Towards top-quark pair production and decay at NNLO QCD

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RadCor 2017-09-26
Introduction

Polarised $t\bar{t}$ production amplitudes

Master integrals

Finite remainder function

Summary
Introduction
Why top-quark physics?

- Top mass
- Top mass difference
- Top charge
- Lifetime
- Top width

- Production cross section
- Production kinematics
- Production via resonance
- New particles

- Branching ratios $|V_{tb}|$
- Anomalous coupling
- New/Rare decays

- Spin correlation
- Charge asymmetry
- Color Flow

- s- & t- channel production, properties and searches in single top events
Theoretical developments

Stable onshell tops and spin summed:

- Total inclusive cross sections @ NNLO+NNLL accuracy
  [Czakon, Fiedler, Mitov ’13]
- Fully differential distributions @ NNLO
  [Czakon, Fiedler, Heymes, Mitov ’16]
- + EW corrections
  [Czakon, Heymes, Mitov, Pagani, Tsinikos, Zaro ’17]

Unstable tops + spin correlations:

- Approximate NNLO + NNLO decay
  [Gao, Papanastasiou ’17]
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Unstable tops + spin correlations:

- Approximate NNLO + NNLO decay
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Goal: $t\bar{t}$ production and decay at NNLO QCD

Narrow-Width-Approximation

- On-shell top-quarks
- Factorization of top-decay
- Separations of QCD corrections
- Keep spin correlations

→ polarised $t\bar{t}$-production amplitudes
Polarised $t\bar{t}$ production amplitudes
**$t\bar{t}$ production amplitudes**

Contributions to $\mathcal{M}(gg(q\bar{q}) \rightarrow t\bar{t})$: $gg|q\bar{q}$

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<thead>
<tr>
<th>Order</th>
<th>Contributions</th>
<th>Examples</th>
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<td>LO</td>
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- Decomposition into color- and Lorentz-structures → full color- and spin information

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Lorentz structures

Gluon channel

\[ \mathcal{M} = \epsilon_{1\mu}(p_1)\epsilon_{2\nu}(p_2)M^{\mu\nu} \]

\[ M^{\mu\nu} \] is a rank-2 Lorentz tensor
- Momentum conservation
- Transversality
- Equation of motion
- Parity conservation \( \rightarrow \) no \( \gamma_5 \)

Quark channel

- Two disconnected fermion lines
- Connection by gluons+loops

8 independent structures

\( (d = 4 \text{ dimensions}) \)

\[ M^{\mu\nu} = \sum_{j=1}^{8} M_j T_j^{\mu\nu} \]

4 independent structures

\[ \mathcal{M} = \sum_{i=1}^{4} M_j T_j \]

with \( T_j \sim \bar{\nu}_2 \Gamma_j u_1 \bar{u}_3 \Gamma'_j v_4 \)
Color structures

Color decomposition: \( \mathcal{M} = \sum_{ij} c_{ij} C_i M_j \)

Gluon channel color representations

- Gluons: \( a, b \) adjoint
- Quarks: \( c, d \) fundamental

\[
\begin{align*}
C_1 &= (T^a T^b)_{cd} \\
C_2 &= (T^b T^a)_{cd} \\
C_3 &= \text{Tr} \{ T^a T^b \} \delta_{cd}
\end{align*}
\]

Quark channel color representations

- Quarks: \( a, b \) fundamental
- Quarks: \( c, d \) fundamental

\[
\begin{align*}
C_1 &= \delta_{ac} \delta_{bd} \\
C_2 &= \delta_{ab} \delta_{cd}
\end{align*}
\]
Construct projectors: \( P_j = \sum_l B_{jl} (T_l)^\dagger \)

Extracting the \( B_{jl} \):

\[
\sum_{\text{spin/pol,col}} P_j A = A_j
\]

leads to system of equations

\[
\sum_{l,k} B_{jl} A_k \sum_{\text{spin/pol,col}} (T_l)^\dagger T_k = A_j
\]

Inversion \( \rightarrow \) coefficients \( B_{jl} \)

---

**Short summary**

\[
\mathcal{M} = \sum_{ij} c_{ij} C_i M_j
\]

- Gluon: 3 (color) \( \cdot \) 8 (spin)
  Quark: 2 (color) \( \cdot \) 4 (spin)
  \( \rightarrow \) combined 32 structures

- Scalar coefficients \( c_{ij} \):
  - Rational function of \( m_s = m_t^2 / s \),
    \( x = t/s \) and \( \epsilon \)
  - Scalar Feynman integrals
## Evaluation of coefficients

### Integration by parts identities (IBP)

\[ \int d^d k_1 d^d k_2 \frac{\partial}{\partial k_{i\mu}} \left( p_i \prod ( p \cdot k)^{b_i}_i \right) \left( \frac{(p \cdot k)^{b_i}_i}{(q^2_i + m^2_i)^{a_i}} \right) \]

