Flavour Anomalies: Phenomenology and BSM Interpretations

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\( b \rightarrow s \mu^+ \mu^- \)

\( b \rightarrow c \tau \nu \)

Kaon CP violation

Muon \( g - 2 \)

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Flavour physics

studies transitions between fermions of different generations.

\[
\text{flavour} = \text{fermion species}
\]

\[
\begin{pmatrix}
  u_L, u_L, u_L \\
  d_L, d_L, d_L \\
  u_R, u_R, u_R \\
  d_R, d_R, d_R \\
\end{pmatrix}
\begin{pmatrix}
  c_L, c_L, c_L \\
  s_L, s_L, s_L \\
  c_R, c_R, c_R \\
  s_R, s_R, s_R \\
\end{pmatrix}
\begin{pmatrix}
  t_L, t_L, t_L \\
  b_L, b_L, b_L \\
  t_R, t_R, t_R \\
  b_R, b_R, b_R \\
\end{pmatrix}
\begin{pmatrix}
  \nu_{e,L} \\
  e_L \\
  e_R \\
\end{pmatrix}
\begin{pmatrix}
  \nu_{\mu,L} \\
  \mu_L \\
  \mu_R \\
\end{pmatrix}
\begin{pmatrix}
  \nu_{\tau,L} \\
  \tau_L \\
  \tau_R \\
\end{pmatrix}
\]
Standard Model (SM): Flavour changes are governed by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$.

$$V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}$$
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- The Yukawa interaction of the Higgs field is the only source of transitions between quarks of different generations (flavour violation) in the SM.
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- The Yukawa interaction of the Higgs field is the only source of transitions between quarks of different generations (flavour violation) in the SM.
- Diagonalising the Yukawa matrices moves the flavour violation into the couplings of the $W$ boson. The strength is encoded in $V$, i.e. the piece of the Lagrangian describing the $W\bar{u}-b$ couplings reads

$$L_{W\bar{u}b} = \frac{g_2}{\sqrt{2}} \left[ \bar{u}_L V_{ub} \gamma^{\mu} b_L \ W^{+}_\mu + \bar{b}_L V_{ub}^{*} \gamma^{\mu} u_L \ W^{-}_\mu \right]$$
Some flavoured mesons

charged:

\[ K^+ \sim \bar{s}u, \quad D^+ \sim \bar{c}d, \quad D_s^+ \sim \bar{c}s, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c, \]
\[ K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c}, \]

neutral:

\[ K \sim \bar{s}d, \quad D \sim c\bar{u}, \quad B_d \sim \bar{b}d, \quad B_s \sim \bar{b}s, \]
\[ \bar{K} \sim s\bar{d}, \quad \bar{D} \sim \bar{c}u, \quad \bar{B}_d \sim bd, \quad \bar{B}_s \sim b\bar{s}, \]

The neutral \( K, D, B_d \) and \( B_s \) mesons mix with their antiparticles, \( \bar{K}, \bar{D}, \bar{B}_d \) and \( \bar{B}_s \) thanks to the weak interaction (quantum-mechanical two-state systems).
Flavour-changing neutral current (FCNC) processes

A process describing transitions between quarks of the same electric charge (e.g. $s \rightarrow d$, $b \rightarrow s$, or $c \rightarrow u$) is called FCNC process. In the SM FCNC processes are a quantum effect, involving a loop Feynman diagram.

Examples:

- $b \rightarrow s \mu^+ \mu^-$
- $b \rightarrow c \tau \nu$
- K$-\bar{K}$ mixing
- Penguin diagram
Flavour physics is sensitive to virtual effects of heavy new physics.

Flavour-changing neutral current (FCNC) interactions probe scales up to 100 TeV and above, because in the Standard Model several suppression factors pile up:

- electroweak loop,
- small CKM elements, e.g. $|V_{ts}| = 0.04$, $|V_{td}| = 0.01$,
- Glashow-Iliopoulos-Maiani (GIM) suppression $\propto (m_c^2 - m_u^2)/M_W^2$, $(m_s^2 - m_d^2)/M_W^2$ in $K$ and $D$ decays.
- helicity suppression $\propto m_b/M_W$ in radiative and leptonic $B$ decays.

