Associated production of a top pair with a heavy boson at the LHC to NLO+NNLL accuracy

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Outline

- Introduction: Top quark pair + Higgs/W/Z production at the LHC
- Factorization of the partonic cross section in the partonic threshold limit and resummation of soft gluon emission corrections
- Results: total cross sections and differential distributions at NLO+NNLL accuracy for tTH, tTW, tTZ processes at the LHC
In collaboration with


Top quark and Higgs boson

the two heaviest Standard Model (SM) particles
mt~173 GeV, mH~125 GeV
Top quark and Higgs boson

- according to the SM, the top quark is the elementary particle which couples most strongly to the Higgs boson

- gluon fusion channel provides the largest Higgs production cross section at the LHC, but tTH process allows direct access to top-quark Yukawa coupling

- measurement of the top-quark Yukawa coupling is one of the goals of the Run II of LHC
Higgs production channels

LHC @ 14 TeV

54.67 pb
4.18 pb
1.50 pb (WH)
0.88 pb (ZH)
0.61 pb

Expected to be seen at LHC 13 TeV, direct measurement of the top Yukawa coupling

\[ \sigma \propto g_{ttH}^2 \]
top pair + Higgs calculations

- Cross section and some distributions computed to NLO QCD (Beenakker, Dittmaier, Kraemer, Plumper, Spira, Zerwas '01-'02 and Dawson, Reina, Wackeroth, Orr, Jackson '01-'03)

- top pair + Higgs benchmark process to test automated NLO multileg codes (Frixione et al. '11; Hirschi et al '11; Garzelli et al '11; Bevilacqua et al. '11)

- EW corrections to the parton level cross section are known (Frixione, Hirshi, Pagani, Shao, Zaro '14; Zhang, Ma, Chen, Guo '14; Frixione, Hirshi, Pagani, Shao, Zaro '15)

- NLO QCD corrections were interfaced with SHERPA and POWHEG BOX (Gleisberg, Hoeche, Krauss, Schonherr, Schaumann '09; Hartanto, Jaeger, Reina, Wackeroth '15)

- NLO QCD corrections to $pp \rightarrow e^+\nu e^-\bar{\nu}_\mu b\bar{b}H$ (Denner, Feger '15)

- NLO+NLL resummation of soft gluon emissions for the total cross section (production threshold limit) (Kulesza, Motyka, Stebel, Theeuwes '15)

- nNLO in the “PIM” threshold limit from NNLL resummation formula (AB, A. Ferroglia, B. Pecjak, A. Signer, L. Yang '15)
and the list continues...

- NLO+NNLL resummation in “PIM” kinematics, RG-evolution in Mellin space (AB, A. Ferroglia, B. Pecjak, A. Signer, L. Yang ’16)

- NLO EW and QCD corrections with off-shell top-antitop pairs (A. Denner, J. Lang, M. Pellen, S. Uccirati ’16)

- NLO+NNLL resummation in “PIM” kinematics with direct QCD approach (invariant mass distribution of the triplet) (Kulesza, Motyka, Stebel, Theeuwes ’17)

- Pseudoscalar couplings at NLO+NLL accuracy (AB, A. Ferroglia, M. Fiolhais, A. Onofre ’17)
Top pair + W or Z boson

- tTW and tTZ are the two heaviest set of particles measured at the LHC with c.o.m. energy of 7, 8, 13 TeV

- 8 TeV
  \[ \sigma_{t\bar{t}W} = 382^{+117}_{-102} \text{ fb (CMS)} \]
  \[ \sigma_{t\bar{t}Z} = 242^{+65}_{-55} \text{ fb (CMS)} \]

- 13 TeV
  \[ \sigma_{t\bar{t}W} = 800^{+176.9}_{-162.8} \text{ fb (CMS)} \]
  \[ \sigma_{t\bar{t}Z} = 1000^{+150}_{-128} \text{ fb (CMS)} \]

- Important to detect anomalies in the top couplings of the Z boson, and can be considered background processes in new physics searches

