Monte-Carlo Top Quark Mass Calibrations

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Motivation: Precision Measurements

- Very precise top quark mass measurements:
  - Final Tevatron (2014): $173.34 \pm 0.64$ GeV
  - World Average (2014): $173.34 \pm 0.75$ GeV
  - CMS Run 1 (2016): $172.44 \pm 0.49$ GeV
  - Atlas Run 1 (2016): $172.84 \pm 0.70$ GeV

- Definitely reached era of sub-GeV top mass measurements!

Open Questions:

- What is the theoretical meaning of the measured quantity?
- How should we interpret the uncertainties?
Top Mass Determinations

- Different methods available
  
  $(t\bar{t})$ production at hadron colliders
  
  ▶ total cross-section measurements
    
    $m_t^{\text{pole}} = 176.7^{+4.0}_{-3.4}$ GeV [K.A.Olive et al. (PDG) 2014]
  
  ▶ leptonic observables [Frixione, Mitov 2014; Kawabata 2016]
  
  ▶ direct reconstruction measurements
  
  ▶ alternative techniques [CMS 2016]
  
- Direct reconstruction: standard technique
  
  ▶ many individual measurements with uncertainty below 1 GeV → CMS combination reaches < 500 MeV
  
  ▶ PDG quotes an uncertainty of $\sim$ 900 MeV

  \[ m_t = 173.21 \pm 0.51(\text{stat}) \pm 0.71(\text{sys}) \text{ GeV} \]

  ▶ relies on Monte Carlo (MC) generators e.g. PYTHIA to determine mass

How should one interpret the “measured” top mass? Is the MC top quark mass $m_t^{\text{MC}}$ universal?
Top Mass Determinations: MC Top Quark Mass

- Historically: all-order identification with $m_{t}^{\text{pole}}$
  - $O(\Lambda_{QCD})$ renormalon ambiguity
    → see C. Lepenik’s talk on Thursday
    [Beneke, Marquard, Nason, Steinhauser '16; Hoang, Lepenik, MP '17]
  - Convergence issues when extracting the pole mass

- Steps in the MC:
  - Hard ME - $t\bar{t}$ production
  - Parton shower - evolution down to the shower cutoff $\Lambda_{\text{cut}} \sim 1\text{GeV}$
  - Hadronization - model dependent

→ related to short distance mass: MSR mass

\[ m_{t}^{\text{MC}} : m_{t}^{\text{MSR}}(R \sim \Lambda_{\text{cut}}) \]

[Hoang, Stewart '08, Hoang '14]
Derived Mass Schemes & MSR Mass

- Define mass scheme by finite difference to other scheme (def by renormalization procedure) PS, 1S, RS, etc.

- $\overline{\text{MS}}$ mass $m(\mu)$: only absorb UV divergences of $\Sigma$

\[
m_{\text{pole}} - \overline{m}(\overline{m}) = \overline{m}(\overline{m}) \sum_{n=1}^{\infty} a_n \left( \frac{\alpha_s(\overline{m}(\overline{m}))}{4\pi} \right)^n
\]

- MSR mass $m^{\text{MSR}}(R)$: derived from $\overline{\text{MS}}$

[Hoang, Jain, Scimemi, Stewart '09; Hoang, Jain, Lepenik, Mateu, MP, Scimemi, Stewart '17]

\[
m_{\text{pole}} - m^{\text{MSR}}(R) = R \sum_{n=1}^{\infty} a_n \left( \frac{\alpha_s(R)}{4\pi} \right)^n
\]

→ with a free scale $R < \overline{m}$, $R$-RGE allows to calculate relation with $\overline{\text{MS}}$ at high precision
→ free of renormalon for $R > \Lambda_{\text{QCD}}$

$R$ effectively interpolates between Pole and $\overline{\text{MS}}$ mass, absorbs fluctuations for $|k_E| > R$

\[
\lim_{R \to 0} m^{\text{MSR}}(R) = m_{\text{pole}} \quad \& \quad m^{\text{MSR}}(R = \overline{m}(\overline{m})) = \overline{m}(\overline{m})
\]
Strategy

