Matrix elements + PYTHIA8

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(DESY)

(with Leif Lönnblad)

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Outline

- Leading-order ME+PS merging revisited.
- NLO multi-jet merging in PYTHIA8.
- NNLO matching?
- Summary

Warning: Selection of topics drenched in personal bias. Little or no mention of MLM, CKKW-L, aMC@NLO or NL^3.

Motivation

I will not talk about CKKW-L or NL^3 (i.e. CKKW-L@NLO), although both of these methods are available in PYTHIA8.

Why?

We want a method for *any* merging scale value.

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We want a method for *any* merging scale value.

To get rid of merging scale dependence:

$\Rightarrow$ Subtract what you add!

$\Rightarrow$ Unitarised MEPS.

Combining multi-jet calculations

ME+PS merging:

Use ME above a cut t_{MS}, and PS below t_{MS}.

- ◇ Apply the same weights (α_s running, Sudakov factors) above and below the cut.
- ◇ Then combine the reweighted sub-samples.

[1] JHEP02(2013)094 (Leif Lönnblad, SP), arXiv:1211.5467 (Simon Plätzer)

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- ◇ In CKKW-L, we then simply add the reweighted sub-samples.

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!
But they should not invalidate the inclusive (0-jet) cross section!

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New approach[1]: Use a (PS) unitarity inspired approach to preserve the inclusive cross section.

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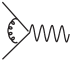
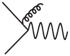
Unitarised ME+PS merging

The sum of virtual and real corrections is finite.

$\sigma\Big($  $\Big) \quad + \quad \int d\sigma\Big($  $\Big) \quad = \quad \text{finite}$

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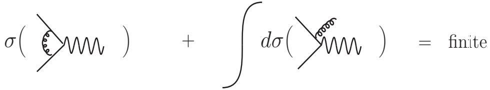
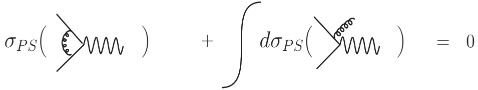
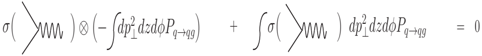


The sum of approximate PS virtual and real corrections vanishes . . .



. . . because the virtual corrections are simply $-1\times$ integrals of splitting kernels



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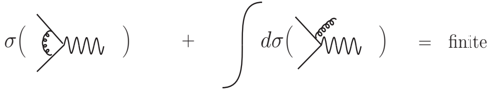

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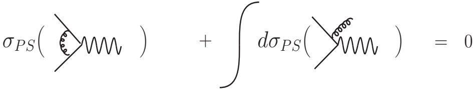

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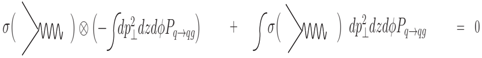

Thus, when including the $+n$-jet calculation, we improve the PS approximate virtual corrections by instead subtracting the full integrated $+n$-jet result . . .

. . . and recover the inclusive cross section.

Comments on UMEPS

This sketch can directly be extended to the case when we have (α_s- or Sudakov-) weighted $+n$-jet states ($\widehat{\mathrm{B}}_n$), e.g. two-jet UMEPS merging:

$$\begin{aligned}\langle\mathcal{O}\rangle \quad = \quad \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \; - \; \int_s \widehat{\mathrm{B}}_{1\to 0} \; - \; \int_s \widehat{\mathrm{B}}_{2\to 0} \right] \\ & + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \; - \; \int_s \widehat{\mathrm{B}}_{2\to 1} \right] \\ & + \int \cdots \int \mathcal{O}(S_{+2j}) \; \widehat{\mathrm{B}}_2 \Bigg\}\end{aligned}$$

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We can get the integrated version of the real-emission matrix elements by projecting onto an underlying Born configuration. Such configurations are available anyway since we need them to perform the Sudakov weighting.

The "subtract what you add" prescription means that this will produce *counter-events* with negative weight.

