$W^+W^- +$ jet – compact analytic results

Tania Robens

.

based on

.

J. Campbell, D. Miller, TR
[Phys.Rev. D92 (2015) 1, 014033]

IKTP, TU Dresden

DESY Theory Workshop
DESY Hamburg
29.9.2015

WW + jet: Motivation from experiment

WW [+**jet(s)**] at the LHC

- Measurement of **WW production cross section** [e.g. ATLAS, JHEP01(2015)049; CMS, Phys. Lett. B 721 (2013)]

- $h \rightarrow W\,W$ measurement [e.g. ATLAS, arXiv:1503.01060; CMS, JHEP01 (2014) 096]

- **spin-/ parity determination** of Higgs [e.g. ATLAS, EPJC75 (2015) 231; CMS, arXiv:1411.3441]

- limits on **anomalous couplings** [e.g. ATLAS, Phys. Rev. D 87, 112001 (2013); CMS, arXiv: 1411.3441]

- background for **BSM searches** (e.g. heavy scalars) [e.g. ATLAS, ATLAS-CONF-2013-067; CMS, arXiv:1504.00936]

- ...

- **K-factors** $\sim 1.2 - 1.8$ [depending on analysis details, cuts, etc...]

in more detail...

Process we are interested in

$$p\,p \to W^+\,W^-\,\textbf{jet} \to (\ell\bar{\nu}_\ell)\,(\bar{\ell}'\,\nu_{\ell'})\,\textbf{jet}$$

at NLO, offshell W's, spin correlations

- **previous results:** Campbell, Ellis, Zanderighi [CEZ] (2007); Dittmaier, Kallweit, Uwer [DKU] (2008/ 2010), Sanguinetti/ Karg [BGKKS] (2008)

- **together with shower merging/ matching:** Cascioli ea (2014) in Sherpa/ OpenLoops framework

- also **"ad hoc" available from automatized tools** (personally tested: MG5/aMC@NLO, others probably similar...)

$\Rightarrow$ our approach: use **unitarity-based techniques**, derive **completely analytic expressions**

- tool/ user-interface: $\Rightarrow$ **implementation in MCFM**

Unitary methods: basic idea

$$\mathcal{A}(\{p_i\}) = \sum_j d_j I_4^j + \sum_j c_j I_3^j + \sum_j b_j I_2^j + R \ .$$

$\Rightarrow$ know that all one-loop calculations can be reduced to **integral basis, + rational terms** [Passarino, Veltman, '78]

$\Rightarrow$ idea: **project out coefficients in front of basis integrals by putting momenta in the loop on mass shell**

(Bern, Dixon, Dunbar, Kosower ('94); Britto, Cachazo, Feng ('04))

- putting $2/3/4$ particles on their mass shell projects out coefficients of a **bubble/ triangle/ box** contribution

Unitarity methods: purely analytic approaches

- as you start with a $d(4)$-dimensional loop integral, **cutting 4 legs is easier than cutting 2**
- **boxes** $\Rightarrow$ straightforward, using **quadruple cuts with complex momenta (BCF)**
- **triangles** $\Rightarrow$ relatively straightforward, using **Fordes method**
- **bubbles**[*] $\Rightarrow$ can get quite complicated, use **spinor integration (BBCF)**
- **rational parts** $\Rightarrow$ long but OK, use **effective mass term (Badger)**
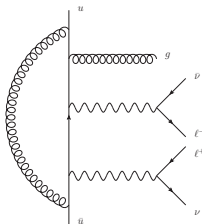
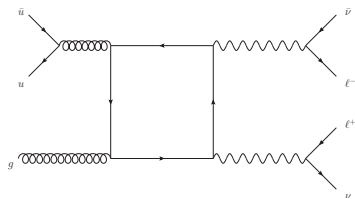[* available in process-independent librarized format]

WWj @ NLO in more detail

We consider

$$q\,\bar{q}\,\rightarrow\,W^+\,W^-\,g\quad[+\text{ permutations}]$$

diagram classes

(a)

(b)

