CoLoRFulNNLO for the LHC

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in collaboration with

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Introduction:

Motivation:

Just take a look at the talks of this conference…

Automated NNLO QCD computations are about to become a reality (slowly but surely)

Double-real and real-virtual kinematic singularities are subtracted or sliced out

More and more methods enter the mature age to form a general prescription which can be coded into general numeric frameworks

CoLoRFulNNLO is such a prescription…

Computations with CoLoRFulNNLO:

In CoLoRFulNNLO kinematic singularities are subtracted (and given back integrated out).

These subtractions:

- are fully local
- derived from first principles: factorization and limits of amps
- several of them share the same PS mapping (grouping!)
- are general
- do not require color-stripped amplitudes
- are process independent
- are worked out for both ISR and FSR
- are implemented in MCCSM (AK) which is also fully general

Computations with CoLoRFulNNLO:

… for an observable J:

$$\sigma[J] = \sigma^{\mathrm{LO}}[J] + \sigma^{\mathrm{NLO}}[J] + \sigma^{\mathrm{NNLO}}[J] + \ldots$$

The NNLO contributions:

$$\sigma^{\mathrm{NNLO}}[J] = \sigma^{\mathrm{NNLO}}_{m+2}[J] + \sigma^{\mathrm{NNLO}}_{m+1}[J] + \sigma^{\mathrm{NNLO}}_{m}[J] =$$

$$= \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

Up to 2 unresolved emissions

2 loops

Up to 1 unres. emission + 1 loop

This talk. Major bottleneck for NNLO computations.

NLO complexity, can be challenging for complex processes (stable loop calculation near kinematic limit…)

Not our business

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Computations with CoLoRFulNNLO:

Subtractions for the m+2 parton line:

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \underline{\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m} - \left[\underline{\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1}} - \underline{\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m} \right] \right\}_{d=4}$$

Subtractions for the m+1 parton line:

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right) J_{m+1} - \left[\underline{\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}}} + \underline{\left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}}} \right] J_m \right\}_{d=4}$$

…and adding 'em back integrated out:

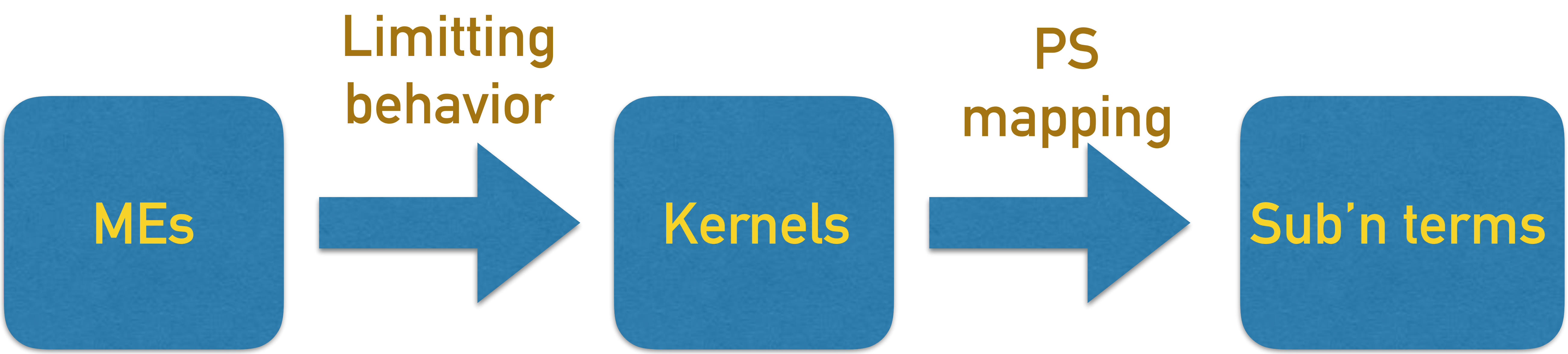
$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\}_{d=4} J_m$$

ISR in CoLoRFulNNLO:

CoLoRFulNNLO was first developed for $e^+ e^-$ collisions (only FSR, no radiation from initial state)

Since everybody fancies LHC processes it includes ISR now

The conventional way to derive subtraction terms:



This approach was used for the original FSR subtraction terms

ISR in CoLoRFulNNLO:

The same approach works even for ISR but care is needed in setting up the Sudakov parametrization!

Luckily all the ISR terms can be derived from FSR ones using crossing

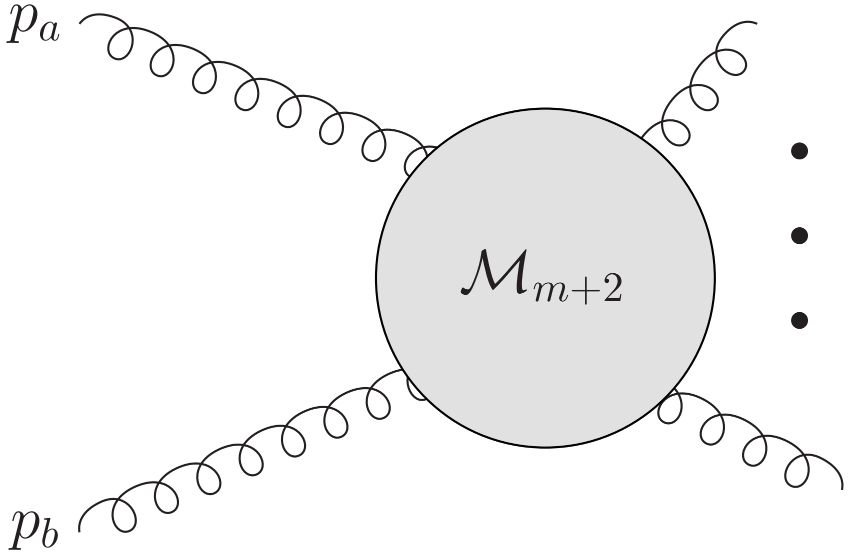
2- and 3-particle Sudakov parameters, transverse momenta and momentum mappings change in a highly non-trivial way!