\( \mathcal{O}(10^4) \) scalar Feynman integrals

→ 422 master integrals

### Master integrals

- Partially canonicalized

  new

analogous to \( [\text{Czakon '08}, \text{Czakon,Fiedler,Mitov '13}] \)

- Differential equations generated by IBPs

- High energy expansion as boundary condition

- Numerical integration for 'bulk' region

  → Interpolation grid

- Threshold expansion for

  \[ \beta = \sqrt{1 - 4m_s} \to 0 \]
Master integrals
**t channel diagrams**

- Divergences for \( \cos \Theta \to \pm 1 \) for high energies → complicates numerical integration
- Improve numerical stability by canonicalization of involved masters
- \( \epsilon - d \log \) form → expect more stable numerical evaluation
Partial canonical basis for master integrals

Idea:

- Perform rational basis transformation \( \vec{f} = \hat{T}(\epsilon, \vec{x}) \vec{f}_{\text{old}} \) such that DEQs have the form

\[
d\vec{f} = \epsilon d\hat{A}(\vec{x}) \vec{f}
\]

- Simple formal solution: \( \vec{f} = \exp \left( \epsilon \int d\hat{A}(\vec{x}) \right) \vec{f}_0 \)

Top-pair case: 422 masters

- Canonical basis for subset of master integrals:
  - No elliptic integrals involved
  - No transformation of kinematic variables needed
  \( \rightarrow 65 \) masters directly canonicalizable

- Using CANONICA [Meyer '16, '17]
Differential equations for master integrals

- Differential equations with respect to $m_s$ and $x$:

$$ m_s \frac{d}{dm_s} l_i = \sum c_k l_k $$

$$ x \frac{d}{dx} l_i = \sum d_k l_k $$

Boundaries

- Expansion around the high energy limit $m_s = \frac{m_t^2}{s} \to 0$
- Using Mellin-Barns representations and a lot of handwork to extract a series in $\epsilon$ and $m_s$ for each integral
- Expanding the differential equations in $\epsilon$
- Deep power-log expansions in $m_s$ for fixed $x$
  $\to$ algebraic system
Numerical evaluation of master integrals

- Using the differential equations to integrate numerically from the pre-calculated boundary conditions
- Leaving the real numbers and integrate in a complex plane to grid points

The grid

Choice of points:
- $\beta = \sqrt{1 - 4m_s} = i/80$ for $i = 1, \ldots, 79$
- 42 points for $x$: Gauss-Kronrod points in available phase-space
Threshold expansion

- Express DEQs in $\beta = \sqrt{1 - 4m_s}$
- Solve with pow-log ansatz for fixed $x = x_0$ (points given by interpolation grid)

$$l_i(\beta, x_0) = \sum \sum c_{imn} \beta^n \ln^m \beta$$

$n \in [-15, 51], m \in [0, 8]$

- Constrains by DEQ → eliminates most of the coefficients
- Match free coefficients to result from numerical integration
Finite remainder function
IR divergences and the finite remainder function

\[ |\mathcal{M}_n\rangle = \mathbf{Z}(\epsilon, \{p_i\}, \{m_i\}, \mu_R) |\mathcal{F}\rangle \]

- Complete factorization of IR structure $\rightarrow$ $\mathbf{Z}$ operator

\[
|\mathcal{M}_n^{(0)}\rangle = |\mathcal{F}_n^{(0)}\rangle \\
|\mathcal{M}_n^{(1)}\rangle = \mathbf{Z}^{(1)} |\mathcal{M}_n^{(0)}\rangle + |\mathcal{F}_n^{(1)}\rangle \\
|\mathcal{M}_n^{(2)}\rangle = \mathbf{Z}^{(2)} |\mathcal{M}_n^{(0)}\rangle + \mathbf{Z}^{(1)} |\mathcal{F}_n^{(1)}\rangle + |\mathcal{F}_n^{(2)}\rangle
\]

- $\mathbf{Z}$ can be calculated by its anomalous dimension equation

\[
\frac{d}{d \ln \mu} \mathbf{Z} = -\Gamma \mathbf{Z}
\]

- Depends on kinematics and operator on color space $\rightarrow$ Projection on color and spin structures
Finite remainder for polarised tops

2-Loop finite remainder for:
\[ q\bar{q} \rightarrow t_L \bar{t}_R \]

(work in progress)
### Summary of progress

#### Finished
- Projection LO, NLO, NNLO amplitudes
- Finite remainder of all coefficients
- Improved set of master integrals
- Kinematical expansions of coefficients
- Implementation of coefficients, color and spin structures in STRIPPER