- Both processes were calculated to NLO QCD accuracy by several groups (A. Lazopoulos, T. McElmurry, K. Melnikov, F. Petriello ’07 - ’08, M.V. Garzelli, A. Kardos, C.G. Papadopoulos, Z. Trocsanyi ’12, J.M. Campbell, R.K. Ellis ’12, F. Maltoni, M.L. Mangano, I. Tsinikos, M. Zaro ‘14)

- EW corrections are also known (Frixione, Hirshi, Pagani, Shao, Zaro ’15)

- NLO+NNLL for tTW in momentum space (Li, Li and Li ’14)
“Triplet” Invariant Mass kinematics

Tree Level subprocesses

\[ q(p_1) + \bar{q}(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + H(p_5) \]
\[ g(p_1) + g(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + H(p_5) \]

\[ \hat{s} = (p_1 + p_2)^2 = 2p_1 \cdot p_2 \]
\[ M^2 = (p_3 + p_4 + p_5)^2 \]

Partonic center of mass energy squared

Invariant mass of the tTH final state

When real radiation is present in the final state \( \hat{s} \neq M^2 \)

\[ z = \frac{M^2}{\hat{s}} \rightarrow 1 \]

“PIM” soft limit

In the soft emission limit a scale hierarchy emerges

\[ \hat{s}, M^2, m_t^2, m_H^2 \gg \hat{s}(1 - z)^2 \gg \Lambda_{QCD}^2 \]

Hard scales

Soft scale
Large logarithmic corrections

- The partonic cross section for top pair + H (or W or Z) production receives potentially large corrections from soft gluon emission diagrams.

- The partonic cross section depends on logarithms of the ratio of two different scales:

\[ L \equiv \ln \left( \frac{\text{“hard” scale}}{\text{“soft” scale}} \right) \]

- It can be that \( \alpha_s L \sim 1 \)

- One needs to reorganise the perturbative series: Resummation

- This can be carried out using effective field theory methods (soft-collinear effective theory)
Factorization and Resummation

\[\sigma(s, m_t, m_H) = \frac{1}{2s} \int_{\tau_{\text{min}}}^{1} d\tau \int_{\tau}^{1} \frac{dz}{\sqrt{z}} \sum_{ij} \mathcal{H}_{ij}(\tau, \mu) \mathcal{S}_{ij}\left(\frac{M(1-z)}{\sqrt{z}}, \{p_i\}, \mu\right) \]

\[\int dPS_{t\bar{t}H} \text{Tr} \left[ \mathcal{H}_{ij}(\{p_i\}, \mu) \mathcal{S}_{ij}\left(\frac{M(1-z)}{\sqrt{z}}, \{p_i\}, \mu\right) \right] + \mathcal{O}(1-z)\]

\[\tau = \frac{M^2}{s}, \quad \tau_{\text{min}} = \frac{(2m_t + m_H)^2}{s}\]

\[P_n(z) \equiv \left[ \frac{\ln^n(1-z)}{1-z} \right] + \]

The hard and soft functions satisfy RG equations that can be solved to obtain the resummed hard-scattering kernels

\[C_{ij}(z, \mu_f) = \exp \left[ 4a_{\gamma'}(\mu_s, \mu_f) \right] \text{Tr} \left[ U_{ij}(p, \mu_h, \mu_s) H_{ij}(p, \mu_h) \right] \]

\[\times \ U_{ij}^\dagger(p, \mu_h, \mu_s) \tilde{S}_{ij}\left(\ln \frac{M^2}{\mu_s} + \partial_\eta, \{p\}, \mu_s\right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{z^{1/2-\eta}}{(1-z)^{1-2\eta}}.\]

Alessandro Broglio 28/09/2017
Mellin space

- Resummation can also be carried out in Mellin space by taking the Mellin transform of the factorized cross section, more similar to “direct QCD” resummation.