No sector function is used $\Longrightarrow$ all terms are active all the time

$\Longrightarrow$ delicate cancellation of spurious singularities happen

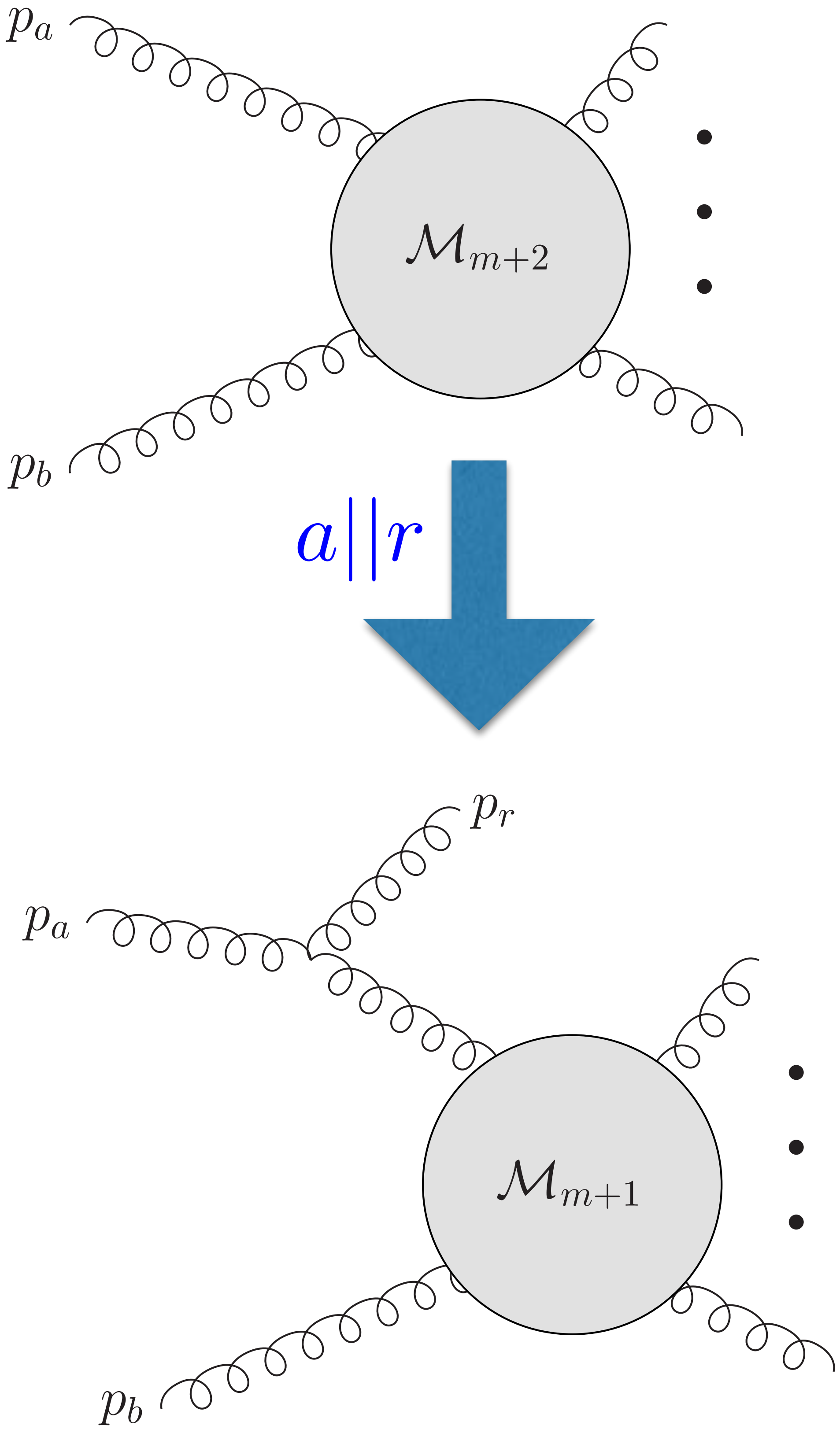
Extreme care is needed to choose parameters in order to cancel all spurious singularities!

The source of spurious singularities:



$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} \propto \int_{m+2} \Big\{ |\mathcal{M}|^2 \Big\}_{d=4}$$

The source of spurious singularities:



$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{m+2} \propto \int_{m+2} \Big\{ |\mathcal{M}|^2 - \\ - \mathcal{C}_{ar} - \ldots \Big\}_{d=4}$$

