The Unavoidable: Monte Carlos for the LHC

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Outline of the Talk

- Introduction & Motivation
- State-of-the-Art for Multi-leg Processes
- Main Obstacles and Computational Complexity
- Leading Order level MC Programs: HELAC-PHEGAS
- Main Features & Numerical Results
- Matching to Parton Shower
- Summary & Outlook
• Total inelastic pp cross section

\[ \sigma_{\text{tot}} \approx 100 \text{ mb} \]

• Event rate at low/high luminosity

\[ R = \frac{N_{\text{evt}}}{\text{sec}} = \sigma \cdot L \approx 10^8 / s \ (10^9 / s) \]

• Interesting events are rare!

\[ \sigma(pp \rightarrow W) \approx 150 \text{ nb} \approx 10^{-5} \sigma_{\text{tot}} \]

• We need luminosity to be high to maximise number of events
Final States @ LHC

<table>
<thead>
<tr>
<th>Process</th>
<th>Events/s</th>
<th>Events/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet $E_T &gt; 100$ GeV</td>
<td>$10^3$</td>
<td>$10^{10}$</td>
</tr>
<tr>
<td>Jet $E_T &gt; 1$ TeV</td>
<td>$1.5 \times 10^{-2}$</td>
<td>$1.5 \times 10^5$</td>
</tr>
<tr>
<td>$W \rightarrow lv$</td>
<td>20</td>
<td>$2 \times 10^8$</td>
</tr>
<tr>
<td>$bb$</td>
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<td>$5 \times 10^{12}$</td>
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<tr>
<td>$tt$</td>
<td>1</td>
<td>$10^7$</td>
</tr>
<tr>
<td>$WW \rightarrow lv lv$</td>
<td>$6 \times 10^{-3}$</td>
<td>$6 \times 10^4$</td>
</tr>
</tbody>
</table>

September 2009 ???

- Low luminosity:
  \[ L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1} \Rightarrow 10 \text{ fb}^{-1} / \text{year} \]
- High luminosity
  \[ L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1} \Rightarrow 100 \text{ fb}^{-1} / \text{year} \]

Provide description of these events, features of new physics processes (rates, distributions, details of the final states, overall multiplicities, etc.)
Monte Carlo Generators

- Physics program of two main LHC experiments ATLAS and CMS:
  - Discovery of Higgs boson(s)
  - Search for signal of new physics beyond the SM
  - Remember: Present day signals = Tomorrow backgrounds

- Signal events dug out from a bulk of background events
- Backgrounds due to SM processes
- Mostly QCD processes, accompanied by additional electroweak bosons
- Final state - high number of jets or identified particles
- Reliable predictions for multi-particle final states needed!
- All this can be described by Monte Carlo generators

Benchmarking against real data turns MC simulation into powerful tool!
ttH event @ LHC

- Hard interaction – big red blob
- Decay of top quarks and Higgs – small red blobs
- **QCD** bremsstrahlung - **ISR** & **FSR**
- Multiple interactions
- **Hadronisation of final state partons**
- Hadrons decays
- QED bremsstrahlung - **Photons radiations**
- Event generators rely on factorisation of such event into different well-defined phases corresponding to different kinematic regimes
- Hard process calculated in fixed order perturbation theory in $\alpha_s$
- QCD evaluation described by parton shower - perturbation theory beyond fixed order - LL
- Hadronisation – $\Lambda_{QCD}$ - phenomenological models with parameters to be fitted to data
- Underlying events - phenomenological models beyond factorisation

Borrowed from **SHERPA** people
Factorisation Theorem

- Cross section for hard scattering process initiated by two hadrons

$$\sigma^{\text{had.}} = \sum_{ij} \int dx_i dx_j f_{i/h_1}(x_1, \mu_F^2) f_{j/h_2}(x_2, \mu_F^2) \hat{\sigma}^{\text{part.}}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), Q^2/\mu^2)$$

