Transverse momentum resummation in Higgs production.

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Outline

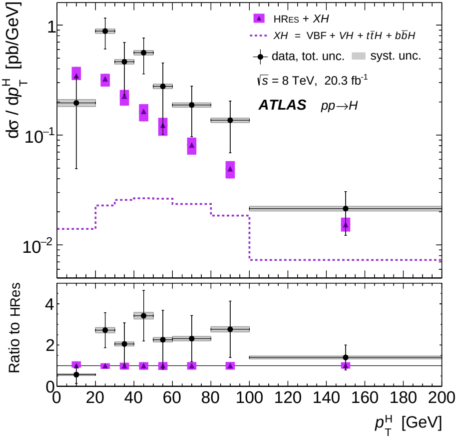
Introduction.

Introduction.

Status of Higgs measurements:

- Important observable of the LHC Higgs program
- Theory and experiment are compatible
- Large uncertainties currently limited by statistics
- ∼20 times more data in the LHC run 2

  → Theory uncertainties important soon



[ATLAS]

Precision predictions for the Higgs p_T-spectrum are needed.

Introduction.

Theory status:

- Spectrum contains large logarithms $\ln \frac{p_T}{m_H}$
- Need to be resummed to all orders for $p_T \ll m_H$
- Current results:
  - [de Florian, Ferrera, Grazzini, Tommasini]: HRes (NNLO + NNLL)
  - [Becher, Lübbert, Neubert, Wilhelm]: CuTe (NLO + NNLL)
    $\rightarrow$ See T. Lübbert's Talk
  - [Neill, Rothstein, Vaidya] (NNLO + NNLL)
  - [Echevarria, Kasemets, Mulders, Pisano] (NNLL)
  - Resummation is performed (partially) in impact parameter space:
    Resums logarithms $\ln(b\, m_h)$ instead of $\ln \frac{p_T}{m_H}$

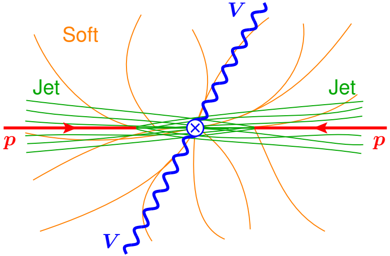
Goal:

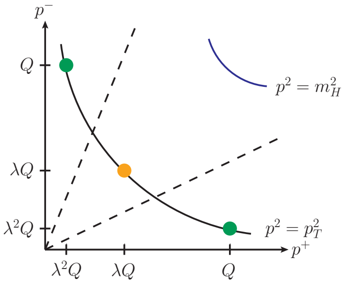
Perform resummation directly in p_T-space

Transverse momentum distribution in SCET.

Higgs production in SCET.

Illustration of factorization in SCET:





$$\sigma = H(B_1 \otimes B_2 \otimes S)(\vec{p}_T)$$

- Hard function: Higgs produced from incoming gluons
- Beam function: collinear radiation along beam axes
- Soft function: anisotropic radiation

RG structure.

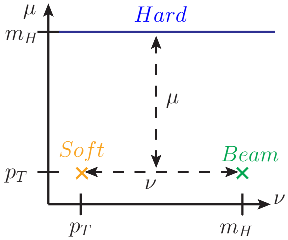
- After regularizing rapidity divergences:

$$\sigma = H(\mu)\left[B_1(\mu,\nu) \otimes B_2(\mu,\nu) \otimes S(\mu,\nu)\right](\vec{p}_T)$$

  - μ: Usual renormalization scale
  - ν: Rapidity renormalization scale
- Logarithms are split into

$$\ln^2 \frac{p_T}{m_H} = \ln^2 \frac{m_H}{\mu} + 2\ln\frac{p_T}{\mu}\ln\frac{\nu}{m_H} + \ln\frac{p_T}{\mu}\ln\frac{\mu\, p_T}{\nu^2}$$

