Examination and improvement of nuclear matrix elements of double-β decay in QRPA approach

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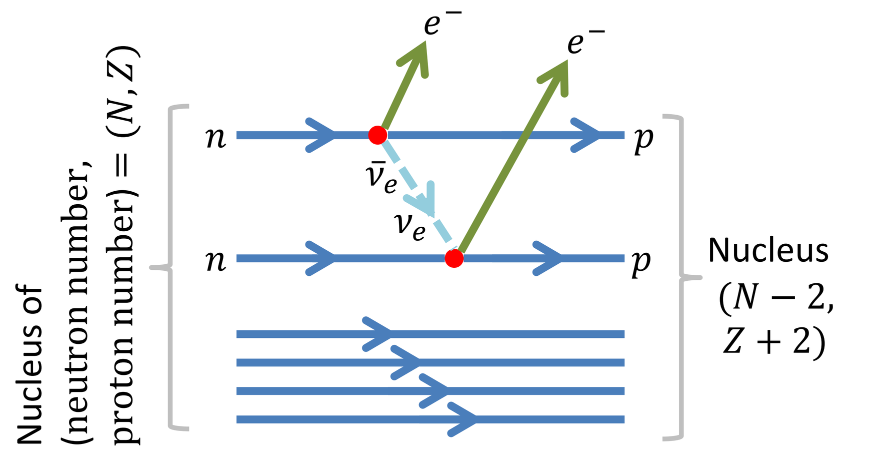
The goal is to determine the effective mass of the neutrino. The double-β decay of nucleus is used for this purpose.



Possible change of two neutrons to two protons in a nucleus emitting two electrons with neutrino exchange (neutrinoless double-β decay). This decay occurs, if the neutrino (ν) is a Majorana particle ($\nu = \bar{\nu}$), and the effective neutrino mass can be determined, see the equations below. Determination of the effective neutrino mass is one of the most important subjects in modern physics.

Why nuclei?

Because E(final state) < E(initial state) is necessary.

Other conditions for the nuclei used in the experiments:

- Single beta decay is suppressed.
- The energy spectrum of two electrons in $2\nu\beta\beta$ decay is well separated from that of $0\nu\beta\beta$.
- Large Q value [≈ E(initial state) − E(final state)].
- The parent nuclei can be produced massively with high purity.

List of nuclei used in the experiments

^{76}Ge→^{76}Se	^{130}Te→^{130}Xe	^{136}Xe→^{136}Ba
^{150}Nd→^{150}Sm	^{48}Ca→^{48}Ti	^{82}Se→^{82}Kr
^{96}Zr→^{96}Mo	^{100}Mo→^{100}Ru	^{110}Pd→^{110}Cd
^{116}Cd→^{116}Sn	^{124}Sn→^{124}Te	and more

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U: Pontecorvo–Maki–Nakagawa–Sakata matrix
m_i: eigen mass (i=1,2,3)

$$1/T_{0\nu}(\mathrm{g.s.} \to \mathrm{g.s.}) = \left|M^{(0\nu)}\right|^2 G_{0\nu} g_A^4 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Half-life ← Experimental measurement; Nuclear matrix element; Phase-space factor; Effective ν mass; Axial-vector current coupling — Theoretical calculation

Phase-space factor ← Wave functions of emitted electrons
Nuclear matrix element ← Nuclear wave functions

Approximation is indispensable.

Accuracy more difficult than the phase-space factor

Nuclear matrix element

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp'|V(r_{12},E_b)|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'}|b\rangle \langle b|c_p^\dagger c_n|0_i^+\rangle$$

Final state, ground state of nucleus(N-2,Z+2); intermediate state, nucleus (N-1,Z+1); Initial state, ground state of nucleus (N,Z)

The transition operator used in my calculation is

$$V(r_{12},E_b) \cong h_+(r_{12})\{-\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2) + g_V^2/g_A^2\}\tau^+(1)\tau^+(2)$$

Neutrino potential; Double-Gamow-Teller+ Double-Fermi

Status:
The calculated nuclear matrix elements by various approximation methods and groups are distributed in a range of factor of 2–3. The nuclear matrix element cannot be obtained by experiment. Thus, examination and improvement of the calculation are essential.

Approximation to the method to obtain nuclear wave function in my study:

Quasiparticle random-phase approximation (QRPA)

Nuclear excitation is described as the superposition of two-quasiparticle excitations.

What is good?

