

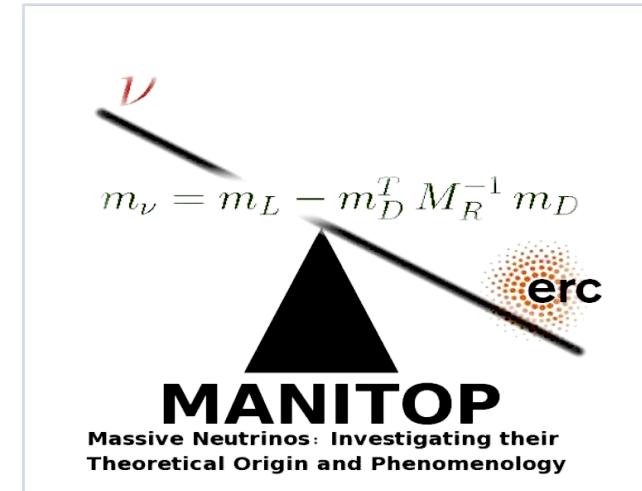


# Sterile Neutrinos for Warm Dark Matter and the Reactor Anomaly in Flavour Symmetry Models

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FLASY12, Dortmund, 2 July 2012



Based on JB, W. Rodejohann, H. Zhang,  
JHEP 1107, 091 (2011); JCAP 1201, 052 (2012)





# Outline

- Motivations for keV & eV scale sterile neutrinos
  - ◆ *keV WDM*
  - ◆ *SBL anomalies,  $N_{\text{eff}}$  from cosmology*
- $0\nu\beta\beta$  phenomenology
- Sterile neutrinos in  $A_4 \times Z_3 \times U(1)_{\text{FN}}$  flavour models
  - ◆ *Effective model*
  - ◆ *Seesaw model*
- NLO corrections and  $\theta_{13}$





# Quick recap: why “sterile”?

- Z-boson decay width means  $N_{\text{active}} = 3$ , i.e. only three active neutrinos with  $m_\nu < m_Z/2$
- Sterile/singlet/right-handed neutrino is SU(2) singlet
  - ◆ *Only interacts via mixing with active sector*
  - ◆ *Can interact with Higgs and/or BSM physics*
- Mass of sterile neutrino is unprotected and can be large (GUT scale) or small (pseudo-Dirac case)
- From phenomenology alone: “low-energy seesaw”?

(de Gouvêa et al)





# WDM and keV sterile neutrinos

- Standard  $\Lambda$ CDM cosmological model:
  - ◆ *non-relativistic DM with WIMP DM candidate*
- keV WDM also compatible with observations, solves problems with small scale structure
  - ◆ *Reduces number of Dwarf satellite galaxies*
  - ◆ *Smoothes cusps in DM halos*
- Sterile neutrino is good candidate
  - ◆ *Appears in usual seesaw model*
  - ◆ *Need a mechanism to produce correct relic abundance, e.g. Dodelson-Widrow scenario:*

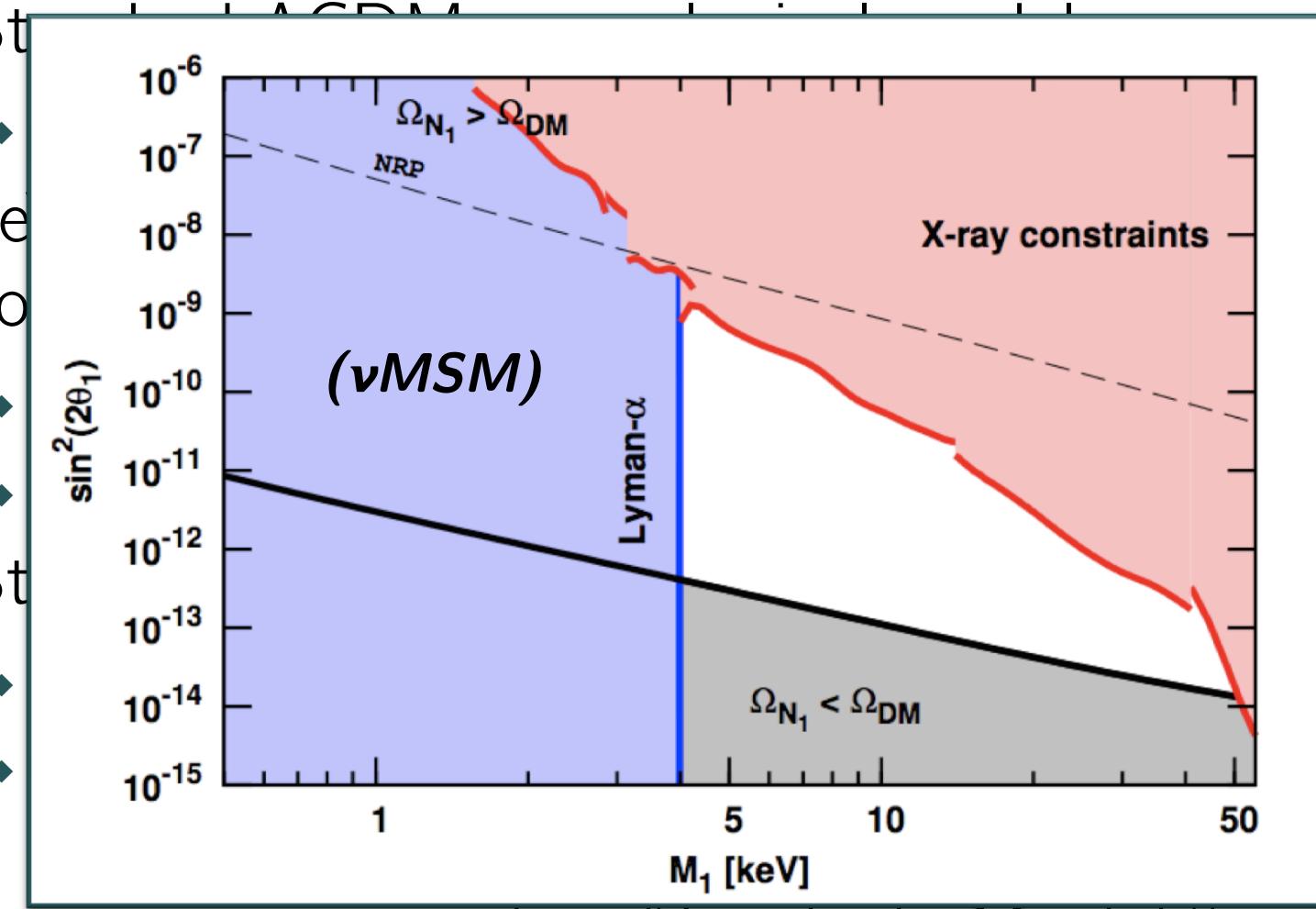
$$\Omega_{\text{DM}} \simeq 0.2 \left( \frac{\theta_s^2}{3 \times 10^{-9}} \right) \left( \frac{M_s}{3 \text{ keV}} \right)^{1.8}$$





# WDM and keV sterile neutrinos

- Standard Model
- keV sterile neutrinos
- SO
- Standard Model
- Sterile neutrino mass