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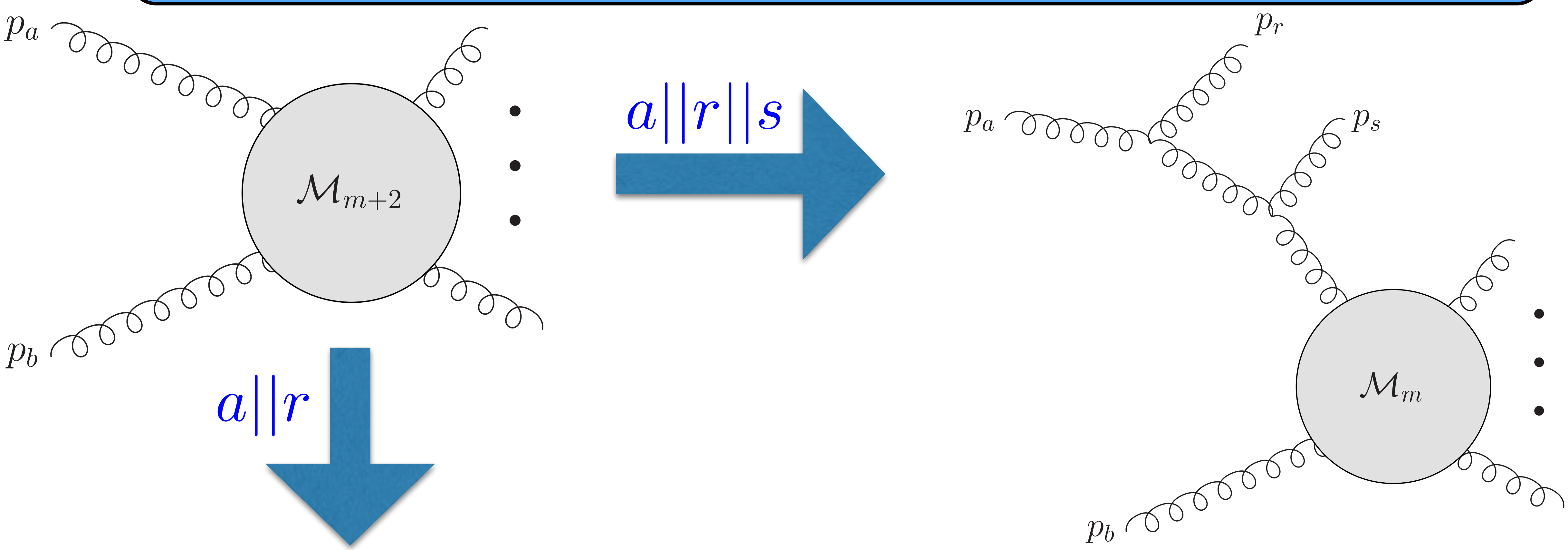