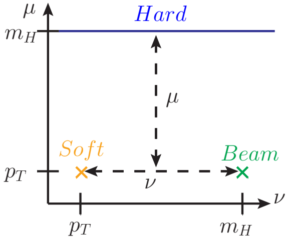
- μ and ν give rise to renormalization group equations (RGE)
- RGEs allow to resum all large logarithms
  - Most general setup
  - Regulator can be arbitrary (we use η-regulator [Chiu, Jain, Neill, Rothstein])
  - Previous results can be recovered through specific scale choices



RG-evolved cross section.

$$\sigma = \underbrace{U(\mu_H, \mu_B, \mu_S)}_{\mu\text{-evolution}} \underbrace{V(\vec{p}_T, \nu_B, \nu_S)}_{\nu\text{-evolution}}$$

$$\otimes \underbrace{H(\mu_H)\left[B^2(\mu_B, \nu_B) \otimes S(\mu_S, \nu_S)\right]}_{\text{Evaluated at natural scales}}(\vec{p}_T)$$



- Natural scale choice:

$$\mu_H = m_H \qquad \mu_B = p_T \qquad \mu_S = p_T$$

$$\nu_B = m_H \qquad \nu_S = p_T$$

- Deviating scale choices in literature:
  - [de Florian, Ferrera, Grazzini, Tommasini]: CSS-formula, corresponding to $\mu_B = \mu_S = \nu_S = 1/b$
  - [Neill, Rothstein, Vaidya]: Choose $\mu_B = \mu_S = \nu_S = 1/b$
  - [Becher, Neubert, Wilhelm]: Corresponds to $\mu_B = \mu_S = q_* + p_T, \qquad \nu_S = 1/b$

Impact parameter space

Goal: Use natural scale choices in momentum space.

The rapidity evolution factor.

$$\sigma = \underbrace{U(\mu_H, \mu_B, \mu_S)}_{\mu\text{-evolution}} \underbrace{V(\vec{p}_T, \nu_B, \nu_S)}_{\nu\text{-evolution}} \otimes \underbrace{H\left[B_1 \otimes B_2 \otimes S\right](\vec{p}_T)}_{\text{Evaluated at natural scales}}$$

- $H\left[B_1 \otimes B_2 \otimes S\right]$: Calculable in fixed-order SCET ✓
- μ-evolution ✓

$$U(\mu_H, \mu_B, \mu_S) =$$
$$\exp\left[\int_{\mu_H}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_H^{(\mu)}(\mu') + 2\int_{\mu_B}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_B^{(\mu)}(\mu') + \int_{\mu_S}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_S^{(\mu)}(\mu')\right]$$

- ν-evolution ?

$$\nu \frac{\mathrm{d}V(\vec{p}_T, \nu, \nu_0)}{\mathrm{d}\nu} = \gamma^{(\nu)}(\vec{p}_T, \mu) \otimes V(\vec{p}_T, \nu, \nu_0) \quad \Rightarrow \quad V(\vec{p}_T, \nu_B, \nu_S) = ?$$

Only complication:

Momentum space solution of rapidity RGE.

Rapidity Renormalization Group.

Rapidity Renormalization Group.

Impact parameter space:

- Problem of resumming the p_T-spectrum reduced to solving

$$\nu \frac{\mathrm{d}V(\vec{p}_T, \nu, \nu_0)}{\mathrm{d}\nu} = \int \frac{\mathrm{d}^2\vec{q}_T}{(2\pi)^2} \gamma^{(\nu)}(\vec{p}_T - \vec{q}_T, \mu) V(\vec{q}_T, \nu, \nu_0)$$

- Easily solved after Fourier transformation (*Impact Parameter Space*):

$$\nu \frac{\mathrm{d}V(b, \nu, \nu_0)}{\mathrm{d}\nu} = \gamma^{(\nu)}(b, \mu) V(b, \nu, \nu_0)$$

$$\Rightarrow \qquad V(b, \nu_B, \nu_S) = \exp\left[\gamma^{(\nu)}(b, \mu) \ln \frac{\nu_B}{\nu_S}\right]$$

- p_T-space solution:

$$V(\vec{p}_T, \nu_B, \nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T \cdot \vec{b}} \exp\left[\gamma^{(\nu)}(b, \mu) \ln \frac{\nu_B}{\nu_S}\right]$$

Rapidity Renormalization Group.