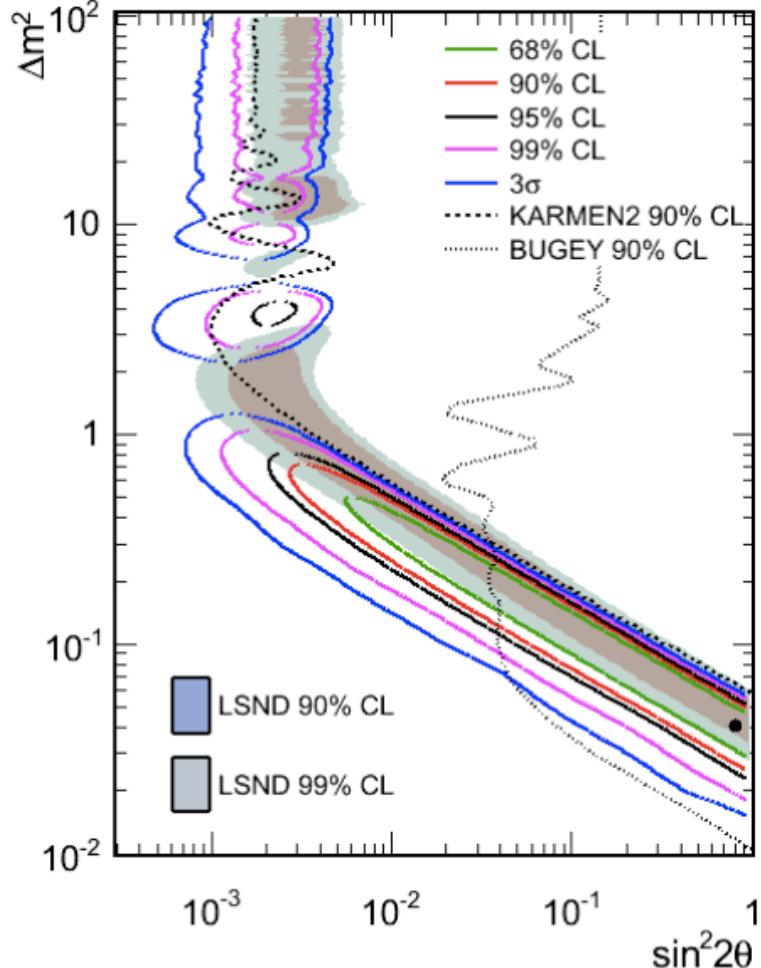
Bezrukov 2011

ence,

$$\Omega_{DM} \simeq 0.2 \left( \frac{\theta_s^2}{3 \times 10^{-9}} \right) \left( \frac{M_s}{3 \text{ keV}} \right)$$



# eV-scale sterile neutrinos



Polly, Neutrino 2012

- LSND

- ◆  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  ( $3.8\sigma$ )
- ◆  $\Delta m^2 \simeq 1 \text{ eV}^2$

⇒ 3+1 scenario

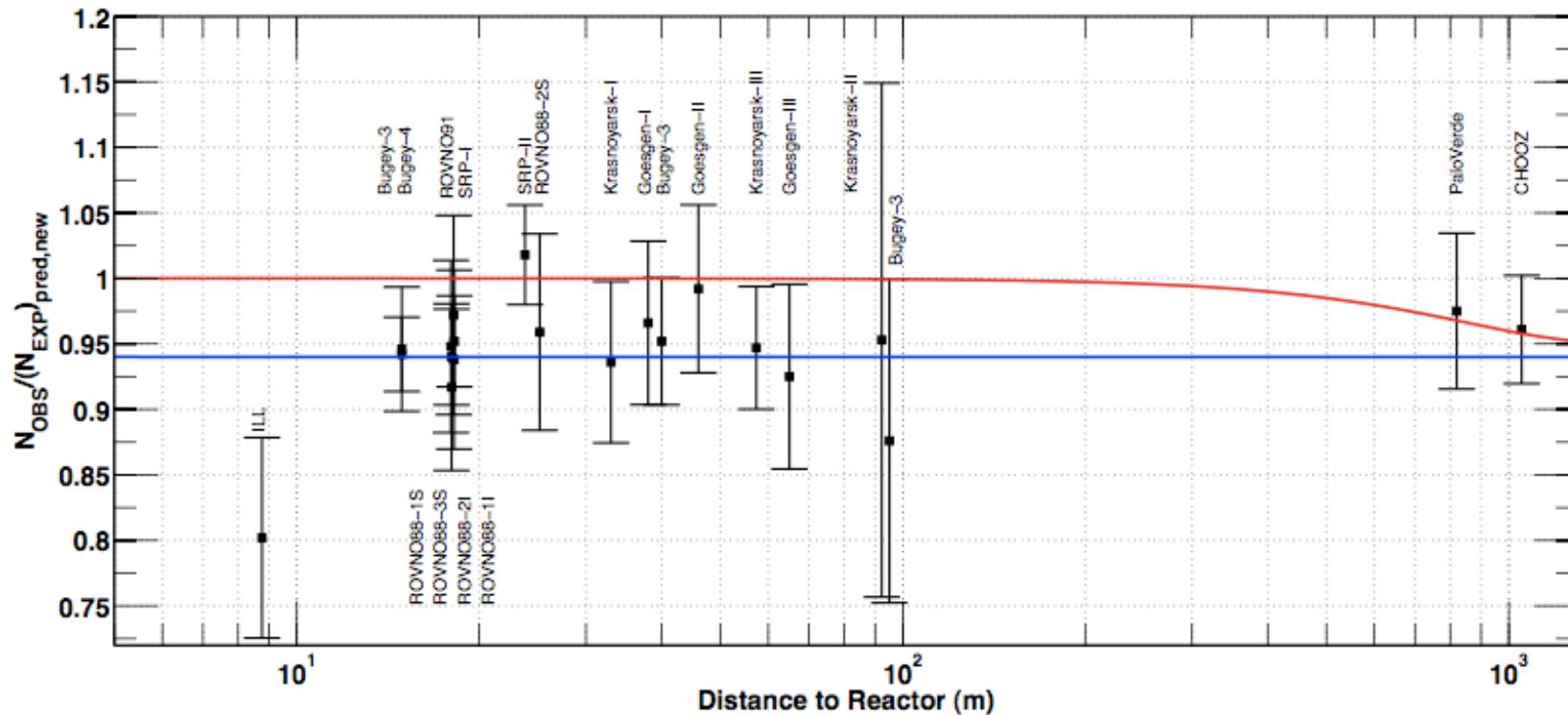
- MiniBooNE ( $\nu_\mu$  and  $\bar{\nu}_\mu$ )

- ◆ *Combined low energy (200-1250 MeV) excess of  $3.8\sigma$*
- ◆ *Excesses not well understood*



# eV-scale sterile neutrinos

- Re-evaluation of beta decay spectra shows a 6% increase in antineutrino flux (reactor anomaly)



	Best-fit	$1\sigma$	$3\sigma$
$ \Delta m_{\text{new}}^2  \text{ (eV}^2)$	2.35	2.25–2.45	> 1.2
$\sin^2 \theta_{\text{new}}$	0.043	0.032–0.054	0.0091–0.067

Mention et al., 2011

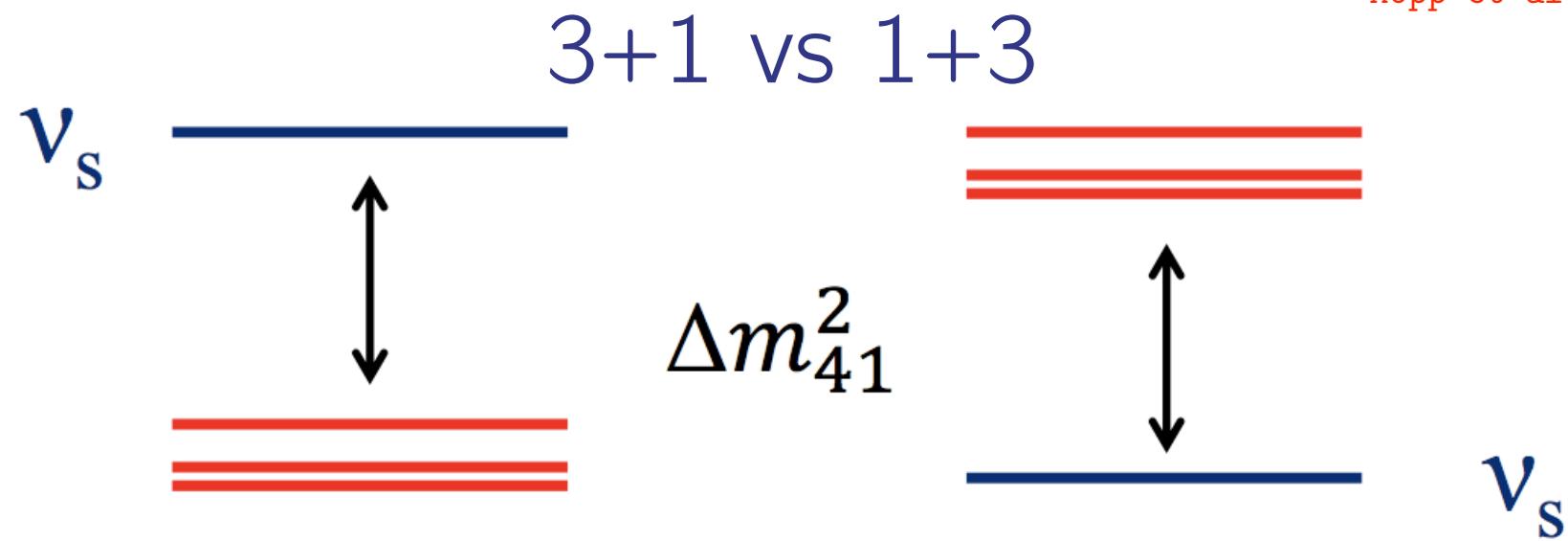


# eV-scale sterile neutrinos

Table 1: Best-fit and estimated  $2\sigma$  values of the sterile neutrino parameters.