UMEPS combines features of CKKW-L and LoopSim.

Now, the merging scale is a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq`/`hfact` in the POWHEG-BOX) is.

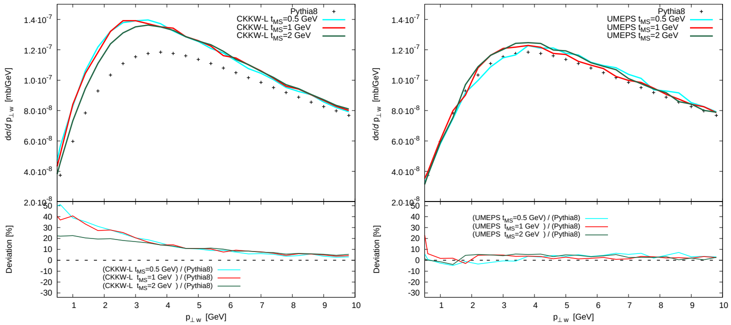

Figure: $p_\perp$ of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV)

⇒ CKKW-L overshoots for (very) low merging scales.

⇒ UMEPS describes the Sudakov peak nicely.

(For jet observables (high-$p_\perp$ tails etc.) UMEPS does as nicely as CKKW-L.)

Is UMEPS enough?

NO

We want to use the full NLO results whenever possible.

Do NLO multi-jet merging for UMEPS ...

$\Rightarrow$ Subtract approximate UMEPS $\mathcal{O}(\alpha_s)$-terms, add back full NLO.

$\Rightarrow$ To preserve the inclusive (NLO) cross section, add approximate NNLO.

$\Rightarrow$ UNLOPS[2].

[2] JHEP03(2013)166 (Leif Lönnblad, SP), contains also NL^3 = CKKW-L@NLO.

The UNLOPS method

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int_s \widehat{\mathrm{B}}_{1\to 0} \qquad - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\qquad \widehat{\mathrm{B}}_1 \qquad - \int_s \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} - \int_s \widehat{\mathrm{B}}_{1\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \widehat{\mathrm{B}}_1 - \int_s \widehat{\mathrm{B}}_{2\to 1} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

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Or for the case of M different NLO calculations and N tree-level calculations:

$$\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$
$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int_s \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$
$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int_s \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$
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$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

This is done internally.

The formula will not be in the exam.

Again, what does it do?

Combine NLO calculations for different jet multiplicities ...

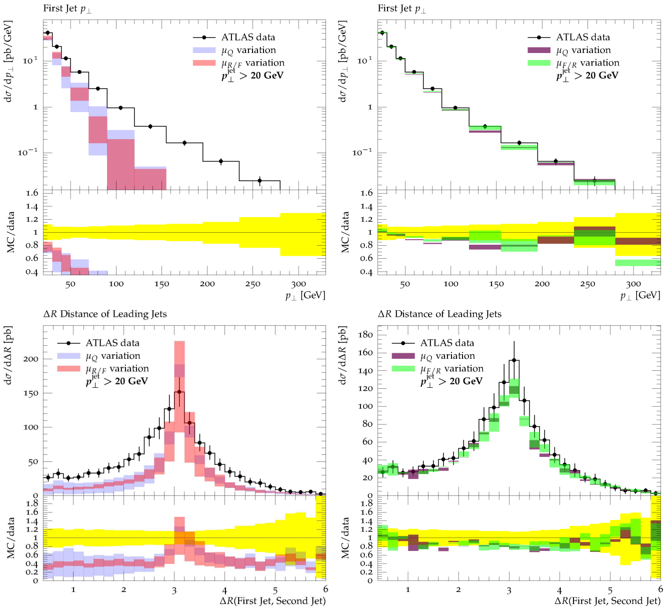
... and add further tree-level calculations on top ...

... and have one single inclusive sample with X+(0,..., M)@NLO and X+(M+1,...,N)@LO

Aim: Use NLO for as many multiplicities as possible,
then use LO for more jets,
and only then use PS.

