GM2Calc
A tool for calculating the muon $(g-2)$ in the MSSM

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J. Park, S. Paßehr, D. Stöckinger, H. Stöckinger-Kim, A. Voigt

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Muon anomalous magnetic moment

- definition: $a_\mu := \dfrac{(g-2)_\mu}{2} = -2\, m_\mu\, F_M(0)$
- high precision in experimental determination and SM calculation
- discrepancy: $a_\mu^{\mathsf{exp}} - a_\mu^{\mathsf{SM}} = (28.7 \pm 8.0) \times 10^{-10} \mathrel{\widehat{=}} 3.6\,\sigma$ [1]

  ⇒ points to BSM physics and also restricts it
- MSSM capable of explaining this deviation in a wide parameter range
- new experiments (Fermilab, J-PARC) with improved accuracy

  ⇒ relevance of higher order corrections in theory predictions

We present: `GM2Calc`

- `C++` code for precise MSSM calculation of

$$a_\mu^{\text{SUSY}} = \left[a_\mu^{\text{1L}} + a_\mu^{\text{2L(a)}} + a_\mu^{\text{2L,photonic}} + a_\mu^{\text{2L,}f\tilde{f}} \right]_{t_\beta\text{-resummed}}$$

- exact formulas implemented for a_μ^{1L} and $a_\mu^{\text{2L,photonic}}$
- adequate approximations implemented for $a_\mu^{\text{2L(a)}}$ and $a_\mu^{\text{2L,}f\tilde{f}}$
- What is new?
  - consideration of two-loop fermion/sfermion corrections
  - $\tan\beta$ resummation for one- and two-loop contributions

Renormalization scheme

- relevant one-loop parameters:

  $$\alpha_{\text{em}}, M_{W,Z}, m_\mu; \tan\beta; M_1, M_2, \mu, M_{L2}, M_{E2}$$

- $M_{W,Z}$ and m_μ defined as pole masses in the on-shell scheme
- parametrization of α_{em} introduces ambiguity at one-loop level

  ⇒ we choose $\alpha(M_Z) = \alpha(0)/(1 - \Delta\alpha(M_Z))$
- $\tan\beta$ defined in $\overline{\text{DR}}$ scheme at SUSY scale Q
- SUSY mass parameters defined in on-shell scheme such that pole masses and tree-level masses of the corresponding particles coincide

  ⇒ given directly or calculated internally from SLHA input

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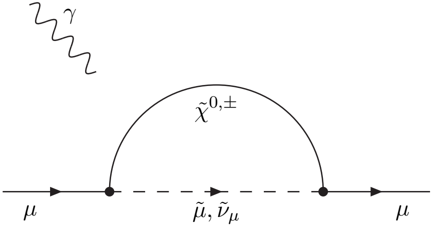
- $\tan\beta$ defined in $\overline{\text{DR}}$ scheme at SUSY scale Q
- SUSY mass parameters defined in on-shell scheme such that pole masses and tree-level masses of the corresponding particles coincide

  $\Rightarrow$ given directly or calculated internally from SLHA input

specific choices minimize theory uncertainties and two-loop corrections

Included SUSY contributions

- a_μ^{1L}



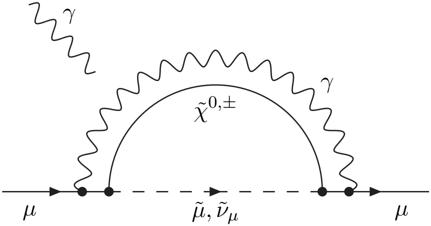
[2]

one-loop smuon-neutralino or sneutrino-chargino diagram

scaling behaviour $\propto \tan\beta / M_{\mathsf{SUSY}}^2$

Included SUSY contributions

- a_μ^{1L}
- $a_\mu^{\text{2L,photonic}}$



[3]

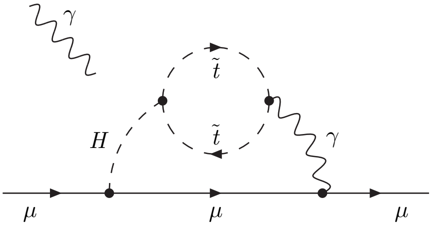
SUSY one-loop diagrams with additional photon exchange

include large QED logarithm $\log(M_{\text{SUSY}}/m_\mu)$

typically negative

Included SUSY contributions

- a_μ^{1L}
- $a_\mu^{\text{2L,photonic}}$
- $a_\mu^{\text{2L(a)}}$



[4, 5]

