Vector Boson Scattering (VBS) at the LHC

\[ pp \rightarrow e^+ \nu_e \mu^+ \nu_\mu jj + X @ \mathcal{O}(\alpha^6 \alpha_s), \]
Approximations, PDF uncertainties, and \( M_H = \infty \)

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In collaboration with:
Stefan Dittmaier, Philipp Maierhöfer and Frank Siegert

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At the LHC it is the process $pp \rightarrow e^+ \nu_e \mu^+ \nu_\mu jj + X$ that contains the VBS subprocess. I will refer to the whole process as VBS, though.

- Constrain anomalous quartic gauge couplings (AQGC),
- probe EW symmetry breaking

For longitudinal gauge boson scattering (pseudo-goldstone bosons) we know there are

- gauge cancellations between QGC diagram (A) and TGC diagrams (B);
- further cancellations between remainders and diagrams containing Higgs(es) (C, restores perturbative unitarity)

- violation of perturbative unitarity, maximum effect of different Higgs sector

$\rightarrow$ different Higgs sector/AQGC can change this
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$W^+ W^+ \rightarrow W^+ W^+$

@ $M_H = \infty$

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→ different Higgs sector/AQGC can change this
Why study the same sign (SS) W scattering case?

→ SS VBS processes at the LHC are ∼ fb, others are smaller or have large backgrounds

0 Dinosaur plot from https://twiki.cern.ch/twiki/bin/view/AtlasPublic/StandardModelPublicResults
● How does one distinguish between Signal ($\mathcal{O}(\alpha^6)$, containing VBS diagrams) and Backgrounds ($\mathcal{O}(\alpha^5 \alpha_s^1)$ and $\mathcal{O}(\alpha^4 \alpha_s^2)$, interferences or no VBS diagrams)? → Use VBS cuts

● What approximations are possible and what can we learn from them about VBS? → Use Pole and t-u approximation

● How large are PDF uncertainties? → Calculate them for NLO QCD $\mathcal{O}(\alpha^6 \alpha_s)$

● What are good/bad scale choices? → Use static and dynamic scales

● How does an extended Higgs sector affect the differential distributions? → Compare with $M_H = \infty$ to see maximum effect of perturbative non-unitarity
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How does an extended Higgs sector affect the differential distributions? → Compare with $M_H = \infty$ to see maximum effect of perturbative non-unitarity
How does one distinguish between signal and background?

Inclusive cross sections (two $R = 0.4$ anti-$k_T$ jets with $p_T > 30$ GeV: taggings jets $j_1$ and $j_2$):

\[ \frac{d\sigma}{dM(j_1j_2)} \quad [\text{fb GeV}^{-1}] \]

\[ \frac{d\sigma}{dy(j_1)} \quad [\text{fb}] \]

$\mathcal{O}(\alpha_6^0) \quad \mathcal{O}(\alpha_5^1) \quad \mathcal{O}(\alpha_4^2) \quad \mathcal{O}(\alpha_6^0)$

$\mathcal{O}(\alpha_5^1) \quad \mathcal{O}(\alpha_4^2)$

$\mathcal{O}(\alpha_6^0)$

$\rightarrow \mathcal{O}(\alpha_6^0)$ similar in size as $\mathcal{O}(\alpha_4^2)$

$\rightarrow$ interference $\mathcal{O}(\alpha_5^1 \alpha_s)$ is suppressed by colour and kinematics

- ATLAS$^1$ 8 TeV/CMS$^2$ 13 TeV analyses: $M(j_1j_2) > 500$ GeV
- $\mathcal{O}(\alpha_6^0)$ peaks at higher rapidity because of (two) space-like $W$ propagators of the VBS subprocess

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$^1$arXiv:1405.06241, ATLAS Collaboration

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$$d\sigma/dM(j_1j_2) \ [fb \ GeV^{-1}]$$

$$d\sigma/d(y(j_1) - y(j_2)) \ [fb]$$

$\rightarrow O(\alpha^6)$ similar in size as $O(\alpha^4 \alpha^2)$

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- ATLAS$^1$ 8 TeV/CMS$^2$ 13 TeV analyses: $M(j_1j_2) > 500$ GeV, and $|y(j_1) - y(j_2)| > 2.4/2.5$

