Correlation functions in N=4 super-Yang-Mills theory

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LAPTH, Annecy, France

1 October 2015, DESY

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Outline

- Four-point correlation function of half-BPS operators with **different** weights in $\mathcal{N} = 4$ SYM in the **three-loop** approximation

$$\langle \mathcal{O}_{\rm BPS}(x_1)\mathcal{O}_{\rm BPS}(x_2)\mathcal{O}_{\rm BPS}(x_3)\mathcal{O}_{\rm BPS}(x_4)\rangle$$

  - 1 loop: $\mathcal{N} = 1$ Feynman supergraphs
  - 2 loops: $\mathcal{N} = 2$ harmonic supergraphs + superconformal symmetry
  - **3 loops**: are not accessible by a Feynman graph calculation
- Motivation
- General properties of the four-point correlators
- Integrands of correlators and Lagrangian insertions
- Amplitude/correlator duality
- Fixing coefficients in the ansatz
- Conclusion

Motivation

- Correlators are master objects in any CFT. They are finite and conformal, need no regularization
- The integrand of the four-point correlation function of the half-BPS operator $\mathcal{O}_{\mathbf{20'}}$, which is at the bottom of the stress-tensor multiplet, has been found up to 6 loops in the planar limit [Eden, Heslop, Korchemsky, Sokatchev '11 '12] Five-loop anomalous dimension of the Konishi [Eden, Heslop, Korchemsky, V. Smirnov, Sokatchev '12] Three-loop structure constants of the twist 2 operators [Eden '12]
- Duality correlators/amplitudes

$$\lim_{x_{i,i+1}^2 \to 0} G(x_1, \dots, x_n) = \left(\mathcal{A}(p_1, \dots, p_n)\right)^2, \qquad p_i = x_i - x_{i+1}$$

works at tree-level and for integrands of loop corrections.
Extensively tested on the stress-tensor supermultiplet [Eden, Heslop, Korchemsky, Sokatchev '10 '11] and the Konishi operator [Adamo, Bullimore, Mason, Skinner '11]
What about other correlators?

- Integrability based predictions for **structure constants** $\langle \mathcal{O}_{BPS} \mathcal{O}_{BPS} \mathcal{O}_S \rangle$ [Vieira, Wang '13] [Basso, Komatsu, Vieira '15]

$$\sum_{n=0}^{\infty} g^{2n} \sum_{m=0}^{n} y^m \, \mathcal{P}_S^{(n,m)} = \sum_{\substack{\text{primaries} \\ \text{with spin } S \\ \text{twist } L}} C_{\text{BPS;BPS};i} \times \widetilde{C}_{\text{BPS;BPS};i} \exp\left(\gamma_i y\right)$$

Four-point correlators

$$\mathcal{O}^{(k)}(x,y) = Y^{I_1}\dots Y^{I_k}\mathrm{Tr}(\phi^{I_1}\dots\phi^{I_n})$$

The lowest component of a half-BPS multiplet in $\mathcal{N}=4$. Dimension $=k$, irrep. $[0,k,0]$ of the R symmetry $SO(6)$. Lightlike auxiliary vectors $Y^I Y^I = 0$

$$\mathcal{G}_{k_1k_2k_3k_4} = \langle \mathcal{O}^{(k_1)}(x_1,Y_1)\,\mathcal{O}^{(k_2)}(x_2,Y_2)\,\mathcal{O}^{(k_3)}(x_3,Y_3)\,\mathcal{O}^{(k_4)}(x_4,Y_4)\rangle$$

The R-charge conservation implies $k_1+k_2+k_3+k_4 = 2n$, $k_i \le \sum_{j\neq i} k_i$

Extremal $k_1 = k_2+k_3+k_4$, next-to-extremal $k_1 = k_2+k_3+k_4-2$ do NOT have quantum corrections

$$\mathcal{G}_{k_1k_2k_3k_4} = \mathcal{G}^0_{k_1k_2k_3k_4} + \mathcal{G}^{\rm loop}_{k_1k_2k_3k_4}$$