- The total cross section can be recovered with an inverse Mellin transform

\[
\sigma(s, m_t, m_H) = \frac{1}{2s} \int_{\tau_{\text{min}}}^{1} \frac{d\tau}{\tau} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sum_{ij} \tilde{f}_i j(N, \mu) \int dPS_{t\bar{t}H} \tilde{c}_{ij}(N, \mu)
\]

- Hard and soft functions are evaluated at values of the scale where the large corrections are absent \( \mu_h = M, \quad \mu_s = M/N \)

- RG evolution to obtain the hard-scattering kernels at the factorization scale

\[
\tilde{c}_{ij}(N, \mu_f) = \text{Tr} \left[ \tilde{U}_{ij}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s) \hat{H}_{ij}(\{p\}, \mu_h) \tilde{U}_{ij}^\dagger(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s) \times \tilde{s}_{ij} \left( \ln \frac{M^2}{N^2 \mu_s^2}, \{p\}, \mu_s \right) \right]
\]

\[
\tilde{U}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s) = \exp \left\{ \frac{4\pi}{\alpha_s(\mu_h)} g_1(\lambda, \lambda_f) + g_2(\lambda, \lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi} g_3(\lambda, \lambda_f) + \cdots \right\} \times u(\{p\}, \mu_h, \mu_s),
\]

\[
\lambda = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_s}, \quad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_f}
\]
3.2 Total cross section

For the differential cross section in a given bin) is then evaluated by combining in quadrature the quantities cross section, or the value of a differential cross section at a default value for the scale is estimated by varying each scale in the interval \( \mu_i \in [1, 2] \). While we are mainly interested in NNLL resummation effects, we will also calculate NLO+NLL results, defined as

\[
\sigma_{\text{NLO}+\text{NNLL}} = \sigma_{\text{NLO}} + \left[ \sigma_{\text{NNLL}} - \sigma_{\text{approx. NLO}} \right]
\]

and

\[
\sigma_{\text{NLO}+\text{NLL}} = \sigma_{\text{NLO}} + \left[ \sigma_{\text{NLL}} - \sigma_{\text{NLL expanded to NLO}} \right]
\]
# Total cross section $tTH$

NLO obtained from MadGraph5_aMC@NLO

$\sqrt{s} = 13\text{ TeV} \quad \mu_{f,0} = M/2$

<table>
<thead>
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<th>code</th>
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<td>MG5_aMC</td>
<td>$474.8^{+47.2}_{-51.9}$</td>
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<tr>
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<td>(NLO+NNLL)$_{NNLO}\text{exp.}$</td>
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<td>$482.7^{+10.7}_{-21.1}$</td>
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**Uncertainties**

\[ \Delta O_{i}^{+} = \max\{O(\kappa_{i} = 1/2), O(\kappa_{i} = 1), O(\kappa_{i} = 2)\} - \bar{O}, \]

\[ \Delta O_{i}^{-} = \min\{O(\kappa_{i} = 1/2), O(\kappa_{i} = 1), O(\kappa_{i} = 2)\} - \bar{O}, \]
Total cross section $tTH$

![Graph showing total cross section for different perturbative approximations.]

The results obtained are summarized in table 2, where we set $\mu_{f,0} = M/2$, in table 3, where we set $\mu_{f,0} = M$, and in figure 2, which presents a visual comparison between the main results at the two different scales.

We first compare the approximate NLO corrections generated from NNLL soft-gluon resummation (second row of each table), with the full NLO corrections without (third row of each table) and with (fourth row of each table) the $qg$ channel. Since the approximate NLO results include only the leading-power contributions from the gluon fusion and quark-annihilation channels in the soft limit, the difference between these results and the NLO corrections without the $qg$ channel gives a measure of the importance of power corrections away from this limit. The two results are seen to differ by no more than a few percent, even though the NLO corrections are large. This shows that at NLO the power corrections away from the soft limit for these channels are quite small. Comparing the NLO results with and without the $qg$ channel reveals that this channel contributes significantly to the scale uncertainty, in particular when one chooses $\mu_{f,0} = M/2$. The fact that the leading terms in the soft limit make up the bulk of the NLO correction provides a strong motivation to resum them to all orders. No information is lost by doing this, as both sources of power corrections are taken into account by matching with NLO as discussed above. Since the power corrections are treated in fixed order, the perturbative uncertainties associated with them are estimated through the standard approach of scale variations.

