Real Parton Emission and Automated Dipole Subtraction

K. Hasegawa
(Kouhei.Hasegawa@desy.de)
DESY Zeuthen

Collaboration with
S. Moch (DESY Zeuthen)
P. Uwer (Humboldt-Universität zu Berlin)

(Full paper will appear soon)

Computer Algebra and Particle Physics 2009, DESY Zeuthen, 1 April 2009
Contents

Part 1 : Theory and Package (45 min)

1. Introduction
2. Dipole Subtraction
3. Automatization
4. Outlook

Part 2 : Exercise (45 min)

1. Use of AutoDipole
2. Exercise 1: $e^- e^+ \rightarrow u\bar{u}g$
3. Exercise 2: $gg \rightarrow t\bar{t}g$
4. Outlook
Part 1 : Theory and Package
1. Introduction

- Large Hadron Collider (LHC) at CERN
  - Energy Frontier: \( \sqrt{S} \simeq 14 \text{TeV} \)
    - Direct production of Higgs and new particles beyond the Standard Model
  - Proton-Proton collision: \( pp \rightarrow X \)
    - Events are triggered by the QCD interaction
  - We need estimate the Standard Model predictions to identify New Physics
    \[
    \text{(New Physics)} = \text{(LHC signals)} - \text{(the SM predictions)}
    \]
  - The rate of QCD processes with high momentum transfer can be predicted by the perturbative expansion in the small strong coupling constant
    For example, \( \alpha_s(m_t) \simeq 0.1 \)
**Perturbative QCD**

- Master Formula: Factorization of the hard scattering process

\[
\sigma_{pp \rightarrow X} = \sum_{i,j,\{k\}} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij \rightarrow \{k\}}(\alpha_s, Q) \otimes D_{\{k\} \rightarrow X}
\]

Parton distribution function (Non-perturbative)

Subprocess partonic cross section (Perturbative)

Jet algorithm

Parton shower

Hadronization model

- Perturbative expansion of the partonic cross section

\[
\hat{\sigma}_{ij \rightarrow \{k\}} = \sigma_{LO}(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \cdots)
\]

Leading order (LO)

Next-to-leading order (NLO)

Next-to-next-to-leading order (NNLO)
Leading order (LO)
- LO(Tree level) is well automated

Typical ones: Alpgen, CompHep, FeynArts, GRACE, HELAC/PHEGAS, MadGraph, ... 

Next-to-leading order (NLO)
- LO has a large uncertainty from the renomalization/factorization scale dependences
- NLO is not yet fully automatized
- Process with multi-parton legs are difficult
- LHC priority NLO wish list in Les Houches 2005 (hep-ph/0604120)

<table>
<thead>
<tr>
<th>process ((V \in {Z, W, \gamma}))</th>
<th>relevant for</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (pp \to VV) jet</td>
<td>(t\bar{t}H), new physics</td>
</tr>
<tr>
<td>2. (pp \to t\bar{t}b\bar{b})</td>
<td>(t\bar{t}H)</td>
</tr>
<tr>
<td>3. (pp \to t\bar{t} + 2) jets</td>
<td>(t\bar{t}H)</td>
</tr>
<tr>
<td>4. (pp \to VV)</td>
<td>VBF (\to H \to VV), (t\bar{t}H), new physics</td>
</tr>
<tr>
<td>5. (pp \to VV + 2) jets</td>
<td>VBF (\to H \to VV)</td>
</tr>
<tr>
<td>6. (pp \to V + 3) jets</td>
<td>various new physics signatures</td>
</tr>
<tr>
<td>7. (pp \to VVV)</td>
<td>SUSY trilepton</td>
</tr>
</tbody>
</table>

-These predictions are urgently needed for the successful operation of LHC
- The computation of these radiative corrections is now a very active field
QCD at NLO: \[ \sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virtual}} \]

- Why is the NLO of Multi-parton legs process so difficult?

**Real correction**

- One additional gluon to LO
  Because it can not be resolved to LO event in some phase space region

- Soft and collinear singularities

\[
\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p (1 - \beta \cos \theta_{kp})} \rightarrow \infty
\]

\[ E_k \rightarrow 0 \]

: Soft divergence
  (IR divergence)

\[ m_q = 0 \text{ and } \theta_{kq} \rightarrow 0 \]

: Collinear divergence
  (mass divergence)

Detector can not be aware of it
Detector can not resolve it to quark

- Phase space integral of those singularities

\[
\int \frac{d^3k}{k \cdot k^2} \simeq \int \frac{dk}{\mu_k} \simeq \log \mu_k + \cdots \quad \leftrightarrow \quad \int \frac{d^{D-1}k}{k \cdot k^2} \simeq \int_0 \frac{dk}{k^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots
\]

Soft region:

Collinear region:

\[
\int_{-1}^1 \frac{d \cos \theta}{1 - \cos \theta} \simeq \int \frac{d\theta}{\theta} \simeq \log \mu_\theta + \cdots \quad \leftrightarrow \quad \int_0 \frac{d\theta}{\theta^{1+\epsilon}} \simeq -\frac{1}{\epsilon} + \cdots
\]

Detector can not resolve it to quark
Virtual correction

- One loop diagram
  Because the intermediate state can not be observed

- Include the ultraviolet (UV) and soft/collinear divergences

\[
\int \frac{d^4 l}{(2\pi)^4} \frac{(l^2)^m}{(l^2 - \Delta)^n} = \frac{i(-1)^{n+m}}{16\pi^2} \left(\frac{1}{\Delta}\right)^{n-m-2} \frac{\Gamma(m+2)\Gamma(n-m-2)}{\Gamma(n)} \rightarrow \infty \quad (n-m-2 \leq 0)
\]

Integral over shifted loop momenta

\[
\int dx_1 dx_2 \cdots \left(\frac{1}{\Delta(x_1, x_2, \cdots)}\right)^{n-m-2} \rightarrow \infty
\]

Integral over Feynmann parameters

- First difficulty of NLO with multi-parton leg:
  Evaluation of 1 loop diagram with 5 legs (Pentagon), 6 legs (Hexagon), and more

Still active field 30 years after the pioneer work of Passarino-Veltman
QCD at NLO: Cancellation of soft/collinear singularities

\[ \sigma_{NLO} = \sigma_{\text{real}} + \sigma_{\text{virtual}} \]

Soft/Collinear \hspace{2cm} Soft/Collinear

\[ \downarrow \quad + \quad \downarrow \]