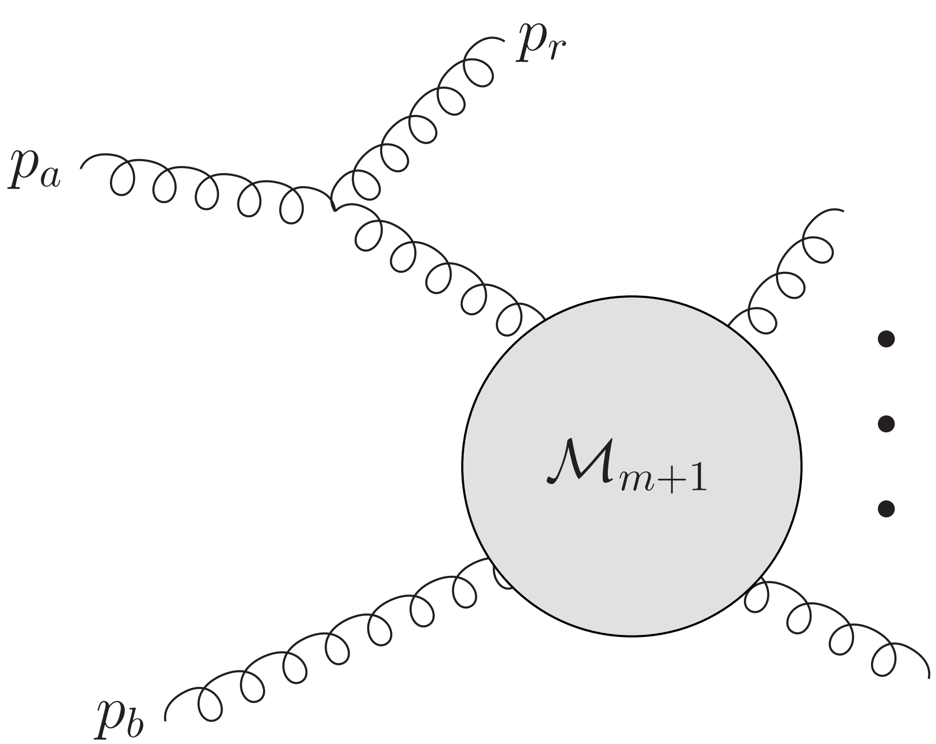
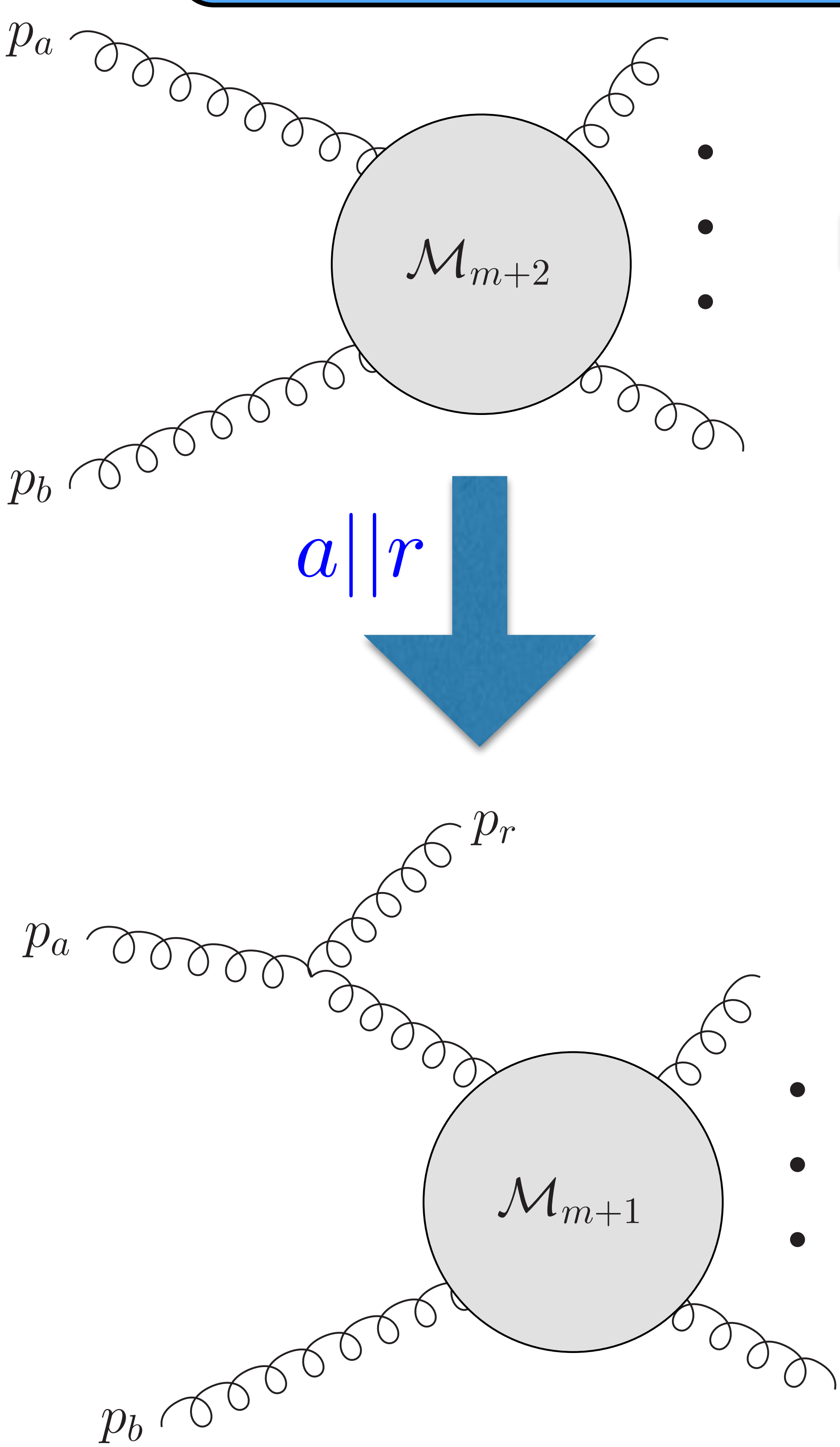
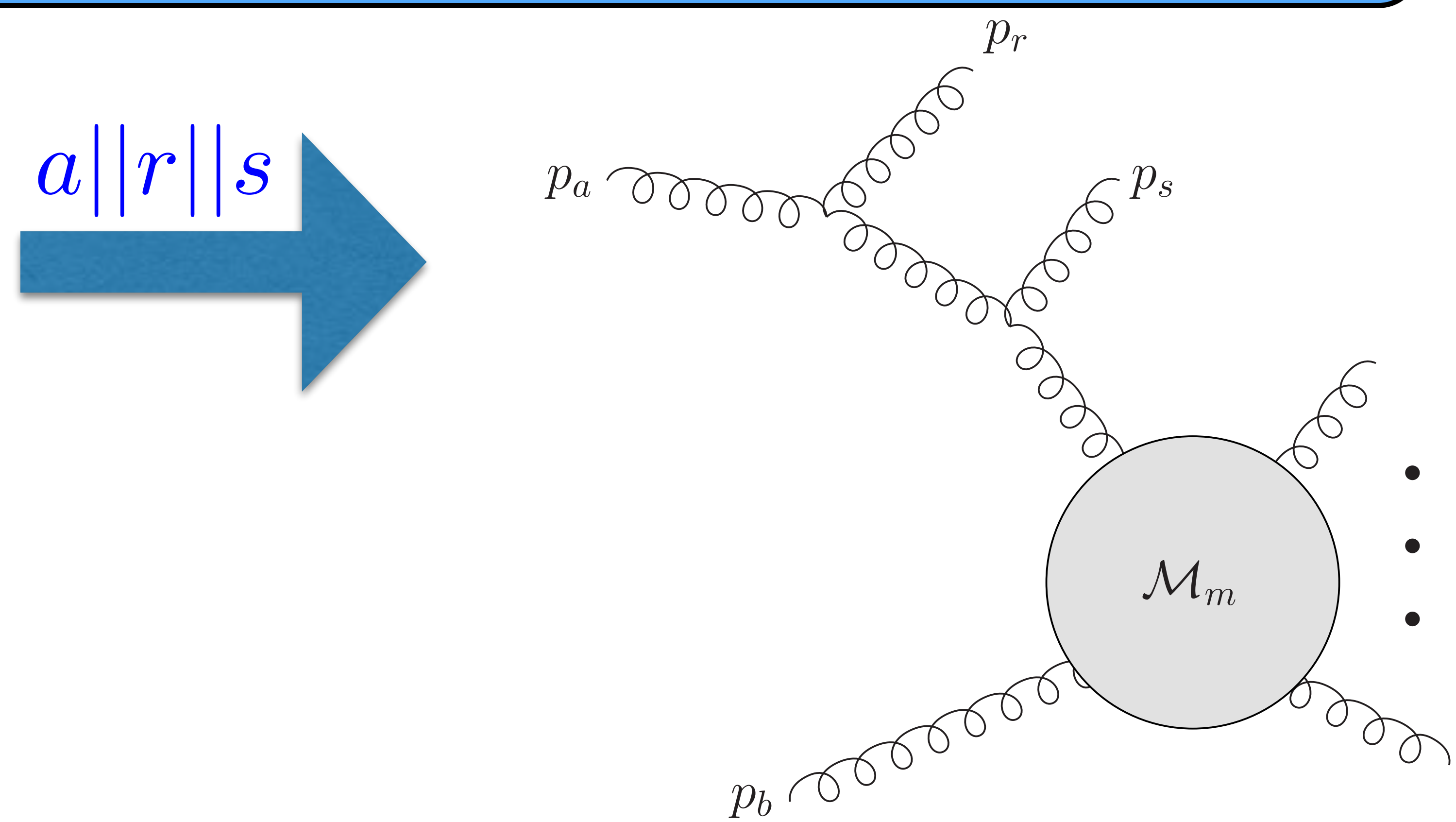




