PRECISION NEUTRINO DATA CONFRONTS $\mu \leftrightarrow \tau$ SYMMETRY (arXiv:1803.04143)

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ABSTRACT

- The neutrino oscillation data indicate that the mixing angle θ_{23} is close to $\pi/4$ and θ_{13} is very small.
- The simplest symmetry, which can explain these features, is the $\mu \leftrightarrow \tau$ exchange symmetry. This symmetry predicts $\theta_{23} = -\pi/4$ and $\theta_{13} = 0$.
- This symmetry is obviously broken since the experimental measurements differ from these predictions.

$\mu \leftrightarrow \tau$ SYMMETRY: INTRODUCTION

- $\mu \leftrightarrow \tau$ symmetry is about invariance under interchange of μ and τ flavors in the mass matrix:

$$M_0 = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = \begin{pmatrix} a & b & b \\ b & c & d \\ b & d & c \end{pmatrix}.$$

- This real symmetric matrix is diagonalized by the orthogonal matrix:

$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{1}{\sqrt{2}}\sin\theta_{12} & \frac{1}{\sqrt{2}}\cos\theta_{12} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sin\theta_{12} & \frac{1}{\sqrt{2}}\cos\theta_{12} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

where

$$\tan 2\theta_{12} = \frac{2\sqrt{2}b}{c+d-a}. \quad (1)$$

- The predictions of the $\mu \leftrightarrow \tau$ symmetry by inspection are:

$$\theta_{23} = -\pi/4 \text{ and } \theta_{13} = 0,$$

leaving θ_{12} as a free parameter like the mass-squared differences.

- The mass-squared differences are:

$$\begin{aligned} \delta m^2 &= m_2^2 - m_1^2 = k(a+c+d) \\ \Delta m^2 &= m_3^2 - \frac{m_1^2+m_2^2}{2} = \frac{1}{2}\left[(c-d)^2 - 4cd - a^2 - 4b^2\right] \end{aligned} \quad (2)$$

where $m_1 = (a+c+d-k)/2$, $m_2 = (a+c+d+k)/2$ and $m_3 = c-d$ with $k = \sqrt{(c+d-a)^2+8b^2}$ are the mass eigenvalues.

SOLVING THE $\mu \leftrightarrow \tau$ SYMMETRIC MASS MATRIX

- To solve the $\mu \leftrightarrow \tau$ symmetric mass matrix exactly, we use the three equations (1) and (2) and take the fourth equation the smallest mass eigenvalue to be negligibly zero. For NH, $m_1 \approx 0$, and for IH, $m_3 \approx 0$.
- By using the neutrino global data best-fit values of the three oscillation parameters, *viz.*, $\delta m^2 = 7.54 \times 10^{-5}\text{eV}^2$, $\Delta m^2 = 2.43 \times 10^{-3}\text{eV}^2$ and $\theta_{12} = 33.7°$, we obtain a set of values for the matrix elements: $a = 0.003274, b = 0.002551, c = 0.02779$ and $d = -0.02151$.
- Now we search for all allowed values of a, b, c and d by using the 3σ range constraints

$$6.99 \times 10^{-5} \le \delta m^2 \le 8.18 \times 10^{-5}, \quad 0.259 \le \sin^2\theta_{12} \le 0.359,$$
$$2.23 \times 10^{-3} \le \Delta m^2 \le 2.61 \times 10^{-3}, \quad |m_1| < 0.1 m_2.$$

- We have the set of ranges for NH:

$$a = 0.0017 - 0.0036,\ b = 0.0025 - 0.0031,$$
$$c = 0.027 - 0.028,\ d = -0.022 - -0.021.$$

$\mu \leftrightarrow \tau$ SYMMETRY BREAKING

- Symmetry breaking through 'ε_1' ($M_{12} = b - \varepsilon_1$ and $M_{13} = b + \varepsilon_1$) :
  - $\sin^2\theta_{13} \approx \varepsilon_1^2/(2c^2)$ (NH) $= 2\varepsilon_1^2/a^2$ (IH) (Analytical estimate)
  - Here, we get acceptable values for θ_{13} but θ_{23} is close to maximality.
- Symmetry breaking through 'ε_2' ($M_{22} = c - \varepsilon_2$ and $M_{33} = c + \varepsilon_2$) :
  - $\delta\theta_{23} \simeq -\varepsilon_2/(2d)$ (Analytical estimate)
  - This symmetry breaking makes θ_{23} deviate from maximality as much as expected but θ_{13} is close to zero.