- Long distance (hadronic) physics factored out and absorbed into PDF
- Short distance (partonic) physics $$Q \gg M_{\text{had}}$$
- Factorisation is not exact but corrections are $$O(M_{\text{had}}/Q)$$
- Asymptotic freedom $$\rightarrow \alpha_s$$ small at high $$Q$$
- $$\hat{\sigma}^{\text{part.}}$$ can be computed as a perturbation series in $$\alpha_s$$
- More terms included in perturbative expansion weaker dependence on $$\mu_F = \mu_R = \mu$$
- Cross section to all orders independent of scales

$$\frac{\partial \sigma}{\partial \mu_F} = \frac{\partial \sigma}{\partial \mu_R} = 0$$
Process Calculation

- For a given process \( gg \to u \bar{u} s \bar{s} gg \)

- With some cuts \( p_T = \sqrt{p_x^2 + p_y^2}, \quad \eta = -\ln \tan(\theta/2), \quad \Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} \)

- We ask to compute cross section \( \hat{\sigma}^{\text{part.}} \) @ LHC

- Steps
  - Find all Feynman diagrams
  - Compute them to get an amplitude
  - Sum over all colour and helicity configurations
  - Square the amplitude
  - Integrate over the phase space
Biggest Obstacles

- Complexity of calculation based on Feynman diagrams \( \sim n! \)
- Flavours of partons never detected (b-tagging)
- For given jet configuration very many contributing subprocesses
- Neither the colour nor the spin of any parton is observed
- Amplitude with \( p \) quark and \( q \) gluons \((2 \times 3)^p (2 \times 8)^q\) contributions
- Amplitude peaks in complicated ways inside the momentum phase space
- Straightforward integration is impractical

Automated calculation of the ME based on recursive equations

MC summation over helicity and colour as well as over flavour

Search for efficient mappings - importance and stratified sampling
### QCD with 1 fermion pairs

<table>
<thead>
<tr>
<th>N=8</th>
<th>N=9</th>
<th>N=10</th>
<th>N=11</th>
<th>N=12</th>
<th>N=13</th>
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<td>231280</td>
<td>4016775</td>
<td>79603720</td>
<td>1773172275</td>
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</table>

### QCD with 3 (identical) fermion pairs

<table>
<thead>
<tr>
<th>N=8</th>
<th>N=9</th>
<th>N=10</th>
<th>N=11</th>
<th>N=12</th>
<th>N=13</th>
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<td>946050</td>
<td>17258640</td>
<td>355273170</td>
<td>8151299520</td>
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</tbody>
</table>

roughly grows like $n!$
Tools @ LHC

- Standard Model and beyond tools @ tree level
- Parton-level tools which are completely self-contained and automated and provide amplitudes and integrators on their own

  **AlpGen, AMEGIC++/Sherpa, Helac-Phegas, MadGraph/MadEvent, O'Mega/WHIZARD, ...**

- General purpose Monte Carlo programs (parton shower, hadronisation, multiple interactions, hadrons decays, ...)

  **Herwig, Herwig++, Pythia 6.4, Pythia 8, Sherpa, ...**

*Generators are not perfect! Shop around and compare several approaches before drawing conclusions. Blind usage of a generator is not encouraged!*

*T. Sjöstrand*
Features of a MC generator

- Complete Standard Model
- Convolution with PDFs
- Summation over subprocesses
- Matching to parton showers and hadronisation
- Reliability
- Extensibility
- Speed
- Flexibility
HELAC-PHEGAS

http://helac-phegas.web.cern.ch/helac-phegas/

Complete Standard Model

Convolution with PDFs

Summation over subprocesses

Matching to parton showers and hadronisation

Leading order without approximations

- Electroweak, QCD and mixed contributions
- Unitary and Feynman gauges
- Fixed width and complex mass schemes for unstable particles
- All correlations (colour, spin) taken into account naturally
- Non-zero fermion masses
- CKM matrix and Running couplings

