XXVIII International Conference on Neutrino Physics and Astrophysics
(NEUTRINO 2018), 4-9 June 2018, Heidelberg, Germany

Neutrino decoherence in matter

Konstantin Stankevich[a], **Alexander Studenikin**[a,b]

[a]Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia;

[b]Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia

E-mail addresses: kl.stankevich@physics.msu.ru, studenik@srd.sinp.msu.ru

1 Introduction

In the present paper we continue the study of the neutrino decoherence effect started in [1].

The phenomenon of neutrino oscillations can proceed only in the case of the coherent superposition of neutrino mass states. An external environment can modify a neutrino evolution in a way that conditions for the coherent superposition of neutrino mass states are violated. Such a violation is called quantum decoherence of neutrino states and leads to suppression of flavor neutrino oscillations. Previously, neutrino quantum decoherence was studied as result of interaction between neutrino system and quantum foam and quantum gravity [2] -[6]. In [7] and [8] matter fluctuations is considered as dissipative source. There are also studies using the Lindblad's Master Equation to evolve the neutrinos as an open system (see, for example [10] - [12]). The advantage of this method is its ability to describe all possible channels of neutrino decoherence and its influence on neutrino oscillations. Anyway, in the Lindblad's theory decoherence parameters are free and can be found only from the experiment data.

In the present paper we consider quantum neutrino decoherence due to radioactive decay in the presence of an electron media and electromagnetic field, and the corresponding damping of neutrino oscillations is calculated. In the present paper the formalism of quantum electrodynamics of open systems [9] is used that gives the possibility to find the exact form of the dissipative term and decoherence parameter. It is shown that the studied phenomenon can be significant for description of neutrino oscillations in extreme conditions of astrophysical environments peculiar to supernovae, neutron stars or quasars.

2 Formalism

To study neutrino decoherence we will use the formalism of quantum electrodynamic of open systems which is described in [9]. Here we present only the main points. We start with the quantum Liouville's equation for density matrix of a system composed of neutrino and electromagnetic field:

$$\frac{\partial}{\partial t}\rho = -i\int d^3x\,[H(x),\rho]\,, \tag{1}$$

where $H(x) = H_\nu(x) + H_f(x) + H_{int}(x)$ is the Hamiltonian density of the system. $H_\nu(x)$ and $H_f(x)$ are the Hamiltonian densities of neutrino system and the electromagnetic field respectively, and $H_{int}(x)$ describes interaction between neutrino and the field

$$H_{int}(x) = j_\alpha(x)A^\alpha(x), \tag{2}$$

where $j_\alpha(x)$ is a current density of neutrino and A_α is the electromagnetic field.

Equation (1) can be formally solved (integrated). Since we are not interested in the evolution of the electromagnetic field its degrees of freedom should be traced out

$$\rho_\nu(t_f) = tr_f\left(Texp\left[\int_{t_i}^{t_f} d^4x\,[H(x),\rho(t)]\right]\right), \tag{3}$$

where $\rho_\nu(t) = tr_f\rho(t)$ is a density matrix which describes the evolution of a neutrino system. Below we will omit the index "ν" in order not to overload formulas. It should be mentioned that the trace makes the equation irreversible and dissipative terms will appear.

After excluding path-ordering (T) and tracing out the degrees of freedom of the electromagnetic field we obtain the following equation for the neutrino density matrix in the second-order approximation

$$\begin{aligned}\frac{\partial}{\partial t}\rho(t) = &-i\,[H_\nu,\rho(t)] - \\ &-\frac{i}{2}\int d^3x\int d^3x'\int_{t_i}^{t_f} dx_0' D(x-x')\left[\vec{j}(x),\{\vec{j}(x'),\rho(t)\}\right] - \\ &-\frac{1}{2}\int d^3x\int d^3x'\int_{t_i}^{t_f} dx_0' D_1(x-x')\left[\vec{j}(x),\left[\vec{j}(x'),\rho(t)\right]\right],\end{aligned} \tag{4}$$

where $D(x-x')_{ij} = i\,[A_i(x),A_j(x')]$ and $D_1(x-x')_{ij} =< \{A_i(x),A_j(x')\} >_f$ are Pauli-Jordan commutator function and anticommutator function respectively. The angular brackets denote the average with respect to the radiation field in a thermal equilibrium state at a certain temperature T: $< O >_f = tr_f\left(O\frac{1}{Z}\exp[-H_f/k_BT]\right)$, where H_f represents the Hamiltonian of the free radiation field.

