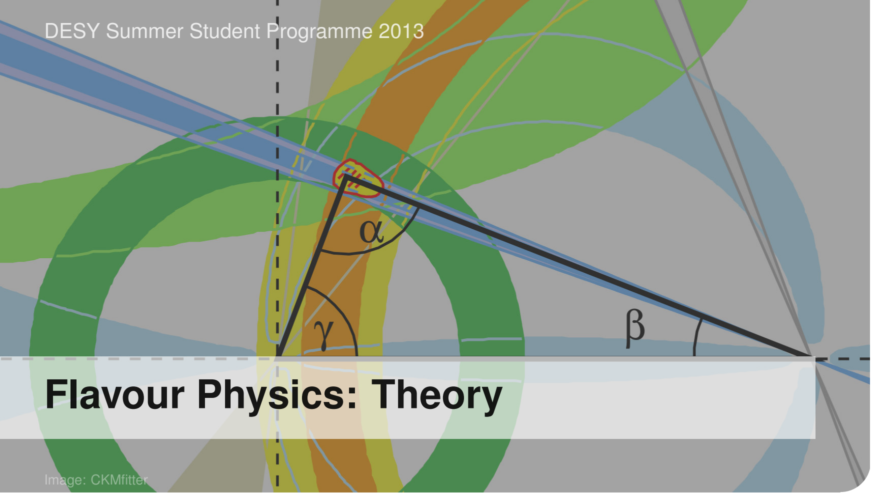
DESY Summer Student Programme 2013

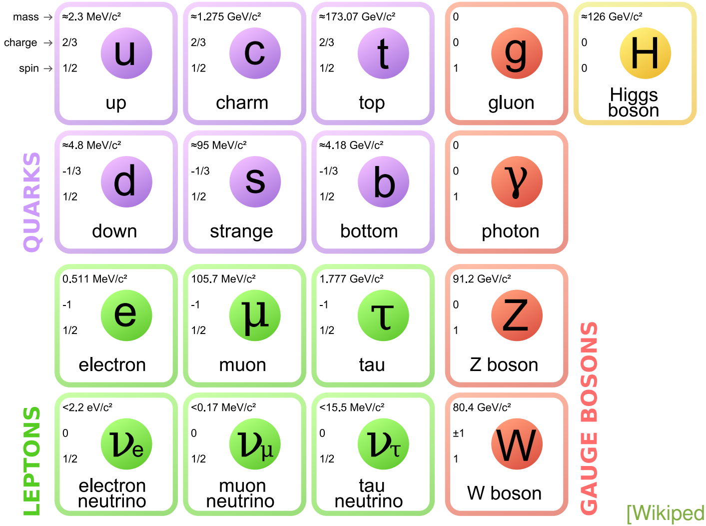


Flavour Physics: Theory

Image: CKMfitter

David M. Straub | Johannes Gutenberg University Mainz

The Standard Model of particle physics



[Wikipedia]

What is flavour?

Flavour is a quantum number used to distinguish particles/fields that have the *same gauge quantum numbers*

- In the SM: quarks and leptons come in three copies with the same colour representation & electric charge

Flavour physics deals with interactions that *distinguish between flavours*

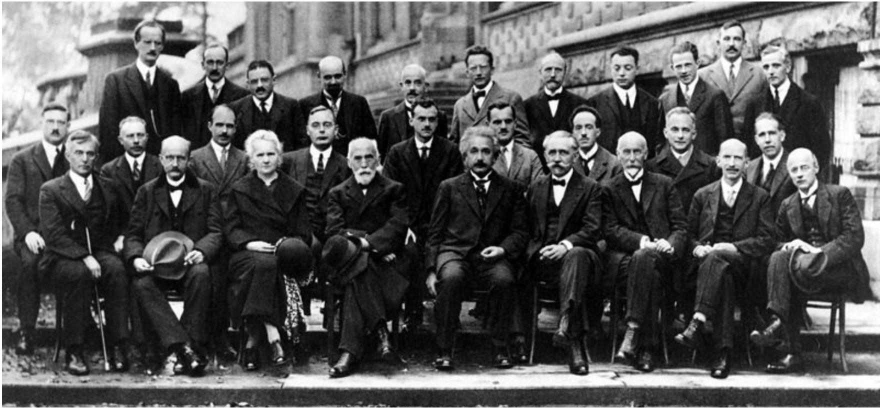
- In the SM: QED and QCD interactions do *not* distinguish between flavours, while the weak interactions and the couplings to the Higgs field do

Today: focus on *quark* flavour physics

Flavour??

In 1971, Harald Fritzsch and Murray Gell-Mann realized that ice cream has another quantum number in addition to colour – just like quarks.

A brief history of fermions



1927 the world consists of p, n, e, and γ

1930 (as well as ν)

A brief history of fermions

1936 discovery of the *muon* (Anderson). “Who ordered that?” (Rabi)

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1977 τ lepton discovered (SLAC)

1995 *top* quark discovered (FNAL)

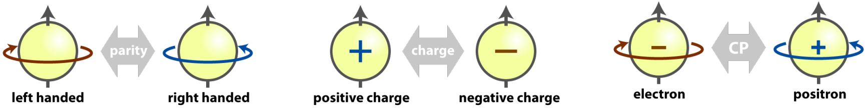
2000 ν_τ discovered (FNAL)

Why is flavour physics interesting?