Cabibbo-Kobayashi-Maskawa (CKM) matrix:

$$
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$$
Generic models of new physics typically have new sources of unsuppressed FCNC transitions.
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Examples:

- **extra Higgses** $\Rightarrow$ Higgs-mediated FCNC’s at tree-level, helicity suppression possibly absent,
- **squarks/gluinos** $\Rightarrow$ FCNC quark-squark-gluino coupling, no CKM/GIM suppression,
- **gauged U(1)′** $\Rightarrow$ FCNC couplings of an extra $Z′$,
- **SU(2)$_R$ gauge bosons** $\Rightarrow$ helicity suppression absent

Most spectacular: **Charged lepton FCNC decays** such as $\mu \rightarrow e\gamma$: If you observe them, it’s new physics!
To predict the decay rate of some meson decay triggered by $b \rightarrow s \bar{q}q$ calculate an effective hamiltonian:

$$H = \frac{4 G_F V_{ub} V_{us}^*}{\sqrt{2}} \sum_{j=1,2} C_j Q^u_j + \frac{4 G_F V_{cb} V_{cs}^*}{\sqrt{2}} \sum_{j=1,2} C_j Q^c_j - \frac{4 G_F V_{tb} V_{ts}^*}{\sqrt{2}} \sum_{j \geq 3} C_j Q_j$$

Here: $G_F$: Fermi constant  
$C_j$: Wilson coefficients = effective couplings  
$Q_j$: effective operators
Matching calculations determine the coefficients $C_j$:

All dependence on the masses of heavy particles such as the top quark, $W$ boson, squarks, $Z'$ bosons, leptoquarks, ... resides in the Wilson coefficients $C_j$. 
For radiative decays $b \rightarrow s\gamma$ and semileptonic decays $b \rightarrow s\ell^+\ell^-$ need more operators.
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Next step: Calculate decay amplitude for $B \to f$ decay with e.g. $f = K\pi, K^*\gamma, K^*\mu^+\mu^-,\ldots$:

$$A(B \to f) = \langle f|H|B\rangle = \frac{4G_F V_{ub} V_{us}^*}{\sqrt{2}} \sum_{j=1,2} C_j \langle f|Q_j^u|B\rangle + \ldots$$

The hadronic matrix element contains strong interaction effects.
Heavy new physics changes the Wilson coefficients and leads to correlated effects in many different $B^+$, $B_d$, $B_s$ decays.

**Bottom-up approach:**

**Step 1:** Fit the Wilson coefficients to all data and identify those which deviate from their SM predictions.
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**Step 1:** Fit the Wilson coefficients to all data and identify those which deviate from their SM predictions.

**Step 2:** Study possible new-physics mechanism which puts effects into desired coefficients (e.g. exchange of leptoquark, $Z'$...).
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**Step 3:** Construct reasonable UV-complete theories.
Introduction

2 Accelerators Find Particles That May Break Known Laws of Physics
The LHC and the Belle experiment have found particle decay patterns that violate the Standard Model of particle physics, confirming earlier observations at the BaBar facility.

By Clara Moskowitz | September 9, 2015 | Visit en español

Hints for New Physics in flavour observables

Democracy suffers a blow—in particle physics
Three independent B-meson experiments suggest that the charged leptons may not be so equal after all.

Steven K. Blau 17 September 2015
Flavour anomaly 1: $b \rightarrow s\mu^+\mu^-$

Decays governed by $b \rightarrow s\mu^+\mu^-$:

- $B \rightarrow K\mu^+\mu^-$
- $B \rightarrow K^*\mu^+\mu^-$
- $B_s \rightarrow \Phi\mu^+\mu^-$
- $B_s \rightarrow \mu^+\mu^-$
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The decay $B \rightarrow K^*\mu^+\mu^-$ permits the measurement of angular observables, defined in terms of angles within and between the $K^* \rightarrow K\pi$ and $\mu^+\mu^-$ decay planes. One of those, called $P'_5$, deviates from the SM prediction by more than $3\sigma$ in the LHCb and Belle experiments.
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The LHCb data for the branching-fraction ratios

\[ \frac{B(B \rightarrow K \mu^+ \mu^-)}{B(B \rightarrow Ke^+ e^-)} \quad \text{and} \quad \frac{B(B \rightarrow K^* \mu^+ \mu^-)}{B(B \rightarrow K^* e^+ e^-)} \]

are too small by $2.3 - 2.6\sigma$ in some bins of the lepton invariant mass $q^2$.