- **Strategy**: compare quark mass-sensitive hadron level QCD calculations with sample data from some MC
  - look into observables with strong kinematic mass sensitivity
  - get accurate hadron level QCD predictions ($\geq$NLO/NNLL) with full control over quark mass scheme dependence
  - fit QCD masses to different values of $m_t^{MC}$

\[
m_t^{MC} = m_t^{MSR}(R \simeq 1\text{GeV}) + \Delta_{t,MC}^{MSR}(R \simeq 1\text{GeV})
\]

\[
m_t^{MC} = m_t^{pole} + \Delta_{t,MC}^{pole}
\]

$\Delta_{t,MC} \simeq O(1\text{GeV})$

Uncertainties one can address in $e^+e^-$ studies
- perturbative uncertainty
- scale uncertainties
- electroweak effects

Additional MC/pp systematics
- PS + UE
- color reconnection
- intrinsic uncertainty
Kinematic Situation & Massive Event Shapes

- **Simplification 1**: Look into **boosted tops** (c.o.m. energy $Q \gg m_t \sim$ high $p_T$)
- **Simplification 2**: Consider $e^+ e^- \rightarrow t\bar{t} \rightarrow$ hadrons
- **Observable**: 2-jettiness $\hat{m} = m/Q$

\[
\tau_2 = 1 - \max \sum_i \frac{|\hat{n} \cdot \vec{p}_i|}{Q}
\]

- peak position

\[
\tau_2^{\text{peak}} \approx \frac{M_1^2 + M_2^2}{Q^2}
\]

with the hemisphere masses $M_{1/2}$

- peak region well suited for mass calibration!
Theory Description - EFT treatment

- Boosted top jets: SCET+bHQET singular terms

\[ n_f = n_l + 1 \]

\[
\frac{d\sigma^{bHQET}}{d\tau} = Q_H(Q, m, \mu_H) U_H^{(n_f)}(Q, \mu_H, \mu_m) H_m^{(n_f)}(Q, \mu_m) U_m^{(n_l)}(Q, m, \mu_m, \mu_B) \\
\times \int ds \, d\ell \, B_e^{(n_l)}(s, m, \mu_B) U_S^{(n_l)}(\ell, \mu_B, \mu_S) S_e^{(n_l)}(Q(\tau - \tau_{\text{min}}) - \frac{s}{Q} - \ell, \mu_S)
\]
Theory Description - EFT treatment

- Developments:
  - VFNS for final state jets (with massive quarks)
    [Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz '13 '14]
  - MSR mass & R-evolution
    [Hoang, Jain, Scimemi, Stewart '10]
  - Non-perturbative power-corrections are included via a shape function
    [Korchemsky, Sterman '99]
    [Hoang, Stewart '07]
    [Ligeti, Stewart, Tackmann '08]
  - Gap-scheme
    NNLL + NLO
    + non-singular + hadronization
    + renormalon-subtraction
    + top quark decay
  - Good convergence
  - Reduction of scale variation (NLL vs. NNLL)
Convergence, Mass Sensitivity

\[ \frac{d\sigma}{d\tau} = f(m_t^{MSR}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t) \]

- any scheme
- non-perturbative
- renorm. scales
- finite lifetime

- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution
- Higher mass sensitivity for lower Q
- Finite lifetime effects included
- Dependence on non-perturbative parameters
Preparing the Fits

• \( \frac{d\sigma}{d\tau} = f(m_t^{MSR}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t) \)

  any scheme non-perturbative renorm. scales finite lifetime

• Generating PYTHIA 8.205 Samples:
  at different energies: \( Q = 600, 700, 800, \ldots, 1400 \) GeV

  ▶ masses: \( m_t^{MC} = 170, 171, 172, 173, 174, 175 \) GeV
  ▶ width: \( \Gamma_t = 1.4 \) GeV
  ▶ tune: 7 (Monash)
  ▶ Statistics: \( 10^7 \) events for each set of parameters

• Feed MC data into Fitting Procedure: all ingredients are there

  Fit parameters: \( m_t^{MSR}, \alpha_s(m_Z), \Omega_1, \Omega_2, \Omega_3, \Omega_4 \)

  ▶ standard fit based on \( \chi^2 \) minimization
  ▶ analysis with 500 sets of profiles (\( \tau_2 \) dependent renorm. scales) for the each MC sample
  ▶ different Q-sets: 7 sets with energies between 600 - 1400 GeV
  ▶ different n-sets: 3 choices for fitranges - \((xx/yy)\)% of maximum peak height
Fit Results: Pythia vs. Theory