Finite in observables

:IR safe (more precisely, soft/collinear safe)

Kinoshita-Lee-Nauenberg theorem
The simplest example: $e^- e^+ \rightarrow u\bar{u}$

- Typical procedure: Dimensional regularization

Calculate all quantity (phase space and matrix element) in dimension: $D = 4 - 2\epsilon$

$$\sigma_{NLO} = \sigma_{\text{real}} + \sigma_{\text{virtual}}$$

$$= \int d\Phi_3 |M(e^- e^+ \rightarrow u\bar{u}g)|^2 \bigg|_{D-d\text{im}} + \int d\Phi_2 |M(e^- e^+ \rightarrow u\bar{u})|^2 \bigg|_{1\text{-loop}} \bigg|_{D-d\text{im}}$$

Phase space

$$\int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{1}{[(1-x_1)(1-x_2)(1-x_3)]^\epsilon}$$

$$|M|^2 = \sigma^{(\epsilon)}_{LO} \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi}{q^2}\right) \frac{4 \Gamma(1-\epsilon)^2}{\Gamma(3-3\epsilon)} \left[\left(\frac{1}{\epsilon^2} - \frac{3}{\epsilon} + \frac{5}{2} + O(\epsilon)\right) + \left(-\frac{1}{\epsilon^2} + \frac{3}{\epsilon} - \frac{7}{4} + O(\epsilon)\right)\right]$$

$$= \sigma_{LO} \cdot \frac{\alpha_s}{\pi} \quad \text{:Finite results}$$

- This method is not practical for the multi-parton leg processes
  - The complexity and the long expression
  - The phase space integral of n and (n+1) particles in D-dimension
Dipole subtraction

- A general and practical procedure to treat soft/collinear divergences at QCD NLO

1. Construct the counter terms which cancel all soft/collinear divergences

2. Subtract it from $\sigma_{\text{real}}$ and add it to $\sigma_{\text{virtual}}$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virtual}}$$

$$= \left(\sigma_{\text{real}} - \sigma_a\right) + \left(\sigma_{\text{virtual}} + \sigma_a\right)$$

$$= \int d\Phi_{m+1} \left[ |M_{\text{real}}|^2 - \sum_i D_i \right]_{D=4} + \int d\Phi_m \left[ |M_{1\text{-loop}}|^2 + \int d\Phi_1 \sum_i D_i \right]_{D=4}$$

- Real correction does not need any regularization
  Calculation (phase space and matrix element) is in 4-dimension

- Dipole term is systematically constructed based on
  the factorization of soft/collinear singuralities
  reduction to Born level

- Integration of dipole term is analytically done once for all

$$D_i \simeq \frac{1}{s_i} V_i \cdot |M_i|_{\text{Born}}$$
Multi-parton leg processes

- Dipole subtraction makes it possible
- It requires many dipole terms and repeats the same kinds of calculation at huge times (Order 50 dipoles)
- The algorithm is a combinatorial way

The automatization is required and it is possible

Our aim

1. Automatize the dipole subtraction
2. Apply it to the QCD backgrounds and the relevant signals in LHC

-There is recent work in the same direction

- M.H. Seymour and C. Tevlin, arXiv0803.2231
- R. Frederix and T. Gehrmann and N. Greiner, JHEP0809:122, arXiv0808.2128
We present today our package of an automated dipole subtraction:

AutoDipole Version 1.0beta

- This version includes the subtracted real emission part: $|M|_{\text{real}}^2 - \sum_i D_i$

$\Rightarrow$ In this talk we treat with only tree level diagrams
2. Dipole Subtraction

- Soft limit

\[ M_{m+1}(p_i, p_j) = \epsilon^{a*}_{\mu} \bar{u}(p_i) i g_s t^a \gamma^{\mu} \frac{i(p_i + p_j)}{(p_i + p_j)^2} M_m(p_i + p_j) \]

Eikonal approximation

\[ |p_j| \ll |p_i| \]

\[ \simeq \epsilon^{a*}_{\mu} g_s \frac{p_i^\mu}{p_i \cdot p_j} \bar{u}(p_i) \alpha(t^a)_{\alpha\beta} M_m(p_i)_{\beta} \]

- No spin correlation
- Color correlation

\[ p_j = \lambda q_j \quad \lambda \to 0 \]

\[ < 1, \cdots, i, \cdots, j, \cdots, m + 1|1, \cdots, i, \cdots, j, \cdots m + 1 >_{m+1} \]

\[ \longrightarrow - \frac{1}{\lambda^2} 4\pi \alpha_s < 1, \cdots, i, \cdots, m + 1| [J^\mu]^\dagger J_\mu |1, \cdots, i, \cdots m + 1 >_m \]

Eikonal current: \( J^\mu = \sum_i T^\mu_i \frac{p_i^\mu}{p_i \cdot q_j} \)

Color operator

\[ T_i|i, u_\alpha >= (t^{a_i})_{\alpha\beta} |i, u_\beta > \]

\[ T_i|i, \bar{u}_\alpha >= - (t^{a_i})_{\beta\alpha} |i, \bar{u}_\beta > \]

\[ T_i|i, g_a >= i f_{a c b} |i, g_b > \]

\[ \longrightarrow - \frac{1}{\lambda^2} 8\pi \alpha_s \sum_i \frac{1}{p_i \cdot q_j} \sum_{k(\neq i)} < 1, \cdots, i, \cdots, m + 1| \frac{p_i \cdot p_k}{(p_i + p_k) \cdot q_j} T_i.T_k |1, \cdots, i, \cdots m + 1 >_m \]

\[ \equiv S_{(j)} \equiv \sum_{i, k(\neq j)} S_{i(j), k} \]

Reduced Born
Collinear limit

- Factorization of the amplitude in the collinear limit is an universal way

\[ p_i^\mu = zp_i^\mu + k_{\perp}^\mu - \frac{k_{\perp}^2}{z} \frac{n^\mu}{2p \cdot n} \]
\[ p_j^\mu = (1-z)p_j^\mu - k_{\perp}^\mu - \frac{k_{\perp}^2}{1-z} \frac{n^\mu}{2p \cdot n} \]
\[ 2p_i \cdot p_j = -\frac{k_{\perp}^2}{z(1-z)} \]

\[ k_{\perp} \to 0 \]

\[ M_{m+1}(p_i, p_j) \to g \frac{1}{\sqrt{p_i \cdot p_j}} f(z) \bar{u}(p_i + p_j) t_{\alpha \mu}^a M_m(p_i + p_j) \beta \]