$B(B_s \rightarrow \Phi \mu^+ \mu^-)$ is smaller than the SM prediction by $2.2\sigma$. 
Effective hamiltonian

\[ H = -\frac{4 G_F V_{tb} V_{ts}^*}{\sqrt{2}} \sum_{\ell, \ell' = e, \mu, \tau} \left[ C_{9,\ell\ell} O_{9,\ell\ell} + C_{10,\ell\ell} O_{10,\ell\ell} \right] + \ldots \]

We are interested in the operators

\[ O_{9,\ell\ell} = \frac{\alpha}{4\pi} [\bar{s}_L \gamma^\mu b_L] [\bar{\ell} \gamma_\mu \ell] \]

\[ O_{10,\ell\ell} = \frac{\alpha}{4\pi} [\bar{s}_L \gamma^\mu b_L] [\bar{\ell} \gamma_\mu \gamma^5 \ell] \]

The Wilson coefficients \( C_{9,\ell\ell} \) and \( C_{10,\ell\ell} \) can be reliably calculated from the \( Z \)-penguin diagram and other diagrams.

In the Standard Model

\[ C_{9,10} \equiv C_{9,10}^{ee} = C_{9,10}^{\mu\mu} = C_{9,10}^{\tau\tau}. \]

Flavour universality of the weak interaction!
A global fit to all relevant observables (including those which comply with the SM prediction) consistently point to new physics with $C_{9\mu\mu}^{\text{NP}} \approx -\frac{1}{4} C_{9\mu\mu}^{\text{SM}}$ and possibly also with NP contributions to $C_{10\mu\mu}^{\mu\mu}$.

Plot from:
Capdevila, Crivellin, Descotes-Genon, Matias, Virto 2017.
Several other analyses in 2017.
Methodology: In a global fit of the Wilson coefficients to all data one performs a likelihood ratio test, comparing the likelihood of the best-fit point to that of the SM scenario.

Result: For scenarios in which the new physics is assumed to be only in $C_{9,10}^{\mu\mu}$ (and possibly in the coefficients $C_{9,10}^{\mu\mu'}$ of the chirality-flipped operators), the statistical significance of the new-physics hypothesis is between $5.0\sigma$ and $5.7\sigma$. The sign and magnitude of the deviation is consistent in all observables, and observables insensitive to $C_{9,10}^{\mu\mu}$ are measured SM-like.

Capdevila, Crivellin, Descotes-Genon, Matias, Virto 2017.

The first evidence for this $b \rightarrow s\mu\mu$ anomaly was found in 2013 by Descotes-Genon, Matias, and Virto as well as Altmannshofer and Straub. Since then the initial significance of $3.9\sigma$ has steadily increased with more data.
Explanation within the SM:

The predictions for $B_s \rightarrow \Phi \mu^+ \mu^-$ and $P'_5$ involve hadronic physics, one needs non-perturbative methods (i) to constrain the contributions from $(c, \bar{c})$ resonances which convert to $(\mu^+, \mu^-)$ through a virtual photon and (ii) to calculate the $B_s \rightarrow \Phi$ and $B \rightarrow K^*$ form factors.

$\Rightarrow$ a theory mistake can fake new physics in $C_{9}^{\ell\ell}$!
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But: $R_K$ and $R_K^*$ are theoretically clean and exhibit deviations from the SM by $3 - 4\sigma$!
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⇒ a theory mistake can fake new physics in $C_{9\ell\ell}^\ell$!

But: $R_K$ and $R_{K^*}$ are theoretically clean and exhibit deviations from the SM by $3-4\sigma$!