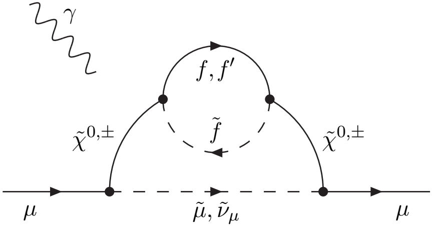
two-loop corrections to SM-like one-loop diagrams

depend on entirely different parameters compared to a_μ^{1L}

small for large masses of sparticles or heavy Higgs bosons

Included SUSY contributions

- a_μ^{1L}
- $a_\mu^{\text{2L,photonic}}$
- $a_\mu^{\text{2L(a)}}$
- $a_\mu^{\text{2L},f\tilde{f}}$



[6, 7]

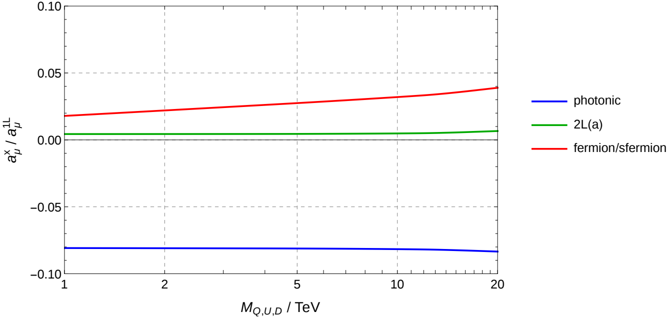
f - $\tilde{f}$ loop insertions into SUSY one-loop diagrams

depend on squark and slepton masses of all generations

no decoupling but log. enhancement for large masses

remove one-loop ambiguity from parametrization of α_{em}

Two-loop contributions



$\overline{\text{DR}}$ parameters at $Q = 1$ TeV:

$\mu = 500$ GeV

$M_2 = 2M_1 = 400$ GeV

$M_3 = 2$ TeV

$M_L = M_E = 500$ GeV

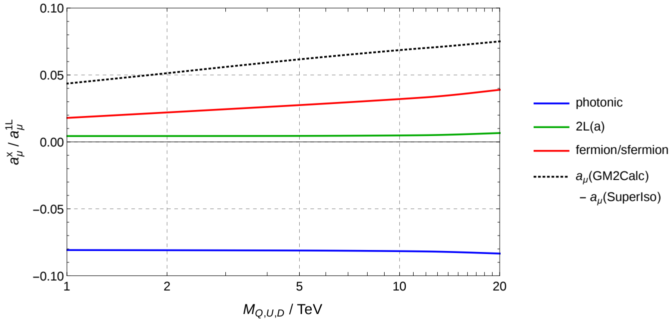
$M_{L3} = M_{E3} = 3$ TeV

$M_A = 1.5$ TeV

$\tan\beta = 40$

- $a_\mu^{\text{2L,photonic}}$ negative and not directly dependent on $M_{Q,U,D}$
- $a_\mu^{\text{2L(a)}}$ small because of large masses of relevant sparticles
- $a_\mu^{\text{2L},f\tilde{f}}$ shows no decoupling but increases with $M_{Q,U,D}$

Two-loop contributions and comparison with `SuperIso`



$\overline{\text{DR}}$ parameters at $Q = 1\ \text{TeV}$:

$\mu = 500\ \text{GeV}$

$M_2 = 2M_1 = 400\ \text{GeV}$

$M_3 = 2\ \text{TeV}$

$M_L = M_E = 500\ \text{GeV}$

$M_{L3} = M_{E3} = 3\ \text{TeV}$

$M_A = 1.5\ \text{TeV}$

$\tan\beta = 40$

In `SuperIso` [8]:

- the two-loop fermion/sfermion corrections are not included
- a different renormalization scheme is used

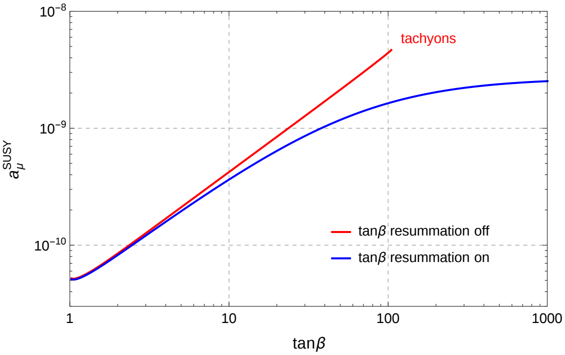
Implementation of $\tan\beta$ resummation

