Scattering Amplitudes and Precision Theory for the LHC

Lance Dixon (SLAC)
Concluding Colloquium
Graduate College on Mass, Spectrum & Symmetry
9 March 2018
• Proton-proton collisions at 13 TeV center-of-mass energy, 6.5 times greater than previous (Tevatron, Fermilab, Illinois)
• Luminosity (collision rate) ~ 100 times greater
• New window into physics at shortest distances, since 2010.
LHC performing exceedingly well

2011-12 @ 7-8 TeV
2015-17 @ 13 TeV
Standard Model

- All elementary forces except gravity in same basic framework
- Matter made of spin ½ fermions
- Forces carried by spin 1 vector bosons: $\gamma$, $W^+$, $W^-$, $Z^0$, $g$
- Add a spin 0 Higgs boson $H$ to explain masses of $W^+$, $W^-$, $Z^0$
  $\rightarrow$ finite, testable predictions for all quantities

electromagnetism (QED)

weak

strong (QCD)
Higgs-like particle discovered!
Is there Physics beyond the Standard Model at the LHC?

There is dark matter in the cosmos

If it is an elementary particle, the LHC could produce it, or produce other particles that decay to it
Beyond the Standard Model

• Hierarchy problem:
In SM, electroweak scale $m_W$ looks fine-tuned as soon as ultraviolet cutoff $\Lambda$ is raised well above $m_W$.
• Many theories predict a host of new massive particles with masses $\sim m_W$ i.e. within reach of the LHC.
• To prevent problems from precision electroweak physics,

such theories often have a discrete symmetry, for which the lightest odd particle is a dark matter candidate.
Searching for BSM at LHC

- Frameworks include:
  - supersymmetry
  - new dimensions of space-time
  - new forces
  - etc.

- Usually some colored particles which can be produced with large cross sections at the LHC.

- Most new massive particles decay rapidly to old, massless particles: quarks, gluons, charged leptons, neutrinos, photons + dark matter?

- How to distinguish new physics from old (Standard Model)?
- From other types of new physics?
New Physics Example: Supersymmetry

- Symmetry between fermions (matter) and bosons (forces)
- Very elegant, solves hierarchy problem
- Lightest supersymmetric particle (LSP) can be dark matter
- Cornucopia of new elementary particles at LHC.

Talk by Philip Dießner
Classic SUSY dark matter signature

Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

→ Missing transverse energy
MET + 4 jets
Not background free: happens in Standard Model too

**MET + 4 jets from** \( pp \rightarrow Z + 4 \) jets, \( Z \rightarrow \text{neutrinos} \)

Neutrinos escape detector. **Irreducible background.**

Plus there are many reducible backgrounds from \( W + \text{jets}, \ tt + \text{jets}, \ldots \)

Precision theory (typically NLO) can help with this, usually when embedded in parton shower Monte Carlos
SUSY searches now very sophisticated

- Also $N_{\text{jet}} = 1,2,3,5,6$
- No significant excesses seen, so set lower limits on masses of superparticles.
Sample exclusion plot

CMS 1802.02110 35.9 fb\textsuperscript{-1} (13 TeV)

- \( p p \rightarrow \tilde{g} \tilde{g} \) NLO+NLL exclusion
- \( \tilde{g} \rightarrow b \bar{b} \chi_1^0 \)
- \( \tilde{g} \rightarrow t \bar{t} \chi_1^0 \)
- \( \tilde{g} \rightarrow q \bar{q} \chi_1^0 \) (Prompt decay)
- Metastable \( \tilde{g} \)
From searches to measurements

• No convincing evidence for SUSY, or any other direct production of new particles.
• Also look for deviations in rates for Standard Model processes, especially involving the brand-new Higgs boson.
• Measurements are hard, take a while to perform.
• More precise theory typically needed.
Precision theory, from NLO to NNLO and even NNNLO required here!
Asymptotic freedom and short-distance calculability

- Gluon self-interactions $\Rightarrow$
  $\alpha_s \rightarrow 0$ asymptotically, but *logarithmically* at short distances (large $Q$)

Talk by Marina Marinkovic

Bethke

confining

calculable
confining

calculable
At short distances, quarks and gluons (partons) in proton are almost free. Sampled “one at a time”