⇒ We need a conspiracy between a malign statistical fluctuation (or a mistake in electron ID) and a theoretical mistake, such that the $b \rightarrow se^+e^-$ data look SM-like!
Flavour anomaly 2: $b \to c\tau\nu$

Decays governed by $b \to c\tau\nu$:
- $B \to D\tau\nu$
- $B \to D^*\tau\nu$
- $B_c \to J/\psi\tau\nu$

$$R(D) = \frac{B(B \to D\tau\nu)}{B(B \to D\mu\nu)}, \quad R(D^*) = \frac{B(B \to D^*\tau\nu)}{B(B \to D^*\mu\nu)}$$

are all measured larger than predicted in the SM.
Uncertainties from form factor calculations efficiently cancel in the SM predictions for these ratios.
$b \rightarrow s\mu^+\mu^-$  $b \rightarrow c\tau\nu$

Kaon CP violation  Muon $g-2$

Summary

![Graph showing $R(D^*)$ versus $R(D)$ for various experiments and theoretical predictions.](image)

$\Delta \chi^2 = 1.0$ contours

- **BaBar, PRL109,101802(2012)**
- **Belle, PRD92,072014(2015)**
- **LHCb, PRL115,111803(2015)**
- **Belle, PRD94,072007(2016)**
- **Belle, PRL118,211801(2017)**
- **LHCb, FPCP2017**
- **Average**

**SM Predictions**

- $R(D) = 0.300(8)$ HPQCD (2015)
- $R(D) = 0.299(11)$ FNAL/MILC (2015)
- $R(D^*) = 0.252(3)$ S. Fajfer et al. (2012)

HFLAV

FPCP 2017

$P(\chi^2) = 71.6\%$
Heavy Flavour Averaging Group (HFLAV):

\[ R(D)^{exp} = 0.403 \pm 0.040 \pm 0.024 \]
\[ > R(D)^{SM} = 0.300 \pm 0.008 \quad \text{by } 2.2\sigma \]

\[ R(D^*)^{exp} = 0.304 \pm 0.013 \pm 0.007 \]
\[ > R(D^*)^{SM} = 0.257 \pm 0.005 \quad \text{by } 3.3\sigma \]

Combined significance: \(4\sigma\)

New: QED corrections could reduce the \(R(D)\) tension to \(1.8\sigma\).

de Boer, Kitahara, Nisandžić, 1803.05881

The LHCb measurement

\[ R(J/\psi) = 0.71 \pm 0.17 \pm 0.18 \quad \text{PRL 120 (2018) 121801} \]

is \(\sim 2\sigma\) above SM estimates.
$B \rightarrow D_{\tau \nu}$ was identified early as a competitive analyser of charged-Higgs effects.

UN, Trine, Westhoff, PRD78 (2008) 015006  
Kamenik, Mescia, PRD78 (2008) 014003

In 2012 a simultaneous explanation of $R(D)$, $R(D^*)$, and $B(B^+ \rightarrow \tau \nu)$ data was possible in a generic two-Higgs doublet model.

Today this explanation is disfavoured by the distribution of the lepton invariant mass squared ($q^2$), the $B_c$ lifetime (Alonso, Grinstein, Camalich, Phys.Rev.Lett. 118 (2017) 081802) and limits from Higgs searches.
Which new physics could simultaneously explain the $b \rightarrow s \mu^+ \mu^-$ and $b \rightarrow c \tau \nu$ anomalies?

Most popular: leptoquarks

Example for an explanation with scalar leptoquarks:

$\Phi_1 + \Phi_3$ need two leptoquarks to suppress excessive contributions to $b \rightarrow s \bar{\nu} \nu$

Crivellin, Müller, Ota 2017
Model predicts sizable enhancement of $B \rightarrow \tau \tau$ and permits the forbidden decays $B \rightarrow K \tau^\pm \mu^\mp$ and $\tau \rightarrow \mu \gamma$.

Crivellin, Müller, Ota 2017
Alternative explanation: $Z'$ boson from some extended gauge symmetry.

E.g. $\text{SU}(3) \times \text{SU}(3) \times \text{U}(1)$ model.

Descotes-Genon, Moscati, Ricciardi, arXiv:1711.03101

A $Z'$ boson can have flavour-changing couplings, like $\bar{s} - b - Z'$ and mediate $b \to s\ell^+\ell^-$ at tree-level.