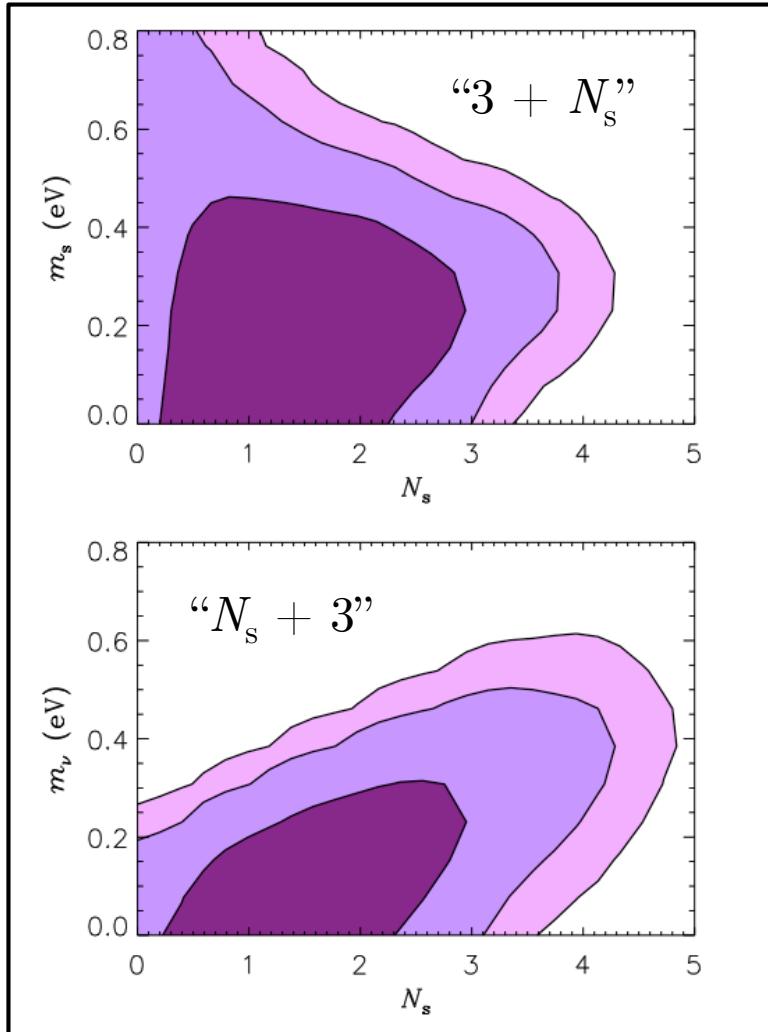
	parameter	$\Delta m_{41}^2$ [eV]	$ U_{e4} ^2$	$\Delta m_{51}^2$ [eV]	$ U_{e5} ^2$
3+1/1+3	best-fit	1.78	0.023		
	$2\sigma$	1.61–2.01	0.006–0.040		
3+2/2+3	best-fit	0.47	0.016	0.87	0.019
	$2\sigma$	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.033
1+3+1	best-fit	0.47	0.017	0.87	0.020
	$2\sigma$	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.035

Kopp et al., 2011





# Cosmology & eV sterile neutrinos



Hamann et al., 2010

- Some extra radiation preferred by CMB, SDSS, HST
- $N_{\text{eff}}$  consistent with 3 (but also with 4) at  $2\sigma$ :
- 3+2 excluded from HDM limit
- Expect PLANCK to clarify the issue soon



# Neutrino mixing with sterile neutrinos

- Phenomenological approach (1 sterile neutrino):

$$U = R_{34} \tilde{R}_{24} \tilde{R}_{14} R_{23} \tilde{R}_{13} R_{12} P$$

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \quad \text{or} \quad \tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix}$$

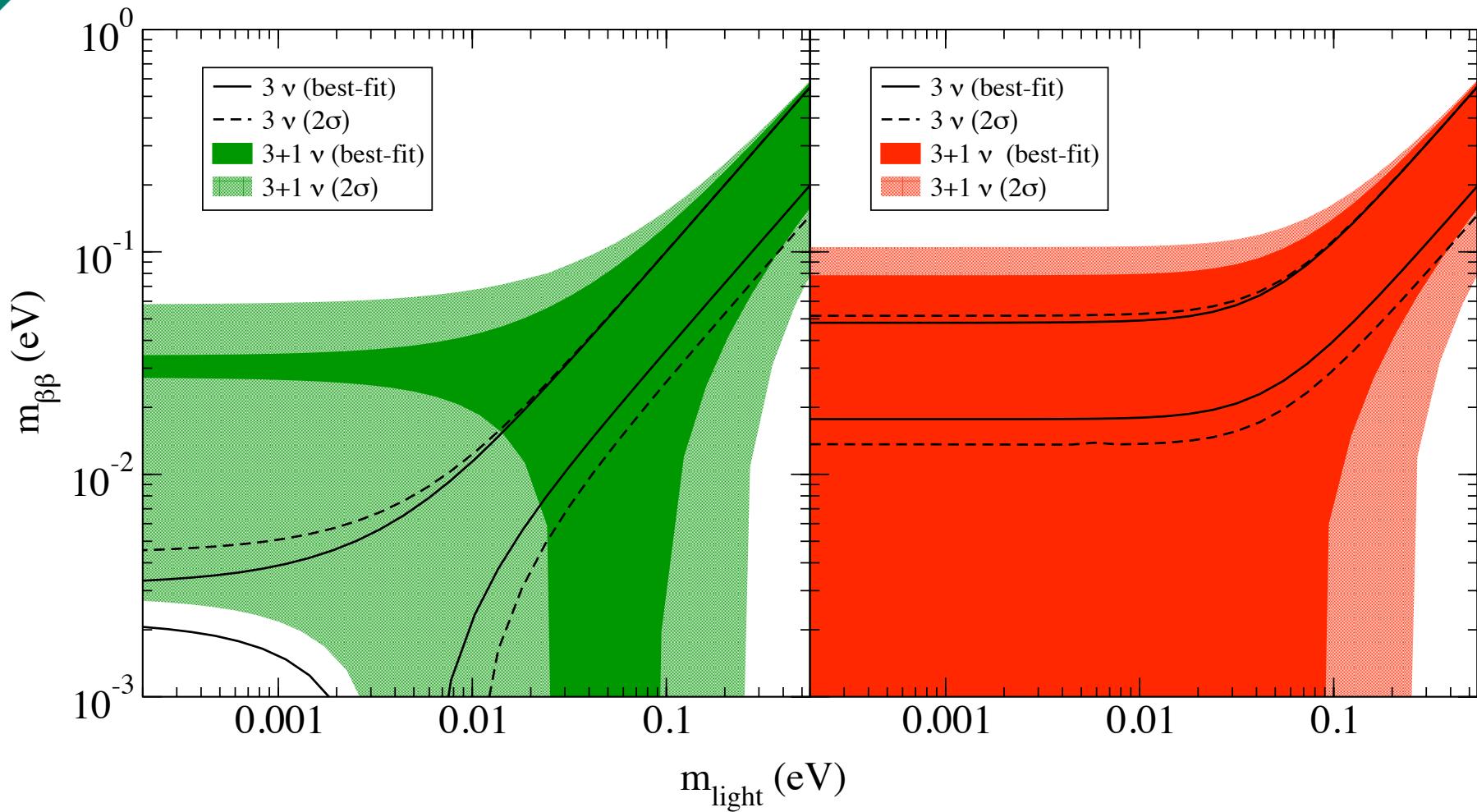
$$P = \text{diag} \left( 1, e^{i\alpha/2}, e^{i(\beta/2 + \delta_{13})}, e^{i(\gamma/2 + \delta_{14})} \right)$$

$\Rightarrow$  3 Dirac, 3 Majorana phases

- New effects in neutrinoless double beta decay...