The free part $\mathcal{G}^0$ is polynomial in the scalar field propagators $d_{ij} = \frac{1}{4\pi^2}\frac{y_{ij}^2}{x_{ij}^2}$; parametrization of Y_I by unconstrained complex 2×2 matrices $y_{a'}^{a}$: $Y_i^I Y_j^I = \det||y_i - y_j|| \equiv y_{ij}^2$

$$\mathcal{G}^0_{k_1k_2k_3k_4} = \sum_{\{a_{ij}\}}\left(\prod_{1\le i<j\le 4}(d_{ij})^{a_{ij}}\right) C_{\{a_{ij}\}}$$

$\sum_j a_{ij} = k_i$ for each $i = 1,\dots,4$

Loop corrections

Factorization of the interacting part according to the 'partial non-renormalization' theorem [Eden, Petkou, Schubert, Sokatchev '00] [Heslop, Howe '02]

weights $k_1 - 2, k_2 - 2, k_3 - 2, k_4 - 2$

$$\mathcal{G}^{\rm loop}_{k_1 k_2 k_3 k_4} = C_{k_1 k_2 k_3 k_4}\, R(1,2,3,4) \times \sum_{\{b_{ij}\}} \left(\prod_{1\le i<j\le 4} (d_{ij})^{b_{ij}} \right) F_{\{b_{ij}\}}(u,v)$$

weights 2,2,2,2

Universal rational factor

$$\begin{aligned} R(1,2,3,4) &= x_{12}^2 x_{34}^2 d_{12}^2 d_{34}^2 + x_{13}^2 x_{24}^2 d_{13}^2 d_{24}^2 + x_{14}^2 x_{23}^2 d_{14}^2 d_{23}^2 \\ &\quad + d_{12} d_{23} d_{34} d_{14} (x_{13}^2 x_{24}^2 - x_{12}^2 x_{34}^2 - x_{14}^2 x_{23}^2) \\ &\quad + d_{12} d_{13} d_{24} d_{34} (x_{14}^2 x_{23}^2 - x_{12}^2 x_{34}^2 - x_{13}^2 x_{24}^2) \\ &\quad + d_{13} d_{14} d_{23} d_{24} (x_{12}^2 x_{34}^2 - x_{14}^2 x_{23}^2 - x_{13}^2 x_{24}^2) \end{aligned}$$

The weights of the renormalized part are lowered $\sum_j b_{ij} = k_i - 2,\ i = 1, \ldots, 4$

Several functions $F_{\{b_{ij}\}}$ describe the quantum corrections

Perturbative expansion in the 't Hooft coupling $a = g^2 N_c/(4\pi^2)$

$$F_{\{b_{ij}\}}(u,v) = \sum_{\ell \ge 1} a^\ell F^{(\ell)}_{\{b_{ij}\}}(u,v)$$

Conformal cross-ratios

$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}\,, \qquad v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

Lagrangian insertions

$$\mathcal{G}^{(\ell)}_{k_1k_2k_3k_4} = \frac{1}{\ell!}\int d^4x_5\dots d^4x_{4+\ell}\,\langle \mathcal{O}^{(k_1)}(1)\,\mathcal{O}^{(k_2)}(2)\,\mathcal{O}^{(k_3)}(3)\,\mathcal{O}^{(k_4)}(4)\,\mathcal{L}(5)\dots\mathcal{L}(4+\ell)\rangle_{\text{Born}}$$

quantum corrections: $F^{(\ell)}_{\{b_{ij}\}}(u,v) = \dfrac{1}{\ell!(-4\pi^2)^\ell}\displaystyle\int d^4x_5\dots d^4x_{4+\ell}\, f^{(\ell)}_{\{b_{ij}\}}(x_1,\dots,x_{4+\ell})$