$\Rightarrow$ Reduce $\mu_F,\ \mu_R$ dependence due to NLO input,
reduce μ_Q dependence because ME's fill most of the phase space.

UNLOPS results



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

Note that in UNLOPS, the lowest-multiplicity cross section is *not* reweighted. Terms entering due to PS weights are $\mathcal{O}(\alpha_s^2)_{\mathrm{PS}} \times \mathcal{O}(\alpha_s^1)_{\mathrm{ME}}$ and $\mathcal{O}(\alpha_s^0)_{\mathrm{PS}} \times \mathcal{O}(\alpha_s^2)_{\mathrm{ME}}$

$$\begin{aligned}
\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \right. \\
& \qquad \left. - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \right) \\
& + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\
& + \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2
\end{aligned}$$

NNLO with UNLOPS

It is simple to remove this single $\mathcal{O}(\alpha_s^2)$-term. Subtracting the term, it is possible to replace the lowest multiplicity cross section by the full NNLO result

$$\begin{aligned}\langle\mathcal{O}\rangle &= \int d\phi_0\Bigg\{\mathcal{O}(S_{+0j})\left(\widetilde{\widetilde{\mathrm{B}}}_0 - \left[\int_s \widehat{\mathrm{B}}_{1\to0}\right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to0}\right]_{-2}\right)\\ &\quad + \int \mathcal{O}(S_{+1j})\left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1\right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to1}\right]_{-2}\right)\\ &\quad + \int\int \mathcal{O}(S_{+2j})\,\widehat{\mathrm{B}}_2\Bigg\}\end{aligned}$$

$\Rightarrow$ The cross section formula becomes simpler again. The inclusive cross section is now

$$\int d\phi_0\mathcal{O}(S_{+0j})\left(\widetilde{\widetilde{\mathrm{B}}}_0 + \int_s \mathrm{B}_{2\to0}\right) + \int d\phi_0 \int \mathcal{O}(S_{+1j})\left(\widetilde{\mathrm{B}}_1 + \int_s \mathrm{B}_{2\to1}\right)$$

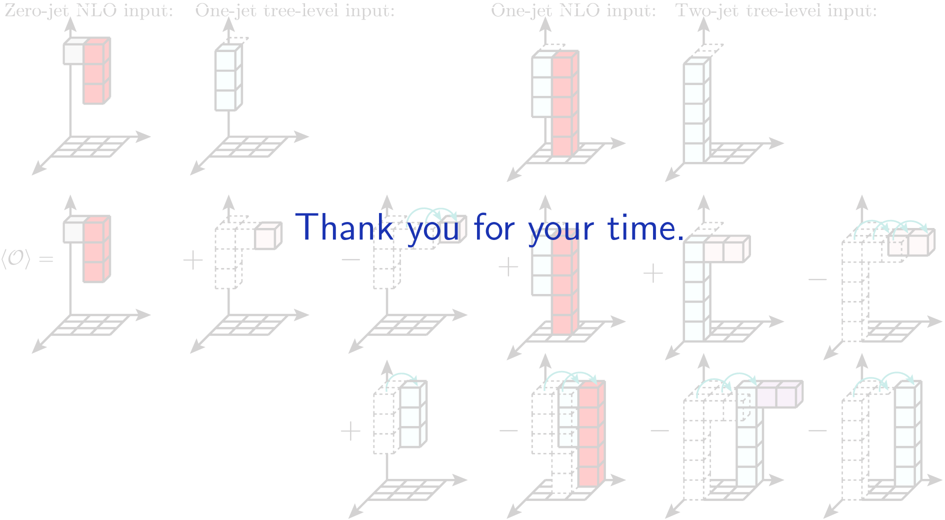
which is just the NNLO result.

We need an NNLO generator to produce $\widetilde{\widetilde{\mathrm{B}}}_0$ or $\overline{\overline{\mathrm{B}}}_0$ (for $gg \to$H, $\overline{\overline{\mathrm{B}}}_0$ is only dependent on the H-rapidity spectrum).