- Self-square of the matrix element has the leading singularity

\[ < 1, \cdots, i, \cdots, j, \cdots, m + 1 | 1, \cdots, i, \cdots, j, \cdots m + 1 >_{m+1} \]

\[ \to \frac{1}{p_i \cdot p_j} 4\pi \alpha_s < 1, \cdots, i, j, \cdots, m + 1 | \hat{P}_{(ij), i}(z, k_{\perp}) | 1, \cdots, i, j, \cdots m + 1 >_m \equiv C_{ij} \]

- Altarelli-Parisi splitting function: \( \hat{P}_{(ij), i}(z, k_{\perp}) \)

Square of the splitting amplitudes

\[ < s|\hat{P}_{qq}(z, k_{\perp})|s' > = \delta_{ss'} C_F \left[ \frac{1 + z^2}{1 - z} \right] \]
\[ < \mu|\hat{P}_{gg}(z, k_{\perp})|\nu > = 2C_A \left[ -g_{\mu \nu} \left( \frac{z}{1 - z} + \frac{1 - z}{z} \right) - 2z(1-z) \frac{k_{\perp}^\mu k_{\perp}^\nu}{k_{\perp}^2} \right] \]
\[ \sum_{\lambda=L,R} \epsilon_\mu^{(\lambda)} \epsilon_\nu^{(\lambda)*} \]

- Gluon spin correlation
- No color correlation
**Construction of dipole terms**

1. Choose emitter pair

   - Initial parton = a, b
   - Final parton = i, j, k
   - Choose all possible leg-pair which matches one of the seven patterns
   - (a, i) or (i, j)

   ![Emitter diagram]

2. Choose spectator

   - Choose a different leg from emitter pair
   - Spectator: \( k \neq i, j \quad b \neq a \)

<table>
<thead>
<tr>
<th>emitter pair</th>
<th>spectator</th>
<th>( k )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (i, j) )</td>
<td>( D_{ij,k} ) ((k \neq i, j))</td>
<td>( D_{ij} )</td>
<td></td>
</tr>
<tr>
<td>( (a, i) )</td>
<td>( D_{a i,k} ) ((k \neq i))</td>
<td>( D_{a i,b} ) ((b \neq a))</td>
<td></td>
</tr>
</tbody>
</table>

Initial parton = a, b
Final parton = i, j, k
3. Use dipole formulae

\[ D_{ij,k}(p_1, \cdots, p_{m+1}) = -\frac{1}{2p_i \cdot p_j} \langle g g \rightarrow \bar{u} \bar{u} \mid T_{\bar{u}} \cdot T_{ulg} V_{13,2} \mid gg \rightarrow \bar{u} \bar{u} \rangle_2 \]

Example: \( g(a)g(b) \rightarrow u(1)\bar{u}(2)g(3) \)

\[ D_{13,2}(p_1, p_2, p_3, p_a, p_b) = -\frac{1}{2p_1 \cdot p_3} \langle g g \rightarrow \bar{u} \bar{u} \mid T_{\bar{u}} \cdot T_{ulg} V_{13,2} \mid gg \rightarrow \bar{u} \bar{u} \rangle_2 \]

- Dipole splitting function: \( V_{13,2}(z, y) = \delta_{ss'} \frac{8\pi\alpha C_F}{2^{1 - z_i(1 - y_{ij,k}) - (1 + z_i)}} \]

- Color linked Born squared (CLBS): \( \langle g g \rightarrow \bar{u} \bar{u} \mid T_{\bar{u}} \cdot T_{ulg} \mid gg \rightarrow \bar{u} \bar{u} \rangle_2 \)

- Reduced momenta satisfy the energy-momentum conservation and on-shell condition

\[ p_a^{\mu} + p_b^{\mu} = \tilde{p}_{13}^{\mu} + \tilde{p}_2^{\mu} \quad \tilde{p}_{13}^2 = \tilde{p}_2^2 = 0 \]

Make it possible to reduce into the physical born amplitude, which can be calculated by the well automated LO softwares

\[ \tilde{p}_{ij}^{\mu} = p_i^{\mu} + p_j^{\mu} - \frac{y_{ij,k} p_k^{\mu}}{1 - y_{ij,k}} \quad \tilde{p}_k^{\mu} = \frac{1}{1 - y_{ij,k}} p_k^{\mu} \]

\[ z_i = \frac{p_i \cdot p_k}{p_j \cdot p_k + p_i \cdot p_k} \]

\[ y_{ij,k} = \frac{p_i \cdot p_j}{p_i \cdot p_j + p_j \cdot p_k + p_k \cdot p_i} \]
Soft/collinear limits of dipole terms

\[ |M|_{\text{real}}^2 - \sum_i D_i : \text{IR safe (Soft/Collinear safe)} \]

\[ \bigcup V_{ij,k} \rightarrow \frac{1}{\lambda} 16\pi\alpha_s T_{ij}^2 \frac{p_i \cdot p_k}{q_j \cdot (p_i + p_k)} \quad \text{(Soft limit)} \]

\[ \otimes \rightarrow 8\pi\alpha_s P_{ij} \quad \text{(Collinear limit)} \]

\[ < 1, \ldots, ij, \ldots, m + 1 |T_{ij} \cdot T_{k}|1, \ldots, ij, \ldots m + 1 >_m \]

- To reproduce the color factors of collinear limits, the identity is used

\[ \sum_{k=1}^{m+1} < 1, \ldots, ij, \ldots, m + 1 |T_{ij} \cdot T_{k}|1, \ldots, ij, \ldots m + 1 >_m = - < 1, \ldots, ij, \ldots, m + 1 |T_{ij} \cdot T_{ij}|1, \ldots, ij, \ldots m + 1 >_m \]

All soft/collinear singularities are cancelled by the dipole terms
Limiting behavior

- We can predict the limiting behavior

$$|M|_{\text{real}}^2 - \sum D_i = \begin{cases} \frac{1}{k^2} (a_0 + a_1 k + a_2 k^2 + \cdots) - \frac{1}{k^2} a_0 = \frac{1}{k} (a_1 + a_2 k + \cdots) \quad \text{Soft} (k \to 0) \\ \frac{1}{s_{ij}} (b_0 + b_1 \sqrt{s_{ij}} + b_2 s_{ij} + \cdots) - \frac{1}{s_{ij}} b_0 = \frac{1}{\sqrt{s_{ij}}} (b_1 + b_2 \sqrt{s_{ij}} + \cdots) \quad \text{Collinear} (\theta_{ij} \to 0) \end{cases}$$

$$\log |M|_{\text{real}}^2 - \sum D_i$$

$$\log (k/\sqrt{s}) , \log \sqrt{s_{ij}/s}$$
- **Final formula**

- **Initial partons**

  - Gluon emission from initial partons produces the collinear singularity which is not cancelled by the virtual correction → Those singularities should be factorized into PDF