Equation(4) is modified under the assumptions of the Marcov and rotating wave approximations. The first approximation consists in the replacement $\int_{t_i}^{t} dx_0' \to \int_{-\infty}^{t} dx_0' = \int_0^\infty d\tau$. The rotating wave approximation is equivalent to an averaging procedure over the rapidly oscillating terms. The density current can be decomposed into eigenoperators of the Hamiltonian. Since neutrino in two-flavour framework is equivalent to two level system with energy difference 2Δ, the decomposition can be written in the following form

$$\vec{j}(x) = e^{-i2\Delta t}\vec{j}_-(\vec{x}) + e^{i2\Delta t}\vec{j}_+(\vec{x}). \tag{5}$$

Finally we get the quantum optical master equation

$$\frac{\partial}{\partial t}\rho(t) = -i\,[H_\nu,\rho(t)] - i\,[H_S,\rho(t)] + D(\rho(t)). \tag{6}$$

The Hamiltonian H_S leads to a renormalization of the system Hamiltonian H_ν which induced by the vacuum fluctuations of the radiation field (the Lamb shift) and by thermally induced processes (the Stark shift). The aim of this paper is to find dissipative terms, so we will omit this part in the following formulas. $D(\rho(t))$ is a dissipator of the equation which can be expressed in the following form

$$\begin{aligned}D(\rho(t)) = &\kappa_0(N(2\Delta)+1)\left(j_-\rho(t)j_+ - \frac{1}{2}j_+j_-\rho(t) - \frac{1}{2}\rho(t)j_+j_-\right) + \\ &+\kappa_0 N(2\Delta)\left(j_+\rho(t)j_- - \frac{1}{2}j_-j_+\rho(t) - \frac{1}{2}\rho(t)j_-j_+\right),\end{aligned} \tag{7}$$

where $j_+ = j_+(\vec{k})$ and $j_- = j_-(\vec{k})$ are the Fourier integral of $j_+(\vec{x})$ and $j_-(\vec{x})$ respectively. $N(2\Delta)$ denotes the Planck distribution at the transition frequency 2Δ which is the energy difference between neutrino states, and

$$\kappa_0 = \frac{\Delta}{4\pi^2}. \tag{8}$$

The first term in equation (7) is responsible for the spontaneous and thermally induced emission process and the second - for thermally induced absorption process.

3 Neutrino radiative decay

One of the mechanisms of neutrino decoherence is radiative decay of neutrino in the presence of the electron media and external electromagnetic field. The decay was first studied in [13]. The Feynman's diagramm of the process is present in the Fig. 1. Note, that the neutrino can also decay in vacuum, but this process is suppressed by the GIM mechanism and will not be discussed in the present paper.

The radiative decay is described by the density current

$$j_\alpha(x) = \overline{\nu'}(x)\Gamma_\alpha\nu(x), \tag{9}$$

where $\nu(x)$ is the neutrino field and Γ_α is an effective electromagnetic vertex

$$\Gamma_\alpha = U^*_{e\nu}U_{e\nu'}\tau_{\alpha\beta}\gamma^\alpha L. \tag{10}$$

In equation (10) U is the lepton mixing matrix and L is the projection operator for the left-handed fermions. We will assume that the four-velocity of the center of mass of the electron background is at rest. Then $\tau_{\alpha\beta}$ can be expressed as

$$\tau^{NR}_{\alpha\beta} = \tau^{NR}P_{\alpha\beta} = -\sqrt{2}\frac{eG_Fn_e}{m_e}P_{\alpha\beta} \tag{11}$$

in the case of a nonrelativistic (NR) background, and

$$\tau^{ER}_{\alpha\beta} = \tau^{ER}P_{\alpha\beta} = -\frac{eG_FT^2}{2\sqrt{2}}P_{\alpha\beta} \tag{12}$$

in the extreme relativistic (ER) case, when the temperature of the background electrons $T \gg m_e$. The tensor $P_{\alpha\beta}$ is a projector onto the transverse component in k-space, e, n_e and m_e are the electron charge, density and mass respectively.