Problem: $B_s - \bar{B}_s$ mixing constrains this coupling and enforces a dangerously tight upper bound on the $Z'$ mass.

di Luzio, Lenz, Kirk, arXiv:1712.06572
The decays $K \rightarrow \pi^+ \pi^-$ and $K \rightarrow \pi^0 \pi^0$ involve the quark decays $s \rightarrow d\bar{u}u$ and $s \rightarrow d\bar{d}d$.

Charge-parity (CP) violation in $K \rightarrow \pi\pi$ decays is characterised by two quantities, $\epsilon_K$ and $\epsilon'_K$.

CP violation (in $K$, $D$, and $B$ physics) is another promising track in the hunt for new physics.
CP violation in $K \rightarrow \pi \pi$

Neutral $K$ mesons:

$K_{\text{long}}$ and $K_{\text{short}}$ (linear combinations of $K$ and $\bar{K}$).

Dominant decay channels:

$K_{\text{long}} \rightarrow \pi \pi \pi$ \hspace{1cm} CP = $-1$

$K_{\text{short}} \rightarrow \pi \pi$ \hspace{1cm} CP = $+1$
CP violation in $K \to \pi\pi$

Neutral $K$ mesons:

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$K_{\text{short}} \to \pi\pi$ \hspace{1cm} $\text{CP} = +1$

1964: Christenson, Cronin, Fitch and Turlay observe

$K_{\text{long}} \to \pi\pi$

and therefore discover CP violation.
CP violation in $K \rightarrow \pi\pi$

Combine decay amplitudes $A(K^0 \rightarrow \pi^+\pi^-)$ and $A(K^0 \rightarrow \pi^0\pi^0)$ into

$$A_0 \equiv A(K^0 \rightarrow (\pi\pi)_{I=0}) \quad \text{and} \quad A_2 \equiv A(K^0 \rightarrow (\pi\pi)_{I=2}),$$

where $I$ denotes the strong isospin.
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Indirect CP violation (from $K - \bar{K}$ mixing):

$$\varepsilon_K \equiv \frac{A(K_{\text{long}} \rightarrow (\pi\pi)_{I=0})}{A(K_{\text{short}} \rightarrow (\pi\pi)_{I=0})} = (2.228 \pm 0.011) \cdot 10^{-3} \cdot e^{i(0.97 \pm 0.02)\pi/4}$$

discovered in 1964
CP violation in $K \to \pi\pi$

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discovered in 1964

Direct CP violation (from decay amplitude):

$$\epsilon'_K \sim \frac{\epsilon_K}{\sqrt{2}} \left[ \frac{\langle (\pi\pi)_{I=2}|K_{\text{long}} \rangle}{\langle (\pi\pi)_{I=0}|K_{\text{long}} \rangle} - \frac{\langle (\pi\pi)_{I=2}|K_{\text{short}} \rangle}{\langle (\pi\pi)_{I=0}|K_{\text{short}} \rangle} \right] = (16.6 \pm 2.3) \cdot 10^{-4} \cdot \epsilon_K$$

discovered in 1999
To predict $\epsilon'_K$ one must calculate $\text{Im} A_0$ and $\text{Im} A_2$.

The calculation of $\text{Im} A_0$ is very challenging and first reliable results employing lattice quantum chromo-dynamics are available only since 2015.

RBC and UKQCD Collaborations, 2015
$\text{Im} A_0$ is dominated by gluon penguins:

Operator: $Q_6 = \bar{s}_L^j \gamma_\mu d_L^k \sum_q \bar{q}_R^k \gamma^\mu q_R^j$

Matrix element: $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle$

$\text{Im} A_2$ is dominated by photon penguin and box diagrams:

Operator: $Q_8 = \frac{3}{2} \bar{s}_L^j \gamma_\mu d_L^k \sum_q e_q \bar{q}_R^k \gamma^\mu q_R^j$

Matrix element: $\langle (\pi\pi)_{I=2} | Q_8 | K^0 \rangle$
\[
\frac{\epsilon'_K}{\epsilon_K} = (16.6 \pm 2.3) \times 10^{-4} \quad \text{(experiments: NA62, KTeV)}
\]

\[
\frac{\epsilon'_K}{\epsilon_K} = (1.1 \pm 4.7_{\text{lattice}} \pm 2.0_{\text{other}}) \times 10^{-4} \quad \text{(SM)}
\]

Kitahara, UN, Tremper, JHEP 1612 (2016) 078


Discrepancy with a significance of 2.8\sigma!
It is well-known for decades that $\epsilon'_K$ is very sensitive to new physics, with possibly large effects in standard extensions of the SM.