- For the purpose, the collinear subtraction term is introduced

\[
\sigma_c(a, \text{Non-parton} \to \{k\}) = -\frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1 - \epsilon)} \sum_b \int_0^1 dz \left[ -\frac{1}{\epsilon} \left( \frac{4\pi\mu_B^2}{\mu_F^2} \right)^{2\epsilon} P^{ab}(z) \right] \sigma_{\text{Born}}(b, \text{Non-parton} \to \{k\})
\]

- **Define jet observable**

\[
F^{(m)}_J(p_1, \cdots, p_m) : \text{Jet defining function}
\]

**Soft limit:**

\[
F^{(m+1)}_J(p_1, \cdots, p_j = \lambda q_j, \cdots, p_{m+1}) \rightarrow F^{(m)}_J(p_1, \cdots, p_{m+1}) \quad (\lambda \rightarrow 0)
\]

**Collinear limit:**

\[
F^{(m+1)}_J(p_1, \cdots, p_i, \cdots, p_j, \cdots, p_{m+1}) \rightarrow F^{(m)}_J(p_1, \cdots, p, \cdots, p_{m+1})
\]

\[
(p_i \rightarrow zp \text{ and } p_j \rightarrow (1 - z)p)
\]

\[
\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virtual}} + \sigma_c
\]

\[
= \left( \sigma_{\text{real}} - \sigma_a \right) + \left( \sigma_{\text{virtual}} + \sigma_c + \sigma_a \right)
\]

\[
= \int d\Phi_{m+1} \left[ |M|_{\text{real}}^2 F^{(m+1)} - \sum_i D_i F^{(m)}_i \right] + \int d\Phi_m \left[ |M|_{1\text{-loop}}^2 + \langle I(\epsilon) \rangle_m \right] F^{(m)} + \int_0^1 dx \int d\Phi_m \left[ \langle (K(x) + P(x)) \rangle_m \right] F^{(m)}
\]

Our package includes only this real emission part
3. Automatization

- Package: AutoDipole Version 1.0beta

K. Hasegawa, S.Moch, and P.Uwer

- Automated Dipole subtraction

- Mathematica code and an interface with MadGraph

- The version 1.0beta includes the subtracted real emission part: $|M|_{\text{real}}^2 - \sum_i D_i$

- Generate the Fortran routines to calculates the subtracted real emission

- Check all soft/collinear safeties

- The version 1.0beta is now available
  The complete package is publicly available soon
Scheme to calculate $|M|^2 - \sum_i D_i$ in AutoDipole

Input process: $gg \rightarrow t\bar{t}gg$

1. Mathematica code

Input momenta: \[ \{p_1, p_2, p_3, p_4, p_5, p_6\} \]

1. Mathematica code

$D(i) = V(i) \cdot A(i)\dagger CF'(i)A(i)$

(FORTRAN code)

2. Interface

$\text{cmatrix}(\{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4, \tilde{p}_5\})$

$\text{allcolor.data}$

$CF'(i) = \begin{pmatrix} 1 & \cdots & 2 \\ \vdots & & \vdots \\ 3 & \cdots & 4 \end{pmatrix}$

3. Checking of all soft/collinear safeties
1. Mathematica code

Input: $gg \rightarrow u\bar{u}g$ (Process at NLO real correction)

Creation of dipole terms  (Write down all $D_i$ except for CLBS)

Show the contents of all dipoles and all soft/collinear limits

Write Fortran files: dipole.f  reducedm.f and interface for MadGraph

- Creation order

Dipole 1  $B_1 = M_0(gg \rightarrow u\bar{u})$

Dipole 4  $B_4 = M_0(ug \rightarrow ug)$

Dipole 2  $B_2 = M_0(gg \rightarrow gg)$

Dipole 3
Process: $gg \rightarrow u\bar{u}g$

- Contents of dipole terms

Number of dipoles

[Dipole1] = 12

1. [Splitting (1); (i,j)=(g,g)] = 6 (0)
   1. (i,j) = (f,g) = (Dij,k) 2 (0)
   2. (i,j) = (g,a) = (Dij,"a) 4 (0)

2. [Splitting (2); (i,j)=(g,g)] = 0 (0)
   1. (i,j) = (gg,k) = (Dij,k) 0 (0)
   2. (i,j) = (g,a) = (Dij,"a) 0 (0)

3. [Splitting (3); (i,j)=(g,g)] = 0 (0)
   1. (i,j) = (f,g) = (D^"i,k) 0 (0)
   2. (i,j) = (g,a) = (D^"i,"a) 0 (0)

4. [Splitting (4); (i,j)=(g,g)] = 6 (0)
   1. (i,j) = (gg,k) = (D^"i,k) 4 (0)
   2. (i,j) = (g,a) = (D^"i,"a) 2 (0)

[Dipole2] = 3

5. (i,j) = (f,ubar)
   (u-ubar splitting) B2u = 3
   1. u*(i,j) = (ubar,k) = (Dij,k) 1 (0)
   2. u-(i,j) = (ubar,a) = (Dij,"a) 2 (0)

6. (i,j) = (g,ubar)
   (d-bar splitting) B2d = 0
   1. d*(i,j) = (ubar,k) = (Dij,k) 0 (0)
   2. d-(i,j) = (ubar,a) = (Dij,"a) 0 (0)

7. (i,j) = (g,ubar)
   (d-bar splitting) B2b = 0
   1. b*(i,j) = (ubar,k) = (Dij,k) 0 (0)
   2. b-(i,j) = (ubar,a) = (Dij,"a) 0 (0)

8. (i,j) = (t,ubar)
   (t-bar splitting) B2t = 0
   1. t*(i,j) = (ubar,k) = (Dij,k) 0 (0)
   2. t-(i,j) = (ubar,a) = (Dij,"a) 0 (0)

[Dipole3] = 12

[Splittings (7); (i,j)=(g,f) or (g,ubar)]

