^\dagger} + \Gamma_{N_i \rightarrow \ell^c\phi}}$$

- Washout term $\mathcal{O}(h^2)$ $(\ell\phi^\dagger \rightarrow N_i)_{tree}$

$$\Gamma_W = \frac{8}{\pi^2} \left(c_\ell + \frac{c_\phi}{2} \right) N_{N_i}^{eq} \Gamma_{N_i} \quad \text{where } c_{\ell(\phi)} = -\frac{N_{\ell(\phi)}}{N_{B-L}}$$

Beyond the vanilla case

- ▶ Other models (type-II seesaw, susy, ... huge amount of literature)

see e.g. review Nir Nardi Davidson 0802.2962; Di Bari 1211.0512

- ▶ Check the mechanism (interference+nonequilibrium)

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- ▶ Check the mechanism (interference+nonequilibrium)
- ▶ Compute coefficients more precisely
 - ▶ Equilibration rate ($\leftrightarrow$ production rate at LO in N-Yukawa h)
 - ▶ Washout

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- ▶ Check the mechanism (interference+nonequilibrium)
- ▶ Compute coefficients more precisely
 - ▶ Equilibration rate ($\leftrightarrow$ production rate at LO in N-Yukawa h)
 - ▶ Washout
- ▶ Take qualitatively new effects into account (extend/scrutinize equations)
 - ▶ Active lepton flavor effects

$$\ell_\tau \leftrightarrow \tau_R \phi \quad \text{vs} \quad (h_{1e}\ell_e + h_{1\mu}\ell_\mu + h_{1\tau}\ell_\tau)\phi \leftrightarrow N_1$$

- ▶ N_i flavor effects (resonant enhancement, oscillations)

$$\epsilon_{N_i}^{\text{wave}} = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

- ▶ Partial flavor/spectator equilibration
- ▶ Kinetic non-equilibrium for RH neutrinos

Techniques

- Compute coefficients more precisely
 - (1) Add processes to Boltzmann eq. ($tN \leftrightarrow \ell\phi$)
Include thermal masses in kinematics
 - (2) Map rates to thermal correlation functions/suszeptibilities
Consistent expansion in g, y_t for $M_N/T \sim g, 1, \gg 1$
 $\rightarrow$ rate when kinematics would lead to thermal blocking,
cancellation of IR div. at $T \neq 0$

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 $\rightarrow$ *rate when kinematics would lead to thermal blocking, cancellation of IR div. at $T \neq 0$*
- Take qualitatively new effects into account (extend/scrutinize equations)
 - (1) Boltzmann eqs.

$$N_{B-L} \rightarrow N_{B-\frac{1}{3}L_\alpha} \quad N_{N_1} \rightarrow N_{N_i}$$

- (2) Density matrix equations

$$N_{B-L} \rightarrow \rho_{\alpha\beta} \quad N_{N_1} \rightarrow \rho_{ij}$$

- (3) Derive kinetic equations based on CTP/Kadanoff-Baym

$$N_{B-L} \rightarrow \langle \ell_\alpha \ell_\beta \rangle \quad N_{N_1} \rightarrow \langle N_i \bar{N}_j \rangle$$

$\rightarrow$ *basis-independent, derive structure of eqs. from 1st principles, no RIS needed*

Equilibration (production) rate

Boltzmann ('naive' NLO in SM couplings (g, y_t); LO in N-yukawa)

$$2 \leftrightarrow 2 \quad AN \leftrightarrow \ell h, tN \leftrightarrow \ell h, \quad A = ad_{SU(2) \times U(1)}$$

$$1 \leftrightarrow 2 \quad \text{vertex+wavefctn virtual}$$

$$1 \leftrightarrow 3 \quad N \leftrightarrow \ell h A, N \leftrightarrow \ell h t$$

→ blocking of $N \leftrightarrow \ell \phi$ for $M_N \ll T$ ('LO → 0'), instead $\phi \leftrightarrow N \ell$

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All orders in SM couplings, LO in N-yukawa,

$$\frac{d\Gamma_{N_1}}{d^3k} = \frac{1}{(2\pi)^3 2E_k} 2f_{FD}(E_k) \text{ImTr}[k \Sigma_R(k)]_{k=(E_k+i0^+, \mathbf{k})}$$

► $M_N \gg T$ NLO

RTF $\mathcal{O}(T^2/M_N^2)$ Lodone, Strumia 1106.2814;

ITF $\mathcal{O}(T^4/M_N^4)$ Laine 1209.2869;

NR-EFT Biondini, Brambilla, Escobedo, Vairo 1307.7680

► $M_N \sim T$ NLO

ITF Laine 1307.4909;

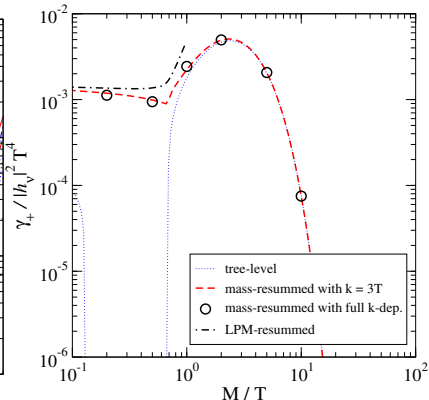
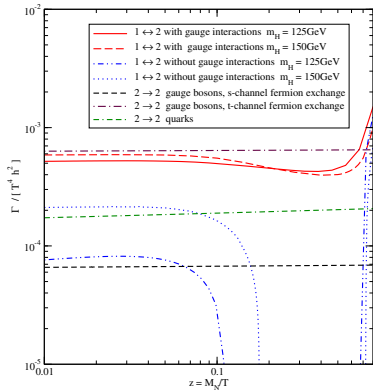
CTP Garbrecht, Glowna, Herranen 1302.0743

► $M_N \sim gT$ LO LPM $N \leftrightarrow \ell \phi, \phi \leftrightarrow N \ell, \ell \leftrightarrow N \phi \quad + \quad 2 \rightarrow 2$

Anisimov, Bodeker, Besak 1012.3784; *Besak Bodeker* 1208.1288;

Garbrecht, Glowna, Schwaller 1303.5498

Equilibration (production) rate



$$\Gamma_N / \Gamma_N^0 = 0 + g^2 f(\ln g)$$

$$\Gamma_N / \Gamma_N^0 = 1 + g^2 (a + bT^2/M^2 + \dots)$$

Besak Bodeker 1208.1288; Laine 1307.4909

cf. Garbrecht, Glowna, Herranen 1302.0743; Garbrecht, Glowna, Schwaller 1303.5498