- terms $\propto (\tan\beta)^n$ are resummed to all orders by extending the tree-level muon Yukawa coupling to

$$y_\mu = \frac{m_\mu}{v\,\cos\beta\,(1+\Delta_\mu)}$$

- analogous formulas for y_τ and y_b in two-loop contributions
- resummation allows for the limit $\tan\beta \to \infty$ [9]
  - viable scenario with radiative muon mass generation
  - large a_μ^{SUSY} even for TeV-scale SUSY masses

Importance of $\tan\beta$ resummation



one-loop on-shell parameters:

$\mu = -M_2 = 30\ \text{TeV}$

$M_1 = M_{L2} = M_{E2} = 1\ \text{TeV}$

two-loop parameters:

$M_3 = 2\ \text{TeV}$

$M_{L1} = M_{E1} = 1\ \text{TeV}$

$M_{L3} = M_{E3} = M_{Q,U,D} = 3\ \text{TeV}$

$M_A = 1.5\ \text{TeV}$

- difference around $15\,\%$ at $\tan\beta = 10$
- small $\tan\beta$: linear increase as $a_\mu^{1\text{L}} \propto \tan\beta$
- limit $\tan\beta \to \infty$: constant behaviour because of large $\Delta_\mu \propto \tan\beta$

Conclusion

`GM2Calc` is the most precise tool for calculating $(g-2)_\mu$ in the MSSM:

- all currently known two-loop contributions included (new: $a_\mu^{\text{2L},f\tilde{f}}$)
- resummation of terms $\propto (\tan\beta)^n$ to all orders (new)
- choice of renormalization scheme minimizes theory uncertainties
- tested against results from relevant literature

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`GM2Calc` is now available at `http://gm2calc.hepforge.org`

Conclusion

`GM2Calc` is the most precise tool for calculating $(g-2)_\mu$ in the MSSM:

- all currently known two-loop contributions included (new: $a_\mu^{\text{2L},f\tilde{f}}$)
- resummation of terms $\propto (\tan\beta)^n$ to all orders (new)
- choice of renormalization scheme minimizes theory uncertainties
- tested against results from relevant literature

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Thanks for your attention!

References

[1] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. C **71** (2011) 1515 [Eur. Phys. J. C **72** (2012) 1874] [arXiv:1010.4180 [hep-ph]]

[2] T. Moroi, Phys. Rev. D **53** (1996) 6565 [Phys. Rev. D **56** (1997) 4424] [hep-ph/9512396]

[3] P. von Weitershausen, M. Schäfer, H. Stöckinger-Kim and D. Stöckinger, Phys. Rev. D **81** (2010) 093004 [arXiv:1003.5820 [hep-ph]]

[4] S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B **690** (2004) 62 [hep-ph/0312264]

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[7] H. Fargnoli, C. Gnendiger, S. Paßehr, D. Stöckinger and H. Stöckinger-Kim, JHEP **1402** (2014) 070 [arXiv:1311.1775 [hep-ph]]

[8] F. Mahmoudi, Comput. Phys. Commun. **180** (2009) 1579 [arXiv:0808.3144 [hep-ph]]

[9] M. Bach, J. h. Park, D. Stöckinger and H. Stöckinger-Kim, arXiv:1504.05500 [hep-ph]

Backup

Definition of the anomalous magnetic moment

- magn. moment of a fermion: $\vec{\mu} = g \, \frac{Q\, e\, \hbar}{2\, m\, c} \, \vec{s}$ (g - gyromagn. ratio)
- interaction with an external photon: $-\mathrm{i}\, Q\, e\, \bar{u}(p')\, \Lambda_\rho\, u(p)$
- decomposition:

  $$\bar{u}(p')\, \Lambda_\rho\, u(p) = \bar{u}(p') \left[\gamma_\rho \, \left(F_E(q^2) - 2\, m\, F_M(q^2) \right) + (p + p')_\rho\, F_M(q^2) + \ldots \right] u(p)$$

- classical limit: $g = 2 \left(1 - 2\, m\, F_M(0) \right)$
- anomalous magnetic moment: $a := \frac{g - 2}{2} = -2\, m\, F_M(0)$