We next turn to the NLO+NLL and NLO+NNLL cross sections, which are the main results of this section. The exact numbers are given in tables 2 and 3, and a pictorial representation is given in figure 2. The results for the default scale choice $\mu_{f,0} = M/2$ converge quite nicely. The scale uncertainties get progressively smaller when moving from NLO to NLO+NLL to NLO+NNLL, and the higher-order results are roughly within the range predicted by the uncertainty bands of the lower-order ones. For $\mu_{f,0} = M$ the convergence is still reasonable but not quite as good, mainly because the NLO and NLO+NLL results are noticeably smaller than at $\mu_{f,0} = M/2$. Interestingly, the NLO+NLL result has...
We want at this point to study results for a different choice of the default factorization scale, namely $\mu_{f,0} = M/2$. As discussed for the case of the total cross section in section 3.2, the numerical impact of the soft emission corrections with the choice $\mu_{f,0} = M$ is significantly larger than the impact of the same corrections with the choice $\mu_{f,0} = M/2$. However, NLO+NNLL predictions obtained with the two choices are in good agreement. For what concerns the differential distributions studied here this can be seen by comparing NLO+NLL calculations carried out with the choice $\mu_{f,0} = M$ or $\mu_{f,0} = M/2$ (figure 8), and NLO+NNLL calculations with $\mu_{f,0} = M$ or $\mu_{f,0} = M/2$ (figure 9). Figure 8 shows that at NLO+NLL the overlap between the distributions evaluated at $\mu_{f,0} = M$ and $\mu_{f,0} = M/2$ is not particularly good, with the band at $\mu_{f,0} = M/2$ slightly larger than the one at $\mu_{f,0} = M$ in all bins. Figure 9 shows instead that the NLO+NNLL distributions...
Distributions $t\bar{t}H$: NLL vs NNLL

Figure 6. Differential distributions $M \mu_f,0 = M/2$ at NLO+NNLL (blue band) compared to the NLO+NLL calculation (red band). The uncertainty bands are generated through scale variations. At $M \mu_f,0 = M$ and $M \mu_f,0 = M/2$ have a large overlap in all bins. The scale uncertainty at NLO+NNLL with $M \mu_f,0 = M$ is larger than the scale uncertainty at $M \mu_f,0 = M/2$ in all bins.

The good agreement between the two bands shown in each panel of figure 9 indicates that NLO+NNLL predictions are quite stable with respect to different (but reasonable) choices of the standard value for the factorization scale.

4 Conclusions

In this paper we evaluated the resummation of the soft emission corrections to the associated production of a top-quark pair and a Higgs boson at the LHC in the partonic threshold limit $z \to 1$. The calculation is carried out to NNLL accuracy and it is matched...
Distributions $t\bar{t}H$: NNLL vs expansions

Figure 7. Differential distributions ratios for $\mu_f$, $0 = M/2$, where the uncertainties are generated through scale variations.

To the complete NLO cross section in QCD. The numerical evaluation of observables at NLO+NNLL was carried out by means of an in-house parton level Monte Carlo code developed for this work, based on the resummation formula derived in [16]. The resummation procedure is however carried out in Mellin space, following the same approach employed in [43, 44, 19] for the calculation of the (boosted) top-quark pair production cross section and in [19] for the calculation of the cross section for the associated production of a top-quark pair and a $W$ boson.