Source Term - Double Counting Problem

Naive contribution from decay/inverse decay

$$|\mathcal{M}|_{N_i \rightarrow \ell \phi^\dagger}^2 = |\mathcal{M}_0|^2(1 + \epsilon_i) \quad |\mathcal{M}|_{\ell \phi^\dagger \rightarrow N_i}^2 = |\mathcal{M}_0|^2(1 - \epsilon_i)$$

$$|\mathcal{M}|_{N_i \rightarrow \ell^c \phi}^2 = |\mathcal{M}_0|^2(1 - \epsilon_i) \quad |\mathcal{M}|_{\ell^c \phi \rightarrow N_i}^2 = |\mathcal{M}_0|^2(1 + \epsilon_i)$$

$$\begin{aligned} \frac{dN_{B-L}}{dt} \propto & (|\mathcal{M}|_{N_i \rightarrow \ell \phi^\dagger}^2 - |\mathcal{M}|_{N_i \rightarrow \ell^c \phi}^2) N_{N_i} \\ & - (|\mathcal{M}|_{\ell \phi^\dagger \rightarrow N_i}^2 - |\mathcal{M}|_{\ell^c \phi \rightarrow N_i}^2) N_{N_i}^{eq} \end{aligned}$$

Source Term - Double Counting Problem

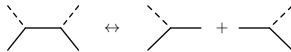
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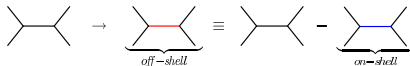
$$\begin{aligned} \frac{dN_{B-L}}{dt} &\propto (|\mathcal{M}|_{N_i \rightarrow \ell \phi^\dagger}^2 - |\mathcal{M}|_{N_i \rightarrow \ell^c \phi}^2) N_{N_i} \\ &\quad - (|\mathcal{M}|_{\ell \phi^\dagger \rightarrow N_i}^2 - |\mathcal{M}|_{\ell^c \phi \rightarrow N_i}^2) N_{N_i}^{eq} \\ &\propto \epsilon_i (N_{N_i} + N_{N_i}^{eq}) \end{aligned}$$

$\Rightarrow$ spurious generation of asymmetry even in equilibrium

Origin: Double Counting Problem



$\rightarrow$ Can be fixed by real intermediate state subtraction in $\ell \phi^\dagger \leftrightarrow \ell^c \phi$ (at least close to equilibrium)



Resonant enhancement

Flanz Paschos Sarkar 94/96; Covi Roulet Vissani 96; Covi Roulet 96

$$\epsilon_{N_i} = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times R, \quad R \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Regulator A determines maximal enhancement $R_{\max} = M_1 M_2 / A$

$$A^2 = \begin{cases} \frac{1}{4}(M_1 + M_2)^4 \left(\frac{\text{Re}(h^\dagger h)_{12}}{16\pi} \right)^2 \\ M_i^2 \Gamma_j^2 \\ (M_1 \Gamma_1 - M_2 \Gamma_2)^2 \\ \dots \end{cases}$$

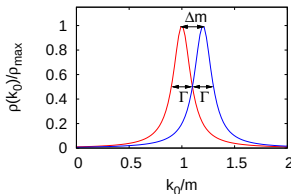
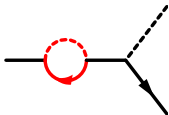
Flanz Paschos Sarkar Weiss 96

Pilaftsis 97; Pilaftsis Underwood 03

Buchmüller Plümacher 97;

Anisimov Broncano Plümacher 05; ...

Γ enters via finite width and dev. from equilibrium

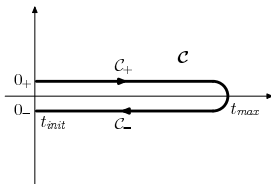


Closed time path / Schwinger-Keldysh / *in-in* formalism

$$\begin{aligned}\langle \mathcal{O}(t) \rangle &= \text{Tr} \left(\rho U_I(t_{init}, t) \mathcal{O}_I(t) U_I(t, t_{init}) \right) \\ &= \text{Tr} \left(\rho \tilde{T} \left[\exp \left(+i \int_{t_{init}}^t dt' H_I(t') \right) \right] \mathcal{O}_I(t) T \left[\exp \left(-i \int_{t_{init}}^t dt' H_I(t') \right) \right] \right)\end{aligned}$$

Closed time path / Schwinger-Keldysh / *in-in* formalism

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 \end{aligned}$$



$$\langle \mathcal{O}(t) \rangle = \text{Tr} \left(\rho T_C \left[\exp \left(+i \int_C dt' H_I(t') \right) \mathcal{O}_I(t) \right] \right)$$

Statistical propagator $S_F^{ij}(x, y) = \langle N_i(x) \bar{N}_j(y) - \bar{N}_j(y) N_i(x) \rangle / 2$

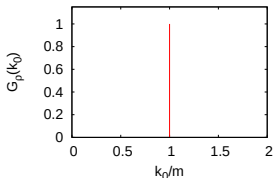
Spectral function $S_\rho^{ij}(x, y) = i \langle N_i(x) \bar{N}_j(y) + \bar{N}_j(y) N_i(x) \rangle$

Statistical propagator $S_F^{ij}(x, y) = \langle N_i(x) \bar{N}_j(y) - \bar{N}_j(y) N_i(x) \rangle / 2$

Spectral function $S_\rho^{ij}(x, y) = i \langle N_i(x) \bar{N}_j(y) + \bar{N}_j(y) N_i(x) \rangle$

Boltzmann limit

- ▶ on-shell quasi-stable particles



$$S_\rho^{ij}(k) \sim \delta^{ij} \delta(k^2 - m_i^2)$$

- ▶ equilibrium-like
fluctuation-dissipation relation

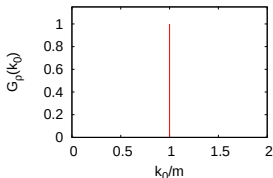
$$S_F^{ij}(t, k) = \left(\frac{1}{2} - f_k^i(t) \right) S_\rho^{ij}(k)$$

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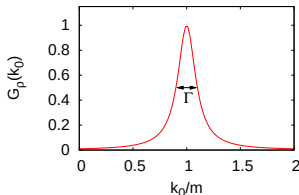
$$S_\rho^{ij}(k) \sim \delta^{ij} \delta(k^2 - m_i^2)$$

- ▶ equilibrium-like fluctuation-dissipation relation

$$S_F^{ij}(t, k) = \left(\frac{1}{2} - f_k^i(t) \right) S_\rho^{ij}(k)$$

More general

- ▶ spectrum with (thermal) width



$$S_\rho^{ij}(t, k) \propto \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \dots$$

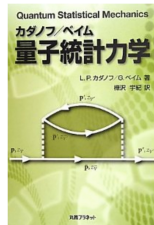
- ▶ coherent $N_1 - N_2$ transitions

$$S_F^{ij}(t, k) = \begin{pmatrix} S_F^{11} & S_F^{12} \\ S_F^{21} & S_F^{22} \end{pmatrix}$$

Kadanoff-Baym equations

$$\begin{aligned}
 ((i\partial_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik}) S_F^{kj}(x, y) &= \int_0^{x^0} dz^0 \int d^3z \Sigma_N^{ik}(x, z) S_F^{kj}(z, y) \\
 &\quad - \int_0^{y^0} dz^0 \int d^3z \Sigma_N^{ik}(x, z) S_F^{kj}(z, y) \\
 ((i\partial_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik}) S_\rho^{kj}(x, y) &= \int_{y^0}^{x^0} dz^0 \int d^3z \Sigma_N^{ik}(x, z) S_\rho^{kj}(z, y)
 \end{aligned}$$

