Improved Estimates for the Parameters of the Heavy Quark Expansion

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based on Nucl.Phys. **B889** (2014) 46-63 [arXiv:1407.4384]
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work in progress [arXiv:151x.xxxx]
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DESY Theory Workshop 2015
September 29th - October 2nd

DFG FOR 1873

GEFÖRDERT VOM Bundesministerium für Bildung und Forschung

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Motivation: Semileptonic B-decays

Decay rate in HQET

$$d\Gamma \sim d\Gamma_0 + \underbrace{\frac{\langle \bar{h}(iD)^2 h\rangle}{m_b^2} d\Gamma_2}_{\text{2 parameters}} + \underbrace{\frac{\langle \bar{h}(iD)^3 h\rangle}{m_b^3} d\Gamma_3}_{\text{2 parameters}} + \underbrace{\frac{\langle \bar{h}(iD)^4 h\rangle}{m_b^4} d\Gamma_4}_{\text{9 parameters}} + \dots$$

- Can only extract some from data, e.g.

$$\langle \bar{b}(iD)^2 b\rangle \sim \mu_\pi^2, \mu_G^2 \quad \text{(kinetic energy, chromomagnetic moment)}$$
$$\langle \bar{b}(iD)^3 b\rangle \sim \rho_D^3, \rho_{LS}^3 \quad \text{(Darwin term, spin-orbit-coupling)}$$

- Many more parameters at higher order: 18 at $1/m_b^5$.

⇒ Can we estimate the higher order parameters?

$$\langle \bar{b}(iD)^4 b\rangle \overset{?}{\sim} \langle \bar{b}(iD)^2 b\rangle^2$$

Idea

- How are matrix elements related to each other?

$\rightarrow$ e.g. $\langle O^2 \rangle = \langle O \rangle^2 + \dots$?

- In quantum mechanics: Insert sum over all states

$$\langle 0|O_1 O_2|0\rangle = \sum_n \langle 0|O_1|n\rangle \langle n|O_2|0\rangle$$
$$= \langle 0|O_1|0\rangle \langle 0|O_2|0\rangle + \langle 0|O_1|1\rangle \langle 1|O_2|0\rangle + \dots$$

- How to do this properly in field theory?

Want something like

$$\langle B|\bar{b}\mathcal{O}_1\mathcal{O}_2 b|B\rangle = \langle B|\bar{b}\mathcal{O}_1 b|B\rangle \langle B|\bar{b}\mathcal{O}_2 b|B\rangle$$
$$+ \langle B|\bar{b}\mathcal{O}_1 b|B^*\rangle \langle B^*|\bar{b}\mathcal{O}_2 b|B\rangle + \dots$$

(systematically elaborate on the work of [Mannel,Turczyk,Uraltsev '10])

Strategy: Overview

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How To

1. Introduce an auxiliary heavy quark field $Q(x)$ and consider

$$\langle B(p_B) \big| \mathcal{T}[\ \bar{b}(x)\mathcal{O}_1 Q(x)\ \bar{Q}(0)\mathcal{O}_2 b(0)\ ] \big| B(p_B)\rangle$$

2. "Insertion of $\mathbb{1}$ as in QM"

$$\langle B|\bar{b}\mathcal{O}_1 Q\bar{Q}\mathcal{O}_2 b|B\rangle \sim \sum_n \langle B|\bar{b}\mathcal{O}_1 Q|n\rangle\,\langle n|\bar{Q}\mathcal{O}_2 b|B\rangle$$

3. Evaluate both sides in the static limit $m_Q \gg m_b \to \infty$

$\rightarrow$ heavy quark symmetry: Replace Q by b.

Technical details

- Consider first only operators $\mathcal{O}$ with spacelike derivatives $iD_\perp$.
- Compute the Fourier transform

$$T(q) = \int d^4x \; e^{iq\cdot x} \langle B(p_B)|\mathcal{T}[[\bar{b}(x)\mathcal{O}_1 Q(x) \; Q(0)\mathcal{O}_2 b(0)]|B(p_B)\rangle$$

at $\vec{q} = 0$, with and without insertion of $\mathbb{1}$.