$f^{(\ell)}_{\{b_{ij}\}}$
- a rational function with conformal weights $+1,+1,+1,+1,\underbrace{+4,\dots,+4}_{\ell\text{ inner points}}$
- permutation symmetry of $\displaystyle\prod_{1\le i<j\le 4}(d_{ij})^{b_{ij}}$

How general can $f^{(\ell)}_{\{b_{ij}\}}$ be ? OPE & protectedness of half-BPS operators lead to

$$f^{(\ell)}_{\{b_{ij}\}}(x_1,\dots,x_{4+\ell}) = \frac{P^{(\ell)}_{\{b_{ij}\}}(x_1,\dots,x_{4+\ell})}{\prod_{i=1}^{4}\prod_{j=5}^{4+\ell}x_{ij}^2\cdot\prod_{5\le j<k\le 4+\ell}x_{jk}^2}$$

$P^{(\ell)}_{\{b_{ij}\}}$ is a polynomial with conformal weights $1-\ell$ at each point

If some $k_i = 2$ then the **permutation symmetry** is enhanced. It swaps the **internal** and **external** points [Eden, Heslop, Korchemsky, Sokatchev '11] Stress-tensor supermultiplet

$$\mathcal{T}(x,y,\rho) = \mathcal{O}^{(2)}(x,y)+\dots+\rho^4\mathcal{L}(x)$$

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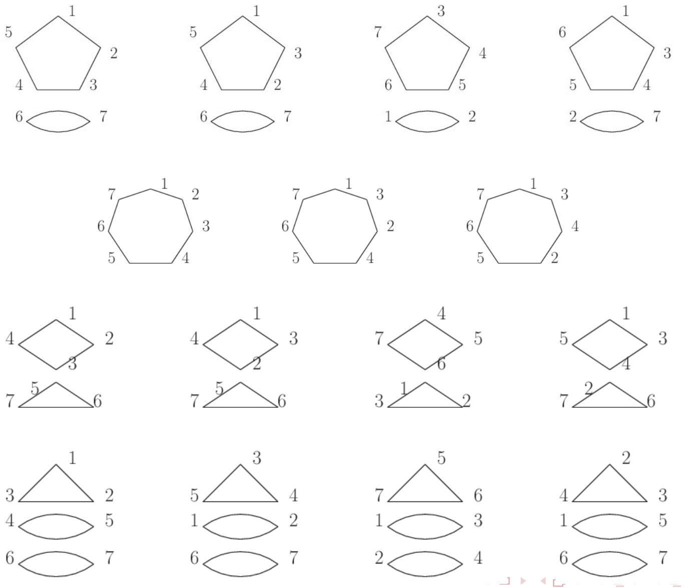
Example. Three-loop polynomial ansatz for the correlator 3322

Permutation symmetry $S_2 \times S_{2+3}$

15 polynomials



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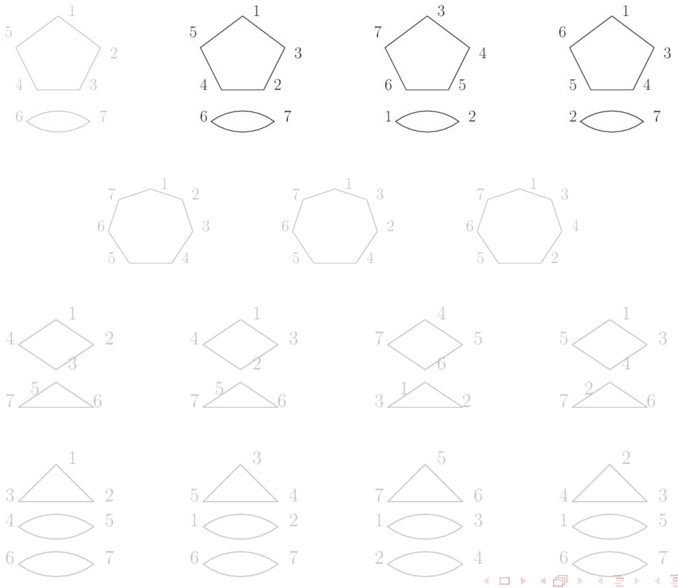
Permutation symmetry $S_2 \times S_{2+3}$