$$
\mathrm{d}\sigma_{m+2}^{\mathrm{NNLO}} \propto \int_{m+2} \Big\{ |\mathcal{M}|^2 - \\
- \mathcal{C}_{ar} - \ldots \\
- \mathcal{C}_{ars} - \ldots \\
\Big\}_{d=4}
$$

The source of spurious singularities:





$$
d\sigma^{\mathrm{NNLO}}_{m+2} \propto \int_{m+2} \Big\{ |\mathcal{M}|^2 - \\
- \mathcal{C}_{ar} - \dots \\
- \mathcal{C}_{ars} - \dots \\
+ \mathcal{C}_{ar}\mathcal{C}_{ars} + \dots \Big\}_{d=4}
$$

The source of spurious singularities:

Subtraction terms for singly unresolved limits can develop singularities in doubly unresolved regions and subtraction terms for doubly unresolved limits can have singularities in singly unresolved regions.

This is possible because all terms are present in the whole phase space (it is possible to confine sub'n terms near their regions using α_{max} [could serve as an important check])

The resulting overlaps are taken care of by the A_{12}-type sub'n terms

MCCSM (Monte Carlo for the CoLoRFulNNLO Method):

CoLoRFulNNLO is just a possible scheme to do and organize NNLO computations $\Longrightarrow$ a numerical program is needed which implements it:

MCCSM:

Main features:

- Fully general
- Can treat radiation from both the final and initial state
- As input it only needs the matrix elements (and spin- and/or color correlated ones)
- Can be used with any PDF provider
- Computation with an arbitrary number of different scales at the same time
- Automatic calculation of PDF and scale uncertainties
- Several built-in checks to test ingredients

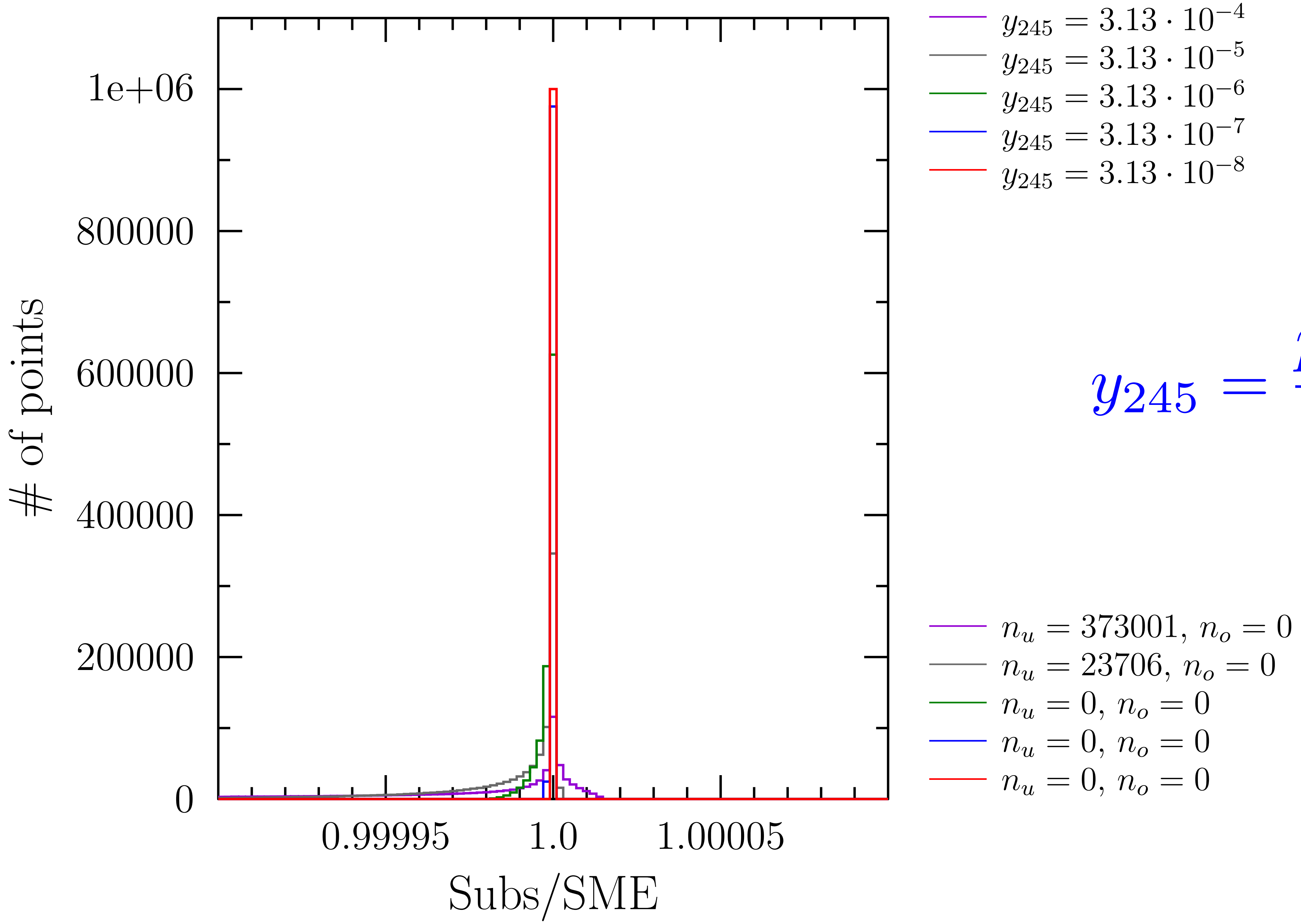
MCCSM:

Checking individual sub'n terms in the deep limit using quad precision:

```
iterm:     2 , g (1) -> g (1) || g (5)   ,   g (4) -> 0
UBorn: g   g   -> H
iexp=    2    1   , CSirs/RR=   0.89440862247853356792390954369905З
iexp=    1    0   , CSirs/RR=   0.98965656945849837824816041226857 9
iexp=    0    0   , CSirs/RR=   0.99898408718357340774466478471308 0
iexp=    0    0   , CSirs/RR=   0.99989908721958698894948319721086 3
iexp=   -1    0   , CSirs/RR=   0.99998993106164601222761837165831 4
iexp=   -2   -1   , CSirs/RR=   0.99999899382120781944210665211646 6
iexp=   -3   -1   , CSirs/RR=   0.99999989940481768878109377922669 9
iexp=   -4   -2   , CSirs/RR=   0.99999998994120035826899461502434 7
iexp=   -5   -2   , CSirs/RR=   0.99999999899414276811130448365796 4
iexp=   -6   -3   , CSirs/RR=   0.99999999989941499575347312985794 8
iexp=   -7   -3   , CSirs/RR=   0.99999999998994152231094306396161 9
iexp=   -8   -4   , CSirs/RR=   0.99999999999899415294965853728274 1
iexp=   -9   -4   , CSirs/RR=   0.99999999999989941532843334643949 5
iexp=  -10   -5   , CSirs/RR=   0.99999999999998994154686781195815 9
iexp=  -11   -5   , CSirs/RR=   0.99999999999999899359825254473035 8
iexp=  -12   -6   , CSirs/RR=   0.99999999999999990218641179942616 0
iexp=  -13   -6   , CSirs/RR=   0.99999999999999992966848183794696 7
iexp=  -14   -7   , CSirs/RR=   0.99999999999999978910222989755487 3
iexp=  -15   -7   , CSirs/RR=   0.99999999999999982179291499339327 8 1
iexp=  -16   -8   , CSirs/RR=    1.0000000000000065119277045433383 5
```

