Towards bouncing
and Genesis cosmologies

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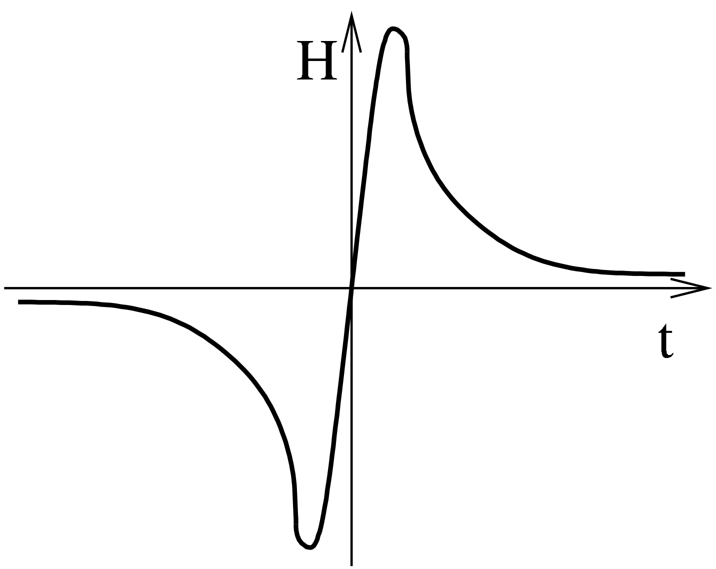
- Bouncing Universe:
  Starts from contracting stage $\Longrightarrow$ bounce $\Longrightarrow$ expansion

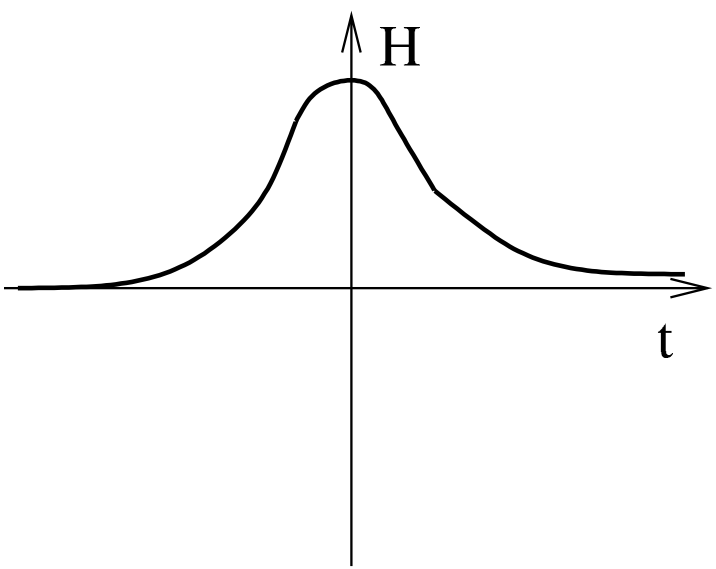
- Genesis

  Creminelli, Nicolis, Trincherini' 2010

  Starts from Minkwski, empty space, then energy density builds up, Universe starts to expand, expansion accelerates.

  Both can be viewed as alternatives to inflation.





What about problems of the Hot Big Bang theory?

- If contracion (or Genesis) stage is long, horizon problem is solved
- Flatness and homogeneity problems are either moved to infinite past, or solved by ekpyrotic contraction (with $p > \rho$)

Steinhardt, Turok, Ijjas,...

- Entropy problem: need exit from exotic epoch to conventional hot epoch

Pro: no initial singularity $\Longleftrightarrow$ geodesic completeness

But very difficult theoretically.

The Null Energy Condition, NEC

$$T_{\mu\nu} n^\mu n^\nu > 0$$

for any null vector n^μ, such that $n_\mu n^\mu = 0$.

- Quite robust
- In the framework of classical General Relativity implies a number of properties
  - Penrose theorem

Penrose' 1965

In cosmology: if the NEC holds, and spatial curvature is negligible, there is initial singularity

No bounce, no Genesis.

- A combination of Einstein equations (spatially flat):

$$\frac{dH}{dt} = -4\pi G(\rho + p)$$

$\rho = T_{00}$ = energy density; $T_{ij} = \delta_{ij} p$ = effective pressure.

- The Null Energy Condition:

$T_{\mu\nu} n^\mu n^\nu > 0$, $n^\mu = (1,1,0,0) \Longrightarrow \rho + p > 0 \Longrightarrow dH/dt < 0$,

Hubble parameter was greater early on. No bounce

Penrose: there was a singularity in the past, $H = \infty$.