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Inclusive cross sections (two $R = 0.4$ anti-$k_T$ jets with $p_T > 30$ GeV: taggings jets $j_1$ and $j_2$):

$$d\sigma/dz^*_{\mu^+}$$

"Zeppenfeld variable"$^3$, divides phase space in two regions:
- $z_{\ell}^* > 0.5 \Rightarrow$ either $y(\ell) > \max\{y(j_1), y(j_2)\}$ or $y(\ell) < \min\{y(j_1), y(j_2)\}$; "lepton outside taggings jets"
- $z_{\ell}^* < 0.5 \Rightarrow \min\{y(j_1), y(j_2)\} < y(\ell) < \max\{y(j_1), y(j_2)\}$; "lepton between taggings jets"
- Advantage of $z_{\ell}^*$: continuous, being more inclusive easy

CMS 13 TeV: $z_{\ell}^* < 0.75$
ATLAS 8 TeV: no cut on $z_{\ell}^*$

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Setup for $pp \rightarrow e^+ \nu_e \mu^+ \nu_\mu jj + X$

Cuts for fiducial cross section (similar to ATLAS’s):

- At least two $R = 0.4$ Anti-$k_t$ jets with $p_T > 30$ GeV and $|y| < 4.5$
- $M(j_1j_2) > 500$ GeV and $|y(j_1) - y(j_2)| > 2.4$
- $p_T(\ell^+) > 27$ GeV and $|y(\ell^+)| < 2.5$
- $p_T(\nu_e\nu_\mu) > 30$ GeV
- $M(e^+\mu^+) > 20$ GeV
- $\Delta R(e^+\mu^+) > 0.3$
- $\Delta R(j_1\ell^+) > 0.3$, $\Delta R(j_2\ell^+) > 0.3$

Other:

- $\sqrt{s} = 13$ TeV
- CT14LO/CT14NLO ($\alpha_s(M_Z) = 0.118$) PDFs

Complex mass scheme\textsuperscript{4,5} input parameters:

- $G_\mu = 1.6637 \times 10^{-5}$ GeV\textsuperscript{-2}
- $M_W = 80.3580$ GeV, $\Gamma_W = 2.0843$ GeV
- $M_Z = 91.1535$ GeV, $\Gamma_Z = 2.4943$ GeV
- $M_H = 125.0$ GeV, $\Gamma_H = 4.07 \times 10^{-3}$ GeV

with coupling calculated as:

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$

Scale choices ($\mu_F = \mu_R = \mu$):

- static: $\mu = M_W$
- dynamic: $\mu = \left( M_W^2 + p_T^2(j_1) \right)^{1/2}$

\textsuperscript{4} arXiv:hep-ph/9904472; A. Denner, S. Dittmaier, M. Roth, D. Wackeroth
\textsuperscript{5} arXiv:hep-ph/0505042; A. Denner, S. Dittmaier, M. Roth, L.H. Wieders
## Setup and cross checks

**BONSAY (C.S.)**

- "BOsoN Scattering with Accuracy"
- MC written from scratch
- Matrix elements from M. Billoni, S. Dittmaier
- Loops calculated by COLLIER\(^6\)

**Comparison against existing**

\(O(\alpha^6 \alpha_s)\) calculation by Denner et. al.\(^7\)

- Comparison of ME and dipoles for a few phase space points, and integrated cross sections (inclusive, fiducial, both for selected partonic channels and all of them)
- Distributions are preliminary

**SHERPA (F. Siegert)**

- One loop matrix elements from OPENLOOPS\(^8\)

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\(^6\) arXiv:1209.2389; A. Denner, L. Hőseková, S. Kallweit

\(^7\) arXiv:1604.06792; A. Denner, S. Dittmaier, L. Hofer

\(^8\) arXiv:1111.5206; F. Cascioli, P. Maierhöfer and S. Pozzorini
Pole approximation for virtual matrix elements

\[ \mathcal{M}_{\text{virt}} = \sum_{\lambda_1, \lambda_2} \mathcal{M}_{q_1 q_2 \rightarrow W_1 W_2}^{\text{virt}} + \ldots \]
Pole approximation for virtual matrix elements

\[ \mathcal{M}_{\text{virt,PA}} = \sum_{\lambda_1, \lambda_2} \mathcal{M}_{\text{virt}}^{q_1 q_2 \rightarrow W_1 W_2 q_3 q_4} \mathcal{M}_{\text{LO}}^{W_1 \rightarrow e^+ \nu_e} \mathcal{M}_{\text{LO}}^{W_2 \rightarrow \mu^+ \nu_\mu} \left| \frac{1}{K_1 K_2} \right| \]