Putting it all together we can write the current in the form

$$j_3 = 2U^*_{e\nu}U^*_{e\nu'}\tau\begin{pmatrix}0 & e^{i2\Delta t}\\ e^{-i2\Delta t} & 0\end{pmatrix}, \tag{13}$$

where τ stands for τ^{ER} or τ^{NR}.

Note that only the third component of the current is responsible for neutrino decay. Moreover, the current can be decomposed into eigenoperators of the Hamiltonian

$$j_3 = 2U^*_{e\nu}U^*_{e\nu'}\tau e^{i2\Delta t}\sigma_+ + 2U^*_{e\nu}U^*_{e\nu'}\tau e^{-i2\Delta t}\sigma_-, \tag{14}$$

where

$$\sigma_+ = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \sigma_- = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}. \tag{15}$$

It should be mentioned that the dissipative operators σ_+ and σ_- in (17) do not commute with the Hamiltonian, which means that there is a loss of energy in neutrino system due to photon emission.

4 Neutrino decoherence parameter

We assume that the background consists of electrons but not of muons or taons. In this case it is necessary to define a new effective neutrino mass basis ($\tilde{\nu}$) with the effective mixing angle $\tilde{\theta}$ in which the Hamiltonian is diagonal (see, for example [14] and [15]). In this basis the energy difference between two neutrino states is expressed as

$$\Delta = \frac{\sqrt{(\delta m^2\cos 2\theta - A)^2 + \delta m^2\sin^2 2\theta}}{2E}, \tag{16}$$

where $A = 2\sqrt{2}G_Fn_eE$ and E is the neutrino energy. The master equation (6) after substituting the current (13) is

$$\begin{aligned}\frac{\partial}{\partial t}\rho(t) = &-i\,[H_\nu,\rho(t)] + \\ &+\kappa_1\left(\sigma_-\rho(t)\sigma_+ - \frac{1}{2}\sigma_+\sigma_-\rho(t) - \frac{1}{2}\rho(t)\sigma_+\sigma_-\right) + \\ &+\kappa_2\left(\sigma_+\rho(t)\sigma_- - \frac{1}{2}\sigma_-\sigma_+\rho(t) - \frac{1}{2}\rho(t)\sigma_-\sigma_+\right),\end{aligned} \tag{17}$$

where the Hamiltonian $H_\nu = diag(\tilde{E}_1, \tilde{E}_2)$ and the decoherence parameters

$$\kappa_1 = 2\sin^2(\tilde{\theta})\cos^2(\tilde{\theta})\tau^2\kappa_0(N(2\Delta)+1), \tag{18}$$

$$\kappa_2 = 2\sin^2(\tilde{\theta})\cos^2(\tilde{\theta})\tau^2\kappa_0 N(2\Delta). \tag{19}$$

Since in our case $N(2\Delta) \gg 1$ we can write $\kappa_1 \approx \kappa_2 = \kappa$. The decoherence parameter depends on composition $\sin\tilde{\theta}\cos\tilde{\theta}$ that means that the parameter undegoes the MSW-effect.

The solution of equation (17) is given by

$$\rho = \begin{pmatrix}1 - sin^2(\tilde{\theta})\cos^2(\tilde{\theta})e^{-\kappa t} & \sin(\tilde{\theta})\cos(\tilde{\theta})e^{i2\Delta t}e^{-\kappa t/2}\\ \sin(\tilde{\theta})\cos(\tilde{\theta})e^{-i2\Delta t}e^{-\kappa t/2} & sin^2(\tilde{\theta})\cos^2(\tilde{\theta})e^{-\kappa t}\end{pmatrix}. \tag{20}$$

From (20) it is easy to find the probability of neutrino oscillations:

$$P_{\nu_e\to\nu_\mu} = \sin^2(2\tilde{\theta})\sin^2(\Delta t)\,e^{-\kappa t/2} + \frac{1}{2}\left(1 - \sin^2(2\tilde{\theta})e^{-\kappa t/2} - \cos^2(2\tilde{\theta})e^{-\kappa t}\right). \tag{21}$$

Nondiagonal elements of the density matrix are responsible for coherence between the neutrino states $\tilde{\nu_1}$ and $\tilde{\nu_2}$. From (20) it follows that nondiagonal elements are decreasing with the rate $\kappa/2$ that leads to damping of the amplitude of neutrino oscillations with the same rate. The diagonal elements ρ_{11} and ρ_{22} denotes probabilities to find $\tilde{\nu_1}$ and $\tilde{\nu_2}$ respectively. In the limit $t \to \infty$ neutrino system tends to the thermal equilibrium which gives the oscillation probability $P_{\nu_e\to\nu_\mu} \to \frac{1}{2}$.