- The masses and mixings of the fermions are arbitrary and unexplained parameters in the SM: the *flavour puzzle*
- Flavour physics allows to *probe new phenomena* indirectly (flavour-changing neutral currents are sensitive to heavy virtual particles)
- In the SM, flavour physics is the only source of *CP violation*

CP invariance

CP is the combined operation of parity inversion *P* and charge conjugation *C*



[Graphics: Ph. Tanedo]

- CP transforms particle $\leftrightarrow$ antiparticle
- until 1964, it was believed to be a symmetry of nature
- if it *were* a symmetry of nature, it wouldn't be possible to generate a matter-antimatter asymmetry dynamically in the universe (Sakharov)

1 Flavour in the Standard Model
- Introduction: What is flavour and why is it interesting?
- Quark mixing in the Standard Model

2 Flavour-changing neutral currents
- FCNCs in the Standard Model
- Meson-antimeson mixing
- Rare decays

The fundamental building blocks of particle physics

...are not particles, but (quantum) *fields*!

- Let us start with 1 generation. The SM then contains three quark fields transforming as

$$Q_L \sim (3, 2, \tfrac{1}{6}) \qquad U_R \sim (3, 1, \tfrac{2}{3}) \qquad D_R \sim (3, 1, -\tfrac{1}{3})$$

  under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$
- They are (massless) Weyl fields

$$i\sigma^\mu \partial_\mu Q_L = 0 \qquad \text{etc.}$$

How do they relate to the quarks u and d?

Quarks & the Higgs field

The quarks have couplings to the Higgs field, the *Yukawa couplings*

When the electroweak symmetry is broken in the Higgs mechanism, the Higgs field obtains a *vacuum expectation value* (VEV) $\langle h \rangle = v$ that gives a *mass* to the quarks.

Dirac quark fields

The *four massless* Weyl fields $(Q_L)_{1,2}$, U_R, D_R combine into *two massive* Dirac fields

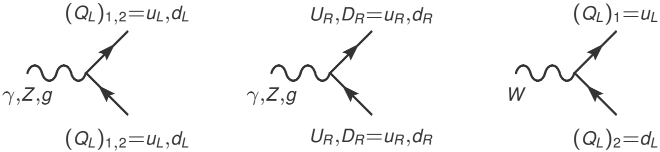
$$u = \begin{pmatrix} (Q_L)_1 \\ U_R \end{pmatrix} \qquad d = \begin{pmatrix} (Q_L)_2 \\ D_R \end{pmatrix}$$

$$\begin{pmatrix} -Y_u v & i\sigma^\mu \partial_\mu \\ i\sigma^\mu \partial_\mu & -Y_u v \end{pmatrix} \begin{pmatrix} (Q_L)_1 \\ U_R \end{pmatrix} = (i\gamma^\mu \partial_\mu - m_u)\, u = 0 \qquad \text{etc.}$$

where $m_{u,d} = Y_{u,d}\, v$.

- We call them *up* and *down* quarks.
- Their electric charge is a combination of the gauge quantum numbers of the SM: weak isospin and hypercharge

Gauge interactions

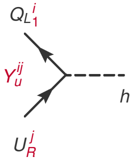


Adding a second generation

We now have the fields

$$Q_L^{1,2} \quad U_R^{1,2} \quad D_R^{1,2}$$

- The Yukawa couplings *distinguish* different flavours.



- After electroweak symmetry breaking, the fields $u^{1,2}$, $d^{1,2}$ are *no longer eigenstates* of the mass operator!
- We have to perform a *field redefinition* to obtain the physical *mass eigenstate* fields:

Field redefinition

We *rotate* the fields *in 2D* generation space

$$(\widehat{\boldsymbol{Q}}_L)_1 = R(\theta_{Q_1})\,(\boldsymbol{Q}_L)_1 \qquad (\widehat{\boldsymbol{Q}}_L)_2 = R(\theta_{Q_2})\,(\boldsymbol{Q}_L)_2$$

$$\widehat{\boldsymbol{U}}_R = R(\theta_U)\,\boldsymbol{U}_R \qquad \widehat{\boldsymbol{D}}_R = R(\theta_D)\,\boldsymbol{D}_R$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\widehat{\boldsymbol{u}} = \begin{pmatrix} (\widehat{\boldsymbol{Q}}_L)_1 \\ \widehat{\boldsymbol{U}}_R \end{pmatrix} \qquad \widehat{\boldsymbol{d}} = \begin{pmatrix} (\widehat{\boldsymbol{Q}}_L)_2 \\ \widehat{\boldsymbol{D}}_R \end{pmatrix}$$

Note that we treated the two weak isospin components of $(\widehat{\boldsymbol{Q}}_L)$ separately: the Higgs VEV has broken $SU(2)_L$ spontaneously, so the symmetry is no longer manifest!