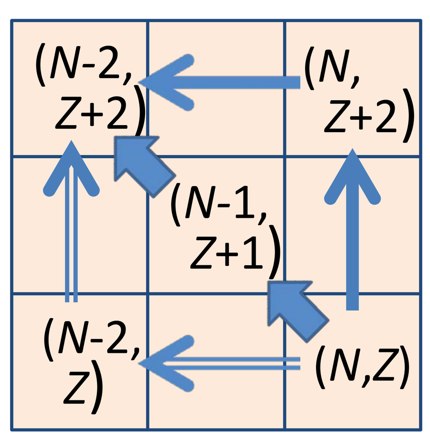
- Transition-strength function can be well reproduced.
- Sum rule is satisfied.
- Widely used in nuclear and condensed-matter physics.

IMPROVEMENT 1

Under the approximation to the equation of the nuclear matrix element by

$$V(r_{12},E_b) \cong V(r_{12},\bar{E}_b)$$

$\bar{E}_b$: average energy of the intermediate states $|b\rangle$,
virtual decay path of two-particle transfers is possible (vertical and horizontal directions in the figure).



The nuclear matrix element of the virtual path must be equal to that of the double-β path. The proton-neutron pairing interaction affects only the calculation of the double-β path

The strength of the isoscalar proton-neutron pairing interaction is determined. J. Terasaki, Phys. Rev. C **91**, 034318 (2015); ibid **93**, 024317 (2016)

NEW

Why important?

This interaction is necessary for calculating the nuclear matrix element of **two-neutrino double-β decay**.

There are exp. data of half-life of this decay for those nuclei used for the neutrinoless double-β exp. For the two-neutrino decay $V(r_{12},E_b)$ cannot be replaced by $V(r_{12},\bar{E}_b)$, thus, the double-β-path calculation is necessary. I use those data for determining the axial-vector current coupling g_A. The vector coupling g_V is 1 as usual.

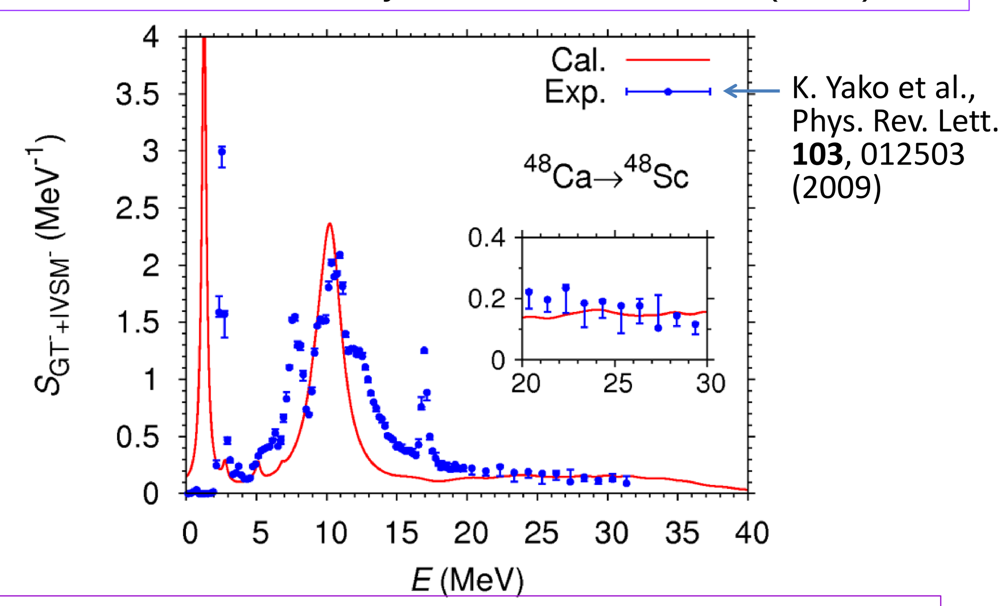
IMPROVEMENT 2

NEW

I included the many-body correlations rigorously in the contribution of the intermediate state $|b\rangle$ [(N–1,Z–1)-nucleus] to the nuclear matrix element.
This improvement is a mathematically complicated story. See J. Terasaki, Phys. Rev. C **87**, 024316 (2013).

EXAMINATION 1

Reproduction of Gamow-Teller strength function of charge-change reaction. This reaction is induced by the strong interaction, but the charge-change transition density is shared by the double-β decay. Thus, the calculated transition density has been confirmed; J. Terasaki Phys. Rev. C **97**, 034304 (2018)



By-product:
It has been clarified that this charge-change reaction was induced by the Gamow-Teller **+ isovector spin monopole** (∝ r^2) operators.