Reliability

Flexibility

Speed

Extensibility
Complete Standard Model

Convolution with PDFs

Summation over subprocesses

Matching to parton showers and hadronisation

- Standalone version with build-in CTEQ6L1
- Interface to the LHAPDF

Reliability

Extensibility

Flexibility

Speed
Complete Standard Model

- Convolution with PDFs
- Summation over subprocesses
- Matching to parton showers and hadronisation

- Lepton colliders, TeVatron, LHC
- Summation over all sub-processes

\[ p_T > 60 \text{ GeV}, \quad \theta_{ij} > 30^\circ, \quad |\eta_i| < 3 \]

<table>
<thead>
<tr>
<th># jets</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(nb) )</td>
<td>91.41</td>
<td>6.54</td>
<td>0.458</td>
<td>2.97 \times 10^{-2}</td>
<td>2.21 \times 10^{-3}</td>
<td>2.12 \times 10^{-4}</td>
</tr>
<tr>
<td>% Gluon</td>
<td>45.7</td>
<td>39.2</td>
<td>35.7</td>
<td>35.1</td>
<td>33.8</td>
<td>26.6</td>
</tr>
</tbody>
</table>

Reliability
Flexibility
Extensibility
Speed
Complete Standard Model

Convolution with PDFs

Summation over subprocesses

Matching to parton showers and hadronisation

- Parton level weighted/unweighted events
- Parton shower and hadronisation via interface to **PYTHIA**
- Merging parton showers and matrix elements via MLM scheme
- Also will be possible via CKKW scheme - **HERWIG++**
- LHA files and interfacing routines


Reliability

Flexibility

Speed

Extensibility
PHEGAS – phase space generator

- Automatic multi-channel phase-space mapping
- Self-adapting procedure to reshape the generated phase space density
- For example: per mille level tt + 0,1,2 jets (6,7,8 final states) with full off-shell and finite width effects
- Straightforward inclusion of new physics effects
- New models: for example MSSM
- New couplings: for example effective $H_{\gamma\gamma}$ and $H_{gg}$ couplings
Convolution with PDFs

Summation over subprocesses

Matching to parton showers and hadronisation

Complete Standard Model

Reliability

Extensibility

Speed

Flexibility

- $3^n$ complexity due to the Dyson-Schwinger recursive algorithm
- Monte Carlo summation over:
  - helicity configurations (completed)
  - color configurations (completed)
  - subprocesses
- Trivial parallelization over clusters (LSF at CERN)
Complete Standard Model

Convolution with PDFs

Summation over subprocesses

Matching to parton showers and hadronisation

- Arbitrary cuts, distributions and scale choices
- Configuration via scripts
- Compilers: Lahey Fujitsu, Intel Fortran, GNU gfortran/g95
- Multiprecision numerics

Flexibility

Speed

Extensibility

Reliability
Dyson-Schwinger Recursion

- Alternative to Feynman Diagrams representation
- Express n-point Green's functions in terms of 1-, 2-,... (n-1)-point functions
- Diagrammatic representation, e.g. QED-like theory
- Interaction of a spinor field to a gauge boson

\[ b^\mu(P) = \sum_{n=1}^n \delta(P=P_i) b^\nu(P_i) + \sum_{P=P_1+P_2} (i\gamma) \prod^\nu \bar{\psi}(P_2) \gamma^\nu \psi(P_1) e(P_1, P_2) \]

\[ b_\mu(P) = \quad \psi(P) = \quad \bar{\psi}(P) = \]

- Sub-amplitude with off-shell boson of momentum P
- Blobs denote sub-amplitude with the same structure
**Dyson-Schwinger Recursion**

- **Fermion with momentum** $P$

\[
\psi(P) = \sum_{n=1}^{n} \delta(P = P_i) \psi(P_i) + \sum_{P = P_1 + P_2} (ig) P b_\mu(P_2) \gamma^\mu \psi(P_1) \epsilon(P_1, P_2)
\]

- **Antifermion with momentum** $P$

\[
\bar{\psi}(P) = \sum_{n=1}^{n} \delta(P = P_i) \bar{\psi}(P_i) + \sum_{P = P_1 + P_2} (ig) \bar{\psi}(P_1) b_\mu(P_2) \gamma^\mu \bar{P} \epsilon(P_1, P_2)
\]
Building Amplitude