- All soft/collinear limits

END
2. MadGraph with our interface

- MadGraph

  - An automated general LO in the Standard Model, MSSM, and some others models
  - Write down the Fortran codes to calculate the matrix element squared
  - Numerical evaluation of the helicity amplitude
  - In order to calculate the helicity amplitude, HELAS library is used


- Color decomposition

  \[ M = \sum_a C_a J_a \]
  \[ J_1 = A_1 - A_3 + \cdots : \text{Joint amplitude} \]

- Our interface

  - Normal Born squared
    \[ < 1, \cdots, m \| 1, \cdots, m >_m \]
  - Color linked Born squared
    \[ < 1, \cdots, m \| T_i \cdot T_k \| 1, \cdots, m >_m \]
  - Different helicity Born squared
    \[ < 1, \cdots, (i, \lambda), \cdots, m \| 1, \cdots, (i, \lambda'), \cdots, m >_m \]

3. Check the soft/collinear limits

- Configure all soft/collinear limits

\[ g(1)g(2) \rightarrow u(3)\bar{u}(4)g(5) \]

\[ p_1 \rightarrow p_2 \]

\[ \theta_{45} \rightarrow 0 \] : Collinear limit of Final-Final splitting

\[ \theta_{15} \rightarrow 0 \] : Collinear limit of Initial-Final splitting

\[ p_5 \rightarrow 0 \] : Soft limit

- Check the cancellation of the singularities

- After the soft/collinear safety are confirmed, an user may go to the MonteCarlo phase space integral with PDF and a jet algorithm
- We checked the cancellation of all soft/collinear singularities in the following real emission processes

<table>
<thead>
<tr>
<th></th>
<th>2 → 3</th>
<th>2 → 4</th>
<th>2 → 5</th>
<th>2 → 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Massless</strong></td>
<td><em>e^+e^- → uuq</em></td>
<td><em>e^-u → e^-ug</em></td>
<td><em>gg → uuuq</em></td>
<td><em>gg → uuuq</em></td>
</tr>
<tr>
<td><strong>(Including lepton)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(Parton only)</strong></td>
<td><em>e^-g → e^-uq</em></td>
<td><em>gg → 3g</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(Including W/Z boson)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Massive</strong></td>
<td><em>e^+e^- → ttg</em></td>
<td><em>gg → ttg</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(Including lepton)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(Parton only)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- *uq → dddgg*
- *gg → W^+W^-gg*
- *gg → W^+uuqg**
- *uq → ttbbqg**
Agreements with the independent results

- $gg \rightarrow t\bar{t}gg \quad u\bar{u} \rightarrow t\bar{t}gg \quad ug \rightarrow ttug \quad \bar{u}g \rightarrow t\bar{t}u\bar{u} \quad gg \rightarrow t\bar{t}u\bar{u} \quad u\bar{u} \rightarrow t\bar{t}gg$

- The modes of NLO real emission process to $pp \rightarrow t\bar{t} + 1\text{jet}$

- All dipoles completely agree with the results in
  
  S. Dittmaier, P. Uwer and S. Weinzierl, arXiv:0810.0452 and

- $u\bar{u} \rightarrow W^+W^- gg$

- One mode of NLO real emission process to $pp \rightarrow W^+W^- + 1\text{jet}$

- All 10 dipoles completely agree with the results in
  
4. Outlook

■ Summary

- Dipole subtraction is a general and practical procedure in NLO QCD
- We automated it in the package: AutoDipole (Version 1.0beta)
  - Mathematica code and an interface with MadGraph
- We apply it to some QCD backgrounds in LHC
  - We checked all soft/collinear safeties in many processes
    Complex ones: \( gg \rightarrow ttggg \quad gg \rightarrow t\bar{t}b\bar{b}g \quad gg \rightarrow W^+\bar{u}dg \)
  - We obtained the complete agreement about all dipoles with the independent results in the processes,
    \( gg \rightarrow t\bar{t}gg \quad \bar{u}d \rightarrow W^+W^-gg \)
- The beta version is now available

■ Plan

- The complete package is publicly available soon
- Compute new and complete NLO QCD predictions for important background at LHC
- Automate the creation of the integrated dipole
Part 2 : Exercise

1. Use of AutoDipole
2. Exercise 1: \( e^- e^+ \rightarrow u\bar{u}g \)
3. Exercise 2: \( gg \rightarrow t\bar{t}g \)
4. Outlook
1. Use of AutoDipole

- **Download package and install**
  - Go to directory: `usr1/tmp`
    ```
    cd /usr1/tmp/
    ```
  - Download the package from the CAPP09 Website
    ```
    https://https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573
    ```
  - Decompress it
    ```
    tar xvf AutoDipole_V1.0beta.tar
    ```
  - Install MadGraph
    ```
    cd ./AutoDipole_V1.0beta/madgraphdip/MG_ME_SA_V4.2.6/MadGraphII/
    make
    ```
  - Go back to the home directory
    ```
    cd ../..
    ```
Directory structure

- **AutoDipole**
  - mathematicadip: Mathematica code
  - interface: For MadGraph
  - madgraphdip: MadGraph (Stand Alone)
  - processes: Created processes

Execution

0. **Set up**: 
   ./[mathematicadip]/parameter.m
   ./[madgraphdip]/MadGraph/
   interactions.dat  param_card.dat

1. **Input to Mathematica code and run**: 
   ./[mathematicadip]/ Realprocess[{g, g}, {t, tbar, g}]

2. **Run of MadGraph with interface**: 
   ./[processes]/ ./[createdir].csh

3. **Checkings**: 
   ./[processes]/ make checkIR
2. Exercise 1: $e^- e^+ \rightarrow u\bar{u}g$

0. Set up

- Set up file for Mathematica code:

```mathematica
(* AutoDipole - Package of an automated dipole subtraction by
Kouhei Hasegawa, Sven Moch, and Peter Uwer, 27.03.2009 *)

ep=0;
kap=2/3;
AlphaS=0.1075205492734706;
topmass=174.00;
bottommass=4.7;
skipdipole={2u,2t};
accucut=10^(-5);
```

$D = 4 - 2\epsilon$

A freedom for non-singular part

$\alpha_s$

$m_{top}$

$m_{bottom}$

Splitting which is skipped

Cut to define soft/collinear safeties
- Set up file for MadGraph:

```
/madgraphdip/MG_ME_SA_V4.2.6/Models/sm/interactions.dat
```

