2-loop MSSM Higgs-mass QCD-EW corrections in real and complex MSSM

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Motivation
The Era of the Large Hadron Collider

Blessing or curse?
Discovery of a scalar particle

- Scalar particle discovered which looks like the Higgs boson of the Standard Model
  - consistent with EW precision bounds
Discovery of a scalar particle

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  - consistent with spin and parity predictions of Standard Model
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  - consistent with EW precision bounds
  - consistent with spin and parity predictions of Standard Model
  - couplings to fermions consistent with SM prediction

ATLAS & CMS Combination Aug ’16
Beyond the Standard Model

- MSSM exclusion bounds are discouraging
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- but other BSM models become better excluded as well
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→ BSM ‘hiding’ at higher energies?
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→ BSM ‘hiding’ at higher energies?

- wait for end of LHC (and HL-LHC) runs

MSSM: low-energy SUSY predictions for LHC

- supersymmetry only known extension to Poincaré algebra
- dark matter candidate
- MSSM particles are within reach of LHC energies
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- supersymmetry only known extension to Poincaré algebra
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- MSSM particles are within reach of LHC energies

→ still a prime candidate for BSM
Higgs sector of the MSSM with real parameters

- The Higgs sector of the MSSM has two scalar doublets

\[
H_1 = \left(\begin{array}{c}
v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - i\chi_1^0) \\
-\phi_1^-
\end{array}\right) \quad H_2 = e^{i\xi} \left(\begin{array}{c}
v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\chi_2^0)
\end{array}\right)
\]

\rightarrow 5 Higgs-bosons: h, H, A, H^{\pm}

- Potential of the Higgs sector (incl. soft SUSY breaking terms)

\[
V = m_1 |H_1|^2 + m_2 |H_2|^2 - m_{12}(\epsilon_{ab} H_1^a H_2^b + h.c.) + \frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2 |H_1^\dagger H_2|^2
\]

\(g_1, g_2\): electro-weak gauge couplings,
\(v_1, v_2\): the v.e.v.’s in \(\tan\beta \equiv \frac{v_2}{v_1}\),
\(m_{12}\): soft SUSY breaking term in \(m_A^2 = m_{12}^2(\tan\beta + \cot\beta)\)
Higgs-boson mass prediction in the MSSM

- feature in the MSSM: light Higgs-boson mass can be predicted!
  \[ \Rightarrow \text{higher-order corrections can be computed} \]

- accurate predictions used to
  - improve exclusions
  - constrain other BSM models
  - more accurate assessment of theoretical uncertainties
MSSM excluded at tree-level

- The tree-level neutral $\mathcal{CP}$-even Higgs-boson masses

\[
M_{\text{Higgs}}^{2, \text{tree}} = \begin{pmatrix}
  m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(m_A^2 + M_Z^2) \sin \beta \cos \beta \\
  -(m_A^2 + M_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta
\end{pmatrix}
\]

are limited to $m_h \leq \min(M_Z, m_A) |\cos(2\beta)|$

- Higher-order corrections shift the Higgs-boson masses considerably

These lead to maximal values for $m_{h_{\text{max}}} \approx 135$ GeV
Higgs-boson masses from self-energies

- include self-energy corrections in inverse Higgs propagator matrix

\[
\Gamma \equiv \Delta^{-1}_{\text{Higgs}} = -i \begin{pmatrix}
  p^2 - m_h^2 + \hat{\Sigma}_h(p^2) & \hat{\Sigma}_{hH}(p^2) \\
  \hat{\Sigma}_{hH}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_H(p^2)
\end{pmatrix}
\]

with renormalized self-energies \( \hat{\Sigma} \)

- The neutral \( CP \)-even masses are the real parts of the poles of the propagator matrix \( \Delta_{\text{Higgs}} \)

- Strategy (in \texttt{FeynHiggs}): Find complex solutions to \( \text{Det}(\Gamma) = 0 \)
Status: Radiative corrections in the MSSM

Higher-order corrections to the Higgs-boson mass in the MSSM:

1-loop  2-loop  3-loop  RGE approach

- Ellis, Ridolfi, Zwirner '91; Okada, Yamaguchi, Yanagida '91; Haber & Hempfling '91; Brignole '92; Chankowski, Pokorski, Rosiek '92 '94; Dabelstein '95; Pilaftsis '98; Demir '99; Heinemeyer '01; Pilaftsis, Wagner '99; Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '07