MCCSM:

Checking rate of cancellation in multiple phase space points:



$$y_{245} = \frac{p_{245}^2}{Q^2}$$

MCCSM:

The code has already been used to get NNLO QCD corrections:

- Precise calculation of event shapes in $e^+ e^-$ collisions at NNLO (even the computation of brand new ones):
  - EEC and Oblateness: Phys.Rev.Lett. 117 (2016) no.15, 152004: Del Duca, Duhr, AK, Somogyi and Trocsanyi
  - JCEF: Phys.Rev. D94 (2016) no.7, 074019: Del Duca, Duhr, AK, Somogyi, Szor, Trocsanyi and Tulipant
- NNLL+NNLO for EEC: Eur.Phys.J. C77 (2017) no.11, 749: Tulipant, AK and Somogyi
- α_S determination using NNLL+NNLO EEC: arXiv:1804.09146: AK, Kluth, Somogyi, Tulipant and Verbytskyi

For details see Z. Tulipant's talk from Tuesday

Preliminary results for the LHC:

To test the scheme with colored initial state(s) the RR contribution is implemented for:

- $p + p \longrightarrow W^{\pm}$: the Born only contains quarks $\Longrightarrow$ no spin correlation is present, the k_T definition was relaxed

- $p + p \longrightarrow H$: The Born only contains gluons $\Longrightarrow$ spin correlations are present, several sub'n terms are sensitive to the choice of transverse momentum (cancellation of spurious singularities).

Preliminary results for the LHC:

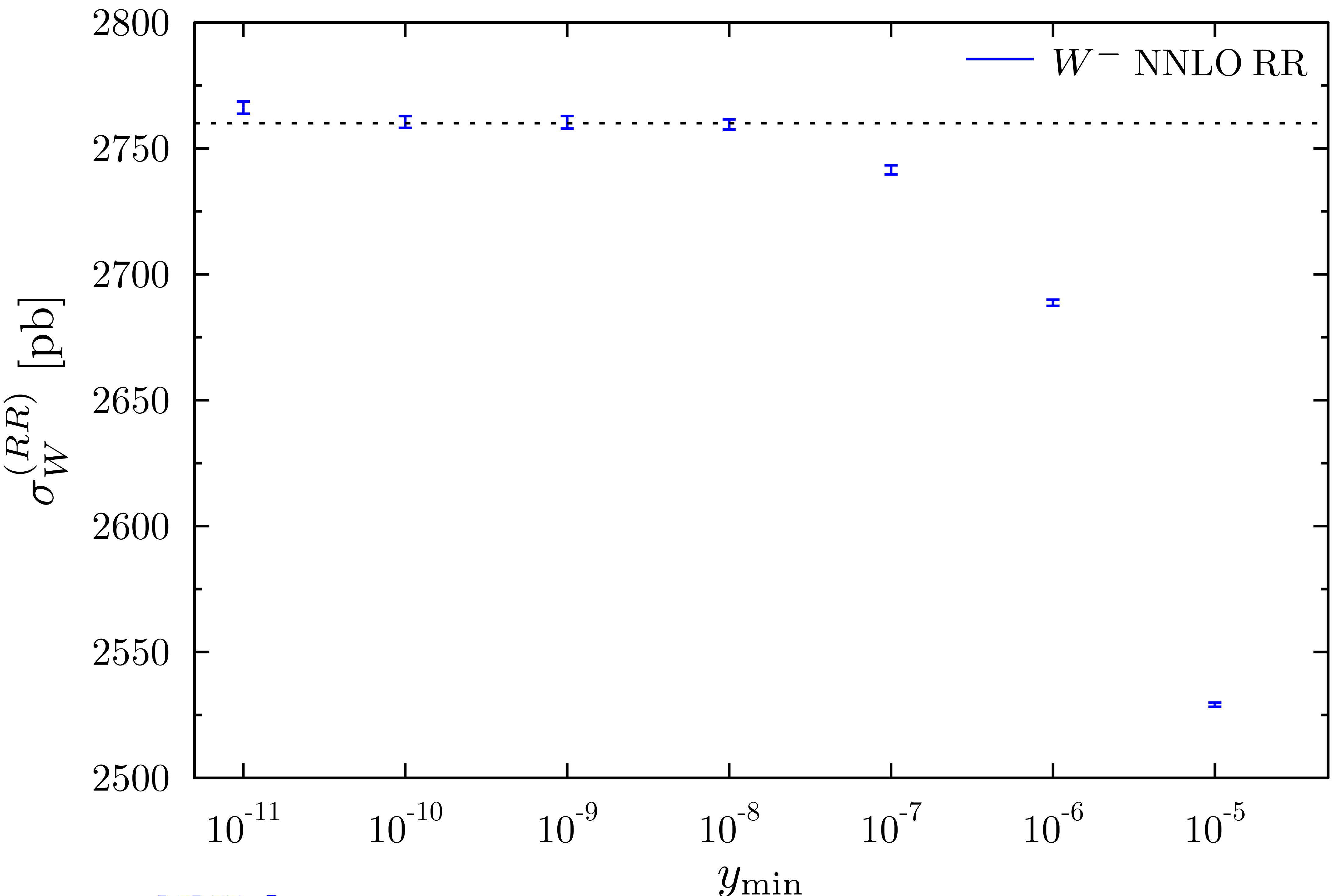
- Every computation beyond LO has a technical cutoff parameter: determining how close we allow particles to get to each other.

$$y_{\min} = \min_{i,\,j} \frac{(p_i + p_j)^2}{Q^2} > y_{\mathrm{cut}}$$

where y_{cut} is between 10^{-6} and 10^{-8}.