In the previous sections we presented predictions for the total cross section for this production process at the LHC operating at a center-of-mass energy of 13 TeV. In addition, we showed results for four different differential distributions depending on the four-momenta of the massive particles in the final state: the differential distributions in the invariant mass of the $t\bar{t}H$ particles, in the invariant mass of the $t\bar{t}$ pair, in the transverse momentum of the Higgs boson, and in the transverse momentum of the top quark. We found that the relative size of the NNLL corrections with respect to the NLO cross section is rather sensitive to the choice of the factorization scale $\mu_f$. In particular, for the two choices which we analyzed in detail, namely $\mu_f, 0 = M/2$ and $\mu_f, 0 = M$, it was found that the NNLL corrections enhance the total cross section and differential distributions in all bins considered.

The NNLL soft emission corrections expressed as a percentage of the NLO observables are larger at $\mu_f, 0 = M$ than they are at $\mu_f, 0 = M/2$. However, by comparing NLO+NNLL – 17 –
Pseudoscalar couplings in tTH

\[ \mathcal{L}^t_0 = -\frac{m_t}{v} \bar{\psi} (\cos \alpha + i \sin \alpha \gamma_5) \psi X_0 \]

[arXiv: 1707.01803]

5 sigmas observation of this channel (on the SM hypothesis) would set the limit

\[ |\cos \alpha| > 0.83^{+0.01}_{-0.02} \]
NNLL resummation formula (numerical dependence on these scales is formally of NNLL order (and is indeed canceled explicit dependence on the matching scales as a result, the constant piece of the NLO expansion of the NLL resummation formula contains NNLL resummation onto the NLO result. However, in contrast to the approximate NLO perturbation theory, we will also calculate NLO+NLL results, defined as

\[ \sigma = \sigma_{\text{NLO}} + \sigma_{\text{NLL}} \]

The di-

The experimental collaborations reported measurements of the $s$ = 13 TeV cross section $\sigma$ for $t\bar{t}$ production at the LHC with the $\mu_f, \mu_R$ = $M/2$. The predictions for $t\bar{t}$ production at energies $\sqrt{s} = 8$ and 13 TeV are in perfect agreement with the measurements at both 8 and 13 TeV, the predictions for $t\bar{t}$ are instead to NLO+NNLL calculations.

<table>
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<td>NLO+NNLL</td>
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<td>$777.8^{+61.3}_{-65.2}$</td>
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<td>in-house MC + MG5_aMC</td>
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Distributions $tT\bar{Z}$

Figure 5. Differential distributions with $\mu_{f,0} = M/2$ at NLO+NNLL (blue band) compared to the NLO calculation (red band). The uncertainty bands are generated through scale variations of $\mu_f, \mu_s$ and $\mu_h$ as explained in the text.

The NLO+NNLL accuracy calculation, which is obtained by varying $\mu_f, \mu_s$, and $\mu_h$ as described above, is smaller than the NLO scale uncertainty band obtained by varying $\mu_f$.

Results at NLO+NLL and NLO+NNLL accuracy are compared in figure 6. The main effect of the NNLL correction with respect to the NLL ones is an increase of the central value of the bins in the tail of the $M$ and $M_{t\bar{t}}$ distributions. The scale uncertainty bands turn out to be of similar size at NLO+NLL and NLO+NNLL in almost all bins shown.

Figure 7 shows the ratio of distributions at various level of precision to the central value of the NLO+NNLL calculation in each bin. In particular, the blue band refers to NLO+NNLL distributions, the dashed red band to nNLO distributions and the dashed black band to distributions obtained from the NNLO expansion of the NLO+NNLL resummation. The NLO+NNLL expanded distributions differ from the NLO+NNLL distributions...
Top pair + W or Z production

Table 3. Total cross section for $t\bar{t}Z$ and $t\bar{t}W$ production at the LHC with $\sqrt{s} = 8$ and 13 TeV and MMHT 2014 PDFs. The default value of the factorization scale is $\mu_f = \frac{M}{2}$, and the uncertainties are estimated through variations of this scale (and of the resummation scales $\mu_s$ and $\mu_h$ when applicable).