Mass eigenstates

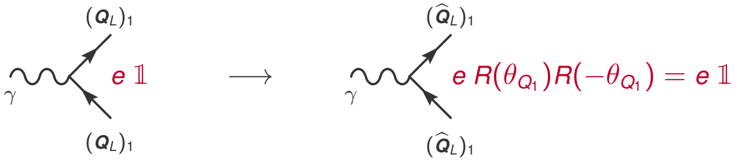
Our new Dirac fields now have *definite masses*.

$$R(\theta_{Q_1})^T \begin{pmatrix} v\, Y_u^{11} & v\, Y_u^{12} \\ v\, Y_u^{21} & v\, Y_u^{22} \end{pmatrix} R(\theta_U) = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}$$

We call $\widehat{u}^1$, $\widehat{u}^2$, $\widehat{d}^1$, $\widehat{d}^2$ the *up* quark, *charm* quark, *down* quark and *strange* quark.

Physical consequences: neutral currents

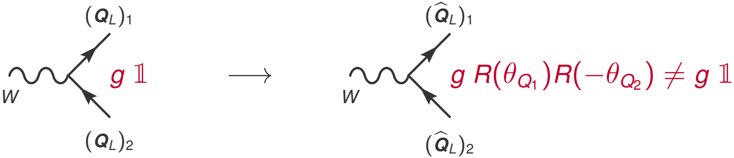
Fundamental vertices of the SM get redefined:



Analogously for the Z, g and the down-type quarks.

- The γ, Z and g do not distinguish flavours.
  There are *no flavour-changing neutral couplings* in the SM.

Physical consequences: charged currents



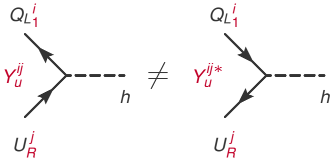
- *W* couplings change flavour!
- Flavour change described by a single physical parameter: the *Cabibbo angle* $\theta_C \equiv \theta_{Q_1} - \theta_{Q_2}$.
- $\theta_{U,D}$ have no physical consequences in the theory and can be chosen at will

Flavour change in gauge interactions?

- In the SM, the only flavour-changing interaction is the W coupling. But weren't gauge interactions to be *blind* to flavour?
- Yes, but the Higgs VEV has broken the $SU(2)_L$ gauge symmetry and the mass terms do not respect the symmetry, which forced us to go to a (mass) basis which is not ($SU(2)_L$) gauge invariant.
- Still, the *source of the flavour-change is the Higgs* sector: in the limit $Y_{u,d} \to 0$, $\theta_C = 0$!

Slight complication: CP violation

- In general, the Yukawa couplings could be *complex*



- Then, they would distinguish between particles and anti-particles: *CP violation*

How would our analysis change?

Complex field redefinition for 2 generations

Roations in *complex* $U(2)$ space

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \to U(\theta, \alpha, \beta, \gamma) = e^{i\gamma} \begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\beta} \\ -\sin\theta e^{-i\beta} & \cos\theta e^{-i\alpha} \end{pmatrix}$$

- We have $3 \times 3 = 9$ phase redefinitions to *remove unphysical* phases.
- There are $2 \times 4 = 8$ phases in $Y_{u,d}^{i,j}$.
- An *overall* phase $\psi \to e^{i\phi}\psi$ has *no effect* on $Y_{u,d}^{i,j}$, so we can only use 8 of the 9 phase redefinitions
- We end up with $8 - (9 - 1) = 0$ physical phases. We were lucky!

The SM with 2 quark generations *conserves CP* symmetry

Adding a third generation

We now have the fields

$$Q_L^{1,2,3} \qquad U_R^{1,2,3} \qquad D_R^{1,2,3}$$

We have to perform a field rotation in complex $U(3)$ space

$$(\widehat{\boldsymbol{Q}}_L)_1 = U_{Q_1}\,\boldsymbol{Q}_L \qquad (\widehat{\boldsymbol{Q}}_L)_2 = U_{Q_2}\,\boldsymbol{Q}_L \qquad \widehat{\boldsymbol{U}}_R = U_U\,\boldsymbol{U}_R \qquad \widehat{\boldsymbol{D}}_R = U_D\,\boldsymbol{D}_R$$

where U_F are unitary 3×3 matrices.

Mass eigenstates

We get 2 × 3 Dirac fields

$$\widehat{\boldsymbol{u}} = \begin{pmatrix} (\widehat{\boldsymbol{Q}}_L)_1 \\ \boldsymbol{U}_R \end{pmatrix} \qquad \widehat{\boldsymbol{d}} = \begin{pmatrix} (\widehat{\boldsymbol{Q}}_L)_2 \\ \boldsymbol{D}_R \end{pmatrix}$$

with definite masses

$$U(\theta_{Q_1})^\dagger \begin{pmatrix} v\,Y_u^{11} & v\,Y_u^{12} & v\,Y_u^{13} \\ v\,Y_u^{21} & v\,Y_u^{22} & v\,Y_u^{23} \\ v\,Y_u^{31} & v\,Y_u^{32} & v\,Y_u^{33} \end{pmatrix} U(\theta_U) = \begin{pmatrix} m_u & 0 & \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \text{etc.}$$

Physical consequences

As in the 2-generation case, there are *no flavour-changing neutral* couplings:

$$e\,\mathbb{1} \to e\,U^\dagger_{Q_1} U_{Q_1} = e\,\mathbb{1}$$

But again, there are *flavour-changing charged* currents:

$$g\,\mathbb{1} \to g\,U^\dagger_{Q_1} U_{Q_2} = g\,V_{\text{CKM}} \neq g\,\mathbb{1}$$

Instead of the Cabibbo angle, we now have 3×3 rotation matrix in charged currents that contains 3 physical parameters: the *Cabibbo-Kobayashi-Maskawa matrix*

CP violation with 3 generations

- Complex 3D rotations (group elements of $U(3)$) have 3 rotation angles and 6 phases.
- We thus have 9 angles and 18 phases to remove any unphysical parameters among the 18 real and 18 imaginary elements of $Y_{u,d}^{ij}$.
- Again, an overall phase does not affect $Y_{u,d}^{ij}$, so the number of physical parameters is

$$(18, 18) - [(9, 18) - (0, 1)] = (9, 1)$$

corresonding to *6 masses*, *3 CKM angles* and *1 physical, CP violating phase* in the CKM matrix.

