Jet Cross Sections and Transverse Momentum Distributions with NNLOJET

Thomas Gehrmann, Universität Zürich

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Benchmark processes at LHC
Benchmark processes: $2 \rightarrow 2$ reactions

- **Large cross sections**
  - Multiple-differential measurements
    - Di-jet production
    - Z$+$jet, W$+$jet
    - H$+$jet

- **Detailed understanding of dynamics**
  - Disentangle production processes and jet definitions
  - Measure fundamental parameters
  - Probe parton distributions

- **Transverse momentum distributions**
  - Continuous transition from hard to soft region
  - Fixed order versus resummation
Z transverse momentum distribution

- Transverse momentum requires partonic recoil

Mismatch of orders in perturbation theory
- NNLO for inclusive Z is only NLO for $p_T$-distribution
- $Z+\text{jet}$ and $Z p_T$ distribution closely related

NLO fails to describe measurements in norm and shape
Ingredients to jet production at NNLO

- **Two-loop matrix elements**
  - Explicit infrared poles from loop integrals

- **One-loop matrix elements**
  - Explicit infrared poles from loop integral
  - Implicit infrared poles from real radiation

- **Tree-level matrix elements**
  - Implicit infrared poles from real radiation

- **Require method to extract singular contributions**
Recent NNLO results for LHC processes

- Calculations with full final state information
  - Can apply experimental selection cuts
  - \( pp \rightarrow V, \ pp \rightarrow H, \ pp \rightarrow VH, \ pp \rightarrow \gamma\gamma \) (C. Anastasiou, K. Melnikov, F. Petriello; S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini, F. Tramontano)
  - \( pp \rightarrow V\gamma, \ pp \rightarrow Z^0Z^0, \ pp \rightarrow W^+W^- \) (F. Cascioli, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi, M. Wiesemann, E. Weihs, TG)
  - \( pp \rightarrow \text{top quark pairs} \) (M. Czakon, D. Heymes, A. Mitov)
  - \( pp \rightarrow V+j \) (R. Boughezal, C. Focke, X. Liu, F. Petriello)
  - \( pp \rightarrow Z^0+j \) (A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, T. Morgan, TG)
  - \( pp \rightarrow \gamma^+ j \) (J. Campbell, K. Ellis, C. Williams)
  - \( pp \rightarrow H+2j \) (VBF) (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)
  - \( pp \rightarrow 2j \) (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, J. Pires, TG)
Antenna subtraction

- Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons

- NNLO antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
  - Four-parton antenna: two unresolved partons
  - Three-parton antenna at one loop
  - Products of NLO antenna functions
  - Soft antenna function
  - Combination with mass factorization (J. Currie, E.W.N. Glover)
NNLOJET code

- **NNLO parton level event generator**
  - Based on antenna subtraction

- **Provides infrastructure**
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing (MPI) support for warm-up and production
  - ApplGrid/fastNLO interfaces in development

- **Processes implemented at NNLO**
  - Z+(0, 1)jet, H+(0, 1)jet, W+0jet
  - DIS-2j, LHC-2j
  - Typical runtimes: 60’000-250’000 core-hours

**NNLOJET project:**
NNLOJET: phase space at NNLO

- **Numerical phase space integrator**
  - Partition final state phase space into wedges
    - Triple collinear: $s_{\text{min}1} = s_{ab}, s_{\text{min}2} = s_{ac}$
    - Double single collinear: $s_{\text{min}1} = s_{ab}, s_{\text{min}2} = s_{cd}$
    - E.g. di-jet $2 \rightarrow 4$ partons: 24 TC and 6 DC wedges
    - Multiple limits per wedge
  - Parametrize phase space
    - Angular variables for unresolved momentum directions
    - Non-linear mapping for unresolved invariants
    - Linear mapping (process-optimized) for resolved invariants
  - Ensure optimal coverage
  - Local cancellation of angular matrix element correlations
Z $p_T$-distribution at NNLO

- Using calculation for $Z+$jet inclusively on partons
  - No jet requirement
  - Including leptonic Z-decay
  - Lower cut on transverse momentum
  - Compute fiducial cross sections

<table>
<thead>
<tr>
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<th>ATLAS</th>
<th>CMS</th>
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<td>leading lepton</td>
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<td>\eta_1</td>
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<td>$p_T^{\ell_2, 2} &gt; 10 \text{ GeV}$</td>
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Z $p_T$-distribution at NNLO

- **NNLO effects**
  - Around 5% corrections, modify shape of $p_T$ distribution
  - Normalization of data not described correctly (both CMS/ATLAS)

A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, T. Morgan, TG
Z $p_T$-distribution at NNLO

- Compute inclusive fiducial cross section at NNLO
- Corresponds to $Z+0j$ calculation
- Observe same discrepancy

Consider normalized $p_T$ distribution

NNLOJET

$\sqrt{s} = 8$ TeV

$p p \rightarrow Z + \geq 0 \text{ jet}$
Z $p_T$-distribution at NNLO

- Double differential distributions
  - $(p_T, m_\|), (p_T, y)$
  - Good agreement for normalized distributions
- Revisit ingredients
  - Luminosity
  - Parton distributions
    (R. Boughezal, A. Guffanti, F. Petriello, M. Ubiali)
Z $p_T$-distribution at NNLO

- Low $p_T$
  - measurements to 1 GeV
  - Challenge for NNLO calculation: stability
  - NNLO reliable to around 5 GeV

- Related observable (purely from lepton directions)

$$\phi^* = \tan \left( \frac{\pi - \Delta \phi}{2} \right) \sin(\theta^*_\eta) \approx \frac{p_T^Z}{2m_{ll}}$$
Z $\phi^*$-distribution at NNLO