- Good agreement of PYTHIA 8.205 with $N^2$LL + NLO QCD description in peak region

- Perturbative uncertainties on theory side estimated via scale variations (profiles)

- MC incompatibility uncertainty estimate intrinsic difference between MC & theory via difference between different Q- & n-sets
Convergence & Stability: MSR vs Pole Mass

500 profiles; $\alpha_s = 0.118$; $\Gamma_t = 1.4$ GeV; tune 7; $Q = 700, 1000, 1400$ GeV; peak(60/80)%

Input: $m_t^{MC} = 173$ GeV

fit to find $m_t^{MSR}(1\text{GeV})$ or $m_t^{pole}$

- Good convergence and stability for $m_t^{MSR}(1\text{GeV})$
- $m_t^{MSR}(1\text{GeV})$ numerically close to $m_t^{MC}$
- Pole mass numerically not at all close to $m_t^{MC}$
- $\sim 1100/700$ MeV difference at NLL/NNLL!
- $m_t^{pole} \neq m_t^{PYTHIA 8.2}$

Similar findings from the other 20 data sets
Final Results for $m_t^{\text{MSR}}$

- All investigated MC top mass values show consistent picture

- MC top quark mass is indeed closely related to MSR mass within uncertainties:
  $m_t^{\text{MC}} \approx m_t^{\text{MSR}} (1 \text{GeV})$

- Calibration for the MSR mass (e.g. for $m_t^{\text{MC}} = 173$ GeV):
  
  \[
  (m_t^{\text{MSR}})_{\text{NLL}} = 172.80 \pm 0.29 \text{ GeV} \\
  (m_t^{\text{MSR}})_{\text{NNLL}} = 172.82 \pm 0.22 \text{ GeV}
  \]
Pole Mass Determinations

1. Pole mass implemented in code

\[ m_{t}^{\text{pole}} < m_{t}^{\text{MSR}(1\text{GeV})} < m_{t}^{\text{MC}} \]

2. Pole mass determined from MSR mass

\[ m_{t}^{\text{pole}} - m_{t}^{\text{MSR}(1\text{GeV})} = \left[ 0.173 + 0.138 + 0.159 + 0.230 + \mathcal{O}(\alpha_s^5) \right] \text{GeV} \]

\[ \alpha_s(M_Z) = 0.118; \ n_f = 5; \]

\[ m_{t}^{\text{MSR}(1\text{GeV})} < m_{t}^{\text{pole}} \]

- Calibration in terms of pole mass involves large higher-order perturbative corrections
  → additional uncertainties for pole mass extraction (for \( m_{t}^{\text{MC}} = 173 \text{ GeV} \))

\[ (m_{t}^{\text{pole}})_{\text{NLL}} = 172.45 \pm 0.52 \text{ GeV} \]

\[ (m_{t}^{\text{pole}})_{\text{NNLL}} = 172.72 \pm 0.40 \text{ GeV} \]
Tune dependence

500 profiles; $\Gamma_t = 1.4,-1$ GeV;tune 1, 3, 7;
diff. Q-sets; peak(60/80)%

$$m_t^{\text{PYTHIA}} = 173 \text{ GeV}$$  (tune 7 $\equiv$ Monash)

- tune dependence:
  $$m_{\text{MSR}}[\text{tune}] - m_{\text{MSR}}[7]$$

- clear sensitivity to tune
- $m_{\text{MC}}$ will depend on tune
- tune dependence is not a calibration uncertainty:
  
  (different tune $\Rightarrow$ different MC $\Rightarrow m_t^{\text{MC}}$)
Top width dependence

500 profiles; tune 1, 3, 7 (colors); diff. Q-sets; peak(60/80)%

- Use different top width values
  \( \Gamma_t = \{0.7, 1.4, 2.0\} \)

- Clear sensitivity to top width value

- Can be interpreted as observable or MC modelling dependence

- Our conclusion: Pythia has problems describing the correct top width dependence \( \rightarrow \Lambda_{cut} \) fixed
Other investigations
Other investigations

• Interesting:
  ▶ look into different observables
  ▶ look into pp collisions

• Massive C-Parameter is defined as:

\[
C_J = \frac{3}{2} \left[ 2 - \frac{1}{Q^2} \sum_{i \neq j} \left( \frac{p_i \cdot p_j}{p_i^0 p_j^0} \right)^2 \right]
\]