In the SM with ≥ 3 generations, the *CP symmetry is violated*!

CKM matrix: standard parametrization

The CKM matrix has 3 mixing angles and 1 phase.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM matrix: standard parametrization

The CKM matrix has 3 mixing angles and 1 phase.

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$; $\theta_{12} \equiv \theta_{\mathrm{C}}$

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$c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$; $\theta_{12} \equiv \theta_{\text{C}}$

Experimentally: $(s_{12}, s_{13}, s_{23}, \delta) \approx (0.225, 0.042, 0.0036, 70^\circ)$

(see the next lecture on how this is measured!)

CKM matrix: Wolfenstein parametrization

Since V_{CKM} turns out to be very hierarchical, it is often very useful to consider a different parametrization, expanding in $\lambda \equiv s_{12} = \sin\theta_C \approx 0.22$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

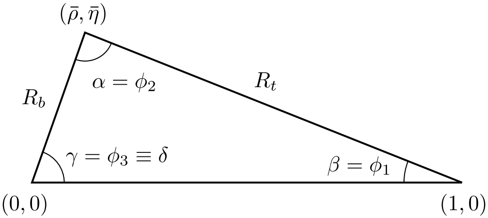
$$(\lambda, A, \bar{\rho}, \bar{\eta}) \approx (0.225, 0.82, 0.13, 0.35)$$

Unitarity triangle

V_{CKM} has to be a *unitary* matrix. This implies certain relations among its elements, in particular

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

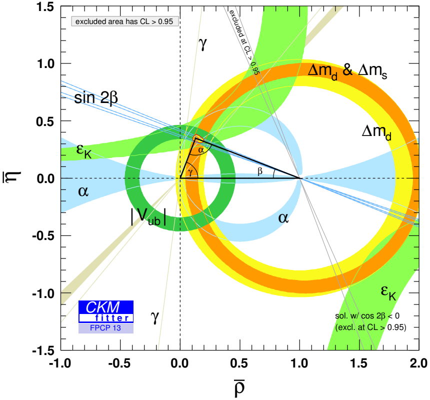
This can be representated as a *triangle* in a complex plane



$$(\alpha, \beta, \gamma) \approx (89^\circ, 22^\circ, 70^\circ)$$

(see handout for definition of $\bar\rho$, $\bar\eta$, R_b, R_t)

Unitarity triangle: scrutiny of the CKM mechanism

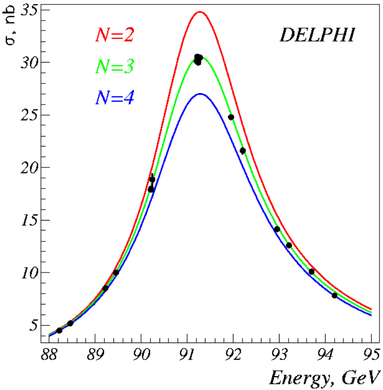


An impressive consistency! $\Rightarrow$  Nobel prize 2008 for Makoto Kobayashi and Toshihide Maskawa (they forgot Nicola Cabibbo!)

Adding a 4th generation?

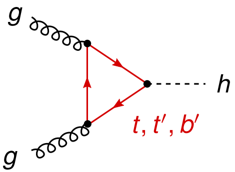
Adding a 4th generation?

Problem 1: Measurements of the decay $Z \to \nu\bar{\nu}$ (actually, $Z \to$invisible) at LEP in the 1990s showed that the number of *neutrinos* with $m_\nu < M_Z/2$ is 3.00 ± 0.08.



Adding a 4th generation?

Problem 2: A 4th generation of *quarks* would have enhanced the dominant production mode of the *Higgs boson* at the LHC by a *factor of 9!*



- There cannot be a (sequential) 4th generation

Summary: what we understand about flavour

- The different masses and mixing of quarks are due to their different *Yukawa couplings* to the Higgs field
- We know from *experiment* that the *Kobayashi-Maskawa mechanism* of flavour and CP violation works to an excellent precision
- We know from *experiment* that there are exactly *three* generations
- Fermions have to occur in *complete generations* of quarks and leptons (otherwise the theory would be inconsistent due to *gauge anomalies*)

Summary: what we *don't* understand about flavour

- What is the *origin* of the *hierarchies* in the Yukawa couplings (which give rise to the hierarchies in quark masses and mixing)?
- *Why 3* generations? (CP violation gives a hint why > 2, but δ_{CKM} is not enough to explain the baryon asymmetry of the universe)
- Is there any relation between mixing in the *neutrino* sector (PMNS) and the *quark* sector (CKM)?
- What explains the *baryon asymmetry* of the universe (CP violation in the SM is too small)?