Without insertion of $\mathbb{1}$

- Do an OPE:

  First term Q-propagator $\langle Q\bar{Q}\rangle \sim \frac{1}{\omega + v\cdot iD}\frac{1+\not{v}}{2}$

With insertion of $\mathbb{1}$

- Compute FT
- Use heavy quark symmetry: $\langle \bar{b}\mathcal{O}_1 Q\rangle \to \langle \bar{b}\mathcal{O}_1 b\rangle$

- Expand both sides in large $\omega = q^0 - m_Q + m_b$.

Results I

The "master equation"

$$\sum_k \langle B \left| \bar{b}\mathcal{O}_1 \left(\frac{v\cdot iD}{\omega}\right)^k \mathcal{O}_2 b \right| B\rangle$$

$$\sim \sum_k \sum_n \left(\frac{-\epsilon_n}{\omega}\right)^k \langle B|\bar{b}\mathcal{O}_1 Q|n\rangle \langle n|\bar{Q}\mathcal{O}_2 b|B\rangle$$

- ϵ_n = excitation energy of state $|n\rangle$
- $k = 0$: only spacelike derivatives $iD_\perp$,
  $k = 1$: exactly one timelike derivatives $(v \cdot iD)$, ...

→ arbitrary number of $(v \cdot iD)$'s next to each other

- To get several $(v \cdot iD)$'s separated by $(iD_\perp)$'s:

→ Insert Q's and "$\mathbb{1}$" several times, i.e.

$$\langle B| \ldots (v \cdot iD) iD_\perp (v \cdot iD) \ldots |B\rangle$$

$$\sim \sum_n \sum_m \epsilon_n \epsilon_m \langle B| \ldots Q_1|n\rangle \langle n|\bar{Q}_1 iD_\perp Q_2|m\rangle \langle m|\bar{Q}_2 \ldots |B\rangle$$

Calculation of B-Meson matrix elements I

- Need to truncate the sum: Let's only take the lowest lying modes, "Lowest lying state approximation" (LLSA)

Calculate the matrix elements using "trace formula"

$$\langle B|\bar{b}\mathcal{O}Q|n\rangle = Tr\,[\bar{\mathcal{M}}_B\,\Gamma_{\mathcal{O}}\,\mathcal{M}_Q\,F^j_{\mathcal{O}}]$$

- $\mathcal{M}$ = Representation of B- or Q-Meson, e.g. $\mathcal{M}_B \sim \left[\frac{1+\not{v}}{2}\gamma_5\right]_{\alpha\beta}$
  - labeled by J^P of Meson and j of light d.o.f. (= "brown muck").
  $\Gamma_{\mathcal{O}}$ = Dirac structure of $\mathcal{O}$
  $F^j_{\mathcal{O}}$ = Rep. of the light d.o.f of $|n\rangle$, depends on $\mathcal{O}$.

$\rightarrow$ Each doublet with same j shares the same function $F^j_{\mathcal{O}}$.
$\Rightarrow$ reduction of free parameters.

Calculation of B-Meson matrix elements II

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- Order calculation by number of derivatives ($\sim 1/m_b^n$)
  - (1) $\langle B|iD|B\rangle = 0.$
  - (1') $\langle B|iD|B^*\rangle$: Two parameters R, R' (for $J^P = 1^+$ and $j_Q = \frac{1}{2}, \frac{3}{2}$).
  - (2) $\langle B|iD^2|B\rangle$: Two parameters μ_π, μ_G

$$2M_B\mu_\pi^2 = -\langle B|\bar{b}_v(iD)^2 b_v|B\rangle$$

$$2M_B\mu_G^2 = -\langle B|\bar{b}_v iD_\mu^\perp iD_\nu^\perp i\sigma_\perp^{\mu\nu} b_v|B\rangle$$

  - (3) $\langle B|iD^3|B\rangle$: Two parameters ρ_D, ρ_{LS}
  - (4) $\langle B|iD^4|B\rangle$: 9 parameters m_i
  - (5) $\langle B|iD^5|B\rangle$: 18 parameters r_i
- Except ($1'$), must always have even number of $iD_\perp$

$\rightarrow$ (3) and (5) will contain $v \cdot iD \sim \epsilon_n$, as will some in (4).