- Hempfling & Hoang '94; Carena et al. '95 '96; Espinosa et al. '95 '00 '01; Heinemeyer, Hollik, Weiglein et al. '98 '99 '00 '00; Zhang '99; Carena, Ellis, Pilaftsis, Wagner '00; Choi, Drees, Lee '00; Degrassi, Slavich et al. '01 '03; Ibrahim, Nath '00 '02; Brignole, Degrassi, Slavich, Zwirner '02; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '05 '07 '13; S. P. Martin '02 '03 '04 '05 '07; SB, Hahn, Heinemeyer, Heinrich, Hollik '14; Degrassi, Di Vita, Slavich '14; Hollik, Paßehr '14; Goodsell, Staub '16; Paßehr, Weiglein '17

- S.P. Martin '07; Harlander, Kant, Mihaila, Steinhauser '08 '10; Harlander, Klappert, Voigt '17

- Binger '04; Giudice, Strumia '11; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '13; Draper, Lee, Wagner '13 '15; Bagnaschi, Giudice, Slavich, Strumia '14; Vega, Villadoro '15; Bahl, Hollik '16; Athron, Park, Steudtner, Stöckinger, Voigt '17; Bahl, Heinemeyer, Hollik, Weiglein '17

Many more contributions...
Are we done?

Unfortunately not:
differences still in range
$1 - 2.5 \text{ GeV}$

Bahl, Heinemeyer, Hollik, Weiglein ’17

Reasons:

▶ precision of prediction depends on code and SUSY scale
▶ perturbative series converges slowly (new channels open up)
▶ renormalization scheme dependence still large at two-loop level
▶ conceptual problems: renormalization scheme ambiguities
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Public codes with fixed-order MSSM corrections

**FeynHiggs** Bahl, Hahn, Heinemeyer, Hollik, Passehr, Rzehak, Weiglein '98 '02 '06 '13 '16 '17

**SoftSusy** Allanach, Bednyakov, Martin, Robertson, Ruiz de Austri et al. '02 '14 '16

**SuSpect** Djouadi, Kneur, Moultaka '02

**SPheno** Porod '03; Porod, Staub '11

**CPsuperH** Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '03

**H3M** Kant, Harlander, Mihaila, Steinhauser '10

Summary of implemented fixed-order real/complex corrections:

1-loop complete

2-loop $\mathcal{O}(\alpha_s \alpha_t), \mathcal{O}(\alpha_t^2), \mathcal{O}(\alpha_s \alpha_b), \mathcal{O}(\alpha_t \alpha_b), \mathcal{O}(\alpha_b^2)$ at $p^2 = 0$

$\mathcal{O}(\alpha_s \alpha_t), \mathcal{O}(\alpha_s \alpha)$ at $p^2 \neq 0$

3-loop $\mathcal{O}(\alpha_s^2 \alpha_t)$ at $p^2 = 0$

NEW: $\mathcal{O}(\alpha_s \alpha_q), \mathcal{O}(\alpha_s \alpha)$ at $p^2 \neq 0$ for complex parameters and $m_b \neq 0$
The Calculation
Higgs-boson self-energy diagrams needed

\[ \Phi_{i,j} = h, H, A \]
Additional diagrams for renormalization
Diagram generation and processing

- Diagrams generated with **FeynArts** Kühlbeck, Böhm, Denner '90; Hahn '01
- Trace evaluation using **TwoCalc** Weiglein et al. '93
- 1-loop tensor reduction with **FormCalc** Hahn et al. '99 '08
- 2-loop tensor reduction with **Reduze** von Manteuffel, Studerus '12

→ Choose topologies producing least artificial mass thresholds in coefficients
Treatment of loop integrals

- 1-loop and factorizable 2-loop integrals known
- different genuine 2-loop mass configurations: 177
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- most difficult topologies:

\[ T_{234} \]
\[ T_{1234} \]
\[ T_{11234} \]
\[ T_{12345} \]
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\[ T_{234} \]

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\[ T_{11234} \]

\[ T_{12345} \]

- big advances on analytical side Adams, Bogner, Weinzierl et al. '13 '15 '16; Remiddi, Tancredi '14 '16
- dedicated numerical codes available S2L Bauberger, Böhm '95, TSIL Martin, Robertson '06
Numerical evaluation of loop integrals

- we compute integrals fully numerically
- use interface to \texttt{FeynHiggs} to link external programs
  