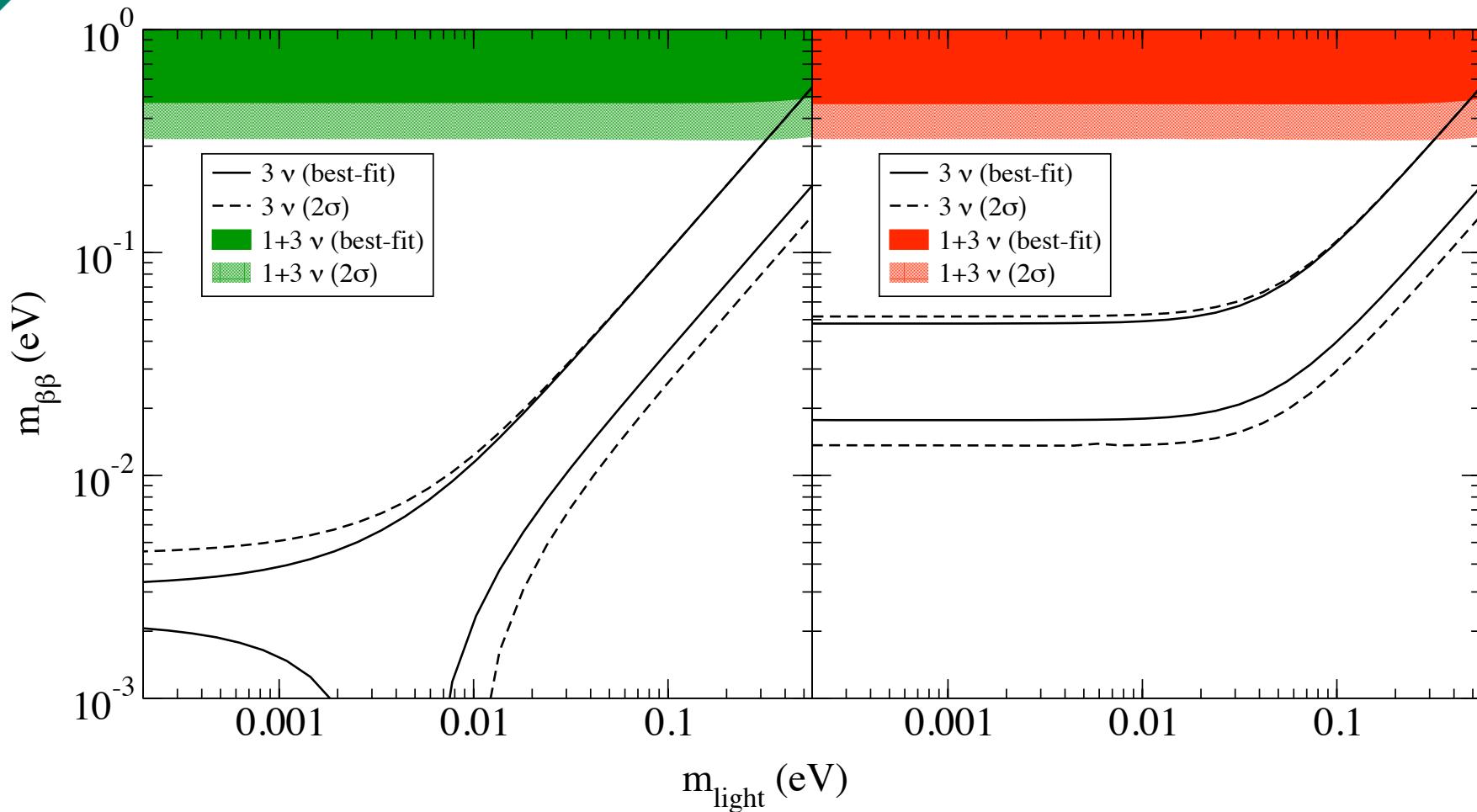
# $0\nu\beta\beta$ phenomenology



$$\langle m_{ee} \rangle_{(3+1)\nu} \simeq \left| c_{14}^2 \langle m_{ee} \rangle_{3\nu} + s_{14}^2 \sqrt{\Delta m_{41}^2} e^{i\gamma} \right|$$



# 0νββ phenomenology



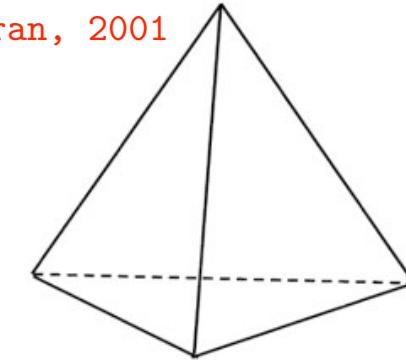
$$\langle m_{ee} \rangle_{(1+3)\nu} \simeq \sqrt{\Delta m_{41}^2} \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha/2}$$



# $A_4$ symmetry

Ma, Rajasekaran, 2001

- Symmetry group of the tetrahedron
- Even permutations of four objects
- Twelve elements
- Four irreducible representations:  $\underline{1}$ ,  $\underline{1}'$ ,  $\underline{1}''$  and  $\underline{3}$
- Product rules:  $\underline{1} \times \underline{1} = \underline{1}$



$$\underline{1}' \times \underline{1}'' = \underline{1}$$

$$\underline{1}'' \times \underline{1}' = \underline{1}$$

$$\underline{1}' \times \underline{1}' = \underline{1}''$$

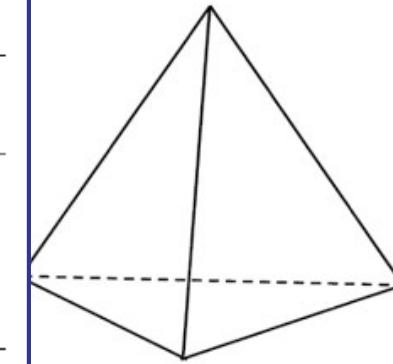
$$\underline{1}'' \times \underline{1}'' = \underline{1}'$$

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3}_{as} + \underline{3}_s$$



- Symmetry
- Even parity
- Twelve
- Four irreps
- Products

Type	$L_i$	$\ell_i^c$	$\nu_i^c$	$\Delta$	References
A1				-	[1–14] [15] <sup>#</sup>
A2	<u>3</u>	1, 1', 1''	-	<u>1</u> , 1', 1'', 3	[16–18]
A3				<u>1</u> , 3	[19]
B1	<u>3</u>	1, 1', 1''	3	-	[4, 20–27] <sup>#</sup> [28–30] <sup>*</sup> [31–48]
B2				<u>1</u> , 3	[49] <sup>#</sup>
C1				-	[2, 50, 51] [52] <sup>#</sup>
C2	<u>3</u>	<u>3</u>	-	1	[53, 54] [55] <sup>#</sup>
C3				<u>1</u> , 3	[56]
C4				<u>1</u> , 1', 1'', 3	[57]
D1				-	[58, 59] <sup>#</sup> [60, 61] <sup>*</sup> [62]
D2	<u>3</u>	<u>3</u>	<u>3</u>	1	[63] [64] <sup>*</sup>
D3				1'	[65] <sup>*</sup>
D4				1', <u>3</u>	[66] <sup>*</sup>
E1	<u>3</u>	<u>3</u>	1, 1', 1''	-	[67, 68]
E2				1	[69]
F	1, 1', 1''	3	3	1 or 1'	[70]
G	<u>3</u>	1, 1', 1''	<u>1</u> , 1', 1''	-	[71]
H	<u>3</u>	<u>1</u> , <u>1</u> , 1	-	-	[72]
I	<u>3</u>	1, 1, 1	1	-	[73] <sup>*</sup>
J	<u>3</u>	<u>1</u> , <u>1</u> , 1	<u>1</u> , <u>1</u>	-	[74] <sup>*</sup> [75]
K	<u>3</u>	<u>1</u> , <u>1</u> , 1	<u>1</u> , <u>1</u> , 1	-	[76] <sup>*</sup>
L	<u>3</u>	1, 1, 1	1, 1', 1''	-	[77]
M	<u>3</u>	<u>1</u> , <u>1</u> , 1	<u>3</u>	-	[12, 39, 78, 79]
N	<u>3</u>	1, 1, 1	1, 1	1	[80] <sup>*</sup>
O	1, 1', 1''	1, 1', 1''	3	-	[81]
P	<u>1</u> , <u>1</u> ', 1''	<u>1</u> , <u>1</u> '', 1'	<u>3</u> , 1	-	[82, 83]
Q	1, 1', 1''	1, 1'', 1'	3, 1', 1''	-	[84]