- They are in MadGraph

```
Block SMINPUTS      # Standard Model inputs
1         1.32506980E+02   # alpha_em(MZ)(-1) SM MSbar
2         1.16639000E-05   # G_Fermi
3         1.18000000E-01   # alpha_s(MZ) SM MSbar
4         9.11880000E+01   # Z mass (as input parameter)
Block MGYUKA       # Yukawa masses m/v=y/sqrt(2)
5         4.20000000E+00   # mbottom for the Yukawa y_b
4         1.42000000E+00   # mcharm for the Yukawa y_c
6         1.64500000E+02   # mtop for the Yukawa y_t
15        1.77700000E+00   # mtau for the Yukawa y_ta
Block MGCKM       # CKM elements for MadGraph
1   1     9.75000000E-01   # Vud for Cabibbo matrix
Block MASS        # Mass spectrum (kinematic masses)
#       PDG       Mass
5     4.70000000E+00   # bottom pole mass
6     1.74300000E+02   # top pole mass
6     1.74000000E+02   # top mass
15    1.77700000E+00   # tau mass
23    9.11880000E+01   # Z mass
24    8.04190000E+01   # W mass
25    1.20000000E+02   # H mass
```

- The values should be consistent with ones in 

```
./mathematicadip/parameter.m
```

- We use the default ones and do not have to do anything here
1. Input to Mathematica code and run

```
mathematica exedip.nb&  

<< driver.m

Realprocess[{e, ebar}, {u, ubar, g}]
```

- Open file exedip.nb
- Includes package
- Input real emission process and run

```
Exit

In[1]:= << driver.m

In[2]:= Realprocess[{e, ebar}, {u, ubar, g}]

NLO: {{e, pa}, {ebar, pb}} —> {{u, p[1]}, {ubar, p[2]}, {g, p[3]}}
Masses: {0,0} —> {0, 0, 0}
```

Dipole 1

```
M0=B1: {e, ebar} —> {u, ubar}
Reduced momenta: {ptil[1], ptil[2]} —> {ptil[3], ptil[4]}
{Splitting (1): (i,j)=(f,g)}
[1.(ij,k)=(fg,k): Dij,k]
```

--Dip(1)--
1. Input to Mathematica code and run - continued

- At the end of Output: Contents of dipole are shown

```
Number of dipoles
[Dipole1] : 2
B1 : 2

{Splitting (1):(i,j)=(f,g)]: 2 (0)
  [1.(ij,k)=(fg,k): Dij,k] 2 (0)
  [2.(ij,a)=(fg,a): Dij^a] 0 (0)

{Splitting (2):(i,j)=(g,g)]: 0 (0)
  [3.(ij,k)=(gg,k): Dij,k] 0 (0)
  [4.(ij,a)=(gg,a): Dij^a] 0 (0)

{Splitting (3):(a,i)=(f,g)]: 0 (0)
  [5.(ai,k)=(fg,k): D^ai,k] 0 (0)
  [6.(ai,b)=(fg,b): D^ai,b] 0 (0)

{Splitting (4):(a,i)=(g,g)]: 0 (0)
  [7.(ai,k)=(gg,k): D^ai,k] 0 (0)
  [8.(ai,b)=(gg,b): D^ai,b] 0 (0)
```

- Type of dipole

\[ D_{ij,k} = D_{\text{quark gluon, something}} \]

We are in processes/ in final state
1. Input to Mathematica code and run - continued

- At the end of Output: All soft/collinear limits and the corresponding dipoles are also shown

**The collinear and soft limits and the corresponding dipoles**

NLO: \[ \{e, p[1]\}, \{\overline{e}, p[2]\} \rightarrow \{u, p[3]\}, \{\overline{u}, p[4]\}, \{g, p[5]\} \]

---

Collinear pairs   Corresponding dipoles
1. \{3, 5\}       \{1\}
2. \{4, 5\}       \{2\}

---

Soft gluon   Collinear assemble  Corresponding dipoles
1. \{5\}   \{1, 2\}   \{1, 2\}

---

END
2. Run of MadGraph with interface

```
cd ../processes/
./createdir.csh
```

- Directory: Proc_e-e+_uuxg is produced

This includes the closed Fortran routines to calculate

\[ |M|_\text{real}^2 - \sum_i D_i \]

3. Checkings

- Go to directory: Proc_e-e+_uuxg

```
cd Proc_e-e+_uuxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
./check
```

| \(|M|^2\) | \(\sum D_i\) | \(\sum D_i/|M|_\text{real}^2\) | \(\left(\frac{|M|_\text{real}^2 - \sum D_i}{|M|_\text{real}^2}\right)\) | Ratio | Accuracy |
|--------|--------|------------------|------------------|--------|-----------|
| 1      | 0.300494427478150E-05 | 0.306483766095212E-05 | 0.101993161293315E+01 | -0.199316129331459E-01 |
| 2      | 0.824571736656559E-05 | 0.831794218053564E-05 | 0.100875906980063E+01 | -0.875906980063354E-02 |
| 3      | 0.108020837472608E-05 | 0.113477335925199E-05 | 0.105051338778941E+01 | -0.505133877894137E-01 |
| 4      | 0.398997680814314E-06 | 0.456062859950099E-06 | 0.104292384945467E+01 | -0.42923849454697E-01 |
| 5      | 0.781169201607950E-06 | 0.814699986512918E-05 | 0.984266071553875E+00 | 0.157339284461249E-01 |
| 6      | 0.348167959120634E-04 | 0.342689909364597E-04 | 0.105415630860717E+01 | -0.541563086071683E-01 |
| 7      | 0.148082090268567E-05 | 0.156101669648346E-05 | 0.105415630860671E+00 | -0.541563086071683E-01 |
| 8      | 0.459595997449995E-06 | 0.531616229332302E-06 | 0.115670334877131E+01 | -0.156703348771315E+00 |
| 9      | 0.722990538823266E-06 | 0.804546570364715E-06 | 0.111280373277663E+01 | -0.112803732776628E+00 |
| 10     | 0.326611878319585E-04 | 0.329691186754487E-04 | 0.100942803565732E+01 | -0.942803565732195E-02 |
- Check all soft/collinear limits

make checkIR

./checkIR

Output: resIRcheck

Cut condition: $(|M|^2 - \text{SumD})/|M|^2 < 0.100000000000000E-04$

----------Collinear limits----------

$\log_{10}(\text{Root}(S_{ij}/S)) < \text{Cut}(i)$ (i=1,2)