[+ diagrams for $q\,\bar{q}\,\rightarrow\,(Z/\gamma)\,g\,\rightarrow\,...$]

$$d_4\left(s_{56}, s_{34}, 0, s_{17}; s_{127}, s_{234}\right) = \frac{1}{s_{34} - m_W^2} \frac{1}{s_{56} - m_W^2} \frac{\langle 12 \rangle^2 [2|P|2\rangle}{\langle 27 \rangle \langle 17 \rangle} \times$$

$$\left([42] - \frac{\langle 2|P|4]}{D_1}\right) \left(\langle 3|2+4|6] - \frac{\langle 23 \rangle \langle 2|P|6]}{D_1}\right) \left(\frac{[71]\langle 15 \rangle}{\langle 2|P|7]} + \frac{\langle 25 \rangle}{D_1}\right)$$

$$P = s_{17}\, p_{34} + s_{234} p_{17}, \ D_2 = [2|(3+4)(1+7)|2], \ D_1 = \langle 2|(3+4)(1+7)|2\rangle.$$

- in principle: also contributions with second deninmonator
  $D_2 = [2|(3+4)(1+7)|2]$ (here: $=0$)
- $D_1 D_2 \sim$ Gram determinant

Implementation: in practise

⇒ **fully implemented in MCFM framework**, i.e. in combination with Born, real radiation, ...

⇒ **MCFM output** (distributions/ cuts implementation/ interfaces/ etc...)

- in practise: handling of expressions ⇒ S@M [Maitre, Mastrolia, 2007]

  [comment: also implemented in multi-core version [Campbell, Ellis, Giele, 2015], now standard]

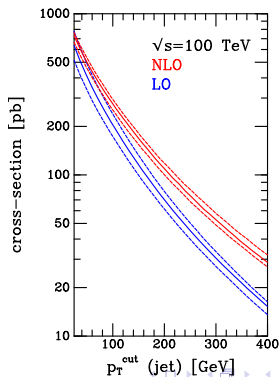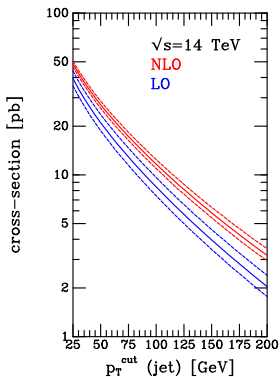**many cross checks: overall agreement:**
amplitude/ coefficient level: 10^{-6} or better
cross section level: always within integration errors
✓

$\sqrt{s}$	σ_{LO} [pb]	σ_{NLO} [pb]
13 TeV	34.9 (-11.0%, +11.4%)	42.9 (-3.7%, +3.7%)
14 TeV	39.5 (-11.0%, +11.7%)	48.6 (-4.0%, +3.8%)
100 TeV	648 (-19.3%, +22.3%)	740 (-9.3%, +4.5%)

More phenomenology: differential distributions at 14 and 100 TeV [More on this at QCD, EW and tools @ 100 TeV WS/ CERN next week...]

$\sigma_{LO} \sim$ 40/ 650/ 30 pb, K-factors $\sim$ 1.23/1.14/1.77
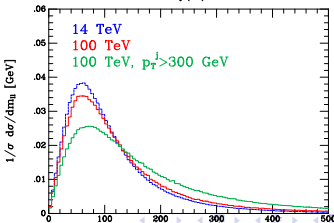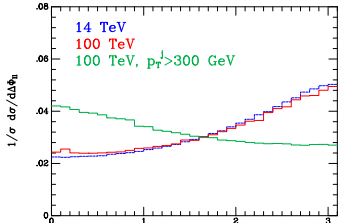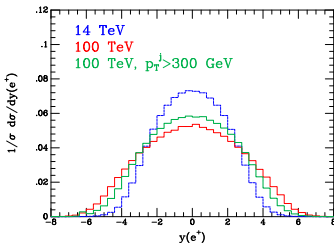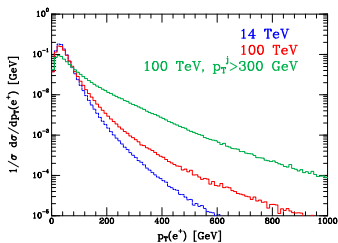
$$q\,\bar{q} \rightarrow W^+ W^- g$$

available and implemented in MCFM, running, rendering stable results [prerelease available upon request]

- virtual contributions: calculated using **unitarity methods** $\Rightarrow$ **available in analytic format**
- $\Rightarrow$ **extensively tested** on coefficient, amplitude, and cross section level $\checkmark$
- $\Rightarrow$ **important ingredient for NNLO calculations**, ready to be used
- $\Rightarrow$ obviously, **similarly useful for stand alone NLO calculations**
- provided sample applications for typical **Higgs spin/ parity studies @ 14 TeV, heavy scalar searches @ 100 TeVpp colliders**

$\Longrightarrow$ **Thanks for listening** $\Longleftarrow$

Appendix

Previous NLO calculations in the SM using analytic expressions from unitarity methods in MCFM

... on the amplitude level ...