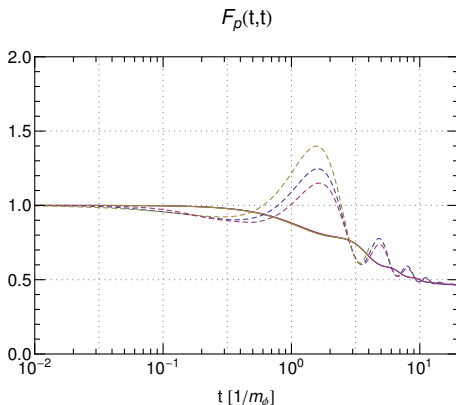
- ▶ **Statistical propagator** encodes time-evolution of the state
- ▶ **Spectral function** includes off-shell effects self-consistently
- ▶ Conserving, non-secular for $\Sigma = \delta\Gamma/\delta S$ (2PI); nPI



Kadanoff-Baym equations

Initial value problem in QFT ($\varphi\chi^2$, $F_p(t, t') = \int e^{ip(x-y)} \langle \{ \varphi(t, x) \varphi(t', y) \} \rangle$)

$$\begin{aligned} F_p(t, t')|_{t=t'=0} &= F_p^{vac} + n_p/\omega_p \\ \partial_t \partial_{t'} F_p(t, t')|_{t=t'=0} &= \partial_t \partial_{t'} F_p^{vac} + n_p \omega_p \end{aligned}$$



$\Lambda = 10, 25, 100 \cdot m_\phi$

MG, Reinos 14xx

- ▶ Study approach to equilibrium
- ▶ Transients
 $\propto \rho_p(t) \rho_p(t') \ln \Lambda$
 sensitive to UV cutoff Λ
- ▶ Hidden assumption:
 Gaussian (free) initial state
- ▶ Can be renormalized by
 extending KB eqs. to
 non-Gaussian initial
 state with three-point
 corr. $i\alpha_3 = i\alpha_3^{vac}$.

CTP/Kadanoff-Baym approach to leptogenesis

$$j_L^\mu(x) = \left\langle \sum_\alpha \bar{\ell}_\alpha(x) \gamma^\mu \ell_\alpha(x) \right\rangle = -\text{tr} \left[\gamma^\mu S_\ell^{\alpha\beta}(x, x) \right]$$

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Lepton asymmetry

$$n_L(t) = \frac{1}{V} \int_V d^3x j_L^0(t, \mathbf{x})$$

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$$n_L(t) = \frac{1}{V} \int_V d^3x j_L^0(t, \mathbf{x})$$

Equation of motion

$$\frac{dn_L}{dt} = \frac{1}{V} \int_V d^3x \partial_\mu j_L^\mu(x) = -\frac{1}{V} \int_V d^3x \text{tr} \left[\gamma_\mu (\partial_x^\mu + \partial_y^\mu) S_\ell^{\alpha\beta}(x, y) \right]_{x=y}$$

CTP/Kadanoff-Baym approach to leptogenesis

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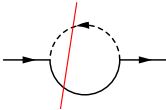
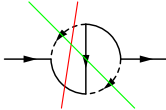
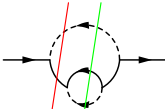
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Use KB equations for leptons on the right-hand side $\Rightarrow$

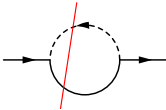
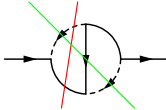
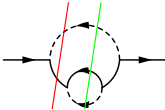
$$\begin{aligned} \frac{dn_L}{dt} = & i \int_0^t dt' \int \frac{d^3p}{(2\pi)^3} \text{tr} \left[\Sigma_{\ell_{\rho\mathbf{p}}}^{\alpha\gamma}(t, t') S_{\ell_{F\mathbf{p}}}^{\gamma\beta}(t', t) - \Sigma_{\ell_{F\mathbf{p}}}^{\alpha\gamma}(t, t') S_{\ell_{\rho\mathbf{p}}}^{\gamma\beta}(t', t) \right. \\ & \left. - S_{\ell_{\rho\mathbf{p}}}^{\alpha\gamma}(t, t') \Sigma_{\ell_{F\mathbf{p}}}^{\gamma\beta}(t', t) + S_{\ell_{F\mathbf{p}}}^{\alpha\gamma}(t, t') \Sigma_{\ell_{\rho\mathbf{p}}}^{\gamma\beta}(t', t) \right] \end{aligned}$$

CTP/Kadanoff-Baym approach to leptogenesis

			
$N \leftrightarrow \ell \phi^\dagger$ $N \leftrightarrow \ell^c \phi$	$ tree ^2$	tree $\times$ vertex-corr.	tree $\times$ wave-corr.
$\ell \phi^\dagger \leftrightarrow \ell^c \phi$		$s \times t$	$s \times s, t \times t$

- unified description of CP-violating **decay**, **inverse decay**, **scattering**

CTP/Kadanoff-Baym approach to leptogenesis

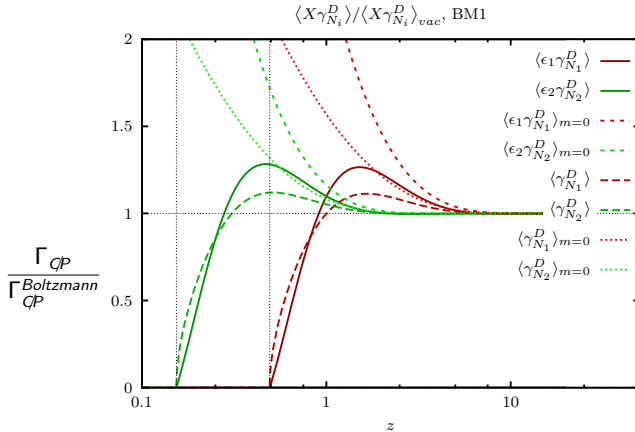
			
$N \leftrightarrow \ell \phi^\dagger$ $N \leftrightarrow \ell^c \phi$	$ tree ^2$	tree $\times$ vertex-corr.	tree $\times$ wave-corr.
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- ▶ unified description of CP-violating **decay**, **inverse decay**, **scattering**
- ▶ dn_L/dt vanishes in equilibrium due to KMS relations

$$S_F^{eq} = \frac{1}{2} \tanh\left(\frac{\beta k^0}{2}\right) S_\rho^{eq} \quad \Sigma_F^{eq} = \frac{1}{2} \tanh\left(\frac{\beta k^0}{2}\right) \Sigma_\rho^{eq}$$