15 polynomials , but only **3** of them survive in the **planar** limit



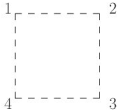
Superamplitude/supercorrelator duality

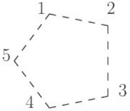
Superamplitude

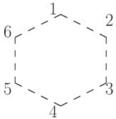
light-cone $x^2_{i\,i+1} \to 0$

$\mathcal{G}_{\mathbf{k_1 k_2 k_3 k_4}} \longrightarrow \mathcal{G}_{2\,2\,2\,2}$

Light-cone limit







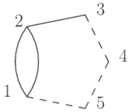
$\cdots$

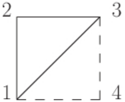
Integrand of $\mathcal{G}^{(\ell)}$ $\to$ ℓ-loop MHV | $(\ell-1)$-loop NMHV | $(\ell-2)$-loop $\mathrm{N}^2\mathrm{MHV}$

Several functions $F_{\{b_{ij}\}}$ describe quantum corrections
The duality works for some of them

$b_{ij} \neq 0 \Longrightarrow x^2_{ij} = 0$ inscribe edges $i \leftrightarrow j$ in the light-like polygon

allowed $\Longrightarrow$





$\Longleftarrow$ forbidden

Fixing coefficients in the polynomial ansatz

Two loops

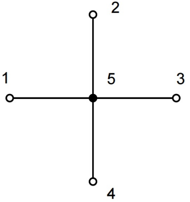
	2222	$kk22,\ k \geq 3$	4332	3333	4444	4433	5533	5544	5555
ansatz	1	2	3	4	$4+4$	$5+7$	$5+7$	$5+7+7+5$	$4+6+2$
planar	1	1	2	3	$3+3$	$3+3$	$3+3$	$3+5+3+1$	$3+2+1$
amp/corr	0	0	0	1	$1+1$	$1+0$	$1+0$	$1+2+1+1$	$1+2+1$

Three loops

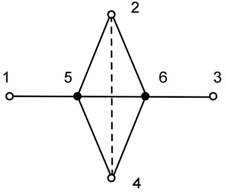
	2222	3322	$kk22,\ k \geq 4$	4332	3333	4444	4433	5533
ansatz	4	15	15	37	41	$41+41$	$64+102$	$64+102$
planar	1	3	2	6	6	$5+4$	$8+11$	$8+9$
amp/corr	0	1	0	1	1	$0+1$	$1+3$	$1+1$

	5544	5555
ansatz	$64+102+102+64$	$41+68+19$
planar	$7+10+6+5$	$5+4+1$
amp/corr	$0+2+2+5$	$0+1+1$

Conformal integrals. One and two loops



$g_{1,2,3,4}$



$h_{1,3;2,4}$

[Usyukina, Davydychev '93]

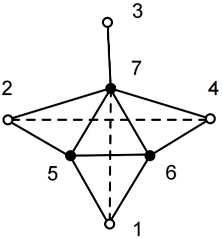
Take into account the identities for conformal integrals that are not obvious at the level of the integrand [Drummond, Henn, V. Smirnov, Sokatchev '06]

$$F^{(1)} = g_{1,2,3,4}$$

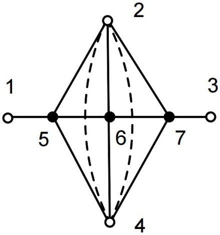
$F^{(2)}$ = linear combination of 6 functions

$$h_{1,2;3,4},\ h_{1,4;2,3},\ h_{1,3;2,4},\ x_{12}^2 x_{34}^2\, g^2,\ x_{13}^2 x_{24}^2\, g^2,\ x_{14}^2 x_{23}^2\, g^2$$