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- Introduction: What is flavour and why is it interesting?
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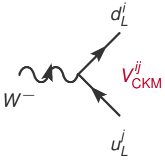
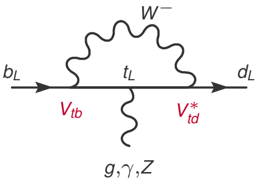
2 Flavour-changing neutral currents
- FCNCs in the Standard Model
- Meson-antimeson mixing
- Rare decays

Flavour-changing neutral currents

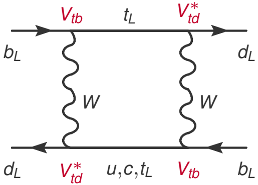
In the SM, the only flavour-changing coupling is the W vertex that changes also the electric charge:



However, flavour-changing neutral currents can be generated at loop level!

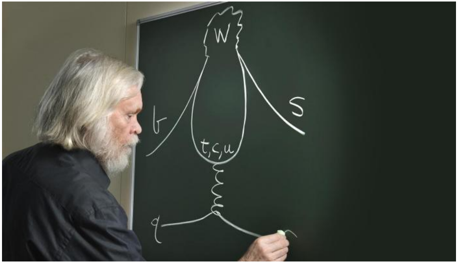


“Penguin diagram”



“Box diagram”

Penguin??

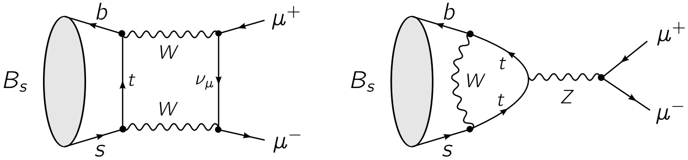


In 1977, John Ellis lost a bet in a game of darts against Melissa Franklin, forcing him to somehow insert the word “penguin” into his next paper

Two classes of FCNC processes

1. Rare meson decays

e.g. $B_s \to \mu^+\mu^-$

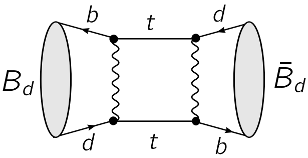


Flavour is changed by one unit: $\Delta F = 1$ *processes*

Two classes of FCNC processes

2. Meson-antimeson mixing

e.g. B^0-$\bar{B}^0$ mixing



Flavour is changed by two units: $\Delta F = 2$ *processes*

From quarks to mesons

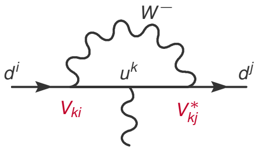
One of the biggest challenges in doing theoretical quark flavour physics is to connect calculable processes at *quark* level to the physical processes involving *mesons*.

Generically,

$$\langle f|\mathcal{H}|i\rangle = \underbrace{C}_{\text{perturbative, short distance}} \times \underbrace{U}_{\text{perturbative, QCD corr.}} \times \underbrace{\langle f|\mathcal{O}(q)|i\rangle}_{\text{non-perturbative}}$$

Glashow-Iliopoulos-Maiani (GIM) mechanism

Generic form of a FCNC amplitude:



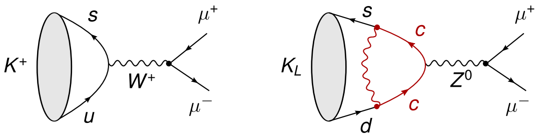
$$
\begin{aligned}
\sum V_{ki} V_{kj}^* F(m_{u^k}) &= V_{ui} V_{uj}^* F(m_u) + V_{ci} V_{cj}^* F(m_c) + V_{ti} V_{tj}^* F(m_t) \\
&\approx \left(V_{ui} V_{uj}^* + V_{ci} V_{cj}^* \right) F(0) + V_{ti} V_{tj}^* F(m_t) \\
&= V_{ti} V_{tj}^* \left[F(m_t) - F(0) \right]
\end{aligned}
$$

FCNC amplitude would be zero if all masses were degenerate!

GIM and the charm quark

Historically, the GIM mechanism led to the prediction of the *charm's existence and mass*

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu_\mu)} \approx 3 \times 10^{-9}$$



GIM: FCNC amplitude suppressed by $\left(\frac{m_c^2 - m_u^2}{M_W^2}\right)^2$

GIM and the charm quark

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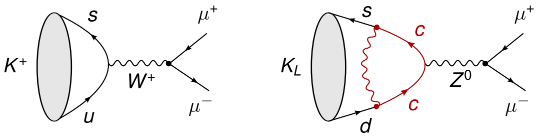


GIM: FCNC amplitude suppressed by $\left(\frac{m_c^2 - m_u^2}{M_W^2}\right)^2$

(a more precise prediction for m_c was obtained from K^0-$\bar{K}^0$ mixing)

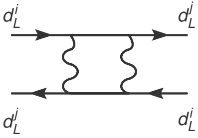
FCNCs are strongly suppressed in the SM

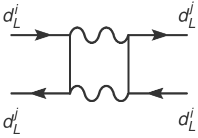
- because they arise only at the *loop* level
- because quark mixing is so *hierarchical* (off-diagonal CKM elements $\ll 1$)
- because of the *GIM* mechanism
- because only the *left-handed* chirality participates in flavour-changing interactions

Any of these conditions could be violated by *physics beyond the SM*. That's why FCNCs are so important!