- $e\,e \to 4$ **quarks**: Bern, Dixon, Kosower, Weinzierl (1996); Bern, Dixon, Kosower (1997)
- **Higgs and four partons** (in various configurations): Dixon, Sofianatos (2009); Badger, Glover, Mastrolia, Williams (2009); Badger, Campbell, Ellis, Williams (2009)
- $t\,\bar{t}$ **production**: Badger, Sattler, Yundin (2011)

... generalized unitarity implemented ...

- **Higgs + 2 jets** Campbell, Ellis, Williams (2010)
- **W + 2 b-jets** Badger, Campbell, Ellis (2011)
- $g\,g \to W\,W$ Campbell, Ellis, Williams (2011, 2014)
- $\gamma\gamma\gamma$ Campbell, Williams (2014)
- $\gamma\gamma\gamma\gamma$ Dennen, Williams (2014)

WWj @ NLO w/ unitarity: complexity

	(a)	(b)
boxes	13	1
triangles	8	4
bubbles	18	2
rational	13	5

Table : **Number of independent** (via singularity structure and/ or symmetries) **coefficients** [neglecting contributions from Z/γ current]

- involving **1,2,3-mass boxes and triangles**,
- **bubbles: 16 different underlying structures**, involving (0/1/2) quadratic poles, e.g. [terms before spinor integration]

$$\frac{[\ell a]^2 \, [\ell b] \, [\ell c]}{[\ell d][\ell e] \, \langle \ell|P|\ell\rangle^4}, \quad \frac{[\ell a] \, [\ell b] \, [\ell c]}{\langle \ell|P|\ell\rangle^4 \, \langle \ell|Q|\ell\rangle}, \quad \frac{[\ell a] \, [\ell b] \, [\ell c] \, [\ell d]}{\langle \ell|P|\ell\rangle^4 \, \langle \ell|Q|\ell\rangle \, \langle \ell|Q_2|\ell\rangle}, \; \cdots$$

Coefficients: easiest bubble and triangle

$I_2^{LC}(s_{156}) \sim$

$$\frac{\langle 65 \rangle [43]}{\langle 27 \rangle} \times \left\{ \frac{\langle 73 \rangle^2 \langle 7|P|6] [76]}{\langle 7|P|1] \langle 7|P|7]} \left[\frac{1}{\langle 7|P|7]} \left(\frac{[7|P|3\rangle}{\langle 37 \rangle} + \frac{[76] s_{156}}{2 \langle 7|P|6]} \right) + \frac{[1|P|3\rangle}{\langle 37 \rangle [1|P|7]} \right. \right.$$

$$\left. \left. - \frac{\langle 15 \rangle [56] [1|P|3\rangle^2}{s_{156} \langle 1|P|1] [1|P|7]} \left[\frac{\langle 15 \rangle [56]}{2 \langle 1|P|1]} + \frac{\langle 7|P|6]}{[1|P|7]} \right] \right\}, \quad P = p_{156}$$

$$I_3^{LC}(s_{34}, s_{27}, s_{156}) \quad \sim \quad \frac{1}{2} \sum_{\gamma=\gamma_{1,2}} \frac{s_{27} [4 K_2^\flat] [72] [65] \langle K_1^\flat 2 \rangle \langle K_1^\flat 3 \rangle \langle 15 \rangle^2}{(\gamma - s_{27}) [7 K_2^\flat] \langle K_1^\flat 1 \rangle \langle K_1^\flat 7 \rangle \langle 27 \rangle}$$

where

$$K_1^\flat = \frac{\gamma [\gamma \, p_{27} + s_{27} \, p_{34}]}{\gamma^2 - s_{27} \, s_{34}}, \; K_2^\flat = -\frac{\gamma [\gamma \, p_{34} + s_{34} \, p_{27}]}{\gamma^2 - s_{27} \, s_{34}},$$

$$\gamma_{1,2} = p_{27} \cdot p_{34} \pm \sqrt{(p_{27} \cdot p_{34})^2 - s_{27} \, s_{34}}$$

Cross checks

- on the **amplitude as well as coefficient level**, i.e. for several ($\sim 20 - 30$) single phase space points against code using D-dimensional unitarity (Ellis, Giele, Kunszt, Melnikov, 2009)