Express all these matrix elements (@LLSA) in terms of

$\mu_\pi, \mu_G, \epsilon_{1/2}$ and $\epsilon_{3/2}$

Calculation of B-Meson matrix elements III

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- Go back recursively to get the parameters
  (1) $\langle B|iD^2|B\rangle \sim \langle B|iD|n\rangle \langle n|iD|B\rangle \to |R|^2, |R'|^2$.
  (2) μ_π, μ_G: ✓
  (3) $\langle B|iD^3|B\rangle \sim \epsilon_n \langle B|iD|n\rangle \langle n|iD|B\rangle \to \rho_D, \rho_{LS}$
  (4) $\langle B|iD^4|B\rangle \sim \langle B|iD^2|n\rangle \langle n|iD^2|B\rangle \to m_i$
  (5) $\langle B|iD^5|B\rangle \sim \epsilon_n \langle B|iD^2|n\rangle \langle n|iD^2|B\rangle \to r_i$
- Example:

$$
\begin{aligned}
m_8 &\sim \langle B \left| \bar{b}(iD_\perp^\mu iD_\mu^\perp)(iD_\alpha^\perp iD_\beta^\perp i\sigma_\perp^{\alpha\beta})b \right| B\rangle \\
&\sim \langle B \left| \bar{b} iD_\perp^\mu iD_\mu^\perp b \right| B\rangle \langle B \left| \bar{b} iD_\alpha^\perp iD_\beta^\perp i\sigma_\perp^{\alpha\beta} b \right| B\rangle \\
&\sim \mu_\pi^2 \, \mu_G^2
\end{aligned}
$$

(including all factors: $m_8 = -8\mu_\pi^2\mu_G^2$)

Results II

	formula	GeV^3
ρ_D^3	$\frac{\epsilon_{1/2}}{3}(\mu_\pi^2 - \mu_G^2) + \frac{\epsilon_{3/2}}{3}(2\mu_\pi^2 + \mu_G^2)$	0.21
ρ_{LS}^3	$\frac{2\epsilon_{1/2}}{3}(\mu_\pi^2 - \mu_G^2) - \frac{\epsilon_{3/2}}{3}(2\mu_\pi^2 + \mu_G^2)$	−0.17
		GeV^4
m_1	$\frac{5}{9}\mu_\pi^4$	0.095
m_2	$-\frac{\epsilon_{1/2}^2}{3}(\mu_\pi^2 - \mu_G^2) - \frac{\epsilon_{3/2}^2}{3}(2\mu_\pi^2 + \mu_G^2)$	−0.082
m_3	$-\frac{2}{3}\mu_G^4$	−0.077
m_4	$\mu_G^4 - \frac{4}{3}\mu_\pi^4$	0.344
m_5	$-\frac{2\epsilon_{1/2}^2}{3}(\mu_\pi^2 - \mu_G^2) + \frac{\epsilon_{3/2}^2}{3}(2\mu_\pi^2 + \mu_G^2)$	0.070
m_6	$\frac{2}{3}\mu_G^4$	0.077
m_7	$-\frac{8}{3}\mu_\pi^2\mu_G^2$	−0.375
m_8	$-8\mu_\pi^2\mu_G^2$	−1.126
m_9	$\mu_G^4 - \frac{10}{3}\mu_\pi^2\mu_G^2$	−0.354

Input

$$\mu_\pi^2 = 0.414\ \text{GeV}^2$$
$$\mu_G^2 = 0.340\ \text{GeV}^2$$
$$\epsilon_{1/2} = 0.390\ \text{GeV}$$
$$\epsilon_{3/2} = 0.476\ \text{GeV}$$

- Similarly for r_i's.
- Cross check (exp.):
  $\rho_D^3 \sim 0.154\ \text{GeV}^3$
  $\rho_{LS}^3 \sim -0.147\ \text{GeV}^3$
  (errors: $\sim$ 0.05-0.1)
  ✓

Error analysis

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What is the error on these estimates?