- Off-shell fields – building blocks of any process
- Used iteratively, at each step two (three) momenta are combined
- Initial conditions for the external particles:

\[
\begin{align*}
    b_\mu(p_i) &= \epsilon_\lambda(p_i), \; \lambda = \pm 1, 0 \\
    \psi(p_i) &= \begin{cases} 
    u_\lambda(p_i) & \text{if } p_i^0 \geq 0 \\
    v_\lambda(-p_i) & \text{if } p_i^0 \leq 0 
    \end{cases} \\
    \bar{\psi}(p_i) &= \begin{cases} 
    \bar{u}_\lambda(p_i) & \text{if } p_i^0 \geq 0 \\
    \bar{v}_\lambda(-p_i) & \text{if } p_i^0 \leq 0 
    \end{cases}
\end{align*}
\]

- Amplitude can be calculated by any of the following relations:

\[
\mathcal{A}(p_1, \ldots, p_n) = \begin{cases} 
    b_0^\mu(P_i)b_\mu(p_i) \\
    \bar{\psi}_0(P_i)\psi(p_i) \\
    \bar{\psi}(p_i)\psi_0(P_i)
\end{cases}
\]
DS equations both for: full and colour ordered amplitudes

- Colour ordered - ordinary approach SU(N) algebra
- Quarks and gluons treated differently

\[
M(\{p_i\}_1^n; \{\varepsilon_i\}_1^n; \{a_i\}_1^n) = \sum_{I \in P(2,\ldots,n)} \text{Tr}(t^{a_{\sigma_i(1)}} t^{a_{\sigma_i(2)}} \cdots t^{a_{\sigma_i(n)}}) A_I(\{p_i\}_1^n; \{\varepsilon_i\}_1^n)
\]

\[
\sum_{\{\varepsilon_i\}_1^n; \{a_i\}_1^n} |M(\{p_i\}_1^n; \{\varepsilon_i\}_1^n; \{a_i\}_1^n)|^2 = \sum_{I} \sum_{J} A_I C_{IJ} A_J^*
\]

\[
C_{IJ} = \sum_{IJ} \text{Tr}(t^{a_1} t^{a_{\sigma_i(2)}} \cdots t^{a_{\sigma_i(n)}}) \text{Tr}(t^{a_1} t^{a_{\sigma_i(2)}} \cdots t^{a_{\sigma_i(n)}})^*
\]
Colour Representation

- Colour ordered - U(N) type colour algebra, gluon = qq pair
- Each colour amplitude proportional to $D_I$
- $\sigma_I$ - I-th permutation of the set 1,2,...,n

$$M\left(\{p_i\}_1^n;\{\epsilon_i\}_1^n;\{a_i\}_1^n\right) = \sum_{I \in P(2,\ldots,n)} D_I A_I(\{p_i\}_1^n;\{\epsilon_i\}_1^n)$$

$$D_I = \delta_{1\sigma_I(1)}\delta_{2\sigma_I(2)}\ldots\delta_{n\sigma_I(n)}$$

$$C_{IJ} = \sum_{IJ} D_ID_J = N_C^{\alpha}, \quad \alpha = \langle \sigma_1, \sigma_2 \rangle$$

- Exact colour treatment, efficient for low colour charge
Simplification for gluon fields

\[ G_{AB} \equiv \sum_{a=1}^{8} t_{AB}^a G^a \quad A, B = 1, 2, 3 \]

New objects traceless 3x3 matrices in colour space

Diagonalization of the colour structure of 3-gluon vertex

\[ f^{abc} t_{AB}^a t_{CD}^b t_{EF}^c = - \frac{i}{4} (\delta_{AD} \delta_{CF} \delta_{EB} - \delta_{AF} \delta_{CB} \delta_{ED}) \]
3-gluon vertex in a new representation