- Drop all diagrams that are not doubly-resonant
- \( \star \): set all widths to zero, project momenta of resonances on-shell \( \rightarrow \) gauge invariance
- Resonant propagator denominators are kept off-shell: \( K_i = \bar{k}_i - M_{W_i}^2 + i M_{W_i} \Gamma_{W_i} \), where \( \bar{k}_1 \) is the off-shell momentum of the first \( W^+ \), \( \bar{k}_2 \) the one of the second \( W^+ \)
- Procedure modifies IR singularities, taken care of by modified insertion operator:

\[ 2 \text{Re}(\mathcal{M}_{\text{LO}}^* \mathcal{M}_{\text{virt}}) + |\mathcal{M}_{\text{LO}}|^2 \otimes I \quad \rightarrow \quad 2 \text{Re}(\mathcal{M}_{\text{LO,PA}}^* \mathcal{M}_{\text{virt,PA}}) + |\mathcal{M}_{\text{LO,PA}}|^2 \otimes I \]
Uncertainty of the PA

Use uncertainty estimation of Biedermann, et. al.\(^9\) and modify it for QCD corrections:

\[
\Delta_{PA}^{NLO} \sim \max \left( \frac{\alpha_s}{2\pi} \frac{\Gamma_W}{M_W} \log(\ldots), |\delta_{PA}| \times \delta_{LO} \right), \quad \delta_{LO} = \frac{\sigma_{LO} - \sigma_{LO}^{PA}}{\sigma_{LO}}
\]

\(\delta_{LO}\) can be large, e.g. di-boson production \(p_T\) distributions PA underestimates by 10–20%

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\( \delta_{\text{LO}} \) can be large, e.g. di-boson production \( p_T \) distributions PA underestimates by 10–20%.

- LO PA differs by 1–3% \( \sim \frac{\Gamma_W}{M_W} \Rightarrow \) No large non-/single-resonant contributions due to same sign W bosons!

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t-/u-channel approximation

\[ |A|^2 \equiv \begin{cases} \begin{array}{c}
\begin{array}{c}
|A|_1^2, \\
|A|_2^2, \\
\vdots
\end{array} \\
\begin{array}{c}
|A|_1^2, \\
|A|_2^2, \\
\vdots
\end{array}
\end{array}
\end{cases} \]

\[ A_{\text{virt}} \equiv \begin{cases} \begin{array}{c}
\begin{array}{c}
A_{\text{virt},1}, \\
A_{\text{virt},2}, \\
\vdots
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A_{\text{virt},1}, \\
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\end{array}
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- \( q\bar{q} \)-Annihilation (only for \( Q = 1 \) processes) suppressed by cut \( M(j_1j_2) > 500 \text{ GeV} \) → for real ME gluon in the initial state must not couple to other initial state quark
- t-u interferences are suppressed in the fiducial volume
- no virtual diagrams connecting different quark lines, because they are zero (squares) or kinematically suppressed (interferences)
**t-/u-channel approximation**

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|A|^2, \\
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\vdots
\end{array} \right\} 

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t-/u-channel approximation

\[ |A|^2 \ni \{ \begin{array}{c} |A|^2, \quad 2 \text{,} \\ |A|^2, \quad 2 \text{,} \\ \vdots \end{array} \} \]

\[ A_{\text{virt}} \ni \{ \begin{array}{c} A_{\text{virt}}, \quad \cdots, \\ A_{\text{virt}}, \quad \cdots, \\ \cdots \end{array} \} \]