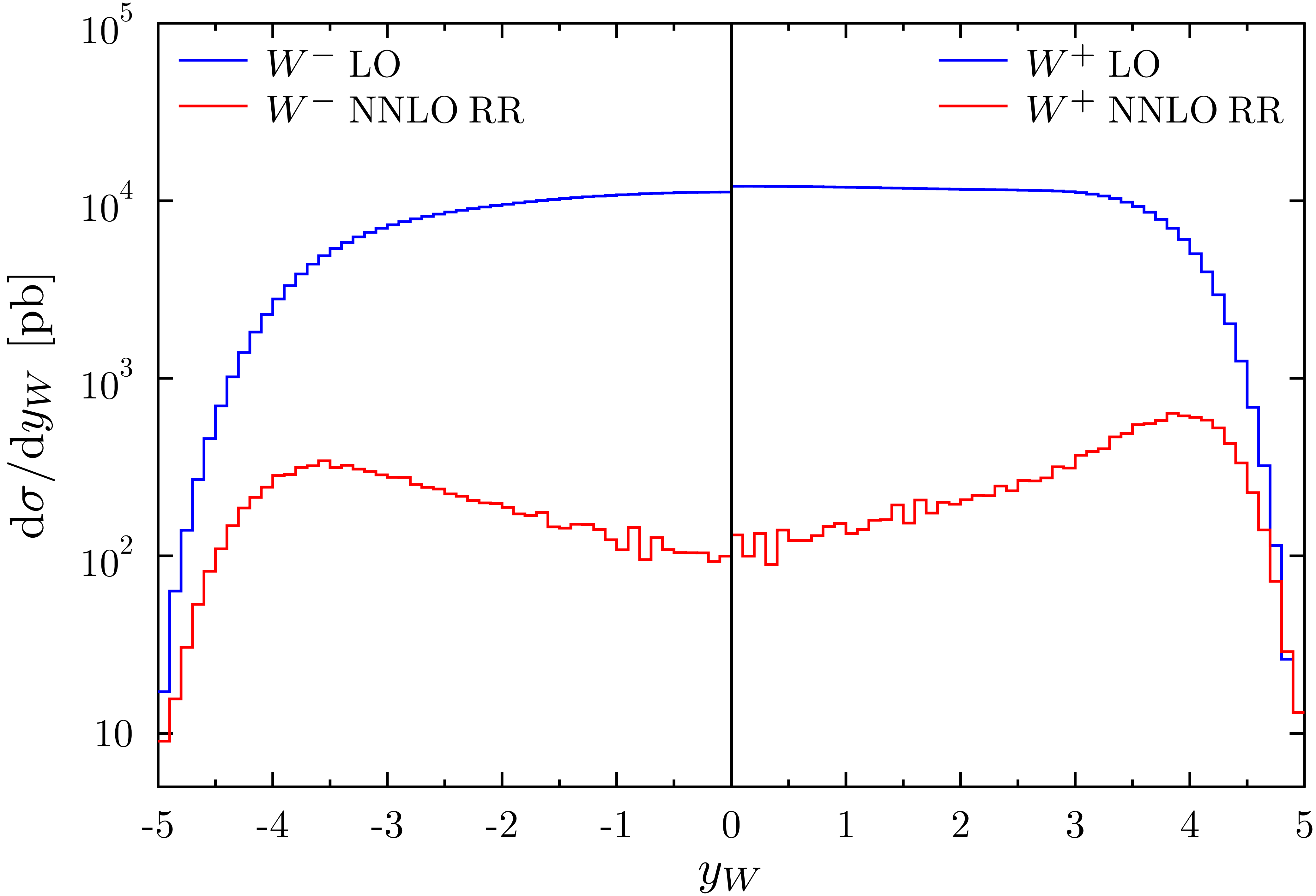
- Note that this is not a slicing but a technical cutoff parameter.
- It is necessitated by floating point arithmetics.
- If the subterms are correct the dependence upon y_{cut} should go away as it is decreased.
- The minimal possible value of y_{cut} depends on the floating point arithmetics

Saturation for W production:



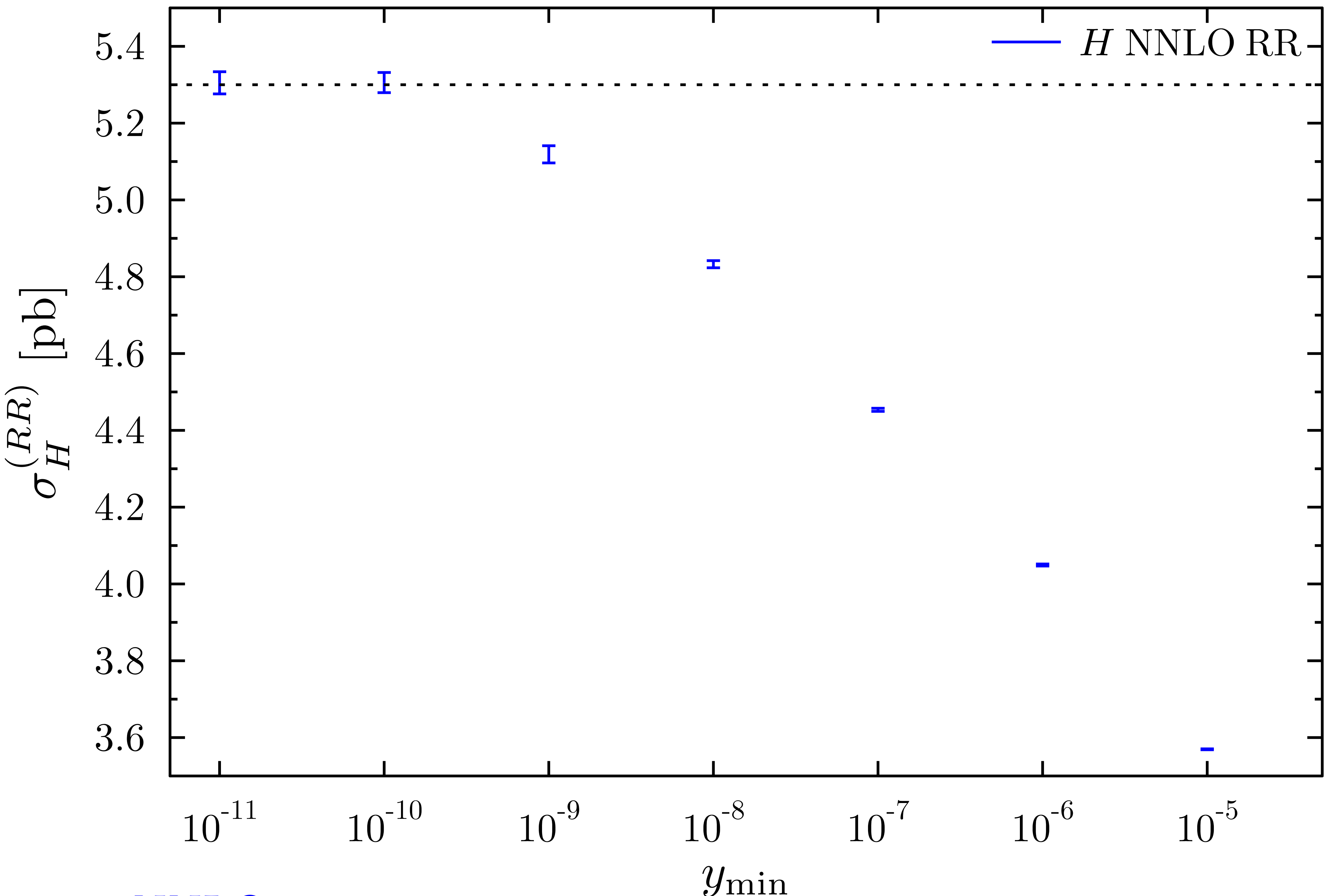
$\sigma_{\mathrm{RR}}^{\mathrm{NNLO}}$ as a function of phase space cutoff parameter