The “femto-universe”
size = factorization scale $\mu_F$ ("arbitrary")

Short-distance cross section
$\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$
predictable using perturbative QCD

"typical" infrared safe final state

Parton distribution functions (from experiment)
Perturbative Short-Distance Cross Section

\[ \tilde{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^{n_\alpha} \left[ \tilde{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \tilde{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \tilde{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right] \]

Leading-order (LO) predictions qualitative:
- poor convergence of expansion in \( \alpha_s(\mu) \)
- Uncertainty bands from varying \( \mu_R = \mu_F = \mu \)

Example: Higgs gluon fusion cross section at LHC vs. CM energy \( \sqrt{s} \)

LO \( \rightarrow \) NNNLO \( \rightarrow \) factor of 2.7 increase!

Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 1503.06056
Short-distance cross sections built out of scattering amplitudes, \( S \)-matrix elements

\[
\sigma \equiv f
\]
LO = Trees

LO cross section uses only Feynman diagrams with no closed loops – tree diagrams.
Here’s a very simple one:

\[ \hat{\sigma}(0) \]

Although there are many kinds of trees, some harder than others, “textbook” methods usually suffice.
NLO = Loops

NLO cross section needs Feynman diagrams with exactly one closed loop

Where the fun really starts – textbook methods quickly fail, even with very powerful computers

- NLO also needs tree-level amplitudes with one more parton
- Both terms infinite(!) – combine them to get a finite result

- One-loop amplitudes were the bottleneck for a long time – but now we know how to mass produce them!
Just one QCD loop can be a challenge

$q\bar{q} \rightarrow W + n$ gluons

$\begin{array}{c}
\text{Zero loops} \\
\text{One loop}
\end{array}$

$\begin{array}{c}
\text{One gluon} \\
\text{Two gluons} \\
\text{Three gluons}
\end{array}$

$= + 256,264$ more

L. Dixon

Humboldt U. 9 March 2018
1960’s Analytic S-Matrix

No QCD, no Lagrangian or Feynman rules for strong interactions.

Bootstrap program: Reconstruct scattering amplitudes directly from analytic properties: “on-shell” information

- Poles

- Branch cuts

Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

Resurrected for computing amplitudes in perturbative QCD – as alternative to Feynman diagrams!
Perturbative information now assists analyticity.
Granularity vs. Fluidity
Helicity Formalism Exposes Tree-Level Simplicity in QCD

Many tree-level helicity amplitudes either vanish or are very short.

right-handed \[ h = +1 \]
left-handed \[ h = -1 \]

\[
\begin{align*}
\mathcal{A}_n & = \mathcal{A}_n \\
\mathcal{A}_n & = 0
\end{align*}
\]

Analyticity makes it possible to recycle this simplicity into loop amplitudes.

\[
\mathcal{A}_n = \frac{\langle i j \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle}
\]

Parke-Taylor formula (1986)
Recycling “Plastic” Amplitudes

Amplitudes fall apart into simpler ones in special limits – pole information

Picture leads directly to BCFW (on-shell) recursion relations:
Reconstruct amplitude from poles in complex plane, where it factorizes into simpler amplitudes
Britto, Cachazo, Feng, Witten, hep-th/0501052

Trees recycled into trees
All Gluon Tree Amplitudes Built From:

\[
\begin{align*}
3^+ & = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle} \\
1^- & \\
2^- & 
\end{align*}
\]

In contrast to Feynman vertices, it’s on-shell, completely physical.

- On-shell recursion \(\rightarrow\) very compact \textit{analytic} formulae, fast \textit{numerical} implementation.
- Can do same sort of thing at \textit{loop level}.
Branch cut information →
Generalized Unitarity (One-loop Plasticity)

Ordinary unitarity: put 2 particles on shell

Generalized unitarity: put 3 or 4 particles on shell

Trees recycled into loops!
One-Loop Amplitude Decomposition

Bern, LD, Dunbar, Kosower (1994)

Missing from the old, nonperturbative analytic $S$-matrix

coefficients can be determined from products of trees using (generalized) unitarity

\[ A^{1\text{-loop}} = \sum_i d_i + \sum_i c_i + \sum_i b_i \]

\[ + R + \mathcal{O}(\epsilon) \]

rational part; from D-dimensional trees, or recursively

Known functions (integrals), same for all amplitudes
Many Automated Programs for One-Loop QCD
→ Many New Processes at NLO