  SB, Hahn, Heinemeyer, Heinrich, Hollik '14

- numerical evaluation using \texttt{SecDec 2} SB, Carter, Heinrich '12; SB, Heinrich '13 and Cuhre in \texttt{CUBA Library} Hahn '04
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  - newest version: **pySecDec** SB, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '17, see talk by Stephan Jahn
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- mass configurations evaluated at
  \[ p^2 = M_{h_1}^2, M_{h_2}^2, M_{h_3}^2, M_{H^\pm}^2, M_W^2, M_Z^2 \]
  \[ \rightarrow 513 \text{ integrals to compute on the fly} \]
- relative accuracies: at least \(10^{-7}\)
2-loop renormalization

- 1-loop counter term insertions
- 2-loop counter terms

- Renormalization procedure consistent with other higher-order corrections in \texttt{FEYNHIGGS} Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '06; SB, Hahn, Heinemeyer, Heinrich, Hollik '14

- Parameter renormalization in the OS scheme:
  \[ \delta m_t^{(1)}, \delta m_{\tilde{t}_1}^{(1)}, \delta m_{\tilde{t}_2}^{(1)}, \delta m_{\tilde{b}_2}^{(1)}, \delta m_{H^\pm}^{2(2)}, \delta M_Z^{2(2)}, \]

- Parameters renormalized \(\overline{DR}\): \(\delta m_b^{(1)}, \delta A_b^{(1)}\)

- Field renormalization only divergent terms, \(\tan\beta\) also finite terms
  \[ \delta Z_{H_1}^{(2)}, \delta Z_{H_2}^{(2)}, \delta \tan\beta^{(2)} = \frac{1}{2} (\delta Z_{H_2}^{(2)} - \delta Z_{H_1}^{(2)} + \delta \tan\beta^{\text{fin}}) \]

\(A_b\): soft SUSY breaking parameter of the sbottom sector
Results
**$m_h^{mod}$-like benchmark scenario**

- Based on scenario in Carena, Heinemeyer, Stal, Wagner, Weiglein ’13
- Parameters:

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{H^\pm}$</td>
<td>1.5 TeV</td>
</tr>
<tr>
<td>$\mu$</td>
<td>500 GeV</td>
</tr>
<tr>
<td>$</td>
<td>M_3</td>
</tr>
<tr>
<td>$</td>
<td>X_t</td>
</tr>
<tr>
<td>$m_{{\tilde{t},\tilde{b}}<em>L} = m</em>{\tilde{Q}_3}$</td>
<td>2.1 TeV</td>
</tr>
<tr>
<td>$m_{{\tilde{t},\tilde{b}}_R}$</td>
<td>2 TeV</td>
</tr>
<tr>
<td>$m_{{\tilde{q},\tilde{l}}_{{L,R}}}$</td>
<td>2.5 TeV</td>
</tr>
</tbody>
</table>

with $|A_b| = |A_t|$, $q \in u, d, s, c$ and $l \in e, \mu, \tau$
Variation of mass shifts with \( \tan \beta \) and \( \mu \)

- evaluation using real parameters

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( m_{H^\pm} ) input</th>
<th>( m_A ) input</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 GeV</td>
<td>solid: ( M_{h_1}^{\text{new}} ); dashed: ( M_{h_1}^{\text{old}} )</td>
<td></td>
</tr>
<tr>
<td>-1500 GeV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- positive \( \mu \): shifts around \( 830 - 850 \) MeV
- negative \( \mu \): shifts decrease with large \( \tan \beta \)
- little difference in shifts between \( m_{H^\pm} \) and \( m_A \) as input
- \( m_{H^\pm} \) input more stable renormalization scheme

Preliminary
Variation of mass shifts with $M_3 = |M_3| e^{i\phi_{M_3}}$

- evaluation using complex parameters, $\tan \beta = 50$

- shifts smallest in the MSSM with real parameters ($\phi_{M_3} = 0$)

- size of phase shifts highly dependent on $|M_3|$
Corrections in other benchmark scenarios