- Shows the colour flow in the real physical process
- Gluon represented by $q\bar{q}$ states in colour space
- Colour remains unchanged on an interrupted colour line
Quarks and antiquarks already in this representation
Additionally representation independent identity is used

\[ \sum_{a=1}^{8} t_{ij}^a t_{kl}^a = \frac{1}{2}(\delta_{il} \delta_{kj} - \frac{1}{3} \delta_{ij} \delta_{kl}) \quad i, j, k, l = 1, 2, 3 \]

Recursion equations modified to reflect the new colour structure

Next step - make the computation of the colour part of an amplitude more efficient!
Leading Colour Approximation

In the limit $N_c \rightarrow \infty$ only diagonal terms survive in colour matrix

Interference between different colour flows vanish in this limit

\[ \sum_{a, \varepsilon} \left| \mathcal{M}(\{p_i\}^n, \{\varepsilon_i\}^n, \{a_i\}^n) \right|^2 = g^{2n-4} N_c^{-2} N_c^2 - 1 \sum_{\varepsilon} \sum_{I} |A_I|^2. \]
Incoherent sum over colour performed via MC over “real” colour (completed, available in the next version)

All possible configurations $N_{CC}^{ALL} = N_{C}^{q\bar{q}}$

MC - one particular colour-anticolour configuration randomly selected

Number of colours and anticolours of each type the same

$|M|^2$ multiplied by the number of non zero colour configurations $N_{CC}$

<table>
<thead>
<tr>
<th>Process</th>
<th>$N_{CC}^{ALL}$</th>
<th>$N_{CC}$</th>
<th>$N_{CC}/N_{CC}^{ALL}$</th>
<th>$N_{CC}^e$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow uu$</td>
<td>729</td>
<td>93</td>
<td>0.1276</td>
<td>93.5</td>
</tr>
<tr>
<td>$gg \rightarrow gu\bar{u}$</td>
<td>6561</td>
<td>639</td>
<td>0.0974</td>
<td>91.6</td>
</tr>
<tr>
<td>$gg \rightarrow 2gu\bar{u}$</td>
<td>59049</td>
<td>4653</td>
<td>0.0788</td>
<td>92.6</td>
</tr>
<tr>
<td>$gg \rightarrow 3gu\bar{u}$</td>
<td>531441</td>
<td>35169</td>
<td>0.0662</td>
<td>94.6</td>
</tr>
<tr>
<td>$gg \rightarrow 4gu\bar{u}$</td>
<td>4782969</td>
<td>272835</td>
<td>0.0570</td>
<td>96.4</td>
</tr>
<tr>
<td>$gg \rightarrow 5gu\bar{u}$</td>
<td>43046721</td>
<td>2157759</td>
<td>0.0501</td>
<td>97.8</td>
</tr>
<tr>
<td>$gg \rightarrow 6gu\bar{u}$</td>
<td>387420489</td>
<td>17319837</td>
<td>0.0447</td>
<td>98.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>$N_{CC}^{ALL}$</th>
<th>$N_{CC}$</th>
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<th>$N_{CC}^e$ (%)</th>
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<td>0.0974</td>
<td>59.1</td>
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<td>$gg \rightarrow 3g$</td>
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<td>4653</td>
<td>0.0788</td>
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<td>3486784401</td>
<td>140668065</td>
<td>0.0403</td>
<td>96.4</td>
</tr>
</tbody>
</table>