1  -0.244942287339502E+01
2  -0.348223120739498E+01

Maximum value of Cut(i)
-0.244942287339502E+01

Corresponding $S_{ij}/S$
0.126227579153667E-04

------------------------------------

-----------Soft limits--------------

$\log_{10}(2E_{soft}/\sqrt{s}) < \text{Softcut}(i)$ (i=1,1)

1  -0.319070974314666E+01

Maximum value of Softcut(i)
-0.319070974314666E+01

Corresponding $(2E_{soft}/\sqrt{s})^2$
0.415509075188428E-06

------------------------------------

Infrared safeties of all collinear and soft limits are confirmed for $S_{ij}/S > 0.126227579153667E-04$

This cut value is set as a parameter ‘acccut’ at parameter.m

We are in processes/Proc_e-e+_uuxg

Confirmation of all soft/collinear safeties
- Plots on all soft/collinear limits

\[ \text{./plotall} \]

- Soft limit

\[ p_4 \]

\[ p_5 \rightarrow 0 \]

- Collinear limit

\[ p_3 \]

\[ \theta_{35} \rightarrow 0 \]

\[ p_4 \]

\[ p_5 \]

\[ \theta_{45} \rightarrow 0 \] limit is also similar to this case

We can easily confirm the soft/collinear safeties by seeing these plots, especially the steep
Available fields and notation in the present version 1.0beta

Notation for input: Realprocess[\{e, ebar\}, \{u, ubar, g\}]

- Parton
  - Quark
    \((u, \bar{u})\) \((u, \bar{u})\)
    \((d, \bar{d})\) \((d, \bar{d})\)
    \((b, \bar{b})\) \((b, \bar{b})\)
    \((t, \bar{t})\) \((t, \bar{t})\)
  - Gluon \(g\) \(g\)
- Non-Parton
  - Lepton
    \((e^-, e^+)\) \((e^-, e^+)\)
  - Gauge boson
    \(\gamma\) \(\gamma\)
    \((W^+, W^-)\) \((W^+, W^-)\)
    \(Z\) \(Z\)

- It is straightforward to include more fields like other quarks and leptons, Higgs boson, and super partners
- Available interactions are same with ones in MadGraph
3. Exercise 2: \( gg \rightarrow t\bar{t}g \)

- **0. Set up**

  - Same with Exercise 1

  \[ \text{skipdipole} = \{2u, 2t\}; \quad \longrightarrow \quad \text{t-bar splitting is skipped} \]

- **1. Input to Mathematica code and run**

```mathematica
Realprocess[{g, g}, {t, tbar, g}]
```

- Input real emission process

```
In[3]:= Exit

In[1]:= << driver.m

In[2]:= Realprocess[{g, g}, {t, tbar, g}]

I am Dipole

NLO: \( \{\{g, pa\}, \{g, pb\}\} \rightarrow \{\{t, p[1]\}, \{tbar, p[2]\}, \{g, p[3]\}\} \)

Masses: \(0, 0\) \rightarrow \(mt, mt, 0\)

Dipole 1

M0=B1: \(\{g, g\} \rightarrow \{t, tbar\}\)

Reduced momenta: \(\{ptil[1], ptil[2]\} \rightarrow \{ptil[3], ptil[4]\}\)

\(\{\text{Splitting (1)}: (i, j) = (f, g)\}\)

\(1. \{ij, k\} = (fg, k): \text{Dij}, k\)

--Dip(1)--
```
1. Input to Mathematica code and run - continued

- At the end of Output: Contents of dipole are shown

Number of dipoles

[Dipole1] : 12

B1 : 12

{Splitting (1): (i, j)=(f, g)}:  6 (6)

[1. (ij, k) = (fg, k) : Dij, k]  2 (2)  (ij, k) = (quark gluon, something in final state)

[2. (ij, a) = (fg, a) : Dij^a]  4 (4)  (ij, a) = (quark gluon, something in initial state)

{Splitting (2): (i, j)=(g, g)}:  0 (0)

[3. (ij, k) = (gg, k) : Dij, k]  0 (0)

[4. (ij, a) = (gg, a) : Dij^a]  0 (0)

{Splitting (3): (a, i)=(f, g)}:  0 (0)

[5. (ai, k) = (fg, k) : D^ai, k]  0 (0)

[6. (ai, b) = (fg, b) : D^ai, b]  0 (0)

{Splitting (4): (a, i)=(g, g)}:  6 (4)

[7. (ai, k) = (gg, k) : D^ai, k]  4 (4)  (ai, k) = (gluon gluon, something in final state)

[8. (ai, b) = (gg, b) : D^ai, b]  2 (0)  (ai, b) = (gluon gluon, something in initial state)

-------------------------------

<table>
<thead>
<tr>
<th>Total</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massive dipoles</td>
<td>10</td>
</tr>
</tbody>
</table>

END

- Gluon radiation from the initial gluon
1. Input to Mathematica code and run - continued

-All soft/collinear limits and the corresponding dipoles are also shown

The collinear and soft limits and the corresponding dipoles

NLO: \{\{g, p[1]\}, \{g, p[2]\}\} \rightarrow \{\{t, p[3]\}, \{tbar, p[4]\}, \{g, p[5]\}\}

<table>
<thead>
<tr>
<th>Collinear pairs</th>
<th>Corresponding dipoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. {3, 5}</td>
<td>{1, 3, 4}</td>
</tr>
<tr>
<td>2. {4, 5}</td>
<td>{2, 5, 6}</td>
</tr>
<tr>
<td>3. {1, 5}</td>
<td>{7, 8, 11}</td>
</tr>
<tr>
<td>4. {2, 5}</td>
<td>{9, 10, 12}</td>
</tr>
</tbody>
</table>

Collinear pairs

1. 
2. 