5 Supernova environment

The decoherence parameter (19) can be significant only in an extreme environment peculiar to supernovas, neutron stars, quasars and other astrophysical objects. Consider supernova environment. The Fig. 2 shows the dependence of the decoherence parameter on the neutrino energy. We take the electron matter to be extremely relativistic $T_e \approx 100$ MeV, the photon temperature $T \approx 10$ MeV and the electron density $n_e \approx 10^{30}$ cm^3. For such an environment the decoherence parameter is vanished for neutrino energies greater than 200 KeV. It means that the neutrino spectrum changes under the decoherence effect in the same range (see Figs. 3 and 4).

6 Conclusion

In the present paper we have proposed a new mechanism of quantum neutrino decoherence in effective mass basis. The decoherence has been studied as a consequence of the entanglement of neutrino system with electromagnetic field. Using the theory of quantum electrodynamic of the open system we have calculated the evolution of neutrino propagating in the electron matter with the electromagnetic field and the transition probability $P_{\nu_e\to\nu_\mu}$. The exact formula for the neutrino decoherence parameter has been obtained.

It has been shown that the studied phenomena can lead to damping of the neutrino spectrum after neutrinos propagate 10 kilometres in extreme environment such as a supernova core. This effect should be taken into account in calculations of the neutrino flux from a supernova to Earth.

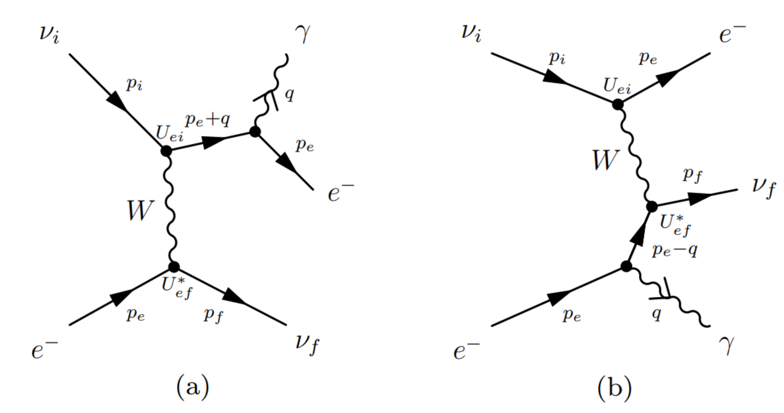

Figure 1: The neutrino radiative decay

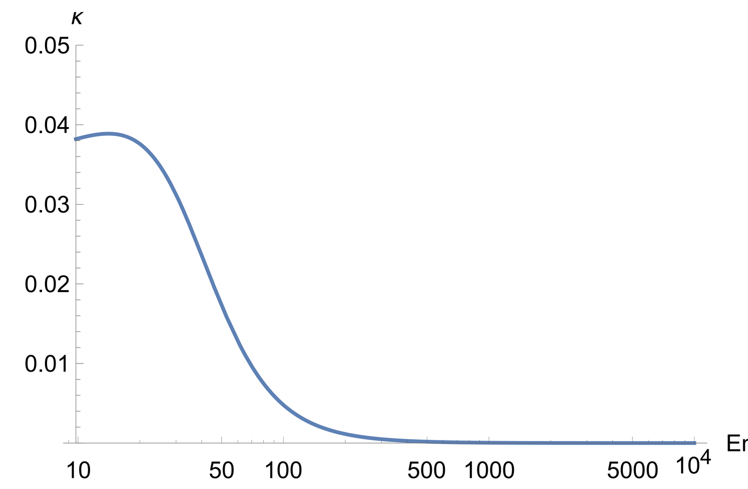

Figure 2: The neutrino decoherence parameter.