Experimental determination of a_μ

- muons from pion decay in a storage ring
- spin precession in magnetic field $\vec{B}$ relative to the momentum with

$$\vec{\omega} = \frac{e}{m_\mu}\left[a_\mu\,\vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

vanishes for "magic" momentum $p = 3.094\,\mathrm{GeV}$

- detection of electrons from muon decay
- result from E821 experiment at Brookhaven National Laboratory:

$$a_\mu^{\mathsf{exp}} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

Standard model prediction for a_μ

- different kinds of contributions
  - QED: massless class, massive class
  - hadronic: vacuum polarization, light-by-light scattering
  - electroweak
- combined result [1]:

$$a_\mu^{\mathsf{SM}} = (11\,659\,180.2 \pm 4.9) \times 10^{-10}$$

- discrepancy between experiment and theory:

$$a_\mu^{\mathsf{exp}} - a_\mu^{\mathsf{SM}} = (28.7 \pm 8.0) \times 10^{-10}$$

new physics?

Muon sneutrino and smuon masses

- muon sneutrino mass: $m_{\tilde{\nu}_\mu}^2 = m_L^2 + \frac{1}{2}\, M_Z^2 \cos 2\beta$
- smuon mass matrix:

$$M_{\tilde{\mu}}^2 = \begin{pmatrix} (m_\mu^0)^2 + m_L^2 + M_Z^2\,(s_W^2 - \frac{1}{2}) \cos 2\beta & m_\mu^0\,(A_\mu^* - \mu \tan\beta) \\ m_\mu^0\,(A_\mu - \mu^* \tan\beta) & (m_\mu^0)^2 + m_R^2 - M_Z^2\, s_W^2 \cos 2\beta \end{pmatrix}$$

- diagonalization: $U^{\tilde{\mu}} M_{\tilde{\mu}}^2\, U^{\tilde{\mu}\dagger} = \mathrm{diag}(m_{\tilde{\mu}_1}^2, m_{\tilde{\mu}_2}^2)$

Usage and general input

```
$ bin/gm2calc.x --slha-input-file=input/slha.in
```

```
$ bin/gm2calc.x --gm2calc-input-file=input/gm2calc.in
```

```
Block GM2CalcConfig
     0      3     # output format (0 = minimal, 1 = detailed,
                  #                2 = NMSSMTools, 3 = SPheno)
     1      2     # loop order (0, 1 or 2)
     2      1     # disable/enable tan(beta) resummation (0 or 1)
     3      0     # force output (0 or 1)
     4      0     # verbose output (0 or 1)
```

```
Block SMINPUTS
     3      0.1184           # alpha_s(MZ) SM MS-bar [relevant at 2L]
     4      91.1876          # M_Z(pole)             [relevant at 1L]
     5      4.18             # m_b(m_b) SM MS-bar    [relevant at 2L]
     6      173.34           # M_top(pole)           [relevant at 2L]
     7      1.777            # M_tau(pole)           [relevant at 2L]
     9      80.385           # M_W(pole)             [relevant at 1L]
    13      0.1056583715     # M_muon(pole)          [relevant at 1L]
```

```
Block GM2CalcInput
     1      0.00775531       # alpha(MZ)
     2      0.00729735       # alpha(0)
```

SLHA input I

```
 Block GM2CalcConfig
     0     3     # output format (0 = minimal, 1 = detailed,
                 #                2 = NMSSMTools, 3 = SPheno)
     1     2     # loop order (0, 1 or 2)
     2     1     # disable/enable tan(beta) resummation (0 or 1)
     3     0     # force output (0 or 1)
     4     0     # verbose output (0 or 1)
Block GM2CalcInput
     1     0.00775531       # alpha(MZ)
     2     0.00729735       # alpha in Thomson limit
Block SMINPUTS
     3     0.1184           # alpha_s(MZ) SM MSbar
     4     9.11876000E+01   # MZ(pole)
     5     4.18000000E+00   # mb(mb) SM MSbar
     6     1.73340000E+02   # mtop(pole)
     7     1.77700000E+00   # mtau(pole)
     8     0.00000000E+00   # mnu3(pole)
    11     0.000510998928   # melectron(pole)
    12     0.00000000E+00   # mnu1(pole)
    13     0.1056583715     # mmuon(pole)
    14     0.00000000E+00   # mnu2(pole)
    21     4.76052706E-03   # md
    22     2.40534062E-03   # mu
    23     1.04230487E-01   # ms
    24     1.27183378E+00   # mc
    Block MASS
        24     8.03933863E+01   # VWm
   1000012     3.52702003E+02   # SveL
   1000014     3.52693688E+02   # SvmL
   1000016     3.51533929E+02   # SvtL
```