<table>
<thead>
<tr>
<th>scenario</th>
<th>( m_h^{\text{max}} ) (GeV)</th>
<th>( m_h^{\text{mod+}} ) (GeV)</th>
<th>( m_h^{\text{mod−}} ) (GeV)</th>
<th>mod. light-stop (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_h^{\text{old}} ) (GeV)</td>
<td>128.313</td>
<td>125.364</td>
<td>124.839</td>
<td>122.681</td>
</tr>
<tr>
<td>( M_h^{\text{old}} + ) ( \mathcal{O}(\alpha_s \alpha_t p^2) ) (GeV)</td>
<td>128.254</td>
<td>125.234</td>
<td>123.828</td>
<td>122.644</td>
</tr>
<tr>
<td>( M_h^{\text{new}} ) (GeV)</td>
<td>128.534</td>
<td>125.754</td>
<td>124.845</td>
<td>122.609</td>
</tr>
<tr>
<td>difference (GeV)</td>
<td>0.221</td>
<td>0.390</td>
<td>0.006</td>
<td>-0.072</td>
</tr>
</tbody>
</table>

- size of corrections largely dependent on stop masses and \( A_t \)
- \( m_h^{\text{mod−}} \) scenario: down-shift from \( \mathcal{O}(\alpha_s \alpha_t p^2) \) corrections approx. compensated by new corrections

Preliminary
Corrections in other benchmark scenarios II

- Increase $M_{\text{SUSY}}$

<table>
<thead>
<tr>
<th>scenario</th>
<th>$m_h^{\text{mod}+}$ (GeV)</th>
<th>$m_h^{\text{mod}−}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\text{SUSY}}$</td>
<td>2 TeV</td>
<td>3 TeV</td>
</tr>
<tr>
<td>$M_h^{\text{old}}$ (GeV)</td>
<td>129.382</td>
<td>126.921</td>
</tr>
<tr>
<td>$M_h^{\text{new}}$ (GeV)</td>
<td>129.922</td>
<td>127.021</td>
</tr>
<tr>
<td>difference (GeV)</td>
<td>0.540</td>
<td>0.414</td>
</tr>
</tbody>
</table>

- Corrections increase with $M_{\text{SUSY}}$
- $O(\alpha_s \alpha_t p^2)$ corrections don’t grow as fast as $O(\alpha_s \alpha)$ corrections
Summary and Outlook

Summary

- $m_h$ is a precision observable in the MSSM
- we have computed two-loop QCD-EW corrections to the light MSSM Higgs mass for complex parameters
- 513 two-point integrals computed numerically with high precision
- corrections can be up to 1 GeV for scenarios within reach of the LHC

Outlook

- inclusion into \textsc{FeynHiggs} and analysis of renormalization scheme dependence
- update two-loop integrator interfaced to \textsc{FeynHiggs} (S2l routine Bauberger '94)
Backup
Two-loop parameter renormalization I

\[ \delta^{(2)} m_h^2 = c_{\alpha-\beta}^2 \delta^{(2)} m_A^2 + s_{\alpha+\beta}^2 \delta^{(2)} m_Z^2 + c_{\beta}^2 \delta^{(2)} t_{\beta} \]

\[ (s_{2(\alpha-\beta)} m_A^2 + s_{2(\alpha+\beta)} m_Z^2) \]

\[ + \frac{e s_{\alpha-\beta}}{2 M_W s_w} \left[ (1 + c_{\alpha-\beta}^2) \delta^{(2)} T_h + s_{\alpha-\beta} c_{\alpha-\beta} \delta^{(2)} T_H \right] , \]

\[ \delta^{(2)} m_H^2 = s_{\alpha-\beta}^2 \delta^{(2)} m_A^2 + c_{\alpha+\beta}^2 \delta^{(2)} m_Z^2 - c_{\beta}^2 \delta^{(2)} t_{\beta} \]

\[ (s_{2(\alpha-\beta)} m_A^2 + s_{2(\alpha+\beta)} m_Z^2) \]

\[ - \frac{e c_{\alpha-\beta}}{2 M_W s_w} \left[ (1 + s_{\alpha-\beta}^2) \delta^{(2)} T_H + c_{\alpha-\beta} s_{\alpha-\beta} \delta^{(2)} T_H \right] , \]
Two-loop parameter renormalization II

\[ \delta^{(2)} m_A^2 = \delta^{(2)} m_{H^\pm}^2 - \delta^{(2)} m_W^2, \]

\[ \delta^{(2)} m_{hH}^2 = \frac{1}{2} \left( s_2(\alpha - \beta) \delta^{(2)} m_A^2 - s_2(\alpha + \beta) \delta^{(2)} m_Z^2 \right) \]

\[ - c_\beta^2 \delta^{(2)} t_\beta \left( c_2(\alpha - \beta) m_A^2 + c_2(\alpha + \beta) m_Z^2 \right) \]