, 1'' and 3

JB, Rodejohann, 2010.  
Updated regularly at  
[www.mpi-hd.mpg.de/  
personalhomes/jamesb/](http://www.mpi-hd.mpg.de/personalhomes/jamesb/)  
(see also Ding, 2011)

$\underline{3}_{as} + \underline{3}_s$





# Model with effective operators





# Effective model with sterile neutrino

Table 2: Altarelli-Feruglio effective  $A_4$  model, including a sterile neutrino  $\nu_s$

Field	$L$	$e^c$	$\mu^c$	$\tau^c$	$h_{u,d}$	$\varphi$	$\varphi'$	$\xi$	$\Theta$	$\nu_s$
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1
$A_4$	3	1	$1''$	$1'$	1	3	3	1	1	1
$Z_3$	$\omega$	$\omega^2$	$\omega^2$	$\omega^2$	1	1	$\omega$	$\omega$	1	1
$U(1)_{\text{FN}}$	-	$F_e$	$F_\mu$	$F_\tau$	-	-	-	-	-1	$F_\nu$

$$\mathcal{L}_{Y,\ell} = \frac{y_e}{\Lambda} \lambda^{F_e} e^c (\varphi L) h_d + \frac{y_\mu}{\Lambda} \lambda^{F_\mu} \mu^c (\varphi L)' h_d + \frac{y_\tau}{\Lambda} \lambda^{F_\tau} \tau^c (\varphi L)'' h_d$$

$$\begin{aligned} \mathcal{L}_{Y,\nu} &= \frac{x_a}{\Lambda^2} \xi (L h_u L h_u) + \frac{x_d}{\Lambda^2} (\varphi' L h_u L h_u) \\ &\quad + \frac{x_e}{\Lambda^2} \lambda^{F_\nu} \xi (\varphi' L h_u) \nu_s + m_s \lambda^{2F_\nu} \nu_s^c \nu_s^c + \text{h.c.} \end{aligned}$$

$$\lambda \equiv \frac{\langle \Theta \rangle}{\Lambda} < 1 \implies \text{FN suppression factor}$$



# Masses & mixing

$\langle \xi \rangle = u$ ,  $\langle \varphi \rangle = (v, 0, 0)$  and  $\langle \varphi' \rangle = (v', v', v')$

$\Rightarrow M_\ell$  diagonal  $\Rightarrow m_\alpha = y_\alpha v_d \frac{v}{\Lambda} \lambda^{F_\alpha}$

$$M_\nu^{4 \times 4} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix}$$

$a$	$=$	$2x_a \frac{uv_u^2}{\Lambda^2}$
$d$	$=$	$2x_d \frac{v'v_u^2}{\Lambda^2}$
$e$	$=$	$\sqrt{2}x_e \frac{uv'v_u}{\Lambda^2}$

Mixing matrix:

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \mathcal{O}\left(\frac{e^2}{m_s^2}\right)$$

$\theta_{13} = 0$



# Numerical example

Mass eigenvalues:

$$m_1 = a + d, \quad m_2 = a - \frac{3e^2}{m_s}, \quad m_3 = -a + d, \quad m_4 = m_s + \frac{3e^2}{m_s}$$

For  $F_\nu = 6$ ,  $\langle \Theta \rangle = v = v' = u = 10^{11}$  GeV,  $\Lambda = 10^{12.5}$  GeV

$$a \sim d \simeq 0.1 \left( \frac{u}{10^{11} \text{ GeV}} \right) \left( \frac{v_u}{10^2 \text{ GeV}} \right)^2 \left( \frac{10^{12.5} \text{ GeV}}{\Lambda} \right)^2 \text{ eV}$$

$$e \simeq 0.1 \left( \frac{\lambda}{10^{-1.5}} \right)^6 \left( \frac{u}{10^{11} \text{ GeV}} \right) \left( \frac{v'}{10^{11} \text{ GeV}} \right) \left( \frac{v_u}{10^2 \text{ GeV}} \right) \left( \frac{10^{12.5} \text{ GeV}}{\Lambda} \right)^2 \text{ eV}$$

Sterile neutrino mass & mixing:

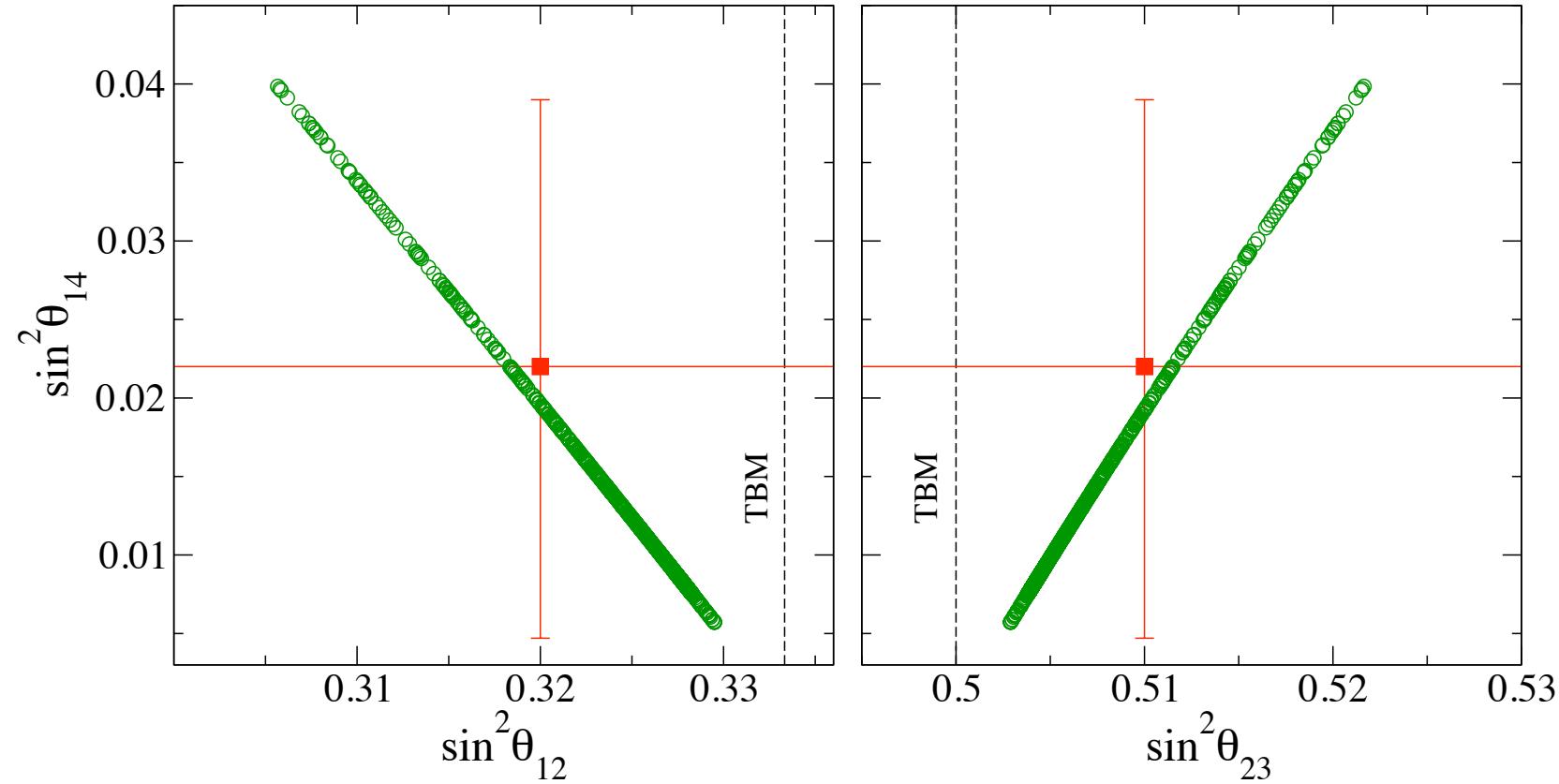
$$\left( \frac{x_s}{\Lambda} (\varphi \varphi) + \frac{x_{s'}}{\Lambda^2} \xi \xi \xi + \frac{x_{s''}}{\Lambda^2} (\varphi' \varphi') \xi \right) \nu_s^c \nu_s^c \implies \left( x_s \frac{v^2}{\Lambda} + x_{s'} \frac{u^3}{\Lambda^2} + x_{s''} \frac{3v'^2 u}{\Lambda^2} \right) \lambda^{2F_\nu}$$