$\Rightarrow$ consistent equations free of double-counting problems

Hierarchical limit $M_1 \lesssim M_{2,3}$



Bose enhancement at $M_N \sim T$, suppression for large T due to thermal masses

Frossard, MG, Hohenegger, Kartavtsev, Mitrouskas 12

Symmetry quantum statistics vs thermal loop corr. important for models where

$\epsilon^{vac} = 0$ *Garbrecht, Ramsey-Musolf 13, cf. also MG, Kartavtsev, Hohenegger, Lindner 09; Beneke,*

Garbrecht, Herranen, Schwaller 10; Anisimov, Buchmüller, Drewes, Mendizabal 11

Flavoured treatment for $N_{1,2}$

- ▶ Statistical propagator S_F and spectral function S_ρ are matrices in N_1, N_2, N_3 flavor space. We consider the sub-space N_1, N_2 of the quasi-degenerate states.

$$S^{ij}(x, y) = \langle T_C N_i(x) \bar{N}_j(y) \rangle = \begin{pmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{pmatrix}$$

$\Rightarrow$ coherent N_1 - N_2 transitions out-of-equilibrium

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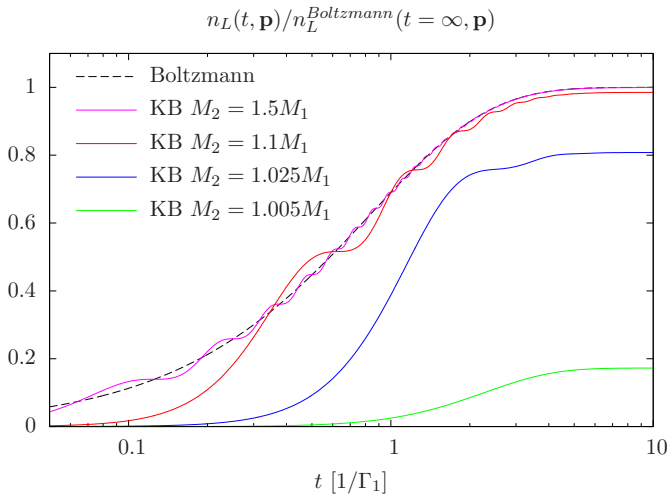
$\Rightarrow$ coherent N_1 - N_2 transitions out-of-equilibrium

- Self-energies for leptons and for Majorana neutrinos

$$\Gamma_2 = \text{diagram} \Rightarrow \underbrace{\text{diagram}}_{\Sigma_\ell^{\alpha\beta}(x, y) = \frac{\partial i\Gamma_2}{\partial S_\ell^{\beta\alpha}(y, x)}} \quad \underbrace{\text{diagram}}_{\Sigma_N^{ij}(x, y) = \frac{\partial i\Gamma_2}{\partial S^{ij}(y, x)}}$$

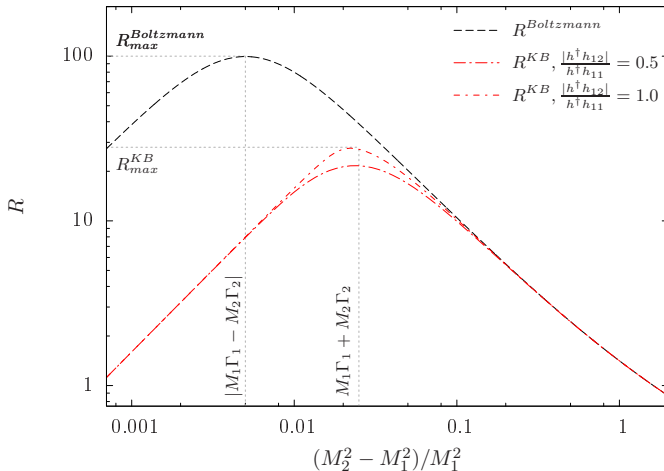
- Important: lepton self-energy contains full Majorana propagator-matrix

Resonant enhancement



$$\Gamma_1 = 0.01M_1, \Gamma_2 = 0.015M_1, \Gamma_{\ell\phi} \rightarrow 0$$

Resonant enhancement

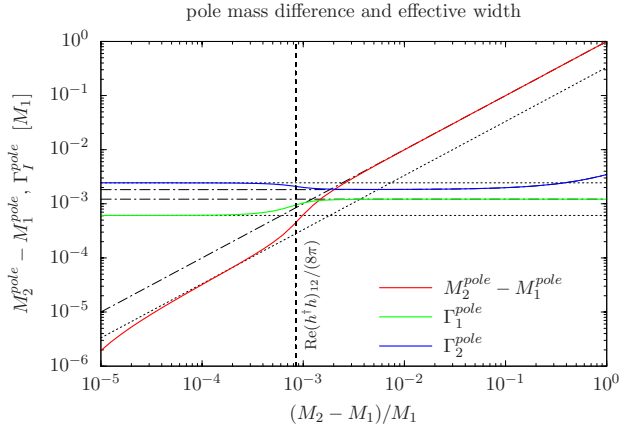


$$\Gamma_2/\Gamma_1 = 1.5$$

MG, Kartavtsev, Hohenegger *Annals Phys.* 328 (2013) 26

related works: Anisimov, Buchmüller, Drewes, Mendizabal *Annals Phys.* 326 (2011) 1998; Iso, Shimada, Yamanaka 1312.7680; Dev, Millington, Pilaftsis, Teresi 1403.1003; Iso Shimada 1404.4806; Garbrecht, Gautier, Klaric 1406.4190

BW approximation for spectral function



Effective masses $M_i^{pole} \equiv \omega_{pl}|_{p=0}$ and widths $\Gamma_i^{pole} \equiv \Gamma_{pl}|_{p=0}$ of the sterile Majorana neutrinos extracted from complex poles of resummed ret/adv prop. for $(h^\dagger h)_{11} = 0.03$, $(h^\dagger h)_{22} = 0.045$, $(h^\dagger h)_{12} = 0.03 \cdot e^{i\pi/4}$ and $T = 0.25M_1$.

Resonant enhancement

Resonant enhancement within the Boltzmann approach

$$R^{Boltzmann} \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

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Resonant enhancement within the Kadanoff-Baym approach, including coherent contributions ($|(h^\dagger h)_{12}| \ll (h^\dagger h)_{ii}$, BW approx.)

$$R^{KB}(t) = \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \times (1 - f_{coherent}(t))$$

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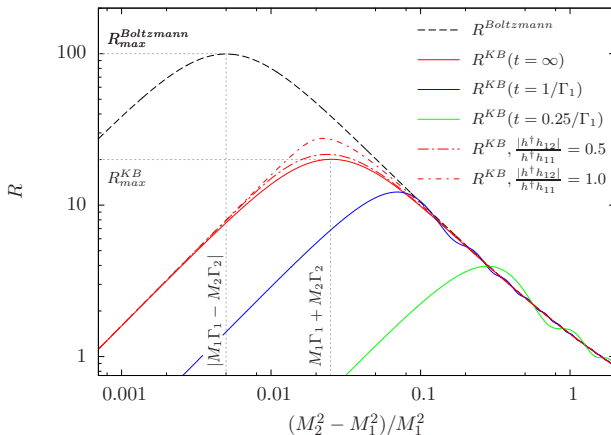
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Partial cancellation of Boltzmann- and coherent contribution cuts off the enhancement in the doubly degenerate limit $M_1 \rightarrow M_2$ and $\Gamma_1 \rightarrow \Gamma_2$

$$R^{KB}(t)|_{t \gtrsim 1/\Gamma_{pl}} \simeq \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 + M_2 \Gamma_2)^2}$$