Meson-antimeson mixing in the SM

In the SM, M-$\bar{M}$ mixing proceeds via box diagrams with W exchange





and occurs in the four neutral meson systems

$$K^0 = (d\,\bar{s}) \qquad B_d^0 \equiv B^0 = (d\,\bar{b}) \qquad B_s^0 \equiv B_s = (s\,\bar{b}) \qquad D^0 = (c\,\bar{u})$$

Meson-antimeson mixing: basics

Consider the time evolution of a meson state $|M\rangle$

$$i\frac{d}{dt}|M(t)\rangle = \left(M_M - i\frac{\Gamma}{2}\right)|M(t)\rangle$$

where M_M is the meson mass and $\Gamma = 1/\tau$ the decay width (inverse lifetime)

$$|M(t)\rangle = e^{-iMt}\, e^{-\Gamma t/2}\, |M(0)\rangle$$

Meson-antimeson mixing: basics

Now: consider a coupled meson-antimeson system

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle\\|\bar{M}(t)\rangle\end{pmatrix}=\begin{pmatrix}M-i\frac{\Gamma}{2} & M_{12}-i\frac{\Gamma_{12}}{2}\\ M_{12}^{*}-i\frac{\Gamma_{12}^{*}}{2} & M-i\frac{\Gamma}{2}\end{pmatrix}\begin{pmatrix}|M(t)\rangle\\|\bar{M}(t)\rangle\end{pmatrix}$$

- The diagonal elements are equal due to CPT symmetry.
- If we switch of the weak interaction, $\Gamma, \Gamma_{12}, M_{12} \to 0$

Diagonalizing the system

Let us start in the limit of CP symmetry: $\delta_{\text{CKM}} \to 0 \Rightarrow M_{12}, \Gamma_{12} \in \mathbb{R}$.

We obtain two mass eigenstates after diagonalization,

$$M_{L,H} = \frac{1}{\sqrt{2}} \left(|M\rangle \pm |\bar{M}\rangle \right)$$

$$i \frac{d}{dt} \begin{pmatrix} |M_L(t)\rangle \\ |\bar{M}_H(t)\rangle \end{pmatrix} = \begin{pmatrix} M_L - i\frac{\Gamma_L}{2} & 0 \\ 0 & M_H - i\frac{\Gamma_H}{2} \end{pmatrix} \begin{pmatrix} |M_L(t)\rangle \\ |\bar{M}_H(t)\rangle \end{pmatrix}$$

The mass and width differences are

$$\Delta M = M_H - M_L = 2|M_{12}| \qquad |\Delta\Gamma| = |\Gamma_H - \Gamma_L| = 2|\Gamma_{12}|$$

Solving the Schrödinger equation

In the meson rest frame, an initially pure flavour eigenstate evolves according to

$$|M(t)\rangle = e^{-iMt}\, e^{-\Gamma t/2} \left[\cos\left(\frac{\Delta M t}{2}\right)|M\rangle + i\sin\left(\frac{\Delta M t}{2}\right)|\bar{M}\rangle\right]$$

(neglecting $\Delta\Gamma$)

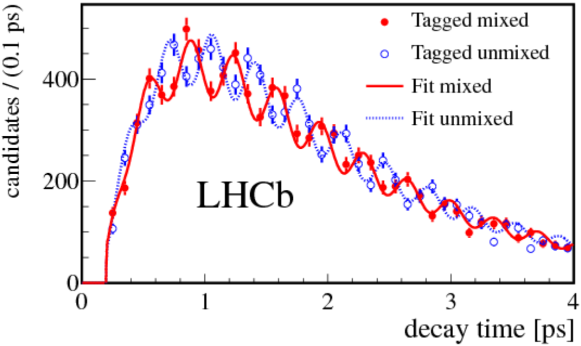
Some numbers

system	$\Delta M/M$	$\Delta M/\Gamma$	$\Delta\Gamma/(2\Gamma)$
K^0	7.0×10^{-15}	0.5^*	1.0
B^0	6.4×10^{-14}	0.8	< 0.02
B_s^0	2.2×10^{-12}	27	0.08
D^0	6.4×10^{-10}	0.006	0.008

Green: can be computed to a reasonable ($O(10\%)$) accuracy within the SM

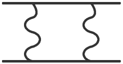
(* this is $\Delta M/\Gamma_S$)

Measurement!



LHCb measurement of B_s-$\bar{B}_s$ mixing, April 2013

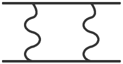
Computing the mass difference

$M_{12} \propto \langle M|$  $|\bar{M}\rangle$

$$\propto \frac{g^2}{m_W^2} \left(V_{ti} V_{tj}^* \right)^2 \ \ldots$$

Computing the mass difference

$$M_{12} \propto \langle M|$$



$$|\bar{M}\rangle$$

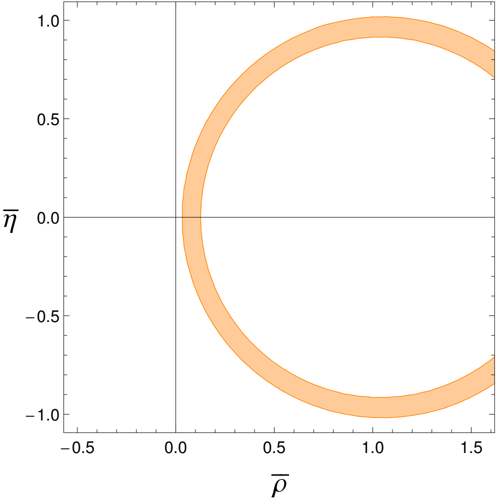
$$\propto \frac{g^2}{m_W^2} (V_{ti} V_{tj}^*)^2 \ \ldots$$