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\[ |A|^2 \supset \begin{cases} |\begin{array}{c} \text{q}q\text{-Annihilaton (only for } Q = 1 \text{ processes) suppressed by cut } M_{j_1j_2} > 500 \text{ GeV} \rightarrow \text{for real ME gluon in the initial state must not couple to other initial state quark} \\ \text{t-u interferences are suppressed in the fiducial volume} \\ \text{no virtual diagrams connecting different quark lines, because they are zero (squares) or kinematically suppressed (interferences)} \end{array}|^2, \end{cases} \]
Integrated cross sections for $pp \to e^+ \nu_e \mu^+ \nu_\mu jj + X$ at $O(\alpha^6 \alpha_s^{0,1})$

<table>
<thead>
<tr>
<th>Scale</th>
<th>LO  $[\text{fb}]$</th>
<th>NLO $[\text{fb}]$</th>
<th>$\delta = \frac{NLO-LO}{LO}$</th>
<th>NLO PDF unc.</th>
<th>LO $M_H = \infty$ $[\text{fb}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>static</td>
<td>$1.373^{+8.3%}_{-7.2%}$</td>
<td>$1.193^{+5.8%}_{-3.8%}$</td>
<td>$-13.1%$</td>
<td>$^{+3.1%}_{-3.2%}$</td>
<td>$^\pm 2.9%$</td>
</tr>
<tr>
<td>dynamic</td>
<td>$1.222^{+7.3%}_{-6.5%}$</td>
<td>$1.208^{+0.0%}_{-0.7%}$</td>
<td>$-1.2%$</td>
<td>$^{+3.1%}_{-3.2%}$</td>
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</tr>
</tbody>
</table>

Perturbative uncertainties estimated from scale variation by factors $\xi \in (1/2, 1, 2)$ for

- static scale: $\mu_R = \mu_F = \xi M_W$, and
- dynamic scale (DS): $\mu_R = \mu_F = \xi \left(M_W^2 + p_T^2(j_1)\right)^{1/2}$

PDF uncertainties for CT14NLO:

\[
\Delta^2_{\pm} = \sum_{i=1}^{28} \left[ \max\{\pm \sigma(z_i^+) \mp \sigma(z^0), \pm \sigma(z_i^-) \mp \sigma(z^0), 0\} \right]^2 \quad z^0 \text{ central PDFs} \\
\Delta^2_{\text{sym}} = \frac{1}{4} \sum_{i=1}^{28} \left[ \sigma(z_i^+) - \sigma(z_i^-) \right]^2 \quad z_i^\pm \text{ PDFs with } i\text{-th fitparameter varied in } \pm \text{ direction}
\]
Leading jet transverse momentum \((\mu = M_W)\)

- LO band given by \(\mu_F\) alone
- PDF errors are uniformly 3%
- very large corrections for high \(p_T\)
- use dynamic scale!
Leading jet transverse momentum (DS)

- smaller corrections for high $p_T$
- overlap of uncertainty bands where cross section is largest
- PDF errors are uniformly 3%
- low $p_T$ region sensitive to modified Higgs sector, large $p_T$ is unaffected
QCD corrections increase the rapidity of the jet(s)  
forward region most sensitive to mod. Higgs sector, difference up 80%
positive QCD corrections for smaller invariants
Long overlap of LO/NLO unc. bands

larger PDF unc. for large inv. mass, \(\sim 7\%\)
\(M_H = \infty\): impact is rather uniform
QCD correction widen the rap. gap
Muon transverse momentum (DS)

$\frac{d\sigma}{dp_T}(\mu^+) [fb GeV^{-1}]$

- **LO**
- **LO unc.**
- **NLO**
- **NLO unc.**
- **PDF unc.**

$M_H = \infty$: large effects in high $p_T$ bins, $\sim 150\%$
Lepton azimuthal angle distance (DS)

- $M_H = \infty$: differences up to 40% in the bins with highest cross section
Summary