SM theory vs ATLAS data
LHC 8 TeV

SM theory vs CMS data
LHC 13 TeV

3.3 Differential distributions
In this section we obtain predictions for four differential distributions which depend on the momenta of the final state massive particles. The distributions are
i) the distribution differential with respect to the $t\bar{t}Z$ invariant mass, $M$, ii) the distribution differential with respect to the $t\bar{t}$ invariant mass, $M_{t\bar{t}}$, iii) the distribution differential with respect to the...
Predictions for the total cross sections together with several differential distributions were obtained at NLO+NNLL for $t\bar{t}H$, $t\bar{t}W$, $t\bar{t}Z$ production at LHC

- Reduction of the theoretical uncertainty

- In principle cuts on the momenta of the final-state particles can be easily applied

- State-of-the-art: combine QCD predictions (NLO+NNLL) with EW corrections

- $t\bar{t}$ pair + photon

- Implement the decay of the final state particles in the NWA
**Distributions: nLO vs NLO**

**tTH**: nLO vs NLO without qg channel

**tTZ**: nLO vs NLO without qg channel
Here we present the C.2 Boosted soft limit resummed result. First, we present the C.1 Soft limit
functions which appear in the evolution factors Eqs. (42) and remind the reader that, by a factor of

\[
g_1 (\lambda_s, \lambda_f) = \frac{\Gamma_0}{2\beta_0^2} \left[ \lambda_s + (1 - \lambda_s) \ln(1 - \lambda_s) + \lambda_s \ln(1 - \lambda_f) \right],
\]

\[
g_2 (\lambda_s, \lambda_f) = \frac{\Gamma_0 \beta_1}{2\beta_0^3} \left[ \ln(1 - \lambda_s) + \frac{1}{2} \ln^2(1 - \lambda_s) \right] - \frac{\Gamma_1}{2\beta_0} \ln(1 - \lambda_s) + \frac{\gamma_0}{\beta_0} \ln \frac{1 - \lambda_s}{1 - \lambda_f}
\]

\[
+ \frac{\Gamma_0}{2\beta_0} L_s \ln \frac{1 - \lambda_s}{1 - \lambda_f} + \frac{\Gamma_0}{2\beta_0} L_h \ln(1 - \lambda_f) + \frac{1}{1 - \lambda_f} \left\{ \frac{\Gamma_0 \beta_1}{2\beta_0^3} \lambda_s [1 + \ln(1 - \lambda_f)] - \frac{\Gamma_1}{2\beta_0^2} \lambda_s \right\},
\]

\[
\lambda_i = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_i}
\]
Distributions $t\bar{t}H$: nLO vs NLO

$nLO$ vs full NLO

$nLO$ vs NLO

without qg channel

We conclude our discussion of the results obtained with the choice $\mu_i^f = \frac{M}{2}$ for the various distributions.

The blue band refers to NLO+NNLL calculations, the dashed red band to nNLO calculations and the dashed black band to the NNLO expansion of the NLO+NNLL resummation. The dashed red band and the dashed black band is due to constant NNLO corrections, which are roughly at the 5% level, and as for the uncertainties contribute roughly at the 5% level, and as for the uncertainty of the all-orders resummation. Numerically, these effects are not visible in the figure.

The ratio, separately for each bin, of the NLO+NNLL results everywhere with the exception of the bins in the far tail of the distributions at NLO+NNLL is to shrink slightly the scale uncertainty bands with respect to the NLO (red band). The default factorization scale is chosen as $\mu_i^f = \frac{M}{2}$.
**Distributions tTZ: nLO vs NLO**

*nLO vs full NLO*

*nLO vs NLO without qg channel*
Distributions $t\bar{t}Z$: nLO vs NLO

In the present work we carried out the resummation of soft gluon emission corrections to the associated production of a top-antitop quark pair and a $Z$ boson. The resummation was studied in the partonic threshold limit $z \rightarrow 1$ and was implemented to NNLL accuracy. Numerical calculations of the total cross section and differential distributions to NNLL accuracy were carried out by means of an in-house partonic Monte Carlo code which we developed for this work. The output of this code was matched with NLO calculations obtained from $\text{MG5\text{-}aMC}$. The final outcome of this work is represented by the NLO+NNLL calculations of the total cross section and differential distributions for the LHC operating at a center-of-mass energy of 13 TeV presented in the previous section. The code can be easily adapted to carry out phenomenological studies which include cuts on the top, antitop and/or $Z$ boson momenta.