In the case of B^0-$\bar{B}^0$ mixing:

$$\Delta M_d = 2|M_{12}| \propto |(V_{tb} V_{td}^*)^2| \approx (A\lambda^3)^2 \left[(1-\rho)^2 + \eta^2\right]^2$$

$\Rightarrow$ a circle in the ρ-η plane

ΔM_d and the unitarity triangle



Enter CP violation

We know that the weak interactions don't respect CP, so we expect $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$

$$M_{L,H} = p|M\rangle \pm q|\bar{M}\rangle$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}$$

- By rephasing $|M\rangle$ or $|\bar{M}\rangle$, we can remove all phases except one in M_{12}, Γ_{12} and q/p.
- We end up with 3 physical *meson mixing parameters*

$$\Delta M = 2|M_{12}| \qquad |\Delta\Gamma| = 2|\Gamma_{12}| \qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Time-dependent CP asymmetry in the B^0 system

Consider the asymmetry in the decays of neutral meson flavour eigenstates to some final state f (that is a CP eigenstate, $f = \bar{f}$)

$$A_{\text{CP}}(t, f) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

$$A_{\text{CP}}(t, f) = A_{\text{CP}}^{\text{dir}}(f) \cos(\Delta M t) + A_{\text{CP}}^{\text{mix}}(f) \sin(\Delta M t)$$

Mixing-induced CP asymmetry

$$\xi_f = \frac{q}{p}\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)}$$

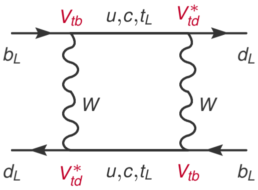
$$A_{\mathrm{CP}}^{\mathrm{dir}}(f) = \frac{1-|\xi_f|^2}{1+|\xi_f|^2} \qquad A_{\mathrm{CP}}^{\mathrm{mix}}(f) = \frac{2\,\mathrm{Im}\xi_f}{1+|\xi_f|^2}$$

In the B^0 system, $\Delta\Gamma \ll \Gamma \Rightarrow |q/p| \approx 1$.

Particular interesting case: decays where the decay is dominated by a single diagram

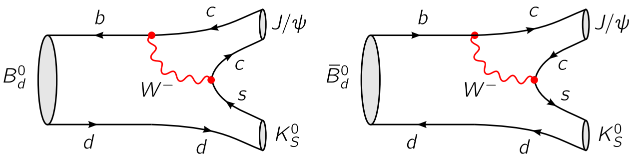
$$\left|\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)}\right| = 1 \qquad A_{\mathrm{CP}}^{\mathrm{dir}}(f) = 0$$

B mixing phase



$$\frac{q}{p} \approx -\frac{M_{12}^*}{|M_{12}|} = -\frac{(V_{td}V_{tb}^*)^2}{|V_{td}^*V_{tb}|^2} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} = e^{-2i\beta} = e^{-2i\phi_1}$$

"Golden mode" $B^0 \to J/\psi K_S$

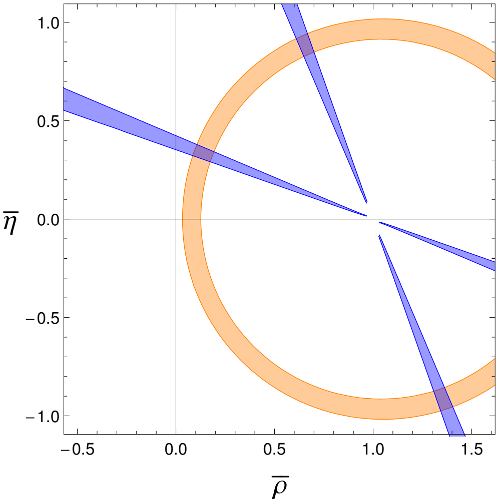


$$\frac{A(\bar{B} \to J/\psi K_S)}{A(B^0 \to J/\psi K_S)} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \approx 1$$

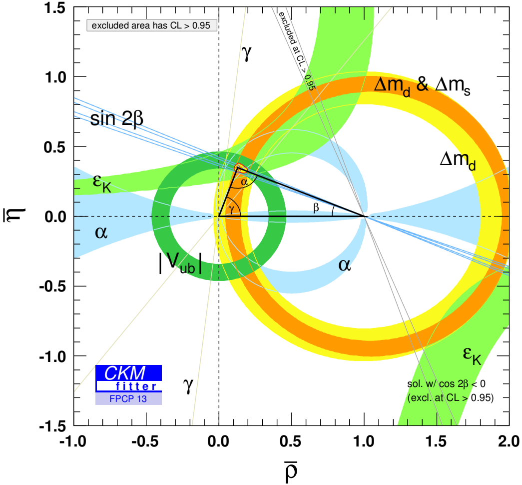
$$\Rightarrow \xi_{J/\psi K_S} = e^{-i2\beta}$$

$$\boxed{A_{CP}^{\text{mix}}(J/\psi K_S) = -\sin(2\beta)}$$

$\sin 2\beta$ and the unitarity triangle



sin 2β and the unitarity triangle



Rare decays at quark level

Similarly to meson-antimeson mixing, there are four different types of quark FCNCs with $\Delta F = 1$

quark FCNC	$s \to d$	$b \to d$	$b \to s$	$c \to u$
decaying meson	$K^{\pm,0}$	$B^{\pm,0}$, B_s	$B^{\pm,0}$, B_s	$D^{\pm,0}$