#### Outlook
- Usage of amplitudes within STRIPPER
  ← Talk by Arnd Behring
- Implementation of decay phase-space and handling of decay products in STRIPPER
- QCD-corrections to decay
Backup
The subtraction scheme

- Method of evaluate the double-real emission radiation contribution to NNLO processes
- Decomposition of the phase-space to factorize the singular limits of the amplitude
- Suitable parameterizations to derive (integrated) subtraction terms

The NNLO event generator

- automated up to small driver program
- fully differential event generation
- several scales simultaneously
- different pdfs simultaneously
- stable tops
- pre-decided binned distributions
Narrow-Width-Approximation

\[ \left( p^2 - m^2 \right)^2 + m^2 \Gamma^2 \]

For cross sections: Integration over phase-space with limit \( \Gamma / m \to 0 \):

\[ \frac{1}{2\pi} \frac{\left( p^2 - m^2 \right)^2 + m^2 \Gamma^2}{\delta \left( p^2 - m^2 \right)} \]

On amplitude level:

\[ M = M_{NWA} + O(\Gamma/m) \]
Narrow-Width-Approximation

\[ \sim \frac{1}{p^2 - M^2 + i\Gamma M} \]

For cross sections: Integration over phase-space limit \( \Gamma / m \to 0 \):

\[ \frac{1}{(p^2 - m^2)^2 + m^2 \Gamma^2} \to \frac{2}{\pi^2 m \Gamma} \delta(p^2 - m^2) \]

On amplitude level:

\[ M = M_{\text{NWA}} + O(\Gamma / m) \]
Narrow-Width-Approximation

• enters matrix element as:
  \[ \sim \frac{1}{(p^2 - m^2)^2 + m^2 \Gamma^2} \]

• For cross sections: Integration over phase-space
  + limit \( \Gamma / m \to 0 \):
  \[ \frac{1}{(p^2 - m^2)^2 + m^2 \Gamma^2} \to \frac{2\pi}{2m\Gamma} \delta(p^2 - m^2) \]

• On amplitude level:
  \[ \mathcal{M} = \mathcal{M}_{\text{NWA}} + \mathcal{O}\left(\frac{\Gamma}{m}\right) \]
Amplitude factorization

\[
\mathcal{M} = \left( \tilde{A}(t \rightarrow bl^+\nu) \frac{i(\not{p}_t + m)}{p_t^2 - m^2 + im\Gamma_t} \right) \cdot \\
\tilde{A}(pp \rightarrow \bar{t}t) \cdot \\
\left( \frac{i(-\not{p}_t + m)}{p_{\bar{t}}^2 - m^2 + im\Gamma_{\bar{t}}} \tilde{A}(\bar{t} \rightarrow b\bar{l}^-\bar{\nu}) \right)
\]
Decay spinors

Narrow-Width-Approximation:

\[
\frac{i(-\not{p}_t + m)}{p_t^2 - m^2 + im\Gamma_t} \tilde{A}(\bar{t} \to \bar{bl}^-\bar{\nu}) \rightarrow \frac{i(-\not{p}_\bar{t} + m)}{\sqrt{2m\Gamma_t}} \tilde{A}(\bar{t} \to \bar{bl}^-\bar{\nu})
\]

Decay spinors

\[
\tilde{U}(p_t) = \tilde{A}(t \to bl^+\nu) \frac{i(p_t + m)}{\sqrt{2m\Gamma_t}}
\]

\[
\tilde{V}(p_{\bar{t}}) = \frac{i(-\not{p}_\bar{t} + m)}{\sqrt{2m\Gamma_t}} \tilde{A}(\bar{t} \to \bar{bl}^-\bar{\nu})
\]

Amplitude:

\[
\mathcal{M} = \tilde{U}(p_t)\tilde{A}(pp \to \bar{t}t)V(p_{\bar{t}}) + O\left(\frac{\Gamma_t}{m}\right)
\]
QCD corrections to decay

Vertex corrections (for massless final state):

\[ \Gamma_{\mu} = g \sqrt{2} \left\{ \gamma_{\mu} \left[ F_{1L}P_{L} + F_{1R}P_{R} \right] + i \sigma_{\mu\nu} q_{\nu} \right\} ^{2} m_{t} \left\{ F_{2L}P_{R} + F_{2R}P_{L} \right\} \]

\[ \times \text{W propagator and decay vertex} \]

\[ \bar{u}(p_{\nu}) ig_{\text{W}} \sqrt{2} \gamma_{\nu} (1 - \gamma_{5}) 2 v_{(p_{l} + p_{n})} \cdot -i (g_{\nu\mu} - q_{\nu} q_{\mu} q^{2}) q^{2} - m_{W}^{2} \text{W} + i \Gamma_{\text{W}} m_{\text{W}} \bar{u}(p_{b}) i \Gamma_{\mu} \]
QCD corrections to decay

Vertex corrections (for massless final state):