$$m_s \simeq 10^{0.5} \left( \frac{\lambda}{10^{-1.5}} \right)^{12} \left( \frac{v}{10^{11} \text{ GeV}} \right)^2 \left( \frac{10^{12.5} \text{ GeV}}{\Lambda} \right) \text{ eV}$$

$$\theta_{14} \simeq e/m_s \simeq 0.1$$



# Deviations from TBM with sterile $\nu$



$$\sin^2 \theta_{12} \approx \frac{1}{3} [1 - 2 \sin^2 \theta_{14}] , \quad \sin^2 \theta_{23} \approx \frac{1}{2} [1 + \sin^2 \theta_{14}]$$



# Non-zero $\theta_{13}$ ?

- Sterile neutrino causes deviations to solar and atmospheric angles, but reactor angle remains zero
- Need charged lepton correction terms from dim-7 operators:

Altarelli, Feruglio, 2006

$$\text{e.g., } \eta_i \propto \langle \varphi \rangle \langle \varphi' \rangle \langle h_u \rangle^2 / \Lambda^3$$

$$\Rightarrow \sin^2 \theta_{13} \simeq \frac{(\eta_1 - \eta_2)^2}{8a^2}$$

$\Rightarrow$  non-zero  $\theta_{13}$  possible with NLO operators



# Seesaw model





# Sterile neutrinos in seesaw models

- Split seesaw mechanism  
*Kusenko; Adulpravitchai et al*
- Symmetries (eg.  $L_e - L_\mu - L_\tau$ )  
*Shaposhnikov; Lindner et al*
- Froggatt-Nielsen mechanism  
*Merle & Niro; JB, Rodejohann, Zhang*
- Extended seesaw mechanisms  
*Mohapatra; Smirnov; Zhang*
- ....?



# NLO seesaw corrections

Full neutrino mass matrix is:

$$M_\nu^{6 \times 6} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$$

$$U_\nu \simeq \begin{pmatrix} 1 - \frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1 - \frac{1}{2}B^\dagger B \end{pmatrix} \begin{pmatrix} V_\nu & 0 \\ 0 & V_R \end{pmatrix}$$

Schechter & Valle, 1982; Grimus & Lavoura, 2000;  
Hettmansperger, Lindner, Rodejohann 2011

$$M_\nu = -M_D M_R^{-1} M_D^T = V_\nu \text{diag}(m_1, m_2, m_3) V_\nu^T$$

$$M_R = V_R \text{diag}(M_1, M_2, M_3) V_R^T$$

NLO corrections governed by

$$B = M_D M_R^{-1} + \mathcal{O}\left(M_D^3 (M_R^{-1})^3\right)$$

$$B \simeq \sqrt{M_\nu / M_R}$$

$\Rightarrow$  can be significant if  $M_R \simeq 1 \text{ eV}$



# Active-sterile mixing & $0\nu\beta\beta$

- Active-sterile mixing is

$$\theta_{\alpha i} \equiv [U_\nu]_{\alpha, 3+i} = [BV_R]_{\alpha i} \simeq \frac{[M_D V_R^*]_{\alpha i}}{M_i}$$

⇒ mixing is a ratio of two scales

- $0\nu\beta\beta$  amplitude vanishes if all sterile neutrinos light

$$\langle m_{ee} \rangle = \left| \sum_{i=1}^3 U_{ei}^2 m_i + \sum_{i=1}^3 U_{e,3+i}^2 M_i \right| = [M_\nu^{6 \times 6}]_{ee} = 0$$

- In addition, in certain flavour symmetry models

$$M_D = V_\nu \text{ diag} \left( \sqrt{-m_1 M_1}, \sqrt{-m_2 M_2}, \sqrt{-m_3 M_3} \right) V_R^T$$

Chen, King 2009; Choubey et al 2010

$$U_{e,3+i}^2 M_i = \left[ - (V_\nu^2)_{ei} \frac{m_i}{M_i} \right] M_i = - U_{ei}^2 m_i, \quad (i = 1, 2, 3)$$

⇒ pairwise cancellation!



# Seesaw model

*A<sub>4</sub> singlets*

Table 3: *A<sub>4</sub>* type I seesaw model, with three right-handed sterile neutrinos

Field	<i>L</i>	<i>e<sup>c</sup></i>	<i>μ<sup>c</sup></i>	<i>τ<sup>c</sup></i>	<i>h<sub>u,d</sub></i>	<i>φ</i>	<i>φ'</i>	<i>φ''</i>	<i>ξ</i>	<i>ξ'</i>	<i>ξ''</i>	<i>Θ</i>	<i>ν<sub>1</sub><sup>c</sup></i>	<i>ν<sub>2</sub><sup>c</sup></i>	<i>ν<sub>3</sub><sup>c</sup></i>
<i>SU(2)<sub>L</sub></i>	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
<i>A<sub>4</sub></i>	$\frac{3}{\omega}$	$\frac{1}{\omega^2}$	$\frac{1''}{\omega^2}$	$\frac{1'}{\omega^2}$	$\frac{1}{\omega}$	$\frac{3}{\omega}$	$\frac{3}{\omega^2}$	$\frac{3}{\omega^3}$	$\frac{1}{\omega^2}$	$\frac{1'}{\omega}$	$\frac{1}{\omega}$	$\frac{1}{\omega}$	$\frac{1}{\omega^2}$	$\frac{1'}{\omega}$	$\frac{1}{\omega}$
<i>Z<sub>3</sub></i>	$\omega$	$\omega^2$	$\omega^2$	$\omega^2$	$\omega^2$	1	$\omega$	$\omega^2$	$\omega^3$	$\omega$	1	1	$\omega^2$	$\omega$	1
<i>U(1)<sub>FN</sub></i>	-	3	1	0	-	-	-	-	-	-	-	-1	<i>F<sub>1</sub></i>	<i>F<sub>2</sub></i>	<i>F<sub>3</sub></i>

$$\begin{aligned}
 -\mathcal{L}_{Y,\nu} = & \frac{y_1}{\Lambda} \lambda^{F_1} (\varphi L h_u) \nu_1^c + \frac{y_2}{\Lambda} \lambda^{F_2} (\varphi' L h_u)'' \nu_2^c + \frac{y_3}{\Lambda} \lambda^{F_3} (\varphi'' L h_u) \nu_3^c \\
 & + \frac{1}{2} [w_1 \lambda^{2F_1} \xi \nu_1^c \nu_1^c + w_2 \lambda^{2F_2} \xi' \nu_2^c \nu_2^c + w_3 \lambda^{2F_3} \xi'' \nu_3^c \nu_3^c] + \text{h.c.}
 \end{aligned}$$

- FN charge drops out of leading order seesaw term
- Different scenarios possible, vary  $F_i$
- Active sterile mixing depends on FN charge, e.g.

$$\theta_{e1} \sim \frac{y_1 v v_u}{w_1 u \Lambda} \lambda^{-F_1}$$



# Reminder: motivations/mass scales

- Three (observed) phenomena to explain, with three distinct mass scales:

- ◆ eV – *SBL oscillation anomalies*
- ◆ keV – *Warm Dark Matter*
- ◆ GeV – *Resonant leptogenesis / vMSM*
- ◆  $> 10^9$  GeV – *Standard leptogenesis*

⇒ with three RH neutrinos one cannot explain all three phenomena!