Due to the multitude of possible initial and final states, the number of independent observables is much larger than for $\Delta F = 2$!

Inclusive and exclusive decays

Depending on the *final state*, we can distinguish three broad classes of rare decays

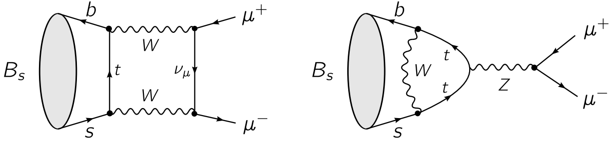
	Non-leptonic	Semi-leptonic, radiative & leptonic	
		exclusive	*inclusive*
	$M \to M' M'' \ldots$	$M \to M' \ell^+\ell^-$	$\sum_X M \to X \ell^+\ell^-$
		$M \to M' \nu\bar{\nu}$	$\sum_X M \to X \nu\bar{\nu}$
		$M \to M' \gamma$	$\sum_X M \to X \gamma$
		$M^0 \to \ell^+\ell^-$	
exper.:			hard
theor.:	very hard	hard	

Rare B and K decays

An incomplete list of rare inclusive and exclusive decays that are sensitive to the existence of physics beyond the SM

	$b \to s\ (\propto \lambda^2)$	$b \to d\ (\propto \lambda^3)$	$s \to d\ (\propto \lambda^5)$
γ	$B \to X_s\gamma$ $B \to K^*\gamma$	$B \to X_d\gamma$ $B \to \rho\gamma$	
$\ell^+\ell^-$	$B \to K\ell^+\ell^-$ $B \to K^*\ell^+\ell^-$ $B \to X_s\ell^+\ell^-$	$B \to \pi\ell^+\ell^-$ $B \to \rho\ell^+\ell^-$ $B \to X_d\ell^+\ell^-$	$K_L \to \pi\ell^+\ell^-$
	$B_s \to \mu^+\mu^-$	$B \to \mu^+\mu^-$	$K_L \to \mu^+\mu^-$
$\nu\bar{\nu}$	$B \to X_s\nu\bar{\nu}$ $B \to K\nu\bar{\nu}$ $B \to K^*\nu\bar{\nu}$	$B \to X_d\nu\bar{\nu}$	$K^+ \to \pi^+\nu\bar{\nu}$ $K_L \to \pi^0\nu\bar{\nu}$

$B_s \to \mu^+\mu^-$

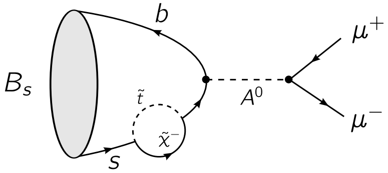


- helicity-suppressed since it vanishes for massless leptons (in addition to the loop- and CKM-suppression) $\Rightarrow$ one of the rarest B decays
- non-hadronic final state $\Rightarrow$ relatively clean theoretically (for an exclusive decay)
- clean experimental signature

$$\mathrm{BR}_{\mathrm{SM}} = (3.2 \pm 0.2) \times 10^{-9} \qquad \mathrm{BR}_{\mathrm{LCHb+CMS\ 2013}} = (2.9 \pm 0.7) \times 10^{-9}$$

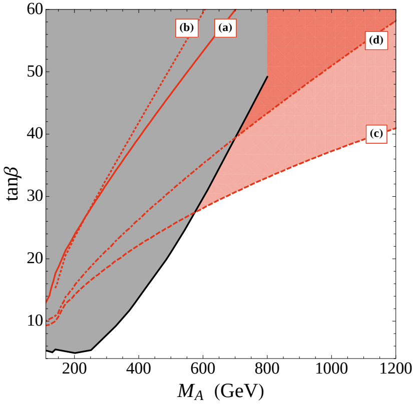
$B_s \to \mu^+\mu^-$ beyond the SM

Example: supersymmetry, heavy Higgs exchange



- In the SM, fermion-Higgs couplings are given by $Y = m/v$ where $v \approx 250$ GeV
- In models with more two Higgs doublets, one can have $Y = m/v_d$ where $v_u^2 + v_d^2 \approx 250$ GeV
- If $\tan\beta = v_u/v_d \gg 1$, the decay rate of $B_s \to \mu^+\mu^-$ can be greatly enhanced

$B_s \to \mu^+\mu^-$ constraint on the MSSM



gray: $A, H \to \tau^+\tau^-$

[Altmannshofer, Carena, Shah (2012)]

- measurement constrains the parameters tan β and M_A
- large tan β + light M_A disfavoured
- constraint is complementary to direct searches for heavy Higgs