- for a **single phase space point as well as total cross section** against comparison in Les Houches proceedings, arXiv:0803.0494 (comparison CEZ, DKU, BGKKS)

- for the latter, also **independent MG5/aMC@NLO run**

overall agreement:
amplitude/ coefficient level: 10^{-6} or better
cross section level: always within integration errors
$\checkmark$

Phenomenology

- **total cross section** as a function of $p_{T,\text{jet}}^{\text{cut}}$ for pp collisions **@ 13/ 14/ 100 TeV**
- **differential distributions,** including more specific cuts
- ... for **spin- parity determination of Higgs @ 14 TeV**
- ... for **searches of extra heavy scalars @ 100 TeV**

jet definitions: anti-k_T, $p_T^{jet} > 25$ GeV ,$|\eta^{jet}| < 4.5$, $R = 0.5$

scales: $\mu_R = \mu_F = \frac{1}{2} \sum_i p_T^i$

More phenomenology: specific studies as background

- e.g. **spin/ parity determination of SM Higgs** (ATLAS, 1503.03643) $\Rightarrow$ @ 14 TeV
- e.g. **searches for additional scalars at high masses** (CMS, 1504.00936) $\Rightarrow$ @ 100 TeV

with cuts roughly following above studies...

Results

order	cm energy	no cuts	K	cuts	K
LO	14 TeV	462.0(2)fb		67.12(4)fb	
NLO	14 TeV	568.4(2)fb	1.23	83.91(5)	1.25
LO	100 TeV	6815(1)fb		1237(1)fb	
NLO	100 TeV	7939(5)	1.16	1471(1)fb	1.19

Cuts

variable	14 TeVanalysis	100 TeVanalysis
$p_{\perp,j}$	> 25 GeV	30 GeV
$|\eta_j|$	< 4.5	4.5
η_ℓ	≤ 2.5	2.5
$p_{\perp,\ell_1}$	> 22 GeV	50 GeV
$p_{\perp,\ell_2}$	> 15 GeV	10 GeV
$m_{\ell\ell}$	$\in [10; 80]\,\mathrm{GeV}$	–
$p_\perp^{\mathrm{miss}}$	$> 20\,\mathrm{GeV}$	20 GeV
$\Delta\Phi_{\ell\ell}$	< 2.8	
m_T	≤ 150 GeV	$\geq 80\mathrm{GeV}$
$\max[m_T^{\ell_1}, m_T^{\ell_2}]$	> 50 GeV	–

$$\left[m_T^2 = 2\, p_T^{\ell\ell} E_T^{\mathrm{miss}} \left(1 - \cos\Delta\Phi(\vec{p}_T^{\,\ell\ell}, \vec{E}_T^{\,\mathrm{miss}}) \right) \right]$$

Results for 14 TeV after all cuts

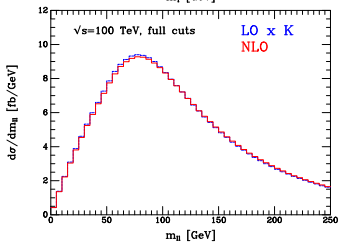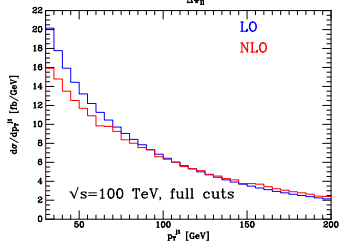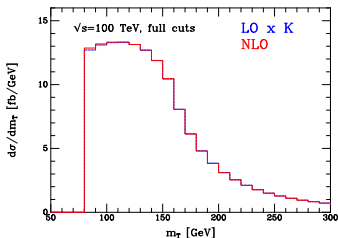$$\sigma_{LO} = 67.128(30)\text{fb}; \ \sigma_{NLO} = 83.923(47)\text{fb}$$
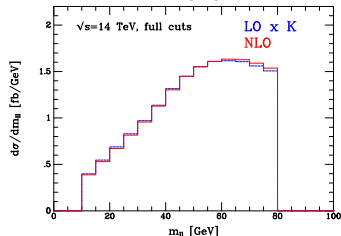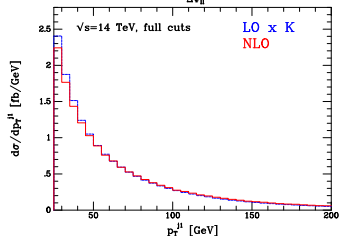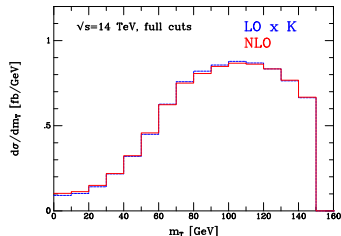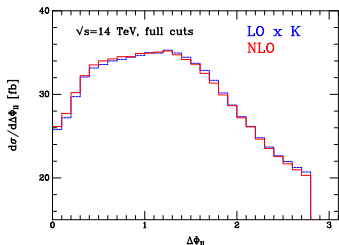
[plot: $\sigma_{LO} \times K$]