Model I: eV, eV, keV

Model II: eV, keV,  $\simeq$  GeV

Model III: keV,  $\gtrsim$  GeV,  $\gtrsim$  GeV



# Seesaw model

Table 4: Summary of possible model scenarios

	$F_1, F_2, F_3$	Mass spectrum	$ U_{\alpha 4} $	$ U_{\alpha 5} $	$\langle m_{ee} \rangle$	Phenomenology
					NO IO	
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$ , $\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11} \text{ GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_A^2}}{3}$
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11} \text{ GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\frac{\sqrt{\Delta m_A^2}}{3}$
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10 \text{ GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\sqrt{\Delta m_A^2}$

Choose  $F_1 = 9 \Rightarrow M_1 \simeq 1 \text{ keV}, \theta_1^2 \simeq 10^{-8}$

Negligible contribution to neutrino mass,  $\theta_1^2 M_1 \simeq 10^{-5} \text{ eV}$

$\Rightarrow$  Decouple  $\nu_1^c$ , get  $5 \times 5$  mass matrix

For example, with  $\langle \varphi' \rangle = (v', v', v')$  and  $\langle \varphi'' \rangle = (0, v'', -v'')$

$$M_D^{(\text{NO})} = \frac{v_u}{\Lambda} \begin{pmatrix} y_2 v' \lambda^{F_2} & 0 \\ y_2 v' \lambda^{F_2} & -y_3 v'' \lambda^{F_3} \\ y_2 v' \lambda^{F_2} & y_3 v'' \lambda^{F_3} \end{pmatrix} \quad M_R = \begin{pmatrix} w_2 u' \lambda^{2F_2} & 0 \\ 0 & w_3 u'' \lambda^{2F_3} \end{pmatrix}$$

$\Rightarrow$  TBM at leading order



# Seesaw model

Table 4: Summary of possible model scenarios

	$F_1, F_2, F_3$	Mass spectrum	$ U_{\alpha 4} $	$ U_{\alpha 5} $	$\langle m_{ee} \rangle$	Phenomenology
I	9, 10, 10	$M_{2,3} = \mathcal{O}(\text{eV})$ ,	$\mathcal{O}(0.1)$	$\mathcal{O}(0.1)$	0	0
IIA	9, 10, 0	$M_2 = \mathcal{O}(\text{eV})$ $M_3 = \mathcal{O}(10^{11} \text{ GeV})$	$\mathcal{O}(0.1)$	$\mathcal{O}(10^{-11})$	0	$\frac{2\sqrt{\Delta m_A^2}}{3}$
IIB	9, 0, 10	$M_2 = \mathcal{O}(10^{11} \text{ GeV})$ $M_3 = \mathcal{O}(\text{eV})$	$\mathcal{O}(10^{-11})$	$\mathcal{O}(0.1)$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\frac{\sqrt{\Delta m_A^2}}{3}$
III	9, 5, 5	$M_{2,3} = \mathcal{O}(10 \text{ GeV})$	$\mathcal{O}(10^{-6})$	$\mathcal{O}(10^{-6})$	$\frac{\sqrt{\Delta m_S^2}}{3}$	$\sqrt{\Delta m_A^2}$

NLO seesaw corrections →

$$|U_{e3}|^2 \simeq \frac{r_1^2}{2} \left[ \left( \frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau} \right)^2 \right] + \frac{1}{2} (\chi - \rho_3)^2 - (\chi - \rho_3) r_1 \left( \frac{y'_\mu}{y_\mu} - \frac{y'_\tau}{y_\tau} \right),$$

$$|U_{e2}|^2 \simeq \frac{1}{3} \left[ 1 - 3\epsilon_1^2 - 2\rho_2 - 2r_1 \left( \frac{y'_\mu}{y_\mu} + \frac{y'_\tau}{y_\tau} \right) \right],$$

$$|U_{\mu 3}|^2 \simeq \frac{1}{2} \left[ 1 - 2\epsilon_2^2 + 2\frac{y'_\tau}{y_\tau} r_1 + \frac{2}{3} \sigma_+^N R \right],$$

$$|U_{e,\mu 4}|^2 \simeq \epsilon_1^2 \left[ 1 \mp 2\rho_2 \mp 2r_1 \left( \frac{y'_\mu}{y_\mu} \pm \frac{y'_\tau}{y_\tau} \right) \right]$$

↑ corrections from NLO operators

⇒ non-zero  $\theta_{13}$  possible from charged leptons

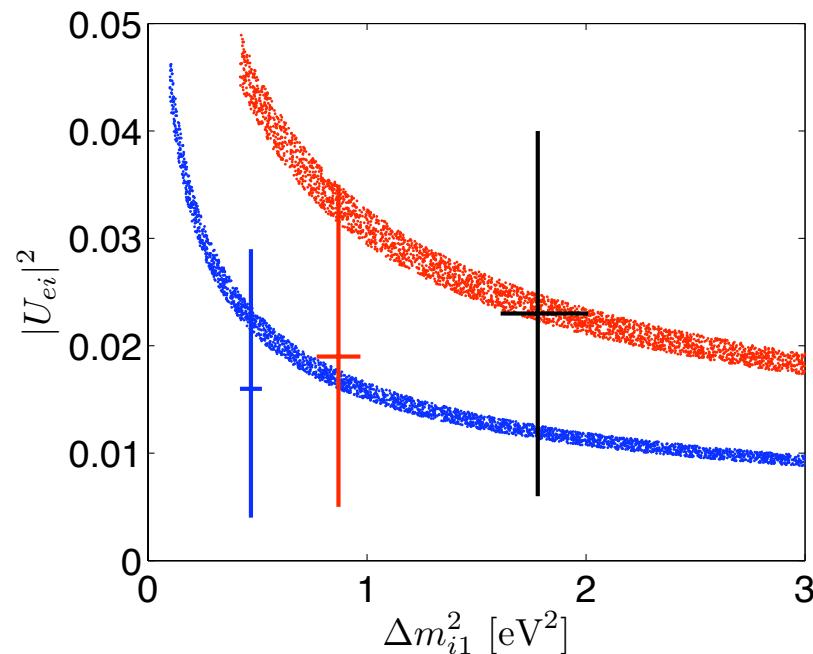


# Seesaw model

- Inverted ordering:

$$|U_{e,\mu 4}|^2 \simeq \epsilon_1^2 \left[ 1 - 2\rho_2 \mp 2r_1 \left( \frac{y'_\mu}{y_\mu} \pm \frac{y'_\tau}{y_\tau} \right) \right],$$

$$\underline{|U_{e5}|^2} \simeq 4\epsilon_2^2 \left[ 1 + r_1 \left( \frac{y'_\mu}{y_\mu} + \frac{y'_\tau}{y_\tau} \right) - (\chi - \rho_3) \right],$$



Scenario I:  
⇒ 3+2 mixing

Scenario II B:  
⇒ 3+1 mixing



# Conclusion

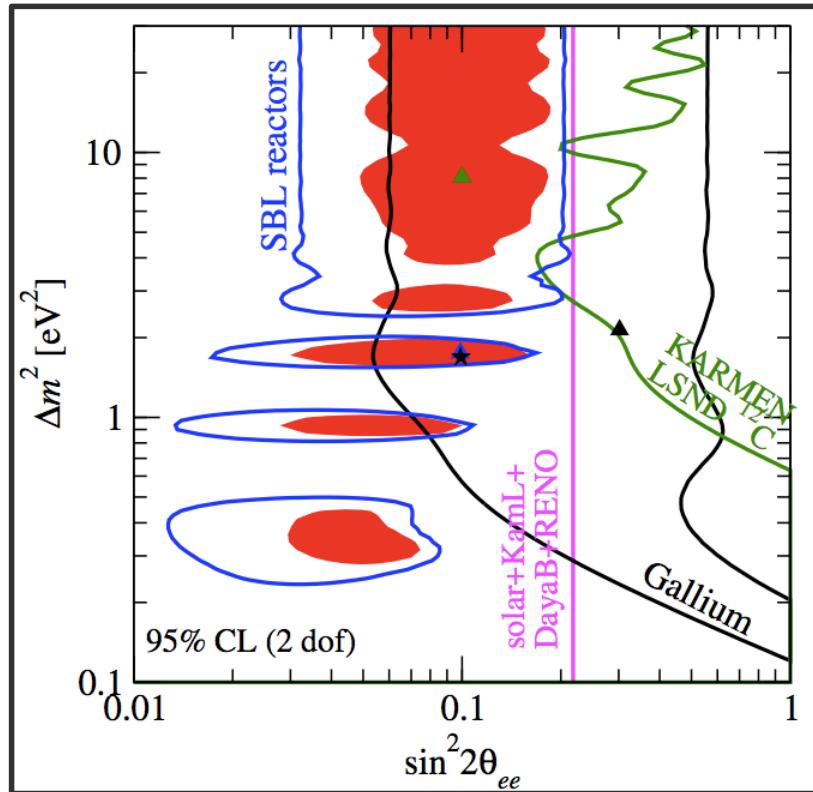
- “Light” sterile neutrinos exhibit distinct phenomenological signatures
- The FN mechanism is one way to suppress the sterile neutrino mass
- Flavour symmetry models can be extended to include light sterile neutrinos
- NLO seesaw terms need to be considered for eV-scale sterile neutrinos
- Deviations from TBM due to sterile neutrinos, but large reactor angle requires NLO operators