Resonant enhancement



Numerical result for $|h^\dagger h_{12}| \sim h^\dagger h_{ii}$ vs BW approximation for $|h^\dagger h_{12}| \ll h^\dagger h_{ii}$

$$R^{Boltzmann}_{max} = M_1 M_2 / (2|\Gamma_1 M_1 - \Gamma_2 M_2|), \quad R^{KB}_{max} \simeq M_1 M_2 / (2(\Gamma_1 M_1 + \Gamma_2 M_2))$$

Conclusions

- ▶ Lots of progress in theory of leptogenesis
- ▶ Washout/Production NLO ($M \gtrsim T$), consistent LO ($M \sim gT$)
- ▶ CTP/Kadanoff-Baym helpful to check saturation of resonant enhancement and useful starting point for deriving (flavoured) kinetic equations
- ▶ Source term at finite T ?
- ▶ Bounds on M_N and m_ν ?
- ▶ Other models ?

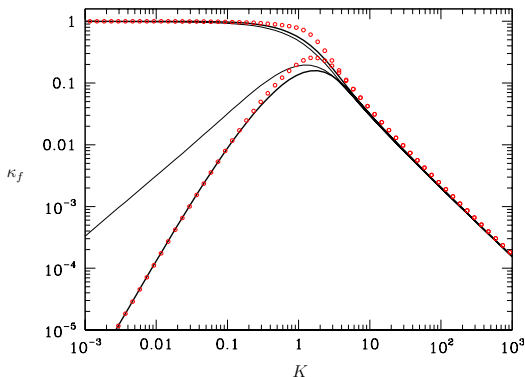
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thank you!

Final asymmetry

$$\eta = \frac{N_B}{N_\gamma} = a_{sph} \frac{N_{B-L}}{N_\gamma} = \frac{3}{4} \frac{a_{sph}}{f} \epsilon_1 \kappa_f = 0.0096 \epsilon_1 \kappa_f$$



$$K = (\Gamma_{N_1}/H)_{T=M_1} = \tilde{m}_1/\text{meV}$$

[N_1 -dominated, D+ID, unflavoured]

Buchmüller, Di Bari, Plümacher 04

$$a_{sph} = \frac{28}{79}, \quad f = \frac{N_\gamma^{rec}}{N_\gamma^{leptog}} = \frac{2387}{86}$$

Flavoured treatment for $N_{1,2}$

- First step: solve KBEs treating lepton and Higgs as a thermal bath (no backreaction); include qualitative damping term for lepton/Higgs (not essential, no consistent treatment of gauge-int. yet;)

MG, Kartavtsev, Hohenegger Annals Phys. 328 (2013) 26

[hierarchical case: Anisimov, Buchmüller, Drewes, Mendizabal Annals Phys. 326 (2011) 1998]

$$\begin{aligned}
 ((i\cancel{\partial}_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_F^{kj}(x, y) &= \int_0^{x^0} dz^0 \int d^3z \Sigma_{N_\rho}^{ik}(x, z)S_F^{kj}(z, y) \\
 &\quad - \int_0^{y^0} dz^0 \int d^3z \Sigma_{N_F}^{ik}(x, z)S_\rho^{kj}(z, y) \\
 ((i\cancel{\partial}_x - M_i)\delta^{ik} - \delta\Sigma_N(x)^{ik})S_\rho^{kj}(x, y) &= \int_{y^0}^{x^0} dz^0 \int d^3z \Sigma_{N_\rho}^{ik}(x, z)S_\rho^{kj}(z, y)
 \end{aligned}$$

see also Garbrecht, Herranen 1112.5954; Garbrecht Gautier Klaric 1406.4190 for approach with grad. exp.

Flavoured treatment for $N_{1,2}$

- Second step: Lepton asymmetry

$$\begin{aligned}
 n_L(t) &= i(h^\dagger h)_{ji} \int_0^t dt' \int_0^t dt'' \int \frac{d^3 p}{(2\pi)^3} \\
 &\quad \text{tr} \left[P_R \left(\underbrace{\delta S_{F\mathbf{p}}^{ij}(t', t'') - \delta \bar{S}_{F\mathbf{p}}^{ji}(t', t'')}_{\propto \text{Deviation from equilibrium, CP-violation}} \right) P_L S_{\ell\phi_{\rho\mathbf{p}}}(t'' - t') \right] \\
 &\propto \text{Deviation from equilibrium, CP-violation}
 \end{aligned}$$

$$\delta \bar{S}_{F\mathbf{p}}^{ji}(t', t'') = CP \delta S_{F\mathbf{p}}^{ij}(t'', t')^T (CP)^{-1}$$

$$\delta S = S - S_{th}$$

$$\text{lepton-Higgs loop } S_{\ell\phi} = S_\ell \Delta_\phi$$

Solution of KBE

Retarded and advanced propagators

$$\begin{aligned}S_R(x, y) &= \Theta(x^0 - y^0) S_\rho(x, y) \\S_A(x, y) &= -\Theta(y^0 - x^0) S_\rho(x, y)\end{aligned}$$

The Kadanoff-Baym equation for the statistical propagator can be written as

$$\begin{aligned}\int_0^\infty d^4 z \left[\left((i \not{\partial}_x - M_i) \delta^{ik} - \delta \Sigma_N^{ik}(x) \right) \delta(x - z) - \Sigma_{NR}^{ik}(x, z) \right] S_F^{kj}(z, y) \\= \int_0^\infty d^4 z \Sigma_{NF}^{ik}(x, z) S_A^{kj}(z, y)\end{aligned}$$

Special solution of the inhomogeneous equation

$$S_F^{ij}(x, y)_{inhom} = - \int_0^\infty d^4 u S_R^{ik}(x, u) \int_0^\infty d^4 z \Sigma_{NF}^{kl}(u, z) S_A^{lj}(z, y)$$

General solution of the homogeneous equation

$$S_F^{ij}(x, y)_{hom} = - \int d^3 u S_R^{ik}(x, (0, \mathbf{u})) \int d^3 v A^{kl}(\mathbf{u}, \mathbf{v}) S_A^{lj}((0, \mathbf{v}), y)$$

Solution of KBE

Instantaneous excitation of $N_{1,2}$ at $t = 0$, lepton/Higgs = thermal bath

$$S_{F\mathbf{p}}^{ij}(t, t') = S_{F\mathbf{p}}^{ij\,th}(t - t') + \underbrace{S_{R\mathbf{p}}^{ik}(t)\gamma_0\delta S_{F\mathbf{p}}^{kl}(0, 0)\gamma_0 S_{A\mathbf{p}}^{lj}(-t')}_{\equiv \delta S_{F\mathbf{p}}^{ij}(t, t')}$$