With the choice of the factorization scale made in this work, we can conclude that the soft emission corrections to $t\bar{t}Z$ production evaluated to NNLL accuracy lead to a moderate increase of the total cross section and differential distributions with respect to NLO calculations of the same observables. The residual perturbative uncertainty at NLO+NNLL accuracy, estimated by varying the soft, hard and factorization scales as explained in the text, is smaller than the NLO scale uncertainty, thus making our evaluations of the cross...
### Total cross section \( t \bar{t}H \)

NLO obtained from MadGraph5_aMC@NLO

\[
\sqrt{s} = 13 \text{ TeV} \quad \mu_{f,0} = M
\]

<table>
<thead>
<tr>
<th>order</th>
<th>PDF order</th>
<th>code</th>
<th>( \sigma ) [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>LO</td>
<td>MG5_aMC</td>
<td>293.5±85.2(^{+61.7}_{-61.7})</td>
</tr>
<tr>
<td>app. NLO</td>
<td>NLO</td>
<td>in-house MC</td>
<td>444.7±28.6(^{+39.2}_{-39.2})</td>
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<tr>
<td>NLO no qg</td>
<td>NLO</td>
<td>MG5_aMC</td>
<td>447.0±35.1(^{+40.4}_{-40.4})</td>
</tr>
<tr>
<td>NLO</td>
<td>NLO</td>
<td>MG5_aMC</td>
<td>423.0±51.9(^{+49.7}_{-49.7})</td>
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<tr>
<td>NLO+NLL</td>
<td>NLO</td>
<td>in-house MC +MG5_aMC</td>
<td>466.2±22.9(^{+26.8}_{-26.8})</td>
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<tr>
<td>NLO+NNLL</td>
<td>NNLO</td>
<td>in-house MC +MG5_aMC</td>
<td>514.3±42.9(^{+39.5}_{-39.5})</td>
</tr>
<tr>
<td>nNLO (Mellin)</td>
<td>NNLO</td>
<td>in-house MC +MG5_aMC</td>
<td>488.4±9.4(^{+8.3}_{-8.3})</td>
</tr>
<tr>
<td>(NLO+NNLL)(\text{NNLO exp.})</td>
<td>NNLO</td>
<td>in-house MC +MG5_aMC</td>
<td>485.7±6.8(^{+8.3}_{-15.0})</td>
</tr>
</tbody>
</table>
Dynamical threshold enhancement

We rewrite the DY cross section introducing the luminosity function

\[
\frac{d\sigma^\text{thresh}}{dM^2} = \frac{4\pi\alpha^2}{3N_cM^2s} \int_\tau^1 \frac{dz}{z} C(z, M, \mu_f) f(f(\tau/z, \mu_f))
\]

\[
z \equiv \frac{M^2}{\hat{s}} \quad \tau \equiv \frac{M^2}{s}
\]

\[
f(y, \mu_f) = \sum_q e_q^2 \int_y^1 \frac{dx}{x} \left[ f_q/N_1(x, \mu_f) f_{\bar{q}}/N_2(y/x, \mu_f) + (q \leftrightarrow \bar{q}) \right]
\]

Does the soft limit \(z \to 1\) provide a good approximation to the exact result?

Two situations in which the threshold region is enhanced:

- the threshold contributions are enhanced near the kinematic limit \(\tau \sim 1\) and \(z \geq \tau\) is near 1

- the relevance of the threshold region arises dynamically due to the steeply falling behaviour of the parton luminosity function