Example: 1-loop

- ν-anomalous dimension:

$$\gamma^{(\nu)}(b,\mu) = -\frac{2C_A\alpha_s}{\pi}\ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)$$

- p_T-space solution:

$$V(\vec{p}_T,\nu_B,\nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T\cdot\vec{b}}\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)^{-\omega_s} \propto e^{-2\gamma_E\omega_s}\frac{\Gamma(1-\omega_s)}{\Gamma(1+\omega_s)}$$

where

$$\omega_s = 2\frac{C_A\alpha_s(\mu)}{\pi}\ln\frac{\nu_B}{\nu_S} \sim 2\frac{C_A\alpha_s(\mu)}{\pi}\ln\frac{m_H}{p_T} = \mathcal{O}(1)$$

- p_T-space solution diverges for $\omega_s \to 1$

Rapidity Renormalization Group.

CSS solution of the divergence:

- Divergence from $b \to 0$:

$$V(\vec{p}_T, \nu_B, \nu_S) = \int \mathrm{d}^2\vec{b}\, e^{i\vec{p}_T\cdot\vec{b}} \left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right)^{-\omega_s} \sim \int \frac{\mathrm{d}b^2}{(\mu^2 b^2)^{\omega_s}}$$

- Simple solution (CSS): $\mu \sim 1/b$
- Problems:
  - Resums logarithms $\ln(b\, m_H)$ instead of $\ln \frac{p_T}{m_H}$
    (In principle fine since $b \sim p_T^{-1}$; in practice affects estimating uncertainties)
  - Complicates matching to fixed-order results

Goal:

Solve $b \to 0$ divergence for $\mu \sim p_T$

- Previous attempts (CSS): [Ellis, Veseli], [Kulesza, Stirling]
- Previous attempts (SCET): [Becher, Neubert, Wilhelm]

Rapidity Renormalization Group.

Solution without b-space:

- Iterative solution to $\nu \frac{dV(\vec{p}_T,\nu,\nu_0)}{d\nu} = \gamma^{(\nu)}(\vec{p}_T,\mu) \otimes V(\vec{p}_T,\nu,\nu_0)$:

$$V(\vec{p}_T,\nu_B,\nu_S) = (2\pi)^2\delta^2(\vec{p}_T) + \sum_{n=1}^{\infty} \frac{1}{n!} \ln^n\left(\frac{\nu_B}{\nu_S}\right) (\gamma^{(\nu)}\otimes^n)(\vec{p}_T)$$

- Multiple convolutions:

$$(\gamma^{(\nu)}\otimes^n)(\vec{p}_T) \;\propto\; \underbrace{n\,\mathcal{L}_{n-1}\left(\frac{p_T^2}{\mu^2}\right)}_{\text{leading log (?)}} - \underbrace{\psi^{(2)}\mathcal{L}_{n-4}\left(\frac{p_T^2}{\mu^2}\right) + \ldots}_{\text{subleading log(?)}}$$

- Removing “subleading logs” by hand removes divergence $\Gamma(1-\omega_s)$
- Conclusion: Divergent part due to incorrectly captured subleading logs
- Already noted in [Frixione, Nason, Ridolfi; 1999]
- Problems:
  - Consistent log counting?
  - Generalization to higher orders?

Rapidity Renormalization Group.