COMPLETE $\mu \leftrightarrow \tau$ SYMMETRY BREAKING

- We need both ε_1 and ε_2 non-zero to obtain the observed value of θ_{13} and observable deviation of θ_{23} from $-\pi/4$.
- Numerical results for the complete symmetry breaking:

Matrix Element	NH	IH
a	0.0027-0.0046	0.048-0.050
$\lvert b\rvert$	0.0026-0.0038	0.0-0.00027
c	0.028	0.023-0.028
d	-0.022	0.0210-0.0270
$\lvert\varepsilon_1\rvert$	0.0043-0.0052	0.0044-0.0058
$\lvert\varepsilon_2\rvert$	0.0-0.0046	0.0-0.0026

- The value of b is always an order of magnitude smaller than those of the other paramters in most cases. In particular, b can be zero for the case of IH.
- In particular, ε is always larger than b by a factor 2 or more. Given the magnitude of θ_{13}, we can not even consider $\mu \leftrightarrow \tau$ symmetry as an even approximate symmetry.

$\mu \leftrightarrow -\tau$ SYMMETRY

- As $b = 0$ is a possible solution for the case of IH, we take this as another constraint to study $\mu \leftrightarrow -\tau$ symmetry. Here, ε_1 is naturally non-zero:

$$M_2 = \begin{pmatrix} a & -\varepsilon_1 & \varepsilon_1 \\ -\varepsilon_1 & c & d \\ \varepsilon_1 & d & c \end{pmatrix}.$$

- After diagonalization, we find $\theta_{23} = -\pi/4$, $\theta_{12} = 0$ and

$$\tan 2\theta_{13} = \frac{2\sqrt{2}\varepsilon_1}{c-d-a}.$$

- This, except for θ_{23}, is exactly opposite to $\mu \leftrightarrow \tau$ symmetry case.

$\mu \leftrightarrow -\tau$ SYMMETRY BREAKING

- Now, we break the $\mu \leftrightarrow -\tau$ symmetry by introducing ε_2:

$$M_5 = \begin{pmatrix} a & -\varepsilon_1 & \varepsilon_1 \\ -\varepsilon_1 & c-\varepsilon_2 & d \\ \varepsilon_1 & d & c+\varepsilon_2 \end{pmatrix}.$$

- Diagonalization of M_5 in the 2-3 sector with $\theta_{23} = -\pi/4 + \delta\theta_{23}$ leads to $\tan 2\delta\theta_{23} = -\varepsilon_2/d$.
- A further simultaneous diagonalization in the 1-3 and 1-2 sectors as

$$(U_{13}U_{12})^T(U_{23}^T M_5 U_{23})(U_{13}U_{12})$$

yields (by demanding off-diagonal terms to be zero)

$$\tan 2\theta_{13} = \frac{2\sqrt{2}\varepsilon_1}{c-d'-a}\cos\delta\theta_{23}$$

$$\tan 2\theta_{12} \approx -\frac{a}{d}\frac{\sqrt{2}\varepsilon_1\varepsilon_2}{a(a-c-d')+2\varepsilon_1^2} \quad \text{where } d' = d\sqrt{1+\frac{\varepsilon_2^2}{d^2}}.$$

- It is possible to fine-tune $(a-c-d') \sim \varepsilon_1^2/a$ to obtain $\sin^2\theta_{12} \approx 0.3$ but leave the values of δm^2 and Δm^2 unchanged.

CONCLUSIONS

- The precision oscillation data suggest $\mu \leftrightarrow \tau$ symmetry must be badly broken.
- With $\mu \leftrightarrow -\tau$ symmetry, charaterized by $b \equiv 0$, it is possible to reproduce all the neutrino oscillation parameters with just five parameters including a sinlge symmetry breaking term ε_2.
- This procedure requires fine-tuning of the mass-matrix parameters to reproduce the measured value of θ_{12}.