\[ + \frac{e}{2 M_W s_w} \left[ s_\alpha^{-\beta} \delta^{(2)} T_H - c_\alpha^{-\beta} \delta^{(2)} T_h \right], \]

\[ \delta^{(2)} m_{hA}^2 = \frac{e}{2 M_W s_w} s_\alpha^{-\beta} \delta^{(2)} T_A, \]

\[ \delta^{(2)} m_{HA}^2 = - \frac{e}{2 M_W s_w} c_\alpha^{-\beta} \delta^{(2)} T_A. \]
Evaluation of neutral $\mathcal{CP}$-even MSSM Higgs-boson masses in $\overline{\text{DR}}$ scheme

\[
\left[ p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_{hh}(p^2) \right] \left[ p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_{HH}(p^2) \right] - \left[ \hat{\Sigma}_{hH}(p^2) \right]^2 = 0
\]

Three steps:

1. Compute $M_{h,0}$ and $M_{H,0}$ from the 1-loop + 2-loop $O(\alpha_s\alpha_t)$ self-energies

2. Compute momentum-dependent renormalized $O(\alpha_s\alpha_t)$ self-energies for $p^2 = M_{h,0}^2$ and $p^2 = M_{H,0}^2$

3. Include new self-energy contributions as constant shifts into FEYNHIGGS and find poles $M_h$ and $M_H$

- corrections available since FEYNHIGGS 2.10.1

The mass shifts are $\Delta M_{\{h,H\}} = M_{\{h,H\}} - M_{\{h,0\},\{H,0\}}$
Non-vanishing $O(\varepsilon)$ terms in $p^2 \neq 0$ calculations

- Lifting the $p^2 = 0$ restriction, momentum-dependent divergent and finite parts appear in the unrenormalized self-energies.
- The non-vanishing $p^2$-dependent divergent terms are cancelled by the field renormalization constants $\delta Z_{H_1}^{(2)}$ and $\delta Z_{H_2}^{(2)}$.
- Whether the additional $p^2$-dependent finite terms are cancelled depends on the renormalization scheme choice.
- We make the choice

\[
\delta Z_{H_1}^{(2)} = \delta Z_{H_1}^{\delta m_{t}^{OS}} \bigg|_{\text{div}} = - \left[ \text{Re} \Sigma_{\phi_i}'^{(2)} \right]^{\text{div}} \bigg|_{p^2=0},
\]

\[
\delta Z_{H_2}^{(2)} = \delta Z_{H_2}^{\delta m_{t}^{OS}} \bigg|_{\text{div}} = \frac{\alpha_s \alpha_t}{2\pi^2} \left( \frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} \right) - \frac{1}{\varepsilon} \frac{3\alpha_t}{2\pi} \delta m_{t}^{\text{fin}} ,
\]

therefore the additional finite $p^2$-dependent terms in the unrenormalized self-energies do not cancel during renormalization.
Analytically: Non-vanishing $O(\varepsilon)$ terms for $p^2 \neq 0$

- For each self-energy non-vanishing $O(\varepsilon)$ terms from $\delta m_t^{OS}$ remain

\[
\hat{\Sigma}^{(2)}_{\phi_1} : \\
- \sin^2 \beta [\delta_A(M_A^2) - \delta_A(0)] = \frac{3\alpha_t}{2\pi} (-\cos^2 \beta \sin^2 \beta M_A^2) \frac{\delta m_t^\varepsilon}{m_t}
\]

\[
\hat{\Sigma}^{(2)}_{\phi_1\phi_2} : \\
\sin \beta \cos \beta [\delta_A(M_A^2) - \delta_A(0)] = \frac{3\alpha_t}{2\pi} (\cos^3 \beta \sin \beta M_A^2) \frac{\delta m_t^\varepsilon}{m_t}
\]

\[
\hat{\Sigma}^{(2)}_{\phi_2} : \\
[\delta_{\Sigma_{22}}(p^2) - \delta_{\Sigma_{22}}(0)] - \cos^2 \beta [\delta_A(M_A^2) - \delta_A(0)] = \\
\frac{3\alpha_t}{2\pi} (p^2 - \cos^4 \beta M_A^2) \frac{\delta m_t^\varepsilon}{m_t}
\]

- $\delta_A(p^2)$ appears as shift in $\delta M_A^{(2)OS} = \delta M_A^{(2)FIN} + \delta_A(m_A^2)$
Numerical verification

- Additional finite parts from $\delta_A(m_A^2)$ and $\delta_{\Sigma 22}(p^2)$