Include a massive quark 

Collinear limits 1 and 2 do not include the collinear divergences

Soft gluon Collinear assemble Corresponding dipoles

1. \{5\} \[1, 2, 3, 4\] \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\] 

END
2. Run of MadGraph with interface

```
cd ../processes/
./createdir.csh
```

- Directory : Proc_gg_ttxg is produced

3. Checkings

```
cd Proc_gg_ttxg
```

- Check the values of the sum of all dipolles on the 10 phase space points

```
make
./check
```

<table>
<thead>
<tr>
<th>IMI^2</th>
<th>SumDipole</th>
<th>Ratio</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0.400893569363976E-03</td>
<td>0.405385268735139E-03</td>
<td>0.101120421906066E+01</td>
<td>-0.112042190606586E-01</td>
</tr>
<tr>
<td>2 0.554612468603335E-03</td>
<td>0.687236683409599E-03</td>
<td>0.123912952252993E+01</td>
<td>-0.239129522529935E+00</td>
</tr>
<tr>
<td>3 0.231759860037041E-03</td>
<td>0.308531272164432E-03</td>
<td>0.133125413570374E+01</td>
<td>-0.331254135703743E+00</td>
</tr>
<tr>
<td>4 0.262017095449925E-03</td>
<td>0.457611638297119E-03</td>
<td>0.174649534798990E+01</td>
<td>-0.746495347989895E+00</td>
</tr>
<tr>
<td>5 0.117434085443178E-03</td>
<td>0.161142425242084E-03</td>
<td>0.137219466251181E+01</td>
<td>-0.372194662511809E+00</td>
</tr>
<tr>
<td>6 0.26751495703035E-02</td>
<td>0.2675149586891E-02</td>
<td>0.999962825413785E+00</td>
<td>0.371745862150187E-04</td>
</tr>
<tr>
<td>7 0.92738018137340E-03</td>
<td>0.113228428952350E-02</td>
<td>0.122100492741344E+01</td>
<td>-0.221004927413437E+00</td>
</tr>
<tr>
<td>8 0.277838316144724E-03</td>
<td>0.509438726318734E-03</td>
<td>0.183357980780941E+01</td>
<td>-0.833579807809412E+00</td>
</tr>
<tr>
<td>9 0.353722424746050E-03</td>
<td>0.706243188852582E-03</td>
<td>0.19960281464943E+01</td>
<td>-0.99602814649425E+00</td>
</tr>
<tr>
<td>10 0.875738991423606E-03</td>
<td>0.882235650188843E-03</td>
<td>0.100741848750468E+01</td>
<td>-0.741848750467973E-02</td>
</tr>
</tbody>
</table>
3. Checkings - continued

- Check all soft/collinear limits

```
make checkIR
./checkIR
```

Output: resIRcheck

more resIRcheck

```
Cut condition: (IMI^2-SumD)/IMI^2 < 0.100000000000000E-04

--------Collinear limits--------
Log10(Root(S_\text{ij}/S)) < Cut(i) (i=1,4)
1  -0.100000000000000E+03
2  -0.100000000000000E+03
3  -0.195941167550387E+01
4  -0.261212258826533E+01

Maximum value of Cut(i)
-0.195941167550387E+01

Corresponding S_\text{ij}/S
0.120552618763782E-03

------------------------------------

-----------Soft limits------------
Log10(2*E_{soft}/\text{root(s)}) < Softcut(i) (i=1,1)
1  -0.130917931411337E+01

Maximum value of Softcut(i)
-0.130917931411337E+01

Corresponding (2*E_{soft}/\text{root(s)})^2
0.240791621767016E-02

------------------------------------

Infrared safeties of all collinear and soft limits are not confirmed
```

Confirmation of all soft/collinear safeties
- Plots on all soft/collinear limits

```
./plotall
```

- Soft limit

\[ p_4 \]

\[ p_5 \rightarrow 0 \]

- Collinear limit

\[ p_4 \]

\[ p_5 \]

\[ \theta_{15} \rightarrow 0 \]

\[ +Z \]

- Initial-Final splitting

We can confirm the soft/collinear safeties
4. Outlook

■ Summary

- AutoDipole Version1.0beta: An automated dipole subtraction for $|M|^2_{\text{real}} - \sum_i D_i$
  - Mathematica code and an interface with MadGraph

- Use
  0. Set up
  1. Input to Mathematica code and run
  2. Run of MadGraph with interface
  3. Checking of all soft/collinear safeties

- We had the exercises for the processes: $e^-e^+ \rightarrow u\bar{u}g$ and $gg \rightarrow t\bar{t}g$
  - We got the all dipoles and confirmed all soft/collinear safeties

■ Further works (if you like)

- Try more complex processes like $gg \rightarrow u\bar{u}gg$
- Try the phase space integral and obtain parton and hadron level cross section

■ Future of AutoDipole

- The complete package is publicly available soon
- Compute new and complete NLO QCD predictions for important background at LHC
- Automate the creation of the integrated dipole
Extra Slide
Unification of soft and collinear limits

\[
\left| M \right|^2_{\text{real}} \rightarrow \sum_{i,j,k} S_{i(j),k} + \sum_{i,j} C_{ij}
\]

\[
= \sum_{i,j,k} (S_{i(j),k} + C'_{ij,k}) = \sum_{i,j,k} U_{ij,k}
\]

- The color conservation can split \( C_{ij} \) into \( C'_{ij,k} \)

\[
\rightarrow C_{ij} = \sum_k C'_{ij,k}
\]

- We can construct IR safe matrix element squared as

\[
\left| M \right|^2_{\text{real}} - \sum_{i,j,k} U_{ij,k}
\]
- Example: NLO $gg \rightarrow u\bar{u}g$

$(i, j) = (1, 3)$

- We can rewrite the diagonal color factor as

$$\Delta_{13,1}X = - (\Delta_{13,2} + \Delta_{13,a} + \Delta_{13,b})X$$
Emitter = gluon case

\[ D_{1}^{a3}(p_1, p_2, p_3, p_a, p_b) = -\frac{1}{2p_a \cdot p_3} x_{31,a}^{(\mu)} \langle \tilde{g}g \rightarrow \tilde{u}\bar{u} \rvert T_u \cdot T_{gg} \rvert V_{a3}(\mu, \nu) \rangle_{2} \]

splitting function:

\[ V_{ak}^{ai}(x, u)_{\mu\nu} = 16\pi\alpha_s C_A \left[ -g^{\mu\nu} \left( \frac{1}{1 - x_{ik,a} + u_i} - 1 + x_{ik,a}(1 - x_{ik,a}) \right) + \frac{1 - x_{ik,a}}{x_{ik,a}} \frac{u_i(1 - u_i)}{p_i \cdot p_k} \left( \frac{p_i^{\mu}}{u_i} - \frac{p_k^{\mu}}{1 - u_i} \right) \right] \]

\times

Color linked Born squared (CLBS)

\[ \epsilon_{\mu}^\lambda \left( f_c^C \right) \rightarrow t^e \rightarrow \nu \left( \epsilon_{\nu}^{\lambda'} \right) \]

(Amputate polarization vector)

Different helicity squared (DHS)