**Special feature** of $\epsilon'_K$: Enhanced sensitivity to new physics breaking strong isospin, i.e. coupling *differently* to up and down quarks.
Sensitivity to new physics

Generic models of heavy new physics typically have a larger impact on $\epsilon_K$ than on $\epsilon'_K$.

⇒ Need clever ideas to suppress $\epsilon_K$. 
Supersymmetry has a mechanism

- to enhance $\text{Re} A_2$, because it permits strong-isospin violation through splittings between right-handed up-squark and down-squark masses (Trojan penguins), Grossman, Kagan, Neubert 1999.

- to suppress the $K - \bar{K}$ mixing amplitude thanks to the Majorana nature of the gluinos, with negative interference of two box diagrams. Crivellin, Davidkov 2010

The supersymmetric contribution to $K - \bar{K}$ mixing vanishes for $M_{\tilde{g}} \sim 1.5 M_{\tilde{q}}$ and stays small for $M_{\tilde{g}} > 1.5 M_{\tilde{q}}$.
Explain $\epsilon'_K$

**x-axis:** generic sparticle mass, $M_{\tilde{g}} = 1.5 M_S$

**y-axis:** right-handed up-squark mass

**red region:** excluded by $\epsilon_K$ if $|V_{cb}|$ from inclusive decays is correct

**blue dashes:** delimit allowed region, if $|V_{cb}|$ from exclusive decays is correct

Teppei Kitahara, UN, Paul Tremper, Phys. Rev. Lett. 117 (2016) 091802
Alternative explanation: $\bar{s}–d–Z$ coupling from new physics, stemming typically from $Z–Z'$ mixing.


A common explanation of anomaly 1 and 3?
The (near) future of Kaon physics:

\[ B(K^+ \to \pi^+ \nu \bar{\nu})^{SM} = (8.3 \pm 0.3) \cdot 10^{-11} \quad \text{for NA62 (CERN)} \]

\[ B(K_L \to \pi^0 \nu \bar{\nu})^{SM} = (2.9 \pm 0.2) \cdot 10^{-11} \quad \text{for KØTØ (J-PARC)} \]

These branching ratios are theoretically extremely clean.

Distinguish MSSM and \( Z' \) scenarios through different footprints on the \( K \to \pi \nu \bar{\nu} \) decays!
Flavour anomaly 4: Muon magnetic dipole moment

The anomalous magnetic moment of the muon, $a_\mu \equiv (g - 2)_\mu$, deviates from the SM prediction by

$$a_\mu^{SM} - a_\mu^{exp} = -(270 \pm 73) \cdot 10^{-11},$$

which corresponds to a discrepancy of $3.7\sigma$.

Keshavarzia, Nomura, Teubner, 1802.02995

SM diagrams:
$a_\mu$ involves the magnetic operator

\[
y_\mu L_\mu \phi \sigma_{\alpha\beta} \mu_R F^{\alpha\beta} \supset y_\mu \nu \mu L \sigma_{\alpha\beta} \mu_R F^{\alpha\beta} = m_\mu \mu_L \sigma_{\alpha\beta} \mu_R F^{\alpha\beta}
\]

Higgs doublet

muon Yukawa coupling

could be replaced by other mass in flavoured NP models
\( a_\mu \) involves the magnetic operator

\[
y_\mu L_\mu \Phi \sigma_{\alpha \beta} \mu_R F^{\alpha \beta} \supset y_\mu \nu_\mu L_\sigma \sigma_{\alpha \beta} \mu_R F^{\alpha \beta} = m_\mu \mu_L \sigma_{\alpha \beta} \mu_R F^{\alpha \beta}
\]

Higgs doublet

Muon Yukawa coupling

could be replaced by other mass in flavoured NP models

In leptoquark models \( a_\mu \propto m_t \) possible.