NEW

EXAMINATION 2

Two sets of the QRPA solutions are used for the intermediate states. One is based on the initial ground state, and another is based on the final ground state. It has been confirmed that this two-way representation had no problem. See J. Terasaki Phys. Rev. C **97**, 034304 (2018)

RESULT

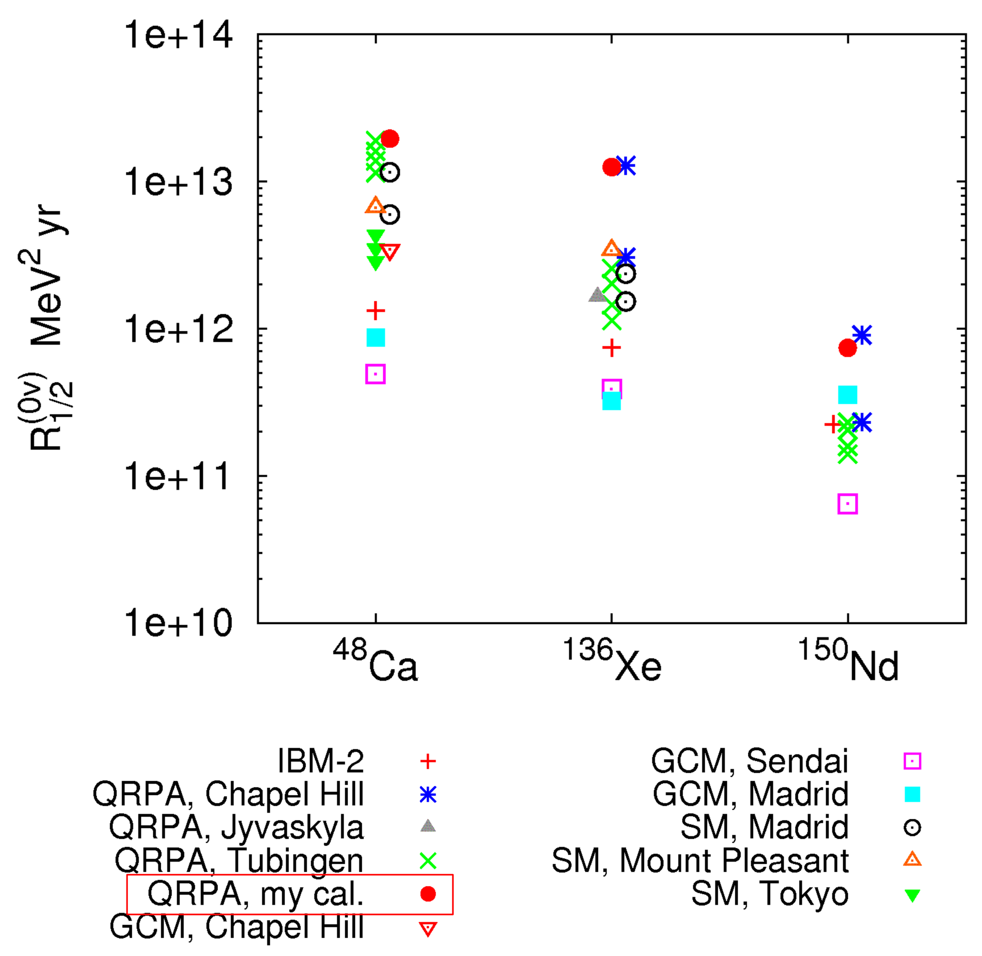
So far I have calculated ^{150}Nd→^{150}Sm, ^{136}Xe→^{136}Ba, and ^{48}Ca→^{48}Ti. Reduced half-life

$$R_{1/2}^{(0\nu)} = (G_{0\nu} g_A^4)^{-1} \left|M^{(0\nu)}\right|^{-2} m_e^2,$$

is shown below. This quantity is the one necessary for determining $\langle m_\nu \rangle$;

$$\langle m_\nu \rangle = \left(R_{1/2}^{(0\nu)}/T_{1/2}^{(0\nu)}\right)^{1/2},$$

suitable for comparing calculations with different effective g_A.



For references, see J. Engel and J. Menéndez, Rep. Prog. Phys. **80**, 046301 (2017).

SUMMARY

Important improvements and examinations have been made of the method to calculate the nuclear matrix element of the double-β decay using the QRPA; the technique to determine g_A and the strength of the isoscalar pairing interactions has been proposed. The unique feature of the QRPA approach in treating the intermediate states was examined and improved. The confirmation of the charge-change transition density used for the calculation of the nuclear matrix element was made using the experimental data of the charge-exchange reactions involving ^{40}Ca and ^{40}Sc (^{40}Ti was also used, but not shown). My reduced half-lives are rather large compared to those of other groups, that is, my half-lives are long.