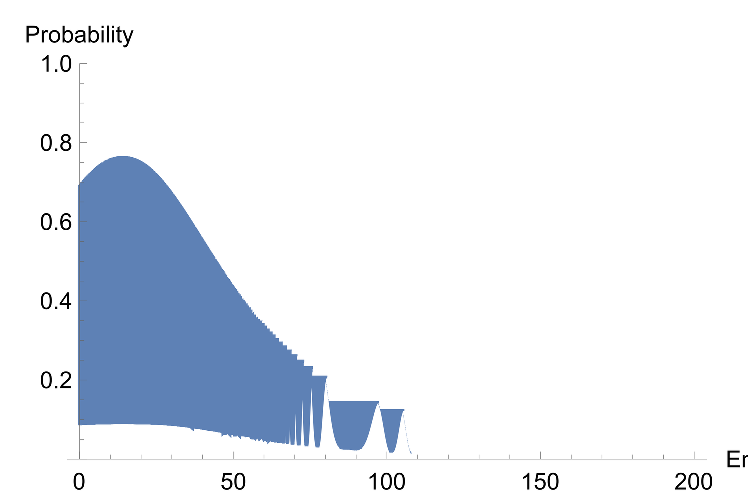

Figure 3: The oscillation probability with the decoherence effect.

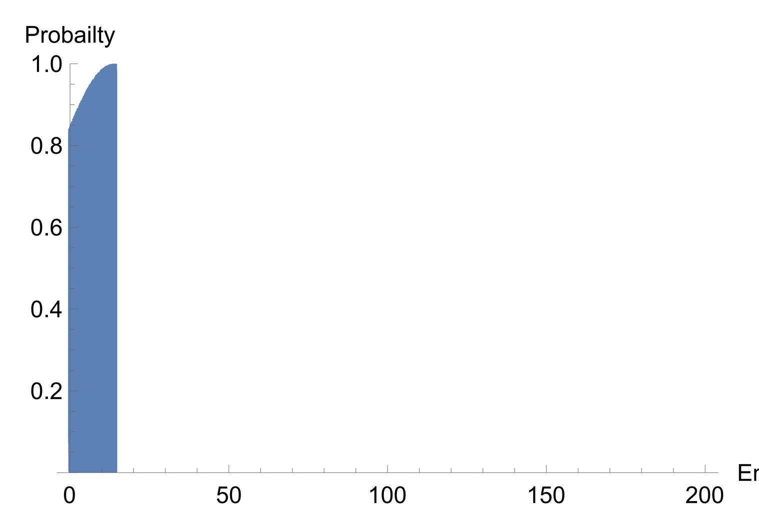

Figure 4: The oscillation probability without the decoherence effect.

This work is supported by the Russian Basic Research Foundation grants No. 16-02-01023 and 17-52-53-133.

References

[1] K. Stankevich, A. Studenikin *"Neutrino quantum decoherence due to entanglement with a magnetic field"* PoS EPS-HEP2017 (2018) 645 arXiv:1801.08896

[2] E. Lisi, A. Marrone, and D. Montanino, Phys. Rev. Lett. **85** (2000) 1166.

[3] J. Ellis et al., Nucl. Phys. **B 241** (1984) 381.

[4] G. Barenboim and N. E. Mavromato, JHEP **01** (2005) 31.

[5] G. Barenboim, N. Mavromatos, S. Sarkar, and A. Waldron-Lauda, Nucl.Phys. **B 758** (2006) 90.

[6] F. Benatti and R. Floreanini, Phys. Rev. **D 64** (2001) 085015.

[7] F. Benatti and R. Florianini, Phys. Rev. **D 71** (2005) 013003.

[8] C. P. Burgess and D. Michaud, Annals. Phys. **256** (1997) 1.

[9] H.P. Breuer, F. Petruccione, *"The theory of open quantum systems"* (Oxford, UK: Univ. Pr. (2002) 625 p).

[10] R. L. N. Oliveira, Eur. Phys. J. **C 76** (2016) 417.

[11] Joo A.B. Coelho, W. Anthony Mann, Phys.Rev. **D 96** (2017) 093009.

[12] G. Balieiro Gomes, M.M. Guzzo, P.C. de Holanda, R.L.N. Oliveira, Phys.Rev. **D 95** (2017) 113005.

[13] J. C. D'Olivo, J. F. Nieves, P. B. Pal,Phys.Rev.Lett. **64** (1990) 1088 .

[14] P. B. Pal, Int.J.Mod.Phys. **A 7** (1992) 5387-5460.

[15] M. Freund, Phys. Rev. **D 64** (2001) 053003.