Rapidity distribution of W:



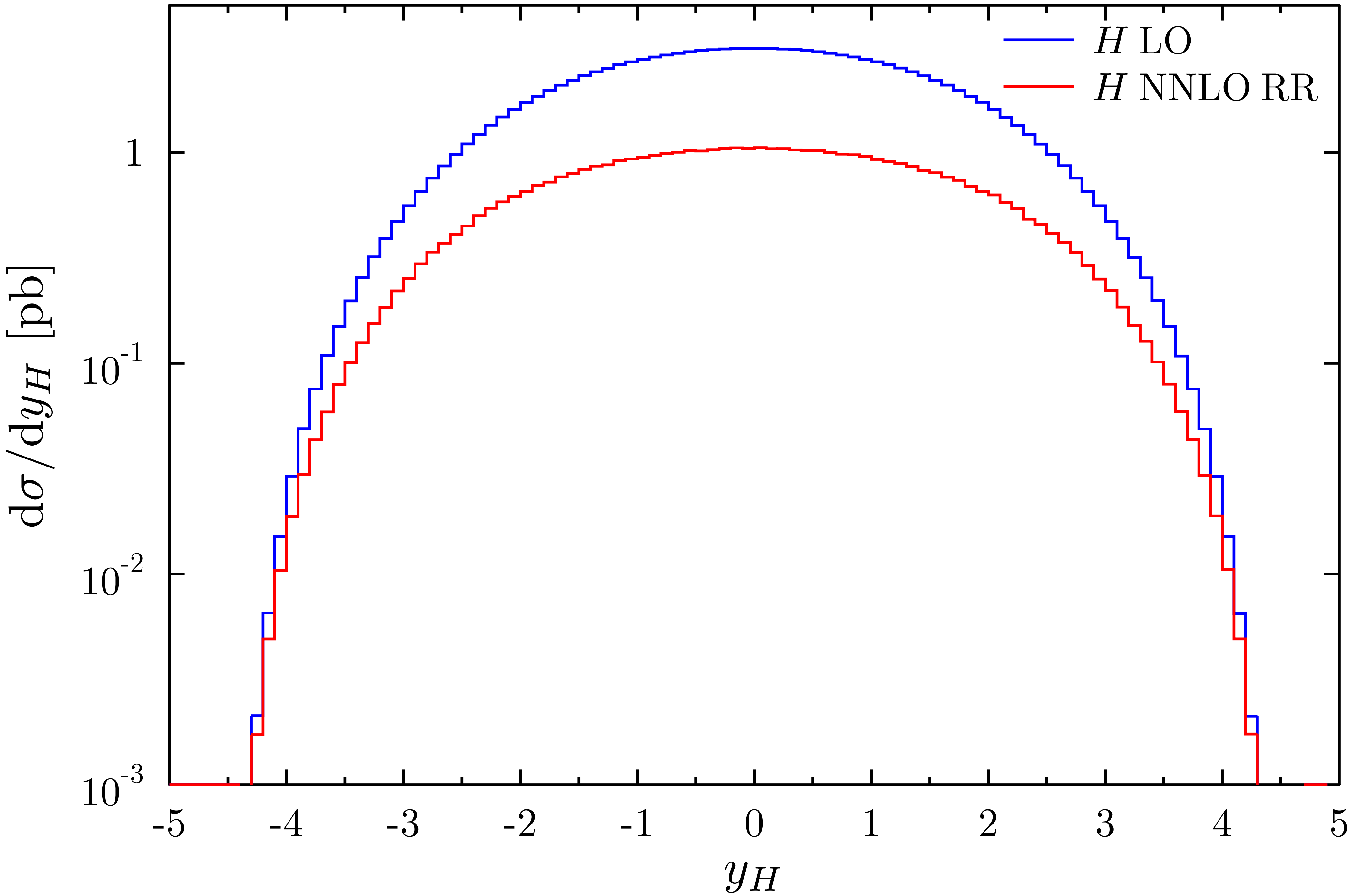
Note that this is only one contribution (RR). To get the total NNLO correction we need two more contributions.

Saturation for Higgs production:



$\sigma_{\rm RR}^{\rm NNLO}$ **as a function of phase space cutoff parameter**

Rapidity distribution of the Higgs:



Note that this is only one contribution (RR). To get the total NNLO correction we need two more contributions.

Conclusions:

- The CoLoRFulNNLO scheme is extended to include colored initial state(s)
- The numerical implementation ($\mathbb{MCCSM}$) can treat LHC processes with multiple PDFs and scale choices.
- W and Higgs production is implemented and saturation shows that the subtraction terms are working
- Still have to do the RV contribution which has NLO complexity
- The subtraction terms have to be integrated out and given back, this is a major step but already done for FSR

Thank you for your attention!