Results for 100 TeV after all cuts

$$\sigma_{LO} = 1237.2(4)\text{fb}; \; \sigma_{NLO} = 1472.0(7)\text{fb}$$
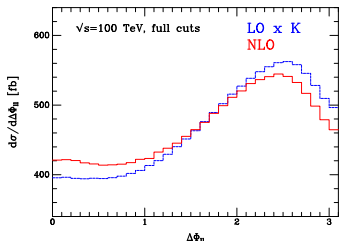
[plot: $\sigma_{LO} \times K$]

One slide of self-commercial

- calculation of **bubble-coefficients: process-independent**
- ⇒ **mathematica-based library,** with (all) librarised poles for ∼ 20 different structures
- ⇒ can **apply these completely straightforward to any other calculation** where the same structures appear in bubble (and can also obviously extend this)
- current interface: **me**
- future plan: **make public** in librarized format

<div align="center">

all tested (✓)

but obviously not every possible structure available at the moment

</div>

Basis integrals, type (a)

$D^{(1)}$	$I_4(0, 0, s_{56}, s_{234}; s_{17}, s_{156})$
$D^{(2)}$	$I_4(s_{34}, s_{56}, 0, s_{27}; s_{127}, s_{156})$
$D^{(3)}$	$I_4(0, 0, s_{34}, s_{156}; s_{27}, s_{234})$
$D^{(4)}$	$I_4(s_{56}, s_{34}, 0, s_{17}; s_{127}, s_{234})$
$D^{(5)}$	$I_4(0, 0, 0, s_{127}; s_{17}, s_{27})$
$C^{(1)}$	$I_3(0, s_{234}, s_{156})$
$C^{(2)}$	$I_3(s_{56}, s_{17}, s_{234})$
$C^{(3)}$	$I_3(s_{34}, s_{27}, s_{156})$
$C^{(4)}$	$I_3(0, s_{27}, s_{127})$
$C^{(5)}$	$I_3(0, s_{17}, s_{127})$
$C^{(6)}$	$I_3(0, 0, s_{17})$
$C^{(7)}$	$I_3(0, s_{56}, s_{156})$
$C^{(8)}$	$I_3(s_{56}, s_{34}, s_{127})$
$C^{(9)}$	$I_3(0, 0, s_{27})$
$C^{(10)}$	$I_3(0, s_{34}, s_{234})$
$B^{(1)}$	$I_2(s_{156})$
$B^{(2)}$	$I_2(s_{234})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(4)}$	$I_2(s_{17})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$

$D^{(6)}$	$I_4(s_{56}, s_{34}, 0, s_{17}; s_{127}, s_{234})$
$D^{(7)}$	$I_4(0, 0, 0, s_{127}; s_{27}, s_{12})$
$D^{(8)}$	$I_4(0, 0, 0, s_{127}; s_{17}, s_{12})$
$D^{(9)}$	$I_4(s_{34}, 0, 0, s_{567}; s_{234}, s_{12})$
$D^{(10)}$	$I_4(s_{34}, s_{12}, s_{56}, 0; s_{567}, s_{347})$
$D^{(11)}$	$I_4(0, 0, s_{56}, s_{347}; s_{12}, s_{156})$
$D^{(12)}$	$I_4(s_{34}, s_{12}, 0, s_{56}; s_{567}, s_{127})$
$D^{(13)}$	$I_4(0, s_{12}, s_{56}, s_{34}; s_{127}, s_{347})$
$C^{(11)}$	$I_3(0, 0, s_{27})$
$C^{(12)}$	$I_3(0, s_{34}, s_{234})$
$C^{(13)}$	$I_3(0, s_{34}, s_{347})$
$C^{(14)}$	$I_3(0, s_{347}, s_{156})$
$C^{(15)}$	$I_3(0, s_{127}, s_{12})$
$C^{(16)}$	$I_3(0, 0, s_{12})$
$C^{(17)}$	$I_3(s_{34}, s_{567}, s_{12})$
$C^{(18)}$	$I_3(s_{56}, s_{347}, s_{12})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table : scalar integrals of type (a) *left* leading colour and *right* additional subleading color amplitude.