Leskow, D’Ambrosio, Crivellin, Müller, PRD95 (2017) 055018
The scalar leptoquark (LQ) explanations only work for two out of the three processes/quantities $b \to s\mu^+\mu^-$, $b \to c\tau\nu$, and $a_\mu$. 
Other avenues:

**Vector leptoquarks** for \( b \to s\mu\mu \) and \( b \to c\tau\nu \):
- Buttazzo,Greljo,Isidori,Marzocca, JHEP 1711 (2017) 044
- Calibbi,Crivellin,Li, 1709.0069
- Blanke,Crivellin, 1801.07256

**Extra (vector-like) fermions and scalars** for \( b \to s\mu\mu \) and \( a_\mu \):
- Arnan,Crivellin,Hofer,Mescia, JHEP 1704 (2017) 043

**Light, feebly coupled \( Z' \)** for \( b \to s\mu\mu \):
- Sala,Straub, PLB774 (2017) 205

...and many more.
Which other manifestations could the physics underlying the flavour anomalies have?

- Spectacular effects in $B \rightarrow K^{(*)} \tau^+ \tau^-$ to be probed by the upcoming Belle II experiment.
- Lepton-flavour violation: $b \rightarrow s \tau^\pm \mu^\mp$, $b \rightarrow s \tau^\pm e^\mp$, $b \rightarrow s \mu^\pm e^\mp$, $\mu \rightarrow e \gamma$, $\tau \rightarrow \mu \gamma$.
- Upcoming measurements of rare decays
  - $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})^{\text{SM}} \equiv (8.3 \pm 0.3) \cdot 10^{-11}$ NA62 (CERN)
  - $B(K_{\text{long}} \rightarrow \pi^0 \nu \bar{\nu})^{\text{SM}} \equiv (2.9 \pm 0.2) \cdot 10^{-11}$ KØTØ (J-PARC)
- ATLAS/CMS discovery of a leptoquark or $Z'$ or ... ????
Summary

• Current data on $b \rightarrow s\mu^+\mu^-$ decays point to a new interaction of the form $[\bar{s}_L\gamma\mu b_L] [\bar{\mu}\gamma\mu\mu]$ or $[\bar{s}_L\gamma\mu b_L] [\bar{\mu}_L\gamma\mu\mu_L]$. See talk by David Straub.

• This evidence for new physics is robust and has steadily grown to more than $5\sigma$ over five years with new measurements. Alternative explanations require a conspiracy of experimental and/or theoretical mistakes.

• Promising for future discoveries: $B \rightarrow K^{(*)}\tau^+\tau^-$, $B \rightarrow K^{(*)}\tau\mu$, $B_s \rightarrow \mu e$, $B \rightarrow K\mu e$, $\mu \rightarrow e\gamma$...

• Data on $b \rightarrow c\tau\nu$ disagree with their SM predictions. Both anomalies hint to the violation of lepton-flavour universality, a cornerstone of the weak interaction. Are there experimental issues with $B \rightarrow D^*\tau\nu$ or $\tau$ reconstruction in general?

• Leptoquark models could link $b \rightarrow s\mu^+\mu^-$ to $b \rightarrow c\tau\nu$. 
Summary

- **CP violation** in $K \rightarrow \pi \pi$ decays disagrees with the SM prediction by $2.8\sigma$. This deviation can be accommodated in the **MSSM** without violating lower bounds on the masses of the supersymmetric particles from LHC searches.
- Alternative explanation: **$U(1)'$** models with $Z'-\bar{s}-d$ couplings.
- Both the **MSSM** and leptoquark models can further explain $a_\mu$.
- Promising for future discoveries: $K^+ \rightarrow \pi^+\bar{\nu}\nu$ and $K_{\text{long}} \rightarrow \pi^0\bar{\nu}\nu$ to discriminate between different explanations.
- $Z'$ models may simultaneously explain $b \rightarrow s\mu\mu$ and $\epsilon'_K$. Do we see first hints of horizontal gauge dynamics?
Penguins in $b \to s\mu^+\mu^-$ or $s \to d\bar{q}q$: 

Wake-up call for New Physics?