Resummed retarded and advanced propagators

$$\begin{aligned} \left((i\cancel{\partial}_x - M_i) \delta^{ik} - \delta\Sigma_N^{ik}(x) \right) S_R^{ik}(x, y) &= i\gamma_0\delta(x - y) \\ &+ \int_{y^0}^{x^0} dz^0 \int d^3z \Sigma_{N\rho}^{ik}(x, z) S_R^{kj}(z, y) \end{aligned}$$

$$\Sigma_{N\rho}^{ij}(x, y) = -2 \left[(h^\dagger h)_{ij} P_L + (h^\dagger h)_{ji} P_R \right] S_{\ell\phi\rho}(x, y)$$

$$S_{\ell\phi\rho}(x, y) = S_{\ell F}(x, y)\Delta_{\phi\rho}(x, y) + S_{\ell\rho}(x, y)\Delta_{\phi F}(x, y)$$

BW approximation

Solve SD equation for $S_{R(A)}(p)$ in Breit-Wigner approximation:

$$S_{R(A)}^{ij}(p) \simeq \frac{Z_{1R(A)}^{ij}}{p^2 - x_1} + \frac{Z_{2R(A)}^{ij}}{p^2 - x_2}$$

with residua $Z_{IR(A)}^{ij}$ and complex poles x_I (basis independent)

$$x_{1,2} = \frac{(V \pm W)^2}{4Q^2} \equiv \left(\omega_{pl} - i \frac{\Gamma_{pl}}{2} \right)^2 - \mathbf{p}^2$$

where

$$\begin{aligned} V &= \sqrt{(\eta_1 M_1 (1 + i\gamma_{22}) - \eta_2 M_2 (1 + i\gamma_{11}))^2 - 4\eta_1 \eta_2 M_1 M_2 (\text{Re}\gamma_{12})^2} \\ W &= \sqrt{(\eta_1 M_1 (1 + i\gamma_{22}) + \eta_2 M_2 (1 + i\gamma_{11}))^2 + 4\eta_1 \eta_2 M_1 M_2 (\text{Im}\gamma_{12})^2} \\ Q &= \det \Omega_{LR} = \det \Omega_{RL} = (1 + i\gamma_{11})(1 + i\gamma_{22}) + |\gamma_{12}|^2 \end{aligned}$$

$$\gamma_{ij} \simeq (h^\dagger h)_{ij} \left[\frac{\Theta(p^2) \text{sign}(p_0)}{16\pi} \left(1 + \frac{2}{e^{|p_0|/T} - 1} \right) + i \left(\frac{\ln \frac{|p^2|}{\mu^2}}{16\pi^2} + \frac{T^2}{6p^2} - \frac{T^2}{6\mu^2} \right) \right]$$

BW approximation

- Regime $(M_2 - M_1)/M_1 \gtrsim \text{Re}(h^\dagger h)_{12}/(8\pi)$

$$M_i^{pole} \simeq M_i ,$$

$$\Gamma_i^{pole} \simeq \Gamma_i \equiv \frac{(h^\dagger h)_{ii}}{8\pi} M_i \left(1 + \frac{2}{e^{M_i/T} - 1} \right)$$

- Regime $(M_2 - M_1)/M_1 \lesssim \text{Re}(h^\dagger h)_{12}/(8\pi)$

$$M_i^{pole} \simeq \frac{M_1 + M_2}{2} \pm \frac{(M_2 - M_1)((h^\dagger h)_{22} - (h^\dagger h)_{11})}{2\sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\text{Re}(h^\dagger h)_{12})^2}} ,$$

$$\Gamma_i^{pole} \simeq \frac{M_i}{16\pi} \left(1 + \frac{2}{e^{M_i/T} - 1} \right) \left((h^\dagger h)_{11} + (h^\dagger h)_{22} \right. \\ \left. \pm \sqrt{((h^\dagger h)_{22} - (h^\dagger h)_{11})^2 + 4(\text{Re}(h^\dagger h)_{12})^2} \right)$$

The relation between the mass- and Yukawa coupling matrices at zero and finite temperature is

$$M(T) = (P_L Z(T)^T + P_R Z(T)^\dagger) M(T=0) (P_L Z(T) + P_R Z(T)^*) ,$$

$$(h^\dagger h)(T) = Z(T)^T (h^\dagger h)(T=0) Z(T)^* ,$$

where $Z_{ij}(T) \equiv V_{ik}(T)(\delta_{kj} + (h^\dagger h)_{kj} T^2/(6\mu^2))$, $V(T)^\dagger V(T) = 1$

Result for the lepton asymmetry

$$n_L(t) = \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 2q 2k} (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \sum_{I,J=1,2} \sum_{\epsilon_n=\pm 1} \mathbf{F}_{JI}^{\epsilon_n} L_{IJ}^{\epsilon_n}(t)$$

- Coefficients $\mathbf{F}$ depend on Yukawa couplings, thermal distributions of lepton and Higgs, resummed ret/adv propagators and initial conditions

$$\begin{aligned} \mathbf{F}_{JI}^{\epsilon_n} = & \sum_{ijkl=1,2} (h^\dagger h)_{ji} \left(\left(\frac{1}{2} + f_\phi(q) \right) + \epsilon_2 \epsilon_3 \left(\frac{1}{2} - f_\ell(k) \right) \right) \text{tr} \left[P_L(|\mathbf{k}| \gamma_0 + \epsilon_2 \mathbf{k} \gamma) \right. \\ & \times \left(S_{RI}^{ik\epsilon_4} \gamma_0 \Delta S_{F\mathbf{p}}^{kl}(0,0) \gamma_0 S_{AJ}^{lj\epsilon_1} - \bar{S}_{RI}^{ik\epsilon_4} \gamma_0 \Delta \bar{S}_{F\mathbf{p}}^{kl}(0,0) \gamma_0 \bar{S}_{AJ}^{lj\epsilon_1} \right) \left. \right] \end{aligned}$$

Result for the lepton asymmetry

$$n_L(t) = \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 2q 2k} (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \sum_{l,J=1,2} \sum_{\epsilon_n=\pm 1} \mathbf{F}_{jl}^{\epsilon_n} L_{lj}^{\epsilon_n}(t)$$

- Coefficients $\mathbf{F}$ depend on Yukawa couplings, thermal distributions of lepton and Higgs, resummed ret/adv propagators and initial conditions

$$\begin{aligned} \mathbf{F}_{jl}^{\epsilon_n} = & \sum_{ijkl=1,2} (h^\dagger h)_{ji} \left(\left(\frac{1}{2} + f_\phi(q) \right) + \epsilon_2 \epsilon_3 \left(\frac{1}{2} - f_\ell(k) \right) \right) \text{tr} \left[P_L (|\mathbf{k}| \gamma_0 + \epsilon_2 \mathbf{k} \gamma) \right. \\ & \left. \times \left(S_{Rl}^{ik\epsilon_4} \gamma_0 \Delta S_{F\mathbf{p}}^{kl}(0,0) \gamma_0 S_{AJ}^{lj\epsilon_1} - \bar{S}_{Rl}^{jk\epsilon_4} \gamma_0 \Delta \bar{S}_{F\mathbf{p}}^{kl}(0,0) \gamma_0 \bar{S}_{AJ}^{li\epsilon_1} \right) \right] \end{aligned}$$