Basis integrals, type (b)

$D^{(10)}$	$I_4(s_{34}, s_{12}, s_{56}, 0; s_{567}, s_{347})$
$D^{(12)}$	$I_4(s_{34}, s_{12}, 0, s_{56}; s_{567}, s_{127})$
$D^{(13)}$	$I_4(0, s_{12}, s_{56}, s_{34}; s_{127}, s_{347})$
$C^{(8)}$	$I_3(0, s_{56}, s_{567})$
$C^{(10)}$	$I_3(s_{56}, s_{34}, s_{127})$
$C^{(13)}$	$I_3(0, s_{34}, s_{347})$
$C^{(15)}$	$I_3(0, s_{127}, s_{12})$
$C^{(17)}$	$I_3(s_{34}, s_{567}, s_{12})$
$C^{(18)}$	$I_3(s_{56}, s_{347}, s_{12})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table : Definitions of the scalar integrals that appear in the calculation of the diagrams of type (b), leading colour only.

Tania Robens   WW + jet @ NLO   DESY, 29.9.15

Appearance of quadratic poles/ square roots

- have a cut leading to a propagator $\sim \frac{1}{s_{\ell_1 p}}$, where $p^2 \neq 0$
  (e.g. $p = p_1 + p_2$)
- for spinor integration $\ell_1 \to \frac{P^2}{[\ell|P|\ell\rangle} \ell$, where P is momentum over the cut
- i.e., use

$$s_{\ell_1, p} = \langle \ell_1 | \not{p} | \ell_1] + p^2 \to \frac{P^2 \langle \ell | \not{p} | \ell]}{\langle \ell | \not{P} | \ell]} + p^2 = \frac{\langle \ell | \not{Q} | \ell]}{\langle \ell | \not{P} | \ell]},$$

- contributions often appear together with factors $\sim \frac{1}{[\ell|P|\ell\rangle}$
$\Rightarrow$ contains poles $\sim \frac{1}{\langle \ell | PQ\ell \rangle}$
  - leads to two possible solutions for $|\ell\rangle$ where $\langle \ell | PQ | \ell \rangle = 0$
    (pole)

Electroweak parameters

m_W	80.385 GeV	Γ_W	2.085 GeV
m_Z	91.1876 GeV	Γ_Z	2.4952 GeV
e^2	0.095032	g_W^2	0.42635
$\sin^2\theta_W$	0.22290	G_F	0.116638×10^{-4}

Effect of neglecting diagrams containing Higgs/ diagrams with top loops

- im MG5/aMC@NLO: top and Higgs included
- check: run with $m_t\, m_H \times 10\,(100)$
- results

calculation	parameters	σ^{NLO} [pb]
MCFM	default	14.571 (18)
MG5	default	14.547 (19)
MG5	$m_h \times 10, m_t \times 10$	14.615 (21)
MG5	$m_h \times 100, m_t \times 100$	14.563 (19)
DKU	default	14.678 (10)

Other (related) (reduction) methods [non-exhaustive listing]

- purely analytic: **generalized unitarity** [Britto, (Buchbinder), Cachazo, Feng (2005, 2006); Britto, Feng, Mastrolia (2006), Forde (2007), Badger (2009), Mastrolia (2009), ...]
- other approaches: **recursion/ reduction methods** [Berends, Giele (1987); del Aguila, Pitta (2004); Ossola, Papadopoulos, Pittau (2006)]
- **numerical implementions** in many (publicly available) codes: (in order of appearance) CutTools (OPP, 2007), Samurai (Mastrolia ea, 2010), Gosam (Cullen ea, 2011), MadLoop/aMC@NLO (Hirschi ea, 2011, Frederix ea, 2011), Helac-NLO (van Hameren ea, 2010; Bevilacqua ea, 2013)
- other numerical implementations (in order of appearance): Blackhat (Berger ea, 2008), Rocket (Giele, Zanderighi, 2008), NGluon (Badger ea, 2011), OpenLoops (Cascioli ea, 2011), Ninja (Peraro, 2014)