1. The MINOS+ Experiment in the NuMI Beam

The NuMI Beam [1]:
- 120 GeV protons from the Main Injector impact on a graphite target
- The resulting hadrons are focused by two magnetic horns and decay into neutrinos and other particles in the decay pipe
- The absorber and the mother, absorb the latter, leaving only the neutrinos to travel towards the detectors

The MINOS+ Experiment:
- MINOS+ was the continuation of the on-axis 738km baseline MINOS experiment and took data from 2013-2016 seeing a medium energy (ME) beam; MINOS+ saw a large statistics on-axis NuMI Beam
- MINOS+ used a Near Detector (ND) at Fermilab and a Far Detector (FD) in the Soudan Mine in Minnesota
- The ND detected the neutrinos before oscillations, and the FD after oscillations

NuMI Target MET-01

The Horn Off Data:
- The horn off data sample provides a unique opportunity to disentangle hadron production from focusing effects
- It allows the checking of a priori hadron production flux calculations, for example PPFX [3], which uses mostly thin-target hadron production data to derive flux predictions
- For the MINOS+ horn off sample, PPFX helps with the Data / Monte Carlo agree below 200eV (validated region for PPFX)

3. Fitting the NuMI Beam Flux - Parametrisation

The NuMI Beam Flux is fit in 2 stages:
1. Parametrise the MC hadron production for pions (positive and negative) with a polynomial
2. Fit the ND data with weights constructed from the parametrised hadron production

Stage 1: Parametrise the Hadron Production in the Flux MC
- Innovations compared to previous work [4]:
  - Flux rate equation has evolved from quadratic to polynomial of exponential term
  - allow the parametrisation to work well for the entire pT range 0 < pT < 1.4 GeV/c
  - Use a parametrisation of the secondary functions (B, C, D, E) uses an empirical parametrisation of flux MC to describe the latter as well as possible (shown for $\nu^-$)

Stage 2: Fit the Near Detector Horn Off Data:
- First fit the hadron production for muon neutrinos from positive hadrons ($\nu^-$) using the same weights as $\nu^-$
- Then fix the $\nu^+$ parameters and fit anti-neutrino negative hadron parents
- Include several systematic errors as a covariance matrix in the fit

4. Fitting the NuMI Beam Hadron Production - Fit to Horn Off Data

Applying the hadron production results to horn on MC:
- After the hadron production horn off fits, we can apply the resulting weights to the horn on MC and compare to the horn on data
- This improves the horn on agreement everywhere except at the falling edge of the peak, a region particularly susceptible to horn focusing effects

5. Focusing - Fit to Horn On Data

Focusing Fitting:
- The remaining data/MC difference can be fitted using two effective focusing parameters
- This is because many of the considered focusing errors are strongly correlated with each other
- For the fit we used horn material to account for effects such as thickness of inner horn conductor and water cooling of the horn
- We also used horn current mis-calibration which favours a value of 10% to account for any unknown focusing effects
- After the focusing fit we are able to achieve good data/MC agreement in horn on beam

Conclusions:
- The horn off data is a very powerful sample that can be used to derive hadron production weights for both the horn on and horn off p-p phase space, away from any other focusing mis-modelling effects
- The results show evidence that there are focusing effects that are at present not well understood
- In a standard muon neutrino disappearance analysis we can correct for such effects due to the power of having two detectors

6. Focusing Errors and Corrections

- White paper focusing errors are needed to fit the data/MC differences
- We model a number of systematic errors to include in the analysis (some examples are shown here)
- Those errors are calculated for the full true neutrino energy space up to 1 TeV
- We also derive a hadron production error band by simulating thousands of alternative flux universes utilising the hadron production parameter error matrix resulting from the horn off data fit