SLHA input II

```
   1000024     3.85988934E+02   # Cha(1)
   1000037     6.50709035E+02   # Cha(2)
        25     1.14863491E+02   # hh(1)
        35     7.19752811E+02   # hh(2)
        37     7.24214922E+02   # Hpm(2)
        36     7.19480165E+02   # Ah(2)
   1000001     1.01544493E+03   # Sd(1)
   2000001     1.05975972E+03   # Sd(2)
   1000003     1.01544114E+03   # Ss(1)
   2000003     1.05975787E+03   # Ss(2)
   1000005     9.70864119E+02   # Sb(1)
   2000005     1.01231331E+03   # Sb(2)
   1000011     2.29840252E+02   # Se(1)
   2000011     3.61609040E+02   # Se(2)
   1000013     2.29789268E+02   # Sm(1)
   2000013     3.61617754E+02   # Sm(2)
   1000015     2.22687306E+02   # Stau(1)
   2000015     3.62761762E+02   # Stau(2)
   1000002     1.01873843E+03   # Su(1)
   2000002     1.05693320E+03   # Su(2)
   1000004     1.01873236E+03   # Sc(1)
   2000004     1.05693385E+03   # Sc(2)
   1000006     8.09088787E+02   # St(1)
   2000006     1.01626608E+03   # St(2)
   1000022     2.04267616E+02   # Chi(1)
   1000023     3.85975573E+02   # Chi(2)
   1000025    -6.36794092E+02   # Chi(3)
   1000035     6.50380139E+02   # Chi(4)
```

SLHA input III

```
Block HMIX Q= 8.79028867E+02
     1      6.31273245E+02   # Mu
     2      9.67313083E+00   # vu/vd
Block Au Q= 8.79028867E+02
  3  3     -8.88841564E+02   # TYu(3,3)/Yu(3,3)
Block Ad Q= 8.79028867E+02
  3  3     -1.31251838E+03   # TYd(3,3)/Yd(3,3)
Block Ae Q= 8.79028867E+02
  2  2     -2.99494259E+02   # TYe(2,2)/Ye(2,2)
  3  3     -2.97870372E+02   # TYe(3,3)/Ye(3,3)
Block MSOFT Q= 8.79028867E+02
     1      2.09353661E+02   # MassB
     2      3.88423131E+02   # MassWB
     3      1.11722397E+03   # MassG
    31      3.54086560E+02   # SignedAbsSqrt(ml2(1,1))
    32      3.54079405E+02   # SignedAbsSqrt(ml2(2,2))
    33      3.53081486E+02   # SignedAbsSqrt(ml2(3,3))
    34      2.21971801E+02   # SignedAbsSqrt(me2(1,1))
    35      2.21948532E+02   # SignedAbsSqrt(me2(2,2))
    36      2.18682727E+02   # SignedAbsSqrt(me2(3,3))
    41      1.02234871E+03   # SignedAbsSqrt(mq2(1,1))
    42      1.02234616E+03   # SignedAbsSqrt(mq2(2,2))
    43      9.42833969E+02   # SignedAbsSqrt(mq2(3,3))
    44      9.84576587E+02   # SignedAbsSqrt(mu2(1,1))
    45      9.84573885E+02   # SignedAbsSqrt(mu2(2,2))
    46      8.11699109E+02   # SignedAbsSqrt(mu2(3,3))
    47      9.80007845E+02   # SignedAbsSqrt(md2(1,1))
    48      9.80005159E+02   # SignedAbsSqrt(md2(2,2))
    49      9.75269422E+02   # SignedAbsSqrt(md2(3,3))
```

GM2Calc Input I

```
Block GM2CalcConfig
     0     1     # output format (0 = minimal, 1 = detailed,
                 #                2 = NMSSMTools, 3 = SPheno)
     1     2     # loop order (0, 1 or 2)
     2     1     # disable/enable tan(beta) resummation (0 or 1)
     3     0     # force output (0 or 1)
     4     0     # verbose output (0 or 1)
Block GM2CalcInput
     0     1000             # ren. scale   [2L]
     1     0.00775531       # alpha(MZ)    [1L]
     2     0.00729735       # alpha in Thompson limit [2L]
     3     100000000        # tan(beta) DR-bar at ren. scale [1L]
     4     30000            # Mu parameter [1L]
     5     1000             # M1           [1L]
     6     -30000           # M2           [1L]
     7     2000             # M3           [2L]
     8     1500             # MA(pole)     [2L]
     9     1000             # msl(1,1)     [2L]
    10     1000             # msl(2,2)     [1L]
    11     3000             # msl(3,3)     [2L]
    12     1000             # mse(1,1)     [2L]
    13     1000             # mse(2,2)     [1L]
    14     3000             # mse(3,3)     [2L]
    15     3000             # msq(1,1)     [2L]
    16     3000             # msq(2,2)     [2L]
    17     3000             # msq(3,3)     [2L]
    18     3000             # msu(1,1)     [2L]
    19     3000             # msu(2,2)     [2L]
    20     3000             # msu(3,3)     [2L]
```