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\Gamma_{\mu} = g \sqrt{2} \left\{ \gamma_{\mu} \left[ F_{1L} P_{L} + F_{1R} P_{R} \right] + i \sigma_{\mu\nu} q_{\nu} \right\}^2 + 2 m_t \left[ F_{2L} P_{R} + F_{2R} P_{L} \right] \}
\]

times W propagator and decay vertex

\[
\bar{u}(p) ig_W \sqrt{2} \gamma_{\nu} \left( 1 - \gamma_5 \right) v(p_l + q) \cdot -i (g_{\nu\mu} - q_{\nu} q_{\mu} q^2) q^2 - m_W^2 + i \Gamma_W m_W \bar{u}(p) i \Gamma_{\mu} R
\]

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QCD corrections to decay

Simplification II

restrict to leptonic top decays

- Vertex corrections (for massless final state):

\[
\Gamma^{\mu} = \frac{g}{\sqrt{2}} \left\{ \gamma^{\mu} [F_{1L} P_L + F_{1R} P_R] + i\sigma^{\mu\nu} q_\nu \frac{2}{2m_t} [F_{2L} P_R + F_{2R} P_L] \right\}
\]

- times W propagator and decay vertex

\[
\bar{u}(p_\nu) \frac{ig_W}{\sqrt{2}} \gamma^{\nu} \left(1 - \gamma_5 \right) \frac{1}{2} v(p_{l^+}) \cdot \frac{-i(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2})}{q^2 - m^2_W + i\Gamma_W m_W} \bar{u}(p_b) i\Gamma^{\mu}
\]

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## Contributions to amplitude

<table>
<thead>
<tr>
<th>Decay</th>
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<th>NLO</th>
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</thead>
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Kinematics and polarization

External Momenta

\[ p_1^2 = p_2^2 = 0 \]
\[ p_3^2 = p_4^2 = m^2 \]

Mandelstamm variables

\[ s = (p_1 + p_2)^2 \]
\[ t = (p_1 - p_3)^2 \]
\[ u = (p_2 - p_3)^2 \]
\[ s + t + u = 2m^2 \]

Polarization sum external gluons (axial gauge)

\[ \sum_{\text{pol}} \epsilon^*_i \epsilon_{i\nu} = -g_{\mu\nu} + \frac{n_{i\mu} p_{i\nu} + n_{i\nu} p_{i\mu}}{n_i \cdot p_i} \]

Equation of motion for external (anti)quarks

\[ (\not{p} - m) U = 0 \]
\[ (\not{p} + m) V = 0 \]
IBP reduction

General two-loop integral:

\[
\int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \prod_i \frac{1}{D_i^{n_i}} \prod_j N_j^{n_j}
\]

with \( D_i = (\sum p + \sum l)^2 - m^2 \) and \( N_i = l \cdot p \)

Basic Idea of Integration-By-Part (IBP) reduction:

\[
\int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{\partial}{\partial q^\mu} q^\mu I(l_1, l_2, \{p_{ext}\}) = 0 \text{ with } q = l_1, l_2, \{p_{ext}\}
\]

- Relations between different integrals
  ⇒ Relate difficult integrals to easy ones
- Reduction to set of master integrals
UV renormalization and decoupling

\[ |M_{g,q}(\alpha_S^0, m^0, \epsilon)\rangle = 4\pi \alpha_S^0 \left[ |M_{g,q}^{(0)}(m^0, \epsilon)\rangle + \left( \frac{\alpha_S^0}{2\pi} \right) |M_{g,q}^{(1)}(m^0, \epsilon)\rangle + \left( \frac{\alpha_S^0}{2\pi} \right)^2 |M_{g,q}^{(2)}(m^0, \epsilon)\rangle \right] \]

UV-renormalized amplitude:

\[ |\mathcal{M}_{g,q}^R \left( \alpha_S^{(n_f)}(\mu), m, \mu, \epsilon \right)\rangle = \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^{-2\epsilon} Z_{g,q} Z_Q |M_{g,q}(\alpha_S^0, m^0, \epsilon)\rangle \]

- \( Z_{g,q}, Z_Q \) : onshell renormalization constants
- \( m^0 = Z_m m \)
- \( \alpha_S^0 = \left( \frac{e^{\gamma_E}}{4\pi} \right)^\epsilon \mu^{2\epsilon} Z_{\alpha_S}^{(n_f)} \alpha_S^{(n_f)}(\mu) \)
  \( \overset{\hat{=}}{=} \) \( \tilde{M}S \)-scheme with \( n_f \) flavours

Decoupling

- \( n_f = n_l + n_h \) is not feasible
- decouple the running of \( \alpha_S \) from the \( n_h \) quarks
- \( \alpha_S^{(n_f)} = \tilde{\zeta}_S \alpha_S^{(n_l)} \)