### MC over Colour

- Total cross sections for processes with gluons

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma_{\text{EXACT}} \pm \varepsilon$ (nb)</th>
<th>$\varepsilon$ (%)</th>
<th>$\sigma_{\text{MC}} \pm \varepsilon$ (nb)</th>
<th>$\varepsilon$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow 2g$</td>
<td>$(0.46572 \pm 0.00258) \times 10^4$</td>
<td>0.5</td>
<td>$(0.46849 \pm 0.00308) \times 10^4$</td>
<td>0.6</td>
</tr>
<tr>
<td>$gg \rightarrow 3g$</td>
<td>$(0.15040 \pm 0.00159) \times 10^3$</td>
<td>1.0</td>
<td>$(0.15127 \pm 0.00110) \times 10^3$</td>
<td>0.7</td>
</tr>
<tr>
<td>$gg \rightarrow 4g$</td>
<td>$(0.11873 \pm 0.00224) \times 10^2$</td>
<td>1.9</td>
<td>$(0.12116 \pm 0.00134) \times 10^2$</td>
<td>1.1</td>
</tr>
<tr>
<td>$gg \rightarrow 5g$</td>
<td>$(0.10082 \pm 0.00198) \times 10^1$</td>
<td>1.9</td>
<td>$(0.09719 \pm 0.00142) \times 10^1$</td>
<td>1.5</td>
</tr>
<tr>
<td>$gg \rightarrow 6g$</td>
<td>$(0.74717 \pm 0.01490) \times 10^{-1}$</td>
<td>2.0</td>
<td>$(0.76652 \pm 0.01862) \times 10^{-1}$</td>
<td>2.4</td>
</tr>
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</table>

- Comparison of the computational time

<table>
<thead>
<tr>
<th>Process</th>
<th>$t_{\text{EXACT}}^\text{CF}$</th>
<th>$t_{\text{MC}}$</th>
<th>$t_{\text{EXACT}}/t_{\text{MC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow 2g$</td>
<td>$0.315 \times 10^0$</td>
<td>$0.554 \times 10^0$</td>
<td>0.57</td>
</tr>
<tr>
<td>$gg \rightarrow 3g$</td>
<td>$0.329 \times 10^1$</td>
<td>$0.143 \times 10^1$</td>
<td>2.30</td>
</tr>
<tr>
<td>$gg \rightarrow 4g$</td>
<td>$0.383 \times 10^2$</td>
<td>$0.372 \times 10^1$</td>
<td>10.29</td>
</tr>
<tr>
<td>$gg \rightarrow 5g$</td>
<td>$0.517 \times 10^3$</td>
<td>$0.105 \times 10^2$</td>
<td>49.24</td>
</tr>
<tr>
<td>$gg \rightarrow 6g$</td>
<td>$0.987 \times 10^4$</td>
<td>$0.362 \times 10^2$</td>
<td>272.65</td>
</tr>
</tbody>
</table>

Helicity Treatment

- Summation over helicity configurations of the external partons
  \[ \rightarrow \text{MC integration over a phase variable} \]

- Polarization vector for gluons:
  \[ \epsilon^\mu_\phi(p) = e^{i\phi} \epsilon^\mu(p,+) + e^{-i\phi} \epsilon^\mu(p,-) \]

- For incoming quarks e.g.:
  \[ u_\phi(p) = e^{i\phi} u_+(p) + e^{-i\phi} u_-(p) \]

- \( \phi \) - random number \( \phi \in (0, 2\pi) \)

- By integrating over phase we can get the correct sum over polarisations
Monte Carlo Integration

- Use random numbers for integration $\bar{x}_i$
- Evaluate, estimate of the integral $I(f)$

$$\langle I(f) \rangle = \frac{1}{N} \sum_{i=1}^{N} f(\bar{x}_i)$$

- Quality of estimate given by error estimator – Variance

$$\langle E(f) \rangle^2 = \frac{1}{N-1} [\langle I^2(f) \rangle - \langle I(f) \rangle^2]$$

- Minimize $E(f)$
- Problem: Large fluctuation in integrand $f$
- Solution: Smart sampling methods!
Importance Sampling

- MC carried out using sets of random points picked from arbitrary distribution
- Uniform distributions give very poor estimates of high-dimensional integrals
- Sampling points from a given probability function!
- Choosing points from a distribution $g(x)$ which concentrates the points where the function $f(x)$ being integrated is large improve convergence behaviour
- Function $g(x)$ is chosen to be a reasonable approximation to $f(x)$
- When $f(x)/g(x)$ is ~ constant $E(f/g)$ is small
Stratified Sampling

