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Neutrino mixing matrices with prescribed singular values

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Aim of studies

- Investigation of the experimentally determined 3×3 neutrino mixing matrix from point of view of matrix theory,
- determination of a number of sterile neutrinos by analyzing singular values of three dimensional mixing matrices,
- construction of the complete unitary mixing matrix via the procedure of matrix dilation.

Common parametrizations of non-unitary mixing matrices [1]

- QR decomposition:
$$V = (I - \alpha)U = TU$$
where α is lower triangular matrix and U is unitary.
- Polar decomposition:
$$V' = (I - \eta)U'$$
where η is a Hermitian matrix and U' is unitary.

Present status [1]

- Current upper bounds on the α matrix for three different neutrino mass scenarios:

	$m > EW$
α_{ee}	$1.3 \cdot 10^{-3}$
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-4}$
$\alpha_{\tau\tau}$	$2.8 \cdot 10^{-3}$
$\lvert\alpha_{\mu e}\rvert$	$6.8 \cdot 10^{-4}$
$\lvert\alpha_{\tau e}\rvert$	$2.7 \cdot 10^{-3}$
$\lvert\alpha_{\tau\mu}\rvert$	$1.2 \cdot 10^{-3}$

$$\alpha \leq \begin{pmatrix} 0.0013 & 0 & 0 \\ |0.00068| & 0.00022 & 0 \\ |0.0027| & |0.0012| & 0.0028 \end{pmatrix}$$

	$\Delta m^2 \gtrsim 100$ eV^2
α_{ee}	$2.4 \cdot 10^{-2}$
$\alpha_{\mu\mu}$	$2.2 \cdot 10^{-2}$
$\alpha_{\tau\tau}$	$1.0 \cdot 10^{-1}$
$\lvert\alpha_{\mu e}\rvert$	$2.5 \cdot 10^{-2}$
$\lvert\alpha_{\tau e}\rvert$	$6.9 \cdot 10^{-2}$
$\lvert\alpha_{\tau\mu}\rvert$	$1.2 \cdot 10^{-2}$

$$\alpha \leq \begin{pmatrix} 0.024 & 0 & 0 \\ |0.025| & 0.022 & 0 \\ |0.069| & |0.012| & 0.1 \end{pmatrix}$$

	$\Delta m^2 \sim 0.1 - 1$ eV^2
α_{ee}	$1.0 \cdot 10^{-2}$
$\alpha_{\mu\mu}$	$1.4 \cdot 10^{-2}$
$\alpha_{\tau\tau}$	$1.0 \cdot 10^{-1}$
$\lvert\alpha_{\mu e}\rvert$	$1.7 \cdot 10^{-2}$
$\lvert\alpha_{\tau e}\rvert$	$4.5 \cdot 10^{-2}$
$\lvert\alpha_{\tau\mu}\rvert$	$5.3 \cdot 10^{-2}$

$$\alpha \leq \begin{pmatrix} 0.01 & 0 & 0 \\ |0.017| & 0.014 & 0 \\ |0.045| & |0.053| & 0.1 \end{pmatrix}$$

Matrix theory: singular values [2]

Singular values σ_i of a given matrix A are positive square roots of the eigenvalues λ_i of the matrix AA'
$$\sigma_i(A) = \sqrt{\lambda_i(AA^\dagger)}, \quad i = 1, 2, 3, \ldots$$
Properties:
- operator norm (spectral norm):
$$\|A\| := \sup_{\|x\|=1} \|Ax\| = \sigma_{\max}(A)$$
- contractions:
$$\|A\| \leq 1$$

General beyond the Standard Model (BSM) scenario [3]

Extensions from three neutrinos setting: $V \to W = \begin{pmatrix} V & V_{lh} \\ V_{hl} & V_{hh} \end{pmatrix}$
- matrix W is unitary,
- operator norm is bounded by unity:
$$\|V\| \leq 1$$
Any deviation of singular value of V from unity $\to$ BSM scenario.

Matrix dilation:

Minimal number of additional sterile neutrinos is determined by the number of singular values strictly less than one.

Strategy: $3+1$ scenario studies

- We are focusing on one additional neutrino model by studying T matrices with prescribed singular values.
- To produce T matrix that fulfill $3+1$ structure, we are setting the singular values to:
$$\sigma(T) = \{1.00, 1.00, \sigma_3\}, \quad \sigma_3 < 1$$
- Conditions given by constrains on α matrix are met through the random generation of T entries within region given by fixed singular values.

Results: $3+1$: diagonal cases

For each mass scheme it is possible to find 3×3 mixing matrices that agrees with data and can describe mixing with only one sterile neutrino. This is simply satisfied by the following diagonal matrices:

$m > EW$
$$T = \begin{pmatrix} 1.0000 & 0 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 0.9972 \end{pmatrix}$$

$\Delta m^2 \gtrsim 100$ eV^2 and $\Delta m^2 \sim 0.1 - 1$ eV^2
$$T = \begin{pmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 0.9 \end{pmatrix}$$

Are these the smallest possible values?

Results: $3+1$: non-diagonal cases

To find non-diagonal cases we involve the idea of an inverse eigenvalue problem, i.e., we construct matrix with prescribed singular values [4]:

- $m > EW$

$$\sigma(T) = \{1.0000, 1.0000, 0.9917\} \qquad T = \begin{pmatrix} 0.9987 & 0 & 0 \\ 0.0007 & 0.9988 & 0 \\ 0.0027 & 0.0012 & 0.9972 \end{pmatrix}$$

- $\Delta m^2 \gtrsim 100$ eV^2

$$\sigma(T) = \{1.000, 1.000, 0.899\} \qquad T = \begin{pmatrix} 1.000 & 0 & 0 \\ 0.001 & 1.000 & 0 \\ 0.012 & 0.012 & 0.900 \end{pmatrix}$$

- $\Delta m^2 \sim 0.1 - 1$ eV^2

$$\sigma(T) = \{1.000, 1.000, 0.888\} \qquad T = \begin{pmatrix} 0.994 & 0 & 0 \\ 0.013 & 0.993 & 0 \\ 0.045 & 0.053 & 0.900 \end{pmatrix}$$

Conclusions

- Every mass scheme allows for one additional sterile neutrino,
- based on Weyl inequalities, errors of T entries are estimated on: **0.003** [3],
- singular values and inverse eigenvalue problem are useful tools for study non-unitary effects in neutrino sector.

Outlook

- Analysis of 3×3 mixing matrix in the context of two additional neutrinos,
- construction of complete unitary mixing matrix through the method of the unitary dilation for $3+1$ and $3+2$ scenarios [3].

[1] M. Blennow, P. Coloma, E. Fernandez-Martinez, J. Hernandez-Garcia, J. Lopez-Pavon, *Non-Unitarity, sterile neutrinos, and Non-standard neutrino interactions*, **JHEP** 1704 (2017) 153.
[2] R. Horn and C. Johnson. *Matrix Analysis*. Cambridge University Press, 2012.
[3] K. Bielas, W. Flieger, J. Gluza and M. Gluza, *Neutrino mixing analysis based on interval matrices*, arXiv:1708.09196.
[4] C. Li, R Mathias, *Construction of matrices with prescribed singular values and eigenvalues*, **BIT** 41 (2001) 1.
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