- Time-dependence: flavor diagonal and off-diagonal contributions:

$$\begin{aligned} L_{ll}^\pm(t) &= \frac{1 - e^{-\Gamma_{pl}t}}{\Gamma_{pl}} \text{Re} \left(\frac{\Gamma_{\ell\phi}}{(\omega_{pl} - k - q + i\Gamma_{pl}/2)^2 + \Gamma_{\ell\phi}^2/4} \right) + \mathcal{O}(\Gamma_{pl}^0) \\ L_{21}^\pm(t) &= \frac{1 - e^{\mp i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} \pm 2i(\omega_{p1} - \omega_{p2})} \text{Re} \left(\frac{\Gamma_{\ell\phi}}{(\omega_{p1} - k - q \pm i\Gamma_{p1}/2)^2 + \Gamma_{\ell\phi}^2/4} \right) \end{aligned}$$

Hierarchical limit

$$n_L(t) = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1}{M_2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_{p1} 2q 2k} 4k \cdot p_1$$

$$(2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \times \text{Re} \frac{\Gamma_{\ell\phi}}{(\omega_{p1} - k - q + i\Gamma_{p1}/2)^2 + \Gamma_{\ell\phi}^2/4}$$

$$\times (1 + f_\phi(q) - f_\ell(k)) f_{FD}(\omega_{p1}) \frac{1 - e^{-\Gamma_{p1}t}}{\Gamma_{p1}}$$

$$n_L^{\text{Boltzmann}}(t) = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1}{M_2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_{p1} 2q 2k} 4k \cdot p_1$$

$$(2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \times 2\pi \delta(\omega_{p1} - k - q)$$

$$\times (1 + f_\phi(q) - f_\ell(k)) f_{FD}(\omega_{p1}) \frac{1 - e^{-\Gamma_{p1}t}}{\Gamma_{p1}} .$$

The 'width' of lepton and Higgs $\Gamma_{\ell\phi} = \Gamma_\ell + \Gamma_\phi$ leads to a replacement of the on-shell delta function in the Boltzmann equations by a Breit-Wigner curve, in accordance with *Anisimov, Buchmüller, Drewes, Mendizabal Annals Phys. 326 (2011) 1998*

The coherent contributions are suppressed with Γ_{p1}/ω_{p2}

Degenerate case

Analytical result for $\text{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$\begin{aligned}
 n_L(t) = & \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p \\
 & \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
 & \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4 \text{Re} \left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 n_L^{\text{Boltzmann}}(t) = & \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p \\
 & \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) 2\pi \delta(\omega_p - k - q) (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
 & \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]
 \end{aligned}$$

Degenerate case

Analytical result for $\text{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$\begin{aligned}
 n_L(t) = & \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p \\
 & \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
 & \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4 \text{Re} \left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 n_L^{\text{Boltzmann}}(t) = & \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p \\
 & \times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) 2\pi \delta(\omega_p - k - q) (1 + f_\phi - f_\ell) f_{FD}(\omega_p) \\
 & \times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]
 \end{aligned}$$

► Regulator $M_1 \Gamma_1 - M_2 \Gamma_2$ is confirmed

Degenerate case

Analytical result for $\text{Re}(h^\dagger h)_{12} \ll (h^\dagger h)_{ii}$ (mass basis $\sim$ 'int. basis')

$$n_L(t) = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p$$

$$\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) \frac{\Gamma_{\ell\phi}}{(\omega_p - k - q)^2 + \Gamma_{\ell\phi}^2/4} (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$

$$\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} - 4 \text{Re} \left(\frac{1 - e^{-i(\omega_{p1} - \omega_{p2})t} e^{-(\Gamma_{p1} + \Gamma_{p2})t/2}}{\Gamma_{p1} + \Gamma_{p2} + 2i(\omega_{p1} - \omega_{p2})} \right) \right]$$

$$n_L^{\text{Boltzmann}}(t) = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi} \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + (M_1 \Gamma_1 - M_2 \Gamma_2)^2} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 \omega_p 2q 2k} 4k \cdot p$$

$$\times (2\pi)^3 \delta(\mathbf{p} - \mathbf{k} - \mathbf{q}) 2\pi \delta(\omega_p - k - q) (1 + f_\phi - f_\ell) f_{FD}(\omega_p)$$

$$\times \left[\sum_{l=1,2} \frac{1 - e^{-\Gamma_{pl} t}}{\Gamma_{pl}} \right]$$

- ▶ Regulator $M_1 \Gamma_1 - M_2 \Gamma_2$ is confirmed
- ▶ Additional oscillating contribution due to **coherent $N_1 \text{---} N_2$ transitions**

Flavoured leptogenesis

$$\mathcal{L}_{int} = -\bar{\ell}_a \tilde{\phi} h_{ai} P_R N_i - y_{ab} \bar{e}_{R,a} \phi \ell_b \quad y_{ab} = \text{diag}(m_e m_\mu m_\tau)/v_{EW}$$

$$q_\ell^{ab} = (\delta n_\ell^+ - \delta n_\ell^-)^{ab}, \quad \delta n_\ell^{ab} = \int \frac{d^3 k}{(2\pi)^3} (f_\ell^{ab} - f_{\ell,eq}^{ab}) = \mu_{ab} \frac{T^2}{12}, \quad \Xi = \dot{U} U^\dagger$$

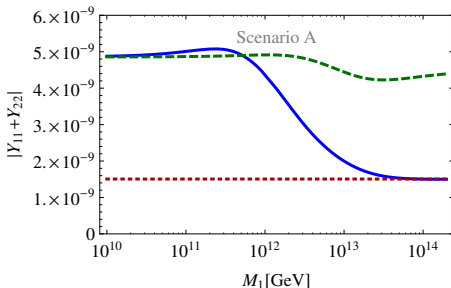
$$\begin{aligned} \partial_t q_\ell + 3H q_\ell &= S + [\Xi, q] - \{W, q\} - \Gamma_{LR}(y^\dagger y q_\ell + q_\ell^\dagger y^\dagger y - y^\dagger q_R y - y^\dagger q_R^\dagger y) \\ \partial_t q_R + 3H q_R &= -\Gamma_{LR}(y y^\dagger q_R + q_R^\dagger y y^\dagger - y q_\ell y^\dagger - y q_\ell^\dagger y^\dagger) \end{aligned}$$