- Decompose integral in $M$ sub-integrals
  \[ \langle I(f) \rangle = \sum_{j=1}^{M} \langle I_j(f) \rangle \]
  \[ \langle E(f) \rangle^2 = \sum_{j=1}^{M} \langle E_j(f) \rangle^2 \]

- Overall variance smallest if equally distributed!
- Sample where the fluctuations are
- Divide interval in bins
- Adjust bin-size or weight per bin such that variance identical in all bins
Multichannel Sampling

- Use sum of functions
  \[ g(\vec{x}) = \sum_{j=1}^{N} \alpha_j g_j(\vec{x}) \]
- Conditions on weights like stratified sampling
- Combination of importance sampling and stratified sampling
- Select \( g_j(\vec{x}) \) with probability \( \alpha_j \rightarrow \vec{x}_j \)
- Calculate total \( g(\vec{x}_j) \) and partial \( g_j(\vec{x}_j) \) weight

- Add \( f(\vec{x}_j)/g(\vec{x}_j) \) to total result and \( f(\vec{x}_j)/g_j(\vec{x}_j) \) to partial (channel-) results
- After N sampling steps update apriori weights

Method for parton level event generators!
Translate Feynman diagrams into channels!
s- and t- channel propagators as building blocks
Take over-estimator \( g(x) \)

\[ g(x) > f(x) \quad \forall x \in [x_{\min}, x_{\max}] \]

- Select \( x \) according to \( g \)
- Accept or reject with \( f(x)/g(x) \)
- Obvious guaranteed \( g(x) \):

\[ g(x) = \text{Max}\{f(x)\} \]

- Compare actual \( f(\bar{x}) \) with maximal value during sampling \( \rightarrow \) unweighted events
Performance

- 87 (3) graphs gg, 40 (1) graph qq
- Higgs contribution is included
- 9 subprocesses
- $\alpha_S^2 \alpha_{EW}^4$

**Time:** few % ~ 10 (4) min.

- 558 (16) graphs gg
- 246 (5) qq, gq, qg
- Higgs contributions is included
- 25 subprocesses
- $\alpha_S^3 \alpha_{EW}^4$

**Time:** few % ~ 106 (34) min.

**Intel Pentium 1.7 GHz**

**Intel Fortran**
How to simulate hard processes with additional hard radiation

**Matrix element:**
- Exact at some given order in $\alpha_s$ - all interferences are included
- High energetic and well separated partons
- Soft/Collinear regions are not adequately described – luck of multiple unresolved gluon emission
- Difficult to match to hadronisation models

**Parton Showers:**
- Include logarithmically enhanced soft and collinear contributions of parton emissions
- Able to connect both hard and fragmentation scales
- Not enough high energetic gluons are emitted that have large angle from the shower initiator

Clearly two descriptions complement each other!
Combining ME & PS

**Goal**
- All jet emissions correct at tree level + LL
- Soft emission correctly resumed in PS

**Problem**
- Double Counting - jet can appear both from relatively hard emission during shower evolution and from inclusion of higher order ME

**Solution**
- Matching algorithm

**Recipe**
- Separate jet production/evolution by $Q_{jet} - k_T$ algorithm
- Produce jets according to LO MEs
- Reweight with Sudakov form factor + running $\alpha_s$ weight
- Veto jet production in parton shower
- Process Independent Implementation!
Combining ME & PS

A few algorithms along these lines:

- **CKKW-L** – for e+e- dependence on the resolution parameter is shifted beyond NLL accuracy, proposal to extend procedure to hadronic collisions but no proof of NLL accuracy
- **MLM** - alternative proposal, LL accuracy

**Differ mainly:**
- Jet definition used for the ME evaluation
- Way the ME rejection weights are constructed
- Details concerning starting conditions of jet vetoing inside PS

**Have similar systematics:**
- Residual dependence on the phase space separation cut $Q_{\text{jet}}$
- Variations with the number of ME legs
- Dependencies on the internal jet algorithm