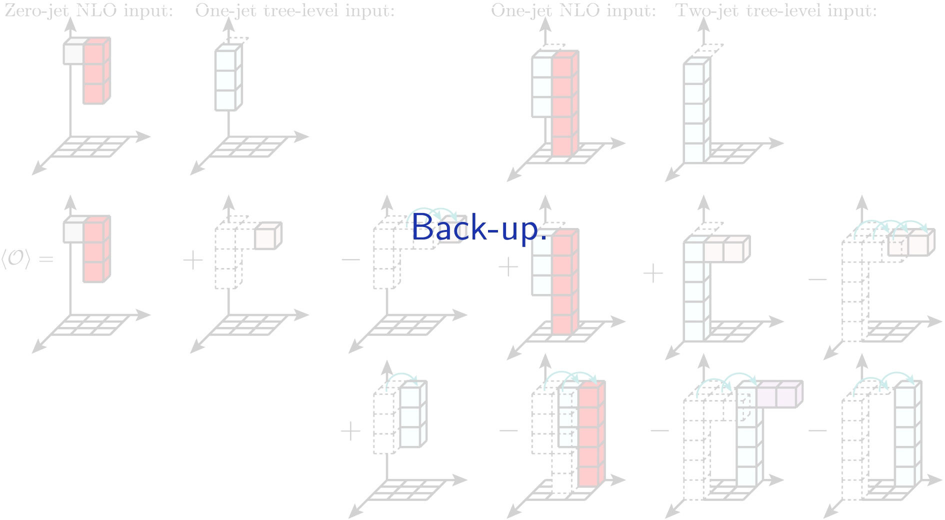
Summary

- To describe data, we need to infuse parton showers with matrix elements.
- CKKW-L tree-level merging is included in PYTHIA8.
  But it does not work for arbitrary small merging scales.
- UMEPS tree-level merging is included in PYTHIA8.
  UMEPS almost cancels the merging scale dependence.
  But it's not NLO.
- Two NLO merging schemes are implemented in PYTHIA8:
  NL^3 and UNLOPS.
- UNLOPS is our preferred choice.
- All merging schemes in PYTHIA8 run on LHEF input, e.g. POWHEG-BOX or MADEVENT input.

Thank you for your time.



Back-up.


Zero-jet NLO input:
One-jet tree-level input:
One-jet NLO input:
Two-jet tree-level input:
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UMEPS definitions

$$
\begin{aligned}
w_n =& \frac{x_n^+ f_n^+(x_n^+,\rho_n)}{x_n^+ f_n^+(x_n^+,\mu_F)} \frac{x_n^- f_n^-(x_n^-,\rho_n)}{x_n^- f_n^-(x_n^-,\mu_F)} \\
& \times \prod_{i=1}^{n} \left[\frac{\alpha_s(\rho_i)}{\alpha_s(\mu_R)} \frac{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_{i-1})}{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+,\rho_i)} \frac{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_{i-1})}{x_{i-1}^- f_{i-1}^-(x_{i-1}^-,\rho_i)} \Pi_{S_{+i-1}}(x_{i-1},\rho_{i-1},\rho_i) \right] \\
\widehat{\mathrm{B}}_n =& \mathrm{B}_n w_n \\
\int_s \widehat{\mathrm{B}}_{n\to m} =& \left[\prod_{a=m+1}^{n-1} \int d\rho_a dz_a d\varphi_a \Theta(\rho_{\mathrm{MS}} - \rho_a) \right] \int d\rho_n dz_n d\varphi_n \mathrm{B}_n w_n \\
\langle \mathcal{O} \rangle =& \sum_{n=0}^{N} \int d\phi_0 \int \dots \int \mathcal{O}(S_{+nj}) \left\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \right\} .
\end{aligned}
$$

In CKKW-L, w_n contains an additonal factor $\Pi_{S_{+n}}(x_n, \rho_n, \rho_{\mathrm{MS}})$.
UMEPS induces this through $\int_s \widehat{\mathrm{B}}_{n+1\to n}$ instead.