Beneke, Garbrecht, Fidler, Herranen, Schwaller 1007.4783

- ▶ Γ_{LR} -term decohere $a \neq b$ terms in $y=\text{diag}$ basis and equilibrate $q_{\ell,ab}, q_{R,ab}$
- ▶ Gauge interactions impose $\delta n^+ + \delta n^- = 0 \Rightarrow$ no oscillations

Flavoured leptogenesis

- ▶ Unflavoured $\Gamma_{LR} \ll H \Rightarrow$ project on flavor that couples to N_1
- ▶ Flavoured $\Gamma_{LR} \gg H \Rightarrow$ project on flavor that couples to τ (and $\perp$ or e, μ)
- ▶ Full $\Gamma_{LR} \sim H$, off-diagonal comp. of q_ℓ^{ab} important



Flavoured leptogenesis

Source term $N_1 \rightarrow \ell_a \phi$ ($\mathcal{O}(h^4)$)

$$S_{ab} = \sum_j \left[\underbrace{(h_{a1} h_{1c}^T h_{cj}^* h_{jb} - h_{b1}^* h_{1c}^\dagger h_{cj} h_{ja}^*)}_{\text{Tr} \propto \epsilon_1} \underbrace{S^{LNV}}_{\propto M_1 M_j} + \underbrace{(h_{a1} h_{1c}^\dagger h_{cj} h_{jb} - h_{b1}^* h_{1c}^T h_{cj}^* h_{ja}^*)}_{\text{Tr} = 0} \underbrace{S^{LFV}}_{\propto T^2} \right]$$

Garbrecht, Drewes 12

Washout $\ell_a \phi \rightarrow N_1$

$$W_{ab} \propto h_{a1} h_{1a}^\dagger$$

- ▶ Potentially large effects for ultrarelativistic N 's, which decay after sphaleron freeze-out *Garbrecht, Drewes 12, cf. also Pilaftsis et. al. 14*
- ▶ Use freedom in h_{ai} to 'hide' asymmetry from washout when $N_{1,2,3}$ have comparable masses $\Rightarrow$ possibility of GeV-scale leptogenesis w/o need for resonant enhancement *Garbrecht, Drewes 14*

Two strategies

1. Derive kinetic equations

$$S(t, k) = \int ds e^{iks} S(\textcolor{blue}{t} + \textcolor{red}{s}/2, \textcolor{blue}{t} - \textcolor{red}{s}/2)$$

$$\text{Gradient expansion } \partial_t \partial_k \sim \frac{\textcolor{blue}{slow}}{\textcolor{red}{fast}} \sim \frac{\textcolor{blue}{\Gamma, H, y^2 T, \Delta M}}{\textcolor{red}{M, T}}$$

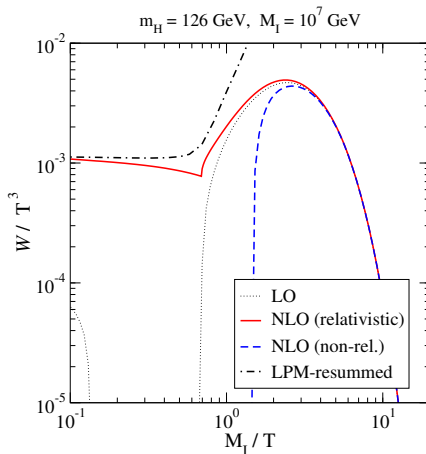
$$\int dz \Sigma(x, z) S(z, y) \rightarrow \Sigma(t, k) S(t, k) + \frac{i}{2} \left(\frac{\partial \Sigma}{\partial t} \frac{\partial S}{\partial k} - \frac{\partial \Sigma}{\partial k} \frac{\partial S}{\partial t} \right)$$

On-shell limit

$$\begin{aligned} S_{\rho}^{ij}(t, k) &\rightarrow U^{in}(t) \delta^{nm} (\not{k} - M_n) \delta(k^2 - M_n^2(t)) U^{\dagger nj}(t) \\ &\rightarrow \delta^{ij} (\not{k} - M_{av}) \delta(k^2 - M_{av}^2) \\ S_F^{ij}(t, k) &\rightarrow \left(\frac{1}{2} \delta^{ij} \delta_{hh'} - f_{hh'}^{ij}(t, \mathbf{k}) \right) u_h(\mathbf{k}) \bar{u}_{h'}(\mathbf{k}) \delta(k^2 - M_{av}^2) \end{aligned}$$

2. Solve two-time KB-eqs. for some simplified setup to study generation of the asymmetry (thermal bath)

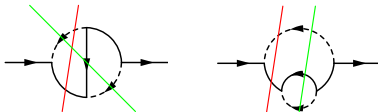
Washout rate



Washout: corr. start already at $\mathcal{O}(g)$ (leading corr. to rel. between μ and n)

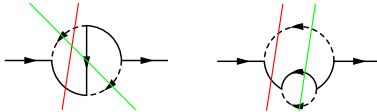
Hierarchical limit $M_1 \lesssim M_{2,3}$

$$\begin{aligned} \partial_t n_L &= 16\pi (h^\dagger h)_{11} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 8\omega_p k^0 q^0} \Theta(p_0) (2\pi)^4 \delta(k - p - q) \\ &\times (f_N(p)(1 - f_\ell(k))(1 + f_\phi(q)) - (1 - f_N(p))f_\ell(k)f_\phi(q)) \\ &\times \epsilon_1(t, k, p, q) \end{aligned}$$



Hierarchical limit $M_1 \lesssim M_{2,3}$

$$\begin{aligned} \partial_t n_L &= 16\pi (h^\dagger h)_{11} \int \frac{d^3 p d^3 q d^3 k}{(2\pi)^9 8\omega_p k^0 q^0} \Theta(p_0) (2\pi)^4 \delta(k - p - q) \\ &\times (f_N(p)(1 - f_\ell(k))(1 + f_\phi(q) - (1 - f_N(p))f_\ell(k)f_\phi(q)) \\ &\times \epsilon_1(t, k, p, q) \end{aligned}$$



$$\begin{aligned} \epsilon_i^{\text{vertex}}(t, k, p, q) &= \sum_j \frac{\text{Im}(h^\dagger h)_{ij}^2}{8\pi(h^\dagger h)_{11}} \int d\Pi_{k_1} d\Pi_{k_2} d\Pi_{k_3} \\ &\times (2\pi)^4 \delta(k + k_1 + k_2) (2\pi)^4 \delta(k_2 - k_3 + q) \\ &\times \text{tr}[D_\rho^\ell(t, k_1) D_F^\phi(t, k_2) D_h^{N_j}(t, k_3) + \{k_1 \leftrightarrow k_2\} \\ &+ D_h^\ell(t, k_1) D_F^\phi(t, k_2) D_\rho^{N_j}(t, k_3) + \{k_1 \leftrightarrow k_2\} \\ &+ D_\rho^\ell(t, k_1) D_F^\phi(t, k_2) D_h^{N_j}(t, k_3) - \{k_1 \leftrightarrow k_2\}] \end{aligned}$$

$\Rightarrow$ consistent equations w/o need for RIS subtractions

works also for arbitrary $f \neq f_{eq}$