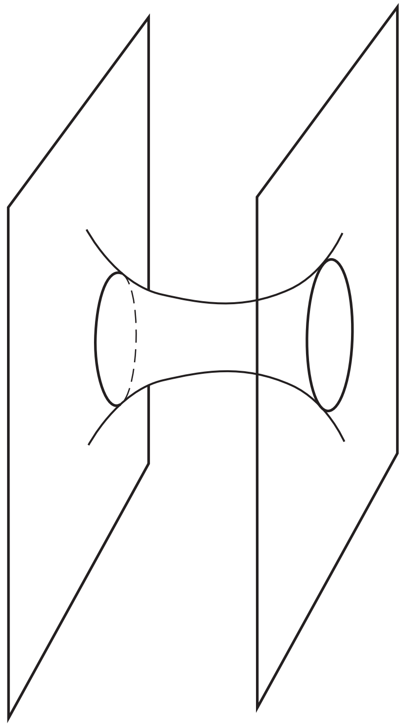
- Another side of the NEC:
Covariant energy-momentum conservation:

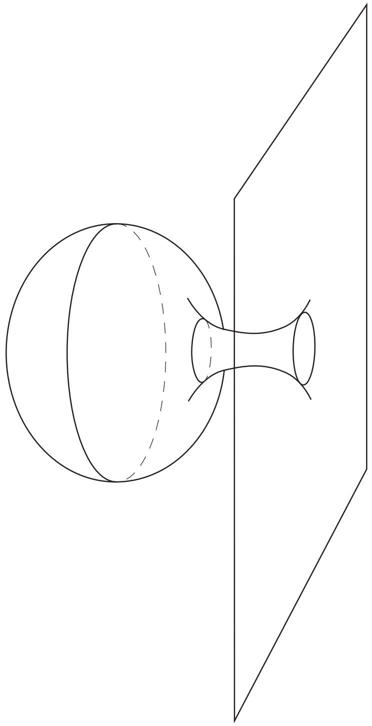
$$\frac{d\rho}{dt} = -3H(\rho + p)$$

NEC: energy density decreases during expansion, except for $p = -\rho$, cosmological constant. No Genesis

Many other facets of the NEC,

- no-go for Lorentzian wormholes





No-go for creation of a universe in the laboratory

- Question raised in mid-80's, right after invention of inflationary theory

Berezin, Kuzmin, Tkachev' 1984; Guth, Farhi' 1986

Idea: create, in a finite region of space, inflationary initial conditions $\Longrightarrow$ this region will inflate to enormous size and in the end will look like our Universe.

- Do not need much energy: pour little more than Planckian energy into little more than Planckian volume.

If NEC holds, no way: initial singularity

Guth, Farhi' 1986; Berezin, Kuzmin, Tkachev' 1987

Can the Null Energy Condition be violated in classical field theory?

- Folklore until recently: NO!

  Pathologies:

  - Ghosts:

$$E = -\sqrt{p^2 + m^2}$$

Example: theory with wrong sign of kinetic term,

$$\mathscr{L} = -(\partial\phi)^2 \quad \Longrightarrow \quad \rho = -\dot{\phi}^2 - (\nabla\phi)^2 \,, \qquad p = -\dot{\phi}^2 + (\nabla\phi)^2$$

$$\rho + p = -2\dot{\phi}^2 < 0$$

Catastrophic vacuum instability

NB: Can be cured by Lorentz-violation

(but hard! – even though Lorentz-violation is inherent in cosmology)

Other pathologies

- Gradient instabilities:

$$E^2 = -(p^2 + m^2) \implies \varphi \propto \mathrm{e}^{|E|t}$$

- Superluminal propagation of excitations

Theory cannot descend from healthy Lorentz-invariant UV-complete theory

Adams et. al.' 2006

No-go theorem for theories with Lagrangians involving first derivatives of fields only (and minimal coupling to gravity)

Dubovsky, Gregoire, Nicolis, Rattazzi' 2006

Buniy, Hsu, Murray' 2006

$$L = F(X^{IJ}, \pi^I)$$

with $X^{IJ} = \partial_\mu \pi^I \partial^\mu \pi^J \Longrightarrow$

$$T_{\mu\nu} = 2\frac{\partial F}{\partial X^{IJ}} \partial_\mu \pi^I \partial_\nu \pi^J - g_{\mu\nu} F$$

In homogeneous background

$$T_{00} \equiv \rho = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} - F$$

$$T_{11} = T_{22} = T_{33} \equiv p = F$$

and

$$\rho + p = 2\frac{\partial F}{\partial X^{IJ}} X^{IJ} = 2\frac{\partial F}{\partial X^{IJ}} \dot{\pi}^I \dot{\pi}^J$$

NEC-violation: matrix $\partial F/\partial X_c^{IJ}$ non-positive definite. But

Lagrangian for perturbations $\pi^I = \pi_c^I + \delta\pi^I$

$$L_{\delta\pi} = A_{IJ}\, \partial_t \delta\pi^I \cdot \partial_t \delta\pi^J - \frac{\partial F}{\partial X_c^{IJ}}\, \partial_i \delta\pi^I \cdot \partial_i \delta\pi^J + \dots$$

Gradient instabilities and/or ghosts

NB. Loophole: $\partial F/\partial X_c^{IJ}$ degenerate.

Higher derivative terms (understood in effective field theory sense) become important and help.