GM2Calc Input II

```
    21     3000             # msd(1,1)      [2L]
    22     3000             # msd(2,2)      [2L]
    23     3000             # msd(3,3)      [2L]
    24     0                # Ae(1,1)       [irrelevant]
    25     0                # Ae(2,2)       [1L]
    26     0                # Ae(3,3)       [2L]
    27     0                # Ad(1,1)       [irrelevant]
    28     0                # Ad(2,2)       [irrelevant]
    29     0                # Ad(3,3)       [2L]
    30     0                # Au(1,1)       [irrelevant]
    31     0                # Au(2,2)       [irrelevant]
    32     0                # Au(3,3)       [2L]
Block SMINPUTS
     3     0.1184           # alpha_s(MZ) SM MSbar [2L]
     4     9.11876000E+01   # MZ(pole)        [1L]
     5     4.18000000E+00   # mb(mb) SM MSbar [2L]
     6     1.73340000E+02   # mtop(pole)      [2L]
     7     1.77700000E+00   # mtau(pole)      [2L]
     8     0.00000000E+00   # mnu3(pole)      [irrelevant]
     9     8.03850000E+01   # MW(pole)        [1L]
    11     0.000510998928   # melectron(pole) [irrelevant]
    12     0.00000000E+00   # mnu1(pole)      [irrelevant]
    13     0.1056583715     # mmuon(pole)     [1L]
    14     0.00000000E+00   # mnu2(pole)      [irrelevant]
    21     4.76052706E-03   # md              [irrelevant]
    22     2.40534062E-03   # mu              [irrelevant]
    23     1.04230487E-01   # ms              [irrelevant]
    24     1.27183378E+00   # mc              [irrelevant]
```

Detailed output I

```
========================================================
   amu (1-loop + 2-loop best) =  2.68729816713390e-09
========================================================

==============================
   amu (1-loop) corrections
==============================

full 1L with tan(beta) resummation:
   chi^0      2.81382411431847e-09
   chi^+-    -2.14343315883469e-12
   -------------------------------
   sum        2.81168068115964e-09 (104.6% of full 1L + 2L result)

full 1L without tan(beta) resummation:
              4.56324981729290e-05

1L approximation with tan(beta) resummation:
   W-H-nu    -2.14916420960056e-12
   W-H-muL    1.05380460353973e-12
   B-H-muL    3.08220768463830e-12
   B-H-muR   -6.16441536927660e-12
   B-muL-muR  2.78006197857453e-09
   -------------------------------
   sum        2.77588441128383e-09

==============================
   amu (2-loop) corrections
==============================
```

Detailed output II

```
2L best with tan(beta) resummation:
             -1.24382514025737e-10 (-4.6% of full 1L + 2L result)

2L best without tan(beta) resummation:
             -4.24627290796466e+05

photonic with tan(beta) resummation:
   chi^0     -2.30379299451942e-10
   chi^+-     2.79887522055950e-13
   -------------------------------
   sum       -2.30099411929886e-10 (-8.6% of full 1L + 2L result)

fermion/sfermion approximation with tan(beta) resummation:
   W-H-nu    -7.41331054883902e-14
   W-H-muL    3.63498552085423e-14
   B-H-muL    6.89134562984302e-14
   B-H-muR   -2.91937296828775e-13
   B-muL-muR  1.06040301855394e-10
   -------------------------------
   sum        1.05779494764583e-10 (3.9% of full 1L + 2L result)

2L(a) (1L insertions into 1L SM diagram) with tan(beta) resummation:
   sfermion   6.34236763361513e-19
   cha^+-    -6.25974946713545e-14
   -------------------------------
   sum       -6.25968604345911e-14 (-0.0% of full 1L + 2L result)

tan(beta) correction:
   amu(1L) * (1 / (1 + Delta_mu) - 1) = -4.56324695548778e-05 (-100.0%)
```