Here, we illustrate how to include the full kinematical information also below the merging scale.

$$d\sigma^u_{+0} \equiv \widehat{\mathrm{B}}_{+0} - \int_s \widehat{\mathrm{B}}_{0+1\to 0} \qquad d\sigma^u_{+1} \qquad \equiv \widehat{\mathrm{B}}_{+1}$$

So far, we have

$$\begin{aligned}\langle\mathcal{O}\rangle = & \, d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\ldots)] \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_1-\rho_{\mathrm{MS}})\end{aligned}$$

But we could write

$$\begin{aligned}\langle\mathcal{O}\rangle = & \, d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \qquad)] \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_1-\rho_{\mathrm{MS}}) \\ & + d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_{\mathrm{MS}}-\rho_1) \\ & - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \\ & - \int \Big\{ d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c) \;+\ldots)] \times \Theta(\rho_{\mathrm{MS}}-\rho_1) \\ & - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c)\Big\}\mathcal{O}_0(S_{+0})\end{aligned}$$

UMEPS ' = UMEPS

This is just

$$\langle\mathcal{O}\rangle = d\sigma^{u}_{+0}\mathcal{O}_0(S_{+0}) - \int d\sigma^{d}_{+1} \times \Theta(\rho_{\text{MS}} - \rho_1)\mathcal{O}_0(S_{+0})$$
$$+ d\sigma^{u}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \ + \ \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \ + \ldots)]$$
$$\times \Theta(\rho_1 - \rho_{\text{MS}})$$
$$+ d\sigma^{d}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \ + \ \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \ + \ldots)]$$
$$\times \Theta(\rho_{\text{MS}} - \rho_1)$$

which for $d\sigma^{d}_{+1} = d\sigma^{u}_{+1}$ becomes

$$\langle\mathcal{O}\rangle = d\sigma^{u}_{+0}\mathcal{O}_0(S_{+0})$$
$$+ d\sigma^{u}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \ + \ \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \ + \ldots)]$$
$$\times \Theta(\rho_1 - \rho_c)$$

i.e. simply the UMEPS result for $\rho_{\text{MS}} = \rho_c$.
Note that this does not depend on the form of $d\sigma^{u}_{+0}$ or $d\sigma^{u}_{+1}$, and can thus also be applied for UNLOPS to add real-emission kinematics with $\rho(S_{+n+1}) < \rho_{\text{MS}}$.

Extreme merging scale values

In unitarised merging, t_{MS} can be varied between the PS cut-off ρ_c and ∞. The t_{MS} dependence is almost exactly cancelled (caveat: jet definition)

For $t_{\mathrm{MS}} \to \infty$, the spectrum of the first add. emission is given by the PS. The below-t_{MS} behaviour can be fixed[3], leading e.g. to the following form:

$$\langle \mathcal{O} \rangle = \left[d\sigma^{u}_{+0} - \int d\sigma^{d}_{+1} \times \Theta(\rho_{\mathrm{MS}} - \rho_1) \right] \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^{u}_{+1} \Theta(\rho_1 - \rho_{\mathrm{MS}}) \right] + \mathbf{PS}\left[d\sigma^{d}_{+1} \Theta(\rho_{\mathrm{MS}} - \rho_1) \right]$$

which for $d\sigma^{d}_{+1} = d\sigma^{u}_{+1}$ becomes

$$\langle \mathcal{O} \rangle = d\sigma^{u}_{+0} \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^{u}_{+1} \Theta(\rho_1 - \rho_c) \right]$$

i.e. simply the UMEPS result for $\rho_{\mathrm{MS}} = \rho_c$.

The merging scale is a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq`/`hfact` in the POWHEG-BOX) is.

[3] See also arXiv:1211.5467 (Simon Plätzer)

UNLOPS results