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338...
+ Sherpa → NLO $W,Z + 3,4,5$ jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO $WWW, WWZ, ...$
NLO $t\bar{t}b\bar{b}, t\bar{t} + 2$ jets, ...
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009

MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO $W + 3$ jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^- + 2$ jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI → GoSAM: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710,…

NGluon: Badger, Biedermann, Uwer, 1011.2900,…
NLO $pp \rightarrow Z + 1,2,3,4 \text{ jets}$ vs. ATLAS 2011 data

ATLAS 1304.7098

BH+S: 1108.2229
NLO timeline
Gavin Salam (2012)

2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al]
2011: NLO $WWjj$ [Rocket: Melia et al]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al]
2011: first automation [MadNLO: Hirschi et al]
2011: first automation [Helac NLO: Bevilacqua et al]
2011: first automation [GoSam: Cullen et al]
2011: $e^+e^- \rightarrow 7j$ [Becker et al, leading colour]

[unitarity]
[unitarity]
[unitarity]
[unitarity]
[unitarity + feyn.diags]
[unitarity]
[feyn.diags(+unitarity)]
[numerical loops]
On to two loops

- State-of-art currently stuck at $2 \rightarrow 2$
  [First 2 -> 3 forays: Henn et al., 1511.05408, Badger et al., 1712.02229, Abreu et al., 1712.03946]

- Why? In part because 2 loop multiscale integrals are typically very hard

- All 1 loop integrals with external legs in $D=4$ are reducible to scalar box integrals + simpler
  $\Rightarrow$ combinations of

\[
\text{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1 - t)
\]

$\Rightarrow$ + simpler

Two loop integrals

- Become “elliptic” very quickly if there are internal particle masses, e.g. the London transport integral
  → sunset integral (sunrise integral?)

\[ P \rightarrow \begin{array}{c}
  \text{LONDON TRANSPORT} \\
  m \\
  m \\
  m \\
\end{array} = \]

Broadhurst-Fleischer-Tarasov, 9304303, Berends-Böhm-Buza-Scharf (1994),
Laporta-Remiddi, 0406160,
Adams-Bogner-Weinzierl, 1302.7004, Bloch-Vanhove, 1309.5865,…
Sunset inside top production

- At subleading color in $gg \rightarrow t\bar{t}$,

- Already done numerically Czakon, Fiedler, Mitov, 1303.6254

- Better analytic understanding will aid in multiscale generalizations
Massless (internal) $2 \rightarrow 2$

- Here, the 2 loop integrals typically belong to a simpler class of multiple polylogarithms, e.g. "G functions" Goncharov, 1105.2076

- Together with advances in handling real radiation, and stable one-loop $2 \rightarrow 3$ amplitudes, made possible a large class of $2 \rightarrow 2$ processes at NNLO
NNLO timeline

G. Zanderighi, CERN Courier (2017)
NNLO beyond $2 \to 2$

- Will require combining multi-loop generalized unitarity (lots of progress already but no time to review) with sophisticated methods for multi-scale multi-loop integration.

- But we know it is possible:

Trees can be recycled into multi loops!
$gg \rightarrow H$ at NNNLO

Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger, 1302.4379, 1311.1425, 1403.4616, 1411.3584, 1503.06056, 1505.04110

- First step, integrate out top quark loop, removes 1 scale and 1 loop:

- Last step, do some very complicated phase space integrals as loop integrals, using “reverse unitarity”:

- Avoid the nastiest special functions by performing a very high order threshold expansion, removing 1 more scale.
The “QCD for LHC” revolution

- Many important hadron collider processes have been computed at NLO and NNLO in the past decade (even one at NNNLO), well beyond what was previously thought possible
- Required a new understanding of scattering amplitudes, at a formal level, as well as efficient, stable implementation
- Many people contributed to this progress
- Parallel progress in understanding supersymmetric gauge & gravity theories
- Revolution is far from over; NNLO 2 $\rightarrow$ 3 awaits!

L. Dixon       Scattering Amplitudes for LHC       Humboldt U.       9 March 2018
Extra Slides
LHC Data Dominoented by Jets

Jets from quarks and gluons.
- $q,g$ from decay of new particles?
- Or from old QCD?