L. Lonnblad JHEP 0205 (2002) 046
F. Krauss JHEP 0208 (2002) 015
Inclusive $E_T$ spectra of leading 4 jets

- **AlpGen** – angular-ordered PS in HERWIG with MLM matching
- **Ariadne** – matrix elements MadGraph, $p_T$ ordered dipole PS with CKKW-L, PYTHIA
- **HELAC** – mass-ordered PS in PYTHIA with MLM matching
- **MadEvent** – mass-ordered PS in PYTHIA with MLM matching
- **SHERPA** – mass-ordered PS with CKKW matching, PYTHIA

di spectra – scale in parton level event where i jets are clustered into i-1 jets using $k_T$ algorithm

ΔR separations

Curves normalized to unit area
Systematics @ LHC

HELAC-PHEGAS

Combining Tools

- $qq \rightarrow W^+ W^- qq$, $qq \rightarrow W^+ Z qq$
- **VBFNLO** - Warped Higgsless Kaluza-Klein model of narrow spin 1 resonances
- **HELAC-PHEGAS** - Most prominent background processes
- Full off-shell and finite width effects for final states with two tagging jets and four leptons
- Double forward jet-tagging techniques
- Dedicated cuts on the observable jets and charged leptons
- Substantial sensitivity to strong interactions in EWSB sector

How to use Helac-Phegas

- Edit once for all the file **myenv**

  - **FC** = Fortran compiler
  - **FORTRAN_LIBRARY** = path to your Fortran libraries
  - **LHAPDFLIB** = path to Les Houches Accord PDF's libraries
  - **LHAPDFSETS** = path to PDF sets

- User interface

  - **run.sh** – shell script that reads input files compiles and runs HELAC
  - **default.inp** – default values (can not be modified)
  - **user.inp** – select process, collider, energy, modify default values
  - **getqcdscale.h** – define QCD scale to be used by PDF and $\alpha_s$

  . run.sh user.inp
  ./run.sh user.inp myenv-xxx
Main Files:

- main_mc.f → main file
- intpar.f → integer arithmetic
- master_new.f → master file for DS solution
- pan1.f → non-dressed vertices and amplitude calculation
- pan2.f → dressed vertices
- physics.f → all couplings

Main common blocks:

- common_int.h → common/helac_int/n,io(20),ifl(20)
- common_helc.h → common/helac_helc/ipol(20),icol(20,2)
- common_phegas_mom.h → common/phegas_mom/pmom(20,5)
From LO to NLO

If one is content with Born level diagrams - possible to go to quite high orders
8-10 partons in the final state
Well separated to avoid phase space regions where divergences become troublesome

To improve accuracy of prediction higher order calculations are needed
Benefits of higher order calculations are well known

- Less sensitivity to unphysical input scales
- First predictive normalisation of observables at NLO
- Confidence that cross sections are under control

There are many programs currently available for predictions

- Author-controlled non-public
- Single process or class of processes
- more generic programs MC@NLO, MCFM, NLOJETS++, VBFNLO,...
**HELAC–PHEGAS:** Framework for high energy phenomenology
- Standard Model fully included - tested
- Ready to be used for LHC, TeVatron, ILC
- High colour charge processes (MC summation over color)
- Multijet production - to be available very soon

**HELAC NEW FEATURES:**
- MSSM - development phase (so far Higgs sector only)
- Effective Hγγ and Hgg (tested phase)
- Anomalous couplings, new resonances, ...

**HELAC GENERAL PLANS:**
- Contribute to ATLAS and CMS generator groups in all stages (interfacing, validation, tuning, installation, configuration, user help, physical analysis...)
- Make HELAC-PHEGAS an option for the LHC!
- Broad range of fully automatic LO Monte Carlo programs
- 8-10 partons in the final state
- LO + LL description for all jet emissions

- Performing NLO calculations on a case-by-case basis is not a way for the future
- Automated approaches combining algebraic and numerical recipes appears both promising and feasible!

First results have already been presented

- **W+3jet**  

- **VVV**  

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**Summary & Outlook**

Lecture by C. Papadopoulos