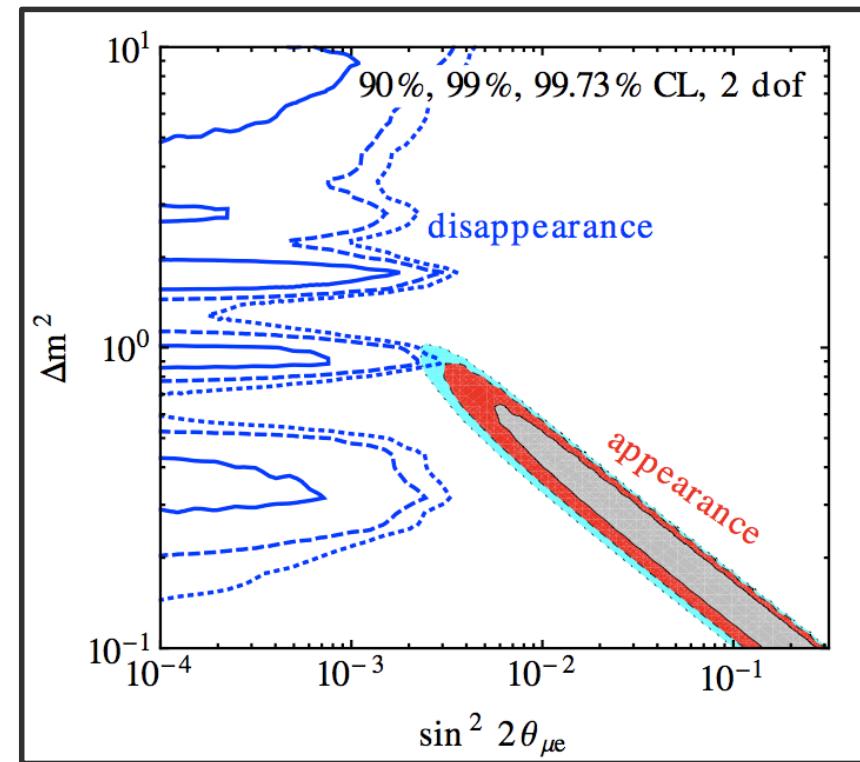


# Latest global fits

$\nu_e$  disappearance data



$\nu_e$  &  $\nu_\mu$  disapp. /  $\nu_\mu$  app.



$$|U_{e4}|^2 = 0.025$$

$$\Delta m_{41}^2 = 1.71 \text{ eV}^2$$

taken from Schwetz, Neutrino 2012

$$|U_{e4}|^2 = 0.022, |U_{\mu 4}|^2 = 0.030$$

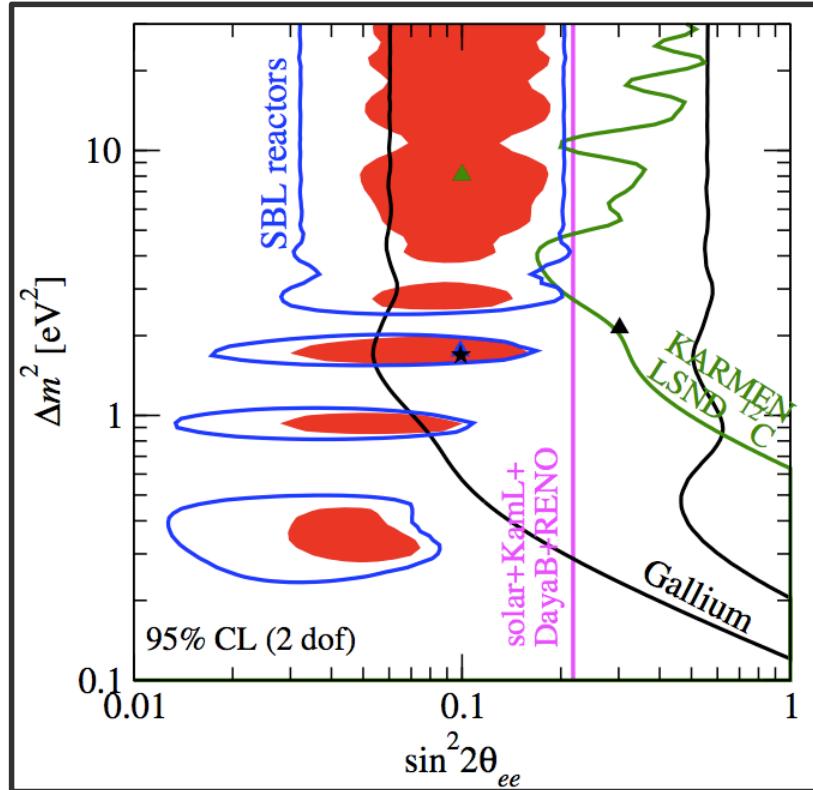
$$\sin^2 2\theta_{\mu e} = 0.0026$$

$$\Delta m_{41}^2 = 0.92 \text{ eV}^2$$



# Latest global fits

$\nu_e$  disappearance data

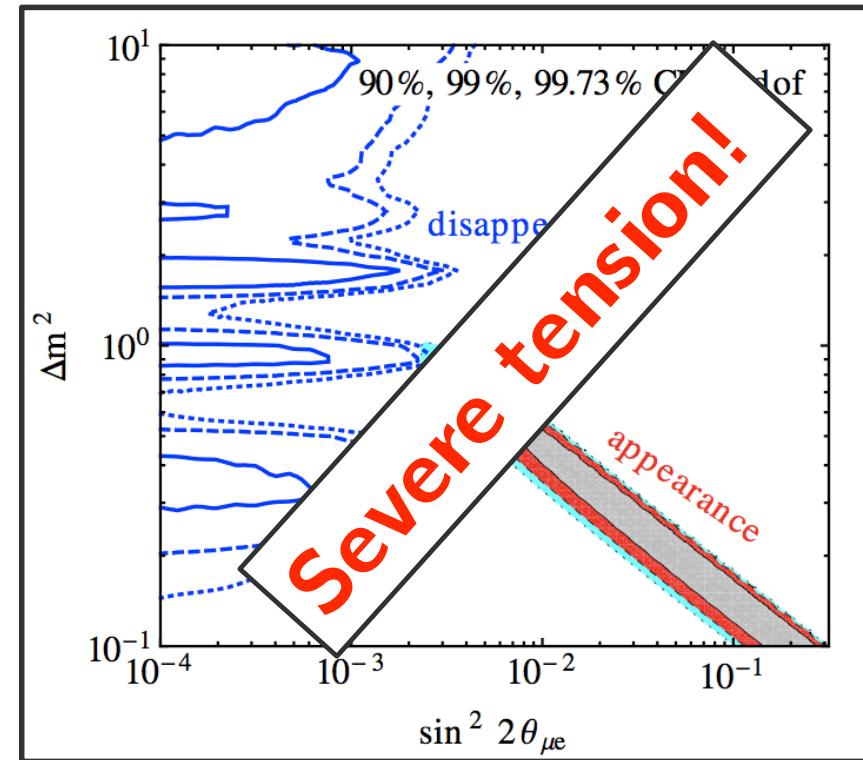


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$\nu_e$  &  $\nu_\mu$  disapp. /  $\nu_\mu$  app.



$$|U_{e4}|^2 = 0.022, |U_{\mu 4}|^2 = 0.030$$

$$\sin^2 2\theta_{\mu e} = 0.0026$$

$$\Delta m_{41}^2 = 0.92 \text{ eV}^2$$