Consistency conditions:

- Commutativity $[\mathrm{d}/\mathrm{d}\mu, \mathrm{d}/\mathrm{d}\nu] = 0$ induces a consistency relation:

$$\mu \frac{\mathrm{d}\gamma^{(\nu)}(b,\mu)}{\mathrm{d}\mu} = -4\Gamma_{\mathrm{C}}[\alpha_s(\mu)]$$

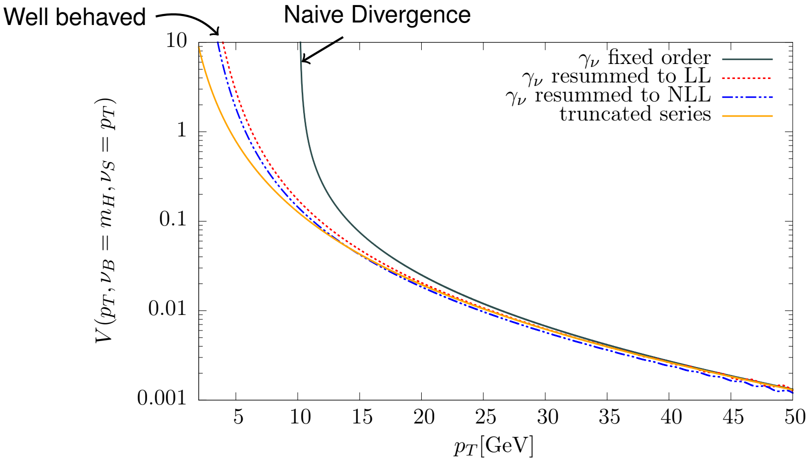
- Solving consistency resums logarithms *inside* $\gamma^{(\nu)}$
  $\to$ should correctly capture subleading logs

Solution:

$$\underbrace{\gamma^{(\nu)}(b,\mu)}_{\text{resummed anom dim}} = -4 \int_{2e^{-\gamma_E}/b}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \Gamma_{\mathrm{C}}[\alpha_s(\mu')] + \underbrace{\gamma^{(\nu)}(b)}_{\text{calculable in fixed order}}$$

$\to$ Regulates the $b \to 0$ divergence ✓

Comparison of current results:



- Rapidity evolution factor well behaved for $p_T \gtrsim \Lambda_{\mathrm{QCD}}$
- $\Rightarrow$ Resummation of $\gamma^{(\nu)}$ allows scale choice in momentum space ✓

Conclusion.

Conclusion.

Transverse momentum spectra:

- Large logarithms $\ln \frac{p_T}{m_H}$ require resummation for $p_T \ll m_H$
- Standard approach: scale setting in impact parameter space
  - Resums logarithms $\ln(b\, m_H)$ instead of $\ln \frac{p_T}{m_H}$
- SCET allows scale setting in momentum space:
  - Directly resums logarithms $\ln \frac{p_T}{m_H}$
  - Requires resummation of anomalous dimension $\gamma^{(\nu)}$ to correctly capture subleading logarithms

Open tasks:

- Numerical comparison to scale choices in literature
- Numerical study of uncertainties

Thank you for your attention!

Backup slides.

Higgs production in SCET.

Light-cone coordinates

- Reference vector: $n^\mu, \bar{n}^\mu = (1, 0, 0, \pm 1)$
- Split any momentum into

$$p^\mu = \underbrace{p^+ n^\mu}_{\text{along } \hat{z}} + \underbrace{p^- \bar{n}^\mu}_{\text{along } -\hat{z}} + \vec{p}_\perp \quad \rightarrow \quad p = (p^+, p^-, \vec{p}_T)$$

Relevant modes:

- Classify modes using $\lambda \sim \frac{p_T}{m_H}$
- Hard mode:
  $p^2 \sim m_H^2, \;\; p \sim Q(1, 1, 1)$
- n-collinear mode:
  $p^2 \sim p_T^2, \quad p \sim Q(1, \lambda^2, \lambda)$
- $\bar{n}$-collinear mode:
  $p^2 \sim p_T^2, \quad p \sim Q(\lambda^2, 1, \lambda)$
- Soft mode:
  $p^2 \sim p_T^2, \quad p \sim Q(\lambda, \lambda, \lambda)$