- Pole approximation differs by 1% to 3% ⇒ no large single/non-resonant phase space regions
- PDF uncertainties ∼ 3%, for large $M(j_1 j_2)$ up to ∼ 7%
- $\mu = M_W$: large negative QCD corrections, not covered by LO uncertainty band
- $\mu = \sqrt{M_W^2 + p_T^2(j_1)}$: small negative corrections; compare with $\mu = \sqrt{p_T(j_1)p_T(j_2)}$?
- $M_H = \infty$: forward jets (low $p_T$, high $y$), high lepton $p_T$, and $\Delta \phi(e^+ \mu^+)$ sensitive to different Higgs sector
- Outlook: Full NLO in DPA in progress
$\mathcal{O}(\alpha^6 \alpha_s)$: Differential distributions (I)

\[ \frac{d\sigma}{dy}(j_1) \quad [\text{fb}] \]

\[ \frac{d\sigma}{d\transverseMomentum}(j_1) \quad [\text{fb GeV}^{-1}] \]

\[ \delta \% \]

\[ y(j_1) \]

\[ \frac{d\sigma}{dy}(j_2) \quad [\text{fb}] \]

\[ \frac{d\sigma}{d\transverseMomentum}(j_2) \quad [\text{fb GeV}^{-1}] \]

\[ \delta \% \]

\[ y(j_2) \]

\[ \delta \% \]

\[ y(j_2) \]
$\mathcal{O}(\alpha^6 \alpha_s)$: Differential distributions (II)
\( \mathcal{O}(\alpha^6 \alpha_s) \): Differential distributions (III)
$\mathcal{O}(\alpha^6 \alpha_S)$: Differential distributions (IV)
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Full LO vs. PA (I)

\[ \frac{d\sigma}{dp_T(j_1)} \text{ [fb GeV}^{-1}] \]

\[ \begin{align*}
\delta \% \\
\text{LO PA}
\end{align*} \]

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\end{align*} \]

\[ \begin{align*}
\delta \% \\
\text{LO PA}
\end{align*} \]
Full LO vs. PA (II)

10^{-3}

\frac{d\sigma}{dM(j_1j_2)} [fb GeV^{-1}]

\frac{d\sigma}{dp_T(e^+)} [fb GeV^{-1}]

\delta [%]

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

0

1000

2000

3000

4000

5000

M(j_1j_2)

\frac{d\sigma}{dp_T(\mu^+)} [fb GeV^{-1}]

\delta [%]

0

100

200

300

400

500

p_T(e^+) [GeV]

p_T(\mu^+) [GeV]
Full LO vs. PA (IV)

\[ \frac{d\sigma}{d\Delta \phi(e^+\mu^+)} \text{[fb]} \]

LO vs. PA

\[ \delta \% \]

\[ \Delta \phi(e^+\mu^+) \]

\[ 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \]

\[ \frac{d\sigma}{dz^*e^+} \text{[fb]} \]

LO vs. PA

\[ \delta \% \]

\[ z^*e^+ \]

\[ 0, 0.2, 0.4, 0.6, 0.8, 1 \]
LO Inclusive Cross Sections (I)
LO Inclusive Cross Sections (II)

\[ \frac{d\sigma}{dM(j_1j_2)} [fb \text{GeV}^{-1}] \]

\[ \frac{d\sigma}{dy(j_1) - y(j_2)} [fb] \]

\[ \frac{d\sigma}{dp_T(e^+)} [fb \text{GeV}^{-1}] \]

\[ \frac{d\sigma}{dp_T(\mu^+)} [fb \text{GeV}^{-1}] \]
LO Inclusive Cross Sections (IV)

\[ \frac{d\sigma}{d\Delta \phi(e^+\mu^+)} \text{ [fb]} \]
\[ O(\alpha^6\alpha^0_s) \quad O(\alpha^5\alpha^1_s) \quad O(\alpha^4\alpha^2_s) \]

\[ \frac{d\sigma}{dz^*_{e^+}} \text{ [fb]} \]
\[ O(\alpha^6\alpha^0_s) \quad O(\alpha^5\alpha^1_s) \quad O(\alpha^4\alpha^2_s) \]

\[ \frac{d\sigma}{d\Delta R(e^+\mu^+)} \text{ [fb]} \]
\[ O(\alpha^6\alpha^0_s) \quad O(\alpha^5\alpha^1_s) \quad O(\alpha^4\alpha^2_s) \]
LO Inclusive Cross Sections (V)

\[
\frac{d\sigma}{d\Delta R(j_1\mu^+)} \text{ [fb]}
\]

\[
\frac{d\sigma}{d\Delta R(j_2\mu^+)} \text{ [fb]}
\]

\[O(\alpha^6\alpha^0)\]
\[O(\alpha^5\alpha^1)\]
\[O(\alpha^4\alpha^2)\]