- Every process shown also with one more jet at $\sim 1/5$ the rate
- Need accurate production rates for $X + 1,2,3,\ldots$ jets in Standard Model

new physics?? →
Fixed order vs. Monte Carlo

- **NLO fixed-order → few partons**: no model of long-distance effects included; cannot pass through a detector simulation
- **Methods available for matching NLO parton-level results to parton showers, accuracy:**
  - **MC@NLO** Frixione, Webber (2002); …; SHERPA implementation
  - **POWHEG** Nason (2004); Frixione, Nason, Oleari (2007)
  - **SHERPA** Krauss et al. (2012,…)
- Recently implemented for increasingly complex final states
Pure QCD: $pp \rightarrow 4 \text{ jets}$ vs. ATLAS data

4 jet events might hide pair production of 2 colored particles, each decaying to a pair of jets.

Detailed study of multi-jet QCD dynamics may help understand other channels.
Top Quark Pairs + Jets

• Like (W,Z) + jets, very important bkgd
• Cross sections large
  – no electroweak couplings
• Jets boost $t\bar{t}$ system, increase MET, provide jets to fake $t\bar{t}H, \ H \rightarrow b\bar{b}$.
• State of art circa 2010:
  • NLO $tt + 1$ jet: Dittmaier, Uwer, Weinzierl, hep-ph/0703120,…
  • + top decays: Melnikov, Schulze, 1004.3284
  • + NLO parton shower: Kardos, Papadopoulos, Trócsányi, 1101.2672

  • NLO $tt + bb$: Bredenstein, Denner, Dittmaier, Pozzorini, 0905.0110, 1001.4006; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723
  • NLO $tt + 2$ jets: Bevilacqua, Czakon, Papadopoulos, Worek, 1002.4009
<table>
<thead>
<tr>
<th>Process</th>
<th>Order</th>
<th>Description</th>
<th>Other Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>dijets</td>
<td>O(3%)</td>
<td>gluon-gluon, gluon-quark</td>
<td>PDFs, strong couplings, BSM</td>
</tr>
<tr>
<td>H+0 jet</td>
<td>O(3-5%)</td>
<td>fully inclusive (N3LO)</td>
<td>Higgs couplings</td>
</tr>
<tr>
<td>H+1 jet</td>
<td>O(7%)</td>
<td>fully exclusive; Higgs decays, infinite mass tops</td>
<td>Higgs couplings, Higgs $p_t$, structure for the $ggH$ vertex.</td>
</tr>
<tr>
<td>tT pair</td>
<td>O(4%)</td>
<td>fully exclusive, stable tops</td>
<td>top cross section, mass, $p_t$, FB asymmetry, PDFs, BSM</td>
</tr>
<tr>
<td>single top</td>
<td>O(1%)</td>
<td>fully exclusive, stable tops, $t$-channel</td>
<td>$V_{tb}$, width, PDFs</td>
</tr>
<tr>
<td>WBF</td>
<td>O(1%)</td>
<td>exclusive, VBF cuts</td>
<td>Higgs couplings</td>
</tr>
<tr>
<td>W+j</td>
<td>O(1%)</td>
<td>fully exclusive, decays</td>
<td>PDFs</td>
</tr>
<tr>
<td>Z+j</td>
<td>O(1-3%)</td>
<td>decays, off-shell effects</td>
<td>PDFs</td>
</tr>
<tr>
<td>ZH</td>
<td>O(3-5%)</td>
<td>decays to $bb$ at NLO</td>
<td>Higgs couplings ($H \rightarrow bb$)</td>
</tr>
<tr>
<td>ZZ</td>
<td>O(4%)</td>
<td>fully exclusive</td>
<td>Trilinear gauge couplings, BSM</td>
</tr>
<tr>
<td>WW</td>
<td>O(3%)</td>
<td>fully exclusive</td>
<td>Trilinear gauge couplings, BSM</td>
</tr>
<tr>
<td>top decay</td>
<td>O(1-2%)</td>
<td>exclusive</td>
<td>Top couplings</td>
</tr>
<tr>
<td>H $\rightarrow$ bb</td>
<td>O(1-2%)</td>
<td>exclusive, massless</td>
<td>Higgs couplings, boosted</td>
</tr>
</tbody>
</table>