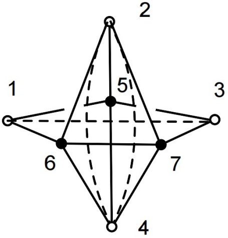
Conformal integrals. Three loops



$T_{1,3;2,4}$

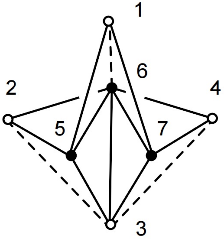


$L_{1,3;2,4}$

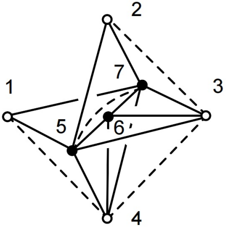


$g \times h_{1,3;2,4}$

[Drummond, Henn, V. Smirnov, Sokatchev '06] [Broadhurst '93]



$E_{1,3;2,4}$



$H_{1,3;2,4}$

Linear combination

of 15 functions

$g \times h_{1,2;3,4}$, $g \times h_{1,3;2,4}$, $g \times h_{1,4;2,3}$,

$L_{1,2;3,4}$, $L_{1,3;2,4}$, $L_{1,4;2,3}$,

$E_{1,2;3,4}$, $E_{1,3;2,4}$, $E_{1,4;2,3}$,

$H_{1,2;3,4}$, $H_{1,3;2,4}$, $H_{1,4;2,3}$,

$1/v\, H_{1,2;3,4}$, $u/v\, H_{1,3;2,4}$, $u\, H_{1,4;2,3}$

[Drummond, Duhr, Eden, Heslop, Pennington, V. Smirnov '13]

How to fix the remaining coefficients ?

- Several unfixed coefficients stand in front of the same conformal integral
- Double short-distance OPE
  Example: Correlator $\mathcal{G}_{3322}$. 1 unfixed coefficients after amp/corr

$$\mathcal{O}^{(2)} \times \mathcal{O}^{(3)} = c_{\mathbf{50}} \frac{y_{12}^2}{x_{12}^2} \mathcal{O}^{(3)} + c_{\mathbf{6}} \frac{y_{12}^4}{x_{12}^2} K_{\mathbf{6}} + \ldots$$

  Non-protected $K_{\mathbf{6}}$ in vector rep. of $SO(6)$, dimension $= 3$

$$\log\left(1 + 4x_{12}^4 \sum_{\ell \geq 1} a^{\ell} F^{(\ell)}_{\{1,0,0,0,0,0\}}\right) \to \frac{1}{2}\gamma_{\mathbf{6}} \log v + O(v^0) \qquad \text{at} \quad v \to 0,\ u \to 1$$

  The constraint works at the level of the **integrand**
- OPE analysis & consistency of different correlators
  Example: $\mathcal{G}_{2222}$, $\mathcal{G}_{3322}$ are already known, $\mathcal{G}_{3333}$ contains 1 unfixed coefficient

$$\mathcal{G}_{3322} \Longrightarrow C_{\mathcal{O}^{(2)}\mathcal{O}^{(2)}K} C_{\mathcal{O}^{(3)}\mathcal{O}^{(3)}K} \ , \ \mathcal{G}_{2222} \Longrightarrow \left(C_{\mathcal{O}^{(2)}\mathcal{O}^{(2)}K}\right)^2 \ , \ \mathcal{G}_{3333} \Longrightarrow \left(C_{\mathcal{O}^{(3)}\mathcal{O}^{(3)}K}\right)^2$$

Conclusion

- We found a number of four-point correlators with **different charges** in the three-loop approximation in the planar limit
- The permutation symmetry is smaller than in the case $\mathcal{G}_{2222}$, so there are much more coefficients in the ansatz but **planarity** excludes most of them
- The light-cone limits of the correlator are not sufficient to fix all the coefficients, so we apply OPE analysis and check the consistency of different correlators with each other
- NO new conformal integrals as compared to $\mathcal{G}_{2222}$
- The method works at higher-loop orders as well and is extremely restrictive. For example, we completely fix $\mathcal{G}_{3322}$ at **four** loops at the level of the integrand (without calculating the integrals !!!)