▶ coincides with the C-parameter for the massless case
▶ basically a mass-sensitive version of C-Parameter & very useful in the context of massive particles

NLO FO with massive gluon [Gardi, Magnea 2003]; NLO FO with massive quarks [Hoang, Mateu, MP in preparation]

• Highly mass sensitive: at parton level we have

\[
C_J^{\text{peak,stable}} = 12 \hat{m}^2 (1 - \hat{m}^2)
\]

\[
C_J^{\text{peak,unstable}} \simeq 12 \hat{m}^2 (1 - 4\hat{m}^2)
\] (for \( t \) to massless particles)

even at high enough boost: big peak-shift!

Top quark case: highly decay sensitive!
Inclusive treatment essentially means dressing the distribution with a Breit-Wigner.

2-jettiness peak:
- same position
- new features below peak

Inclusive setup is **sufficient** for peak position in unstable case!

Massive C-Parameter peak:
- displaced
- distorted

Inclusive setup is **not sufficient** for unstable case!
Simplifications & Decay Factorization

• To treat this we use some simplifications:
  - Top width small & $\Gamma_W \to 0$
  - Neglect spin correlations between top and decay products

→ top production and decay factorizes

• For the mentioned observables (leading in SCET+bHQET power counting):
  \[ e^{\text{unstable}}(X_n, \hat{\phi}_i, \hat{\theta}_i) = e^{\text{stable}}(X_n) + e^{(4)}(\hat{\phi}_i, \hat{\theta}_i) \]

• Final setup for singular contributions ($F^{\text{decay}}$ can be extracted from MC):
  \[ \frac{d\sigma^{\text{sing}}}{de} = \int d\hat{e} \frac{d\sigma^{\text{sing, incl}}}{de} (e - \hat{e}) F^{\text{decay}}(\hat{e}) \]

  - describe radiation from top
  - on-shell top decay

→ work in progress
pp Collisions

- Problems in pp collisions: very complicated environment
- Additional complications as compared with $e^+e^-$ annihilation:
  - Observable
  - Initial state radiation (ISR)
  - Beam remnant
  - Underlying event / MPI
    → no systematic approach

- Can write down factorization formula [Stewart, Tackmann, Waalewijn '09]

$$\frac{d^2\sigma}{dM_{J1}^2 dM_{J2}^2 dT_{\text{cut}}} = \text{tr} \left[ \hat{H}_{Qm} \hat{S}(T_{\text{cut}}, R, \ldots) \otimes F \right] \otimes J_B \otimes J_B \otimes \mathcal{I} \otimes f f$$

Jet Veto in Beam Region

Same Jet Functions!

Initial State Radiation

PDFs

Underlying event / MPI treatment model dependent
→ handled within hadronization model (jet veto dependent) [Stewart, Tackmann, Waalewijn '15]

- Improve on this problem by utilizing jet substructure techniques
soft drop (sd) & sd groomed jet mass

- Take jet and recluster it using Cambridge/Aachen. Go through branching history and remove soft branches until: \[ \frac{\min(E_i, E_j)}{E_i + E_j} > z_{\text{cut}} \theta_{ij} \]

removes soft radiation from jet, **keeps collinear jet core**.
Very effective in removing soft jet contamination from UE/MPI.

- Modified factorization theorem for **sd groomed jet mass (top case)**:
\[ d\sigma \over dM_J = N \times [J_B \otimes S_C] \otimes F \]

- To use this setup for a **precise** top quark mass measurements or a \( m_t^{MC} \) calibration: need to push this to NNLL (consistent mass scheme dependence)

\[ \rightarrow \text{work in progress} \]
Conclusion & Outlook

- First precise MC top quark mass calibration based on $e^+e^-$ 2-jettiness
  QCD calculations at NNLL + NLO based on an extension of the SCET approach to include massive quark effects

- Top mass calibration for PYTHIA 8.205 in terms of Pole and MSR mass.
  For $m_t^{\text{MC}} = 173$ GeV at NNLL:
    - $m_t^{\text{pole}} = 172.72 \pm 0.40$ GeV
    - $m_t^{\text{MSR}}(1\text{GeV}) = 172.82 \pm 0.22$ GeV

Outlook:

- Consistency checks

- Soft-Drop for pp jet-mass at NNLL & calibration