- Leptonic variable $\phi^*$ allows higher resolution
- Observe breakdown of fixed order similar to $p_T$-distribution
- Matching onto NNLL resummation ongoing
Angular coefficients in $Z$ production

- Angular distribution of final state leptons
  - Collins-Soper frame

\[
\frac{d\sigma}{d^4q \, d \cos \theta \, d \phi} = \frac{3}{16\pi} \frac{d\sigma^{\text{unpol.}}}{d^4q} \left\{ (1 + \cos^2 \theta) + \frac{1}{2} A_0 \left(1 - 3 \cos^2 \theta\right) + A_1 \sin(2\theta) \cos \phi + \frac{1}{2} A_2 \sin^2 \theta \cos(2\phi) + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2 \theta \sin(2\phi) + A_6 \sin(2\theta) \sin \phi + A_7 \sin \theta \sin \phi \right\},
\]

- Angular coefficients $A_0..A_7$ are functions of transverse momentum
- Important input to MC tuning for the measurement of $W$ mass
Angular coefficients in Z production

- Coefficients $A_0..A_4$ measured by fixed target, ATLAS, CMS
  - $A_4$ is forward-backward asymmetry: $\sin^2 \Theta_W$
  - Theory: $O(\alpha_s^0): A_4$, $O(\alpha_s^1): A_0..A_3$, $O(\alpha_s^2): A_5..A_7$
- NLO for $O(\alpha_s^1)$-coefficients: from NNLO V+0j
  - (M. Lambertsen, W. Vogelsang)
- NNLO for $O(\alpha_s^1)$ and NLO for $O(\alpha_s^2)$-coefficients: from NNLO V+1j
  - (R. Gauld, A. Gehrmann-De Ridder, N. Glover, A. Huss, TG)
  - require non-local cancellations
  - ease tension with data
Angular coefficients in Z production

- Lam-Tung relation (spin-1/2 quarks): $A_0 - A_2 = 0$
  - Broken by QCD corrections only at $\mathcal{O}(\alpha_s^2)$
- Angular coefficients $A_i$ defined relative to $p_{T,Z}$-distribution
  - Kinematical suppression for $p_{T,Z} \to 0$ (region of large statistics)
- Define Lam-Tung violation: $\Delta^{LT} = 1 - A_2/A_0$

\[ \Delta^{LT} = 1 - \frac{A_2}{A_0} \]
Higgs $p_T$ distribution at NNLO

- Normalized results in good agreement with 8TeV data

- Prepare for precision studies at higher energy

X. Chen, J. Cruz-Martinez, E.W.N. Glover, M. Jaquier, TG
Jet cross sections at NNLO

- **NNLO corrections to di-jet production in DIS**
  - Implemented in NNLOJET (J. Currie, A. Huss, J. Niehues, TG)
  - Substantial NNLO effects
  - Uncovered infrared-sensitive interplay of H1/ZEUS event selection
    - Combination of jet-$p_T$ and di-jet mass restricts LO/NLO phase space

![Graphs showing jet cross sections at NNLO](image)

**RADCOR 2017**
Jet cross sections in DIS

- Single inclusive jet
  - New H1 measurement
- NNLO considerably improves description of data
- First application of NNLOJET/AppGrid interface (with D. Britzger, C. Gwelnan, M. Sutton, K. Rabbertz)

H1 Inclusive jets

- H1 HERA-II
- H1 HERA-II

Systematic uncertainty
Jet cross sections in DIS

- **Determination from H1 jet data**
  - $\alpha_s = 0.1157(20)_{\exp}(29)_{\text{th}}$

- **Fit with PDF**
  - $\alpha_s = 0.1142(28)_{\text{total}}$

- **Potential impact on future PDF**
NNLO corrections to di-jet production

- Four QCD partons at tree level
  - Most complicated process so far
  - Larger number of unresolved limits than in V+jet
- NNLO corrections at leading color $N$ and leading $N_F$
  (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, J. Pires, TG)
  
- Subleading corrections: below 2% of cross section at NLO
- Stabilization of predictions
  - Scale dependence
  - Lower spread between different central scales
  - Many potential applications

![NNLO corrections graph]

ATLAS 7 TeV, anti-kt jets, $R=0.4$, $0.0 < |y^*| < 0.5$
Single jet inclusive production at NNLO

- Scale setting in single jet inclusive production: choice
  - for full event: $p_{T1}$ of hardest jet in event
  - for each jet: $p_T$ of jet
- Scale choice: measure of theory uncertainty?

![Graphs showing scale dependence of NLO prediction and comparison with data](image-url)
NNLO corrections for jet processes

- **Precision phenomenology**
  - Want to compare with multiple data sets
  - Vary theory parameters
    - Renormalization and factorization scale
    - Parton distributions
    - Strong coupling constant

- **Running NNLO parton-level program a large number of times is not realistically feasible**

- **Possible dissemination of NNLO results**
  - K-factors, re-weighting (NNPDF, MMTW)
  - NNLO coefficient functions for each data bin (fastNLO, ApplGrid: used e.g. in HERAPDF)
  - Event n-Tuples (NLO: BlackHat)
Conclusions

- **NNLO corrections to precision observables at LHC**
  - Various methods have been applied successfully
  - Healthy competition between groups

- **Current frontier: $2 \rightarrow 2$ QCD processes**
  - Substantial number of calculations completed in the past two years
  - More results coming
  - Higher multiplicities require new methods for two-loop amplitudes

- **Precision phenomenology with jet observables starting**
  - Run-time of NNLO codes prohibitive: other methods to distribute results
  - Measurements of coupling constants
  - Determination of parton distributions