Approach ist very systematic:

- Uncertainties from input $\mu_\pi, \mu_G, \epsilon_{1/2}$ and $\epsilon_{3/2}$: ✓
- QCD corrections in the OPE: ✓
- BUT error of truncation difficult to account for:
  1. Compare to nonrelativistic QM:
     Spherical box, Coulomb potential, harmonic oscillator, ...

  $\rightarrow$ truncation gives $\sim$ few 10 % error

  2. Better: Make toy model for the dispersion relation

$$\tilde{T}(\omega) = \sum_n g(n) \frac{1}{\omega - n\Lambda} \qquad \text{with} \qquad g(n) \sim n^{-\beta}$$

[In spherical box: $\beta = 2$.]

Compare full sum to first term $\rightarrow \sim$ few 10 % error

$\Rightarrow$ Truncation error $\sim$ 30 - 60 %.

$\rightarrow$ Estimates still good for order of magnitude, sign and correlcations.

[Can also improve by including higher excitations.]

Cross check: work in progress I

- Try to estimate the matrix elements by using sum rules:

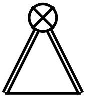
Consider the three point correlation function

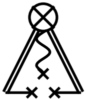
$$T(\omega,\omega') \sim \int d^4x\, e^{i(\omega x^0+\omega' y^0)} \underbrace{\langle 0|\mathcal{T}[\bar{q}(x)i\gamma^5 b(x)\,\bar{b}(0)\mathcal{O}b(0)\,\bar{b}(y)i\gamma^5 q(y)]|0\rangle}_{\sim\langle B|\bar{b}\mathcal{O}b|B\rangle}$$

$$\sim \int \frac{dE}{E-\omega}\int\frac{dE'}{E'-\omega'}\,\rho(E,E')$$

$\rightarrow$ Calculate the spectral density, .g. for $\mathcal{O}\sim(iD_\perp)^2$:

$$\rho(E,E') \sim -\frac{N_c}{2\pi^2}E^4\delta(E-E') + \frac{3}{16}\langle\bar{q}Gq\rangle\,\delta(E)\delta(E') + \dots$$





- Compare to "known" form of the correlation function

$$\hat{T}(\omega,\omega') \sim \frac{F^2 \langle B|\bar{b}\mathcal{O}b|B\rangle}{4(\bar{\Lambda}-\omega)(\bar{\Lambda}-\omega')} + \ldots$$

and we obtain

Finite energy sum rules

$$\Rightarrow \qquad \frac{F^2 \langle B|\bar{b}\mathcal{O}b|B\rangle}{4} \sim \int_0^{E_0} ds \int_0^{E_0} ds' \, \rho(s,s')$$

- For example (incl. normalization/reasonable input)

$$\mu_\pi^2 \sim \frac{\frac{N_c}{10\pi^2} E_0^5 + \frac{3}{16}\langle \bar{q}Gq\rangle}{\frac{N_c}{3\pi^2} E_0^3 - \langle \bar{q}q\rangle} \sim 0.42 \text{ GeV}^2$$

- Now proceed to higher orders → ... to come ...

Summary

What have we done:

- Higher order matrix elements can be estimated systematically

$$\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle \sim \langle B|\mathcal{O}_1|n\rangle \langle n|\mathcal{O}_2|B\rangle + \dots$$

- In lowest state approximation need only a few parameters: 2nd order ME μ_π^2, μ_G^2 and the excitation energies $\epsilon_{1/2}, \epsilon_{3/2}$
- Error of truncation $\sim 30 - 60\%$

$\rightarrow$ still obtain orders of magnitude, sign and correlcations.

What's next:

- Estimate the ME using finite energy sum rules.
- Include QCD corrections in OPE ($\rightarrow$ subtracted dispersion relations).