Ghost condensate

Arkani-Hamed et. al.' 2003

Ways out until fairly recently:

- Large spatial curvature at bounce $\Longleftrightarrow$ possible, but needs inflation to solve curvature problem
- Give up *classical* field theory

Can the Null Energy Condition be violated in a simple and healthy way?

- Folklore until fairly recently: NO!

  Today: YES,

  Senatore' 2004;
  V.R.' 2006;
  Creminelli, Luty, Nicolis, Senatore' 2006

- General properties of non-pathological NEC-violating field theories:

  - Non-standard kinetic terms
  - Non-trivial background

- Candidate NEC-violating theory: Horndeski

  Horndeski' 1974

  aka Euler hierarchies, aka generalized Galileons, aka KGB, aka generalized Fab Four

Example:

Creminelli, Nicolis, Trincherini '2010
Deffayet, Pujolas, Sawicki, Vikman' 2010
Kobayashi, Yamaguchi, Yokoyama' 2010

simplest generalized Galileon theory: cubic Galileon + Einstein–Hilbert ($\kappa = 8\pi G$)

$$L = \frac{1}{2\kappa} R + F(\pi, X) - K(\pi, X) \Box \pi$$

where $$X = \nabla_\mu \pi \nabla^\mu \pi$$

- Second order equations of motion (but L cannot be made first order by integration by parts)
- Generalization: Horndeski theory (1974) rediscovered many times. Four Lagrangians in 4d

  Minkowski: Fairlie, Govaerts, Morozov' 91; Nicolis, Rattazzi, Trincherini' 09, ...

  $$L_n = K_n(X, \pi) \partial^{\mu_1} \partial_{[\mu_1} \pi \cdot \dots \cdot \partial^{\mu_n} \partial_{\mu_n]} \pi$$

  Generalization to GR: L_0, L_1 trivial, $L_{n>1}$ non-trivial (below)

  Horndeski '1974; Deffayet, Esposito-Farese, Vikman' 09

Simple playground

$$L = F(Y) \cdot \mathrm{e}^{4\pi} + K(Y) \cdot \Box\pi \cdot \mathrm{e}^{2\pi}$$

$$\Box\pi \equiv \partial_\mu \partial^\mu \pi \, , \qquad Y = \mathrm{e}^{-2\pi} \cdot (\partial_\mu \pi)^2$$

- Second order equations of motion
- Scale invariance: $\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln\lambda$.

  (technically convenient)

Homogeneous solution in Minkowski space (attractor)

$$e^{\pi_c} = \frac{1}{\sqrt{Y_*}\,|t|}\,, \qquad t < 0$$

- $Y \equiv e^{-2\pi_c} \cdot (\partial_\mu \pi_c)^2 = Y_* =$ const, a solution to

$$Z(Y_*) \equiv -F + 2Y_* F_Y - 2Y_* K + 2Y_*^2 K_Y = 0$$

$$F_Y = dF/dY\,.$$

Energy density

$$\rho = e^{4\pi_c} Z = 0$$

Effective pressure T_{11}:

$$p = e^{4\pi_c}\,(F - 2Y_* K)$$

Can be made negative by suitable choice of $F(Y)$ and $K(Y)$
$\Longrightarrow \rho + p < 0$, violation of the Null Energy Condition.

Turning on gravity

$$p = \mathrm{e}^{4\pi_c}(F - 2Y_* K) = -\frac{M^4}{Y_*^2 |t|^4}, \qquad \rho = 0$$

M: mass scale characteristic of π

- Use $\dot{H} = -4\pi G(p + \rho) \Longrightarrow$

$$H = \frac{4\pi}{3} \frac{M^4}{M_{Pl}^2 Y_*^2 |t|^3}$$

NB:

$$\rho \sim M_{Pl}^2 H^2 \propto \frac{1}{M_{Pl}^2 |t|^6}$$

Genesis.

NB: Early times $\Longrightarrow$ weak gravity, $\rho \ll p$. Expansion, $H \neq 0$, is negligible for dynamics of π.

Perturbations about homogeneous Minkowski solution

$$\pi(x^\mu) = \pi_c(t) + \delta\pi(x^\mu)$$

- Quadratic Lagrangian for perturbations:

$$L^{(2)} = \mathrm{e}^{2\pi_c} Z_Y (\partial_t \delta\pi)^2 - B(\vec{\nabla}\delta\pi)^2 + W(\delta\pi)^2$$

$B = B[Y; F, K, F_Y, K_Y, K_{YY}]$. Absence of ghosts:

$$Z_Y \equiv dZ/dY > 0 \quad \text{at } Y = Y_*$$

Absence of gradient instabilities and of superluminal propagation

$$B > 0\,; \qquad B < \mathrm{e}^{2\pi_c} Z_Y$$

Can be arranged.

- Bounce:

  (1) early contraction dominated by another matter; Galileon takes over and reverses sign of H

  (2) Judicial choice of Lagrangian functions F and K.

- Both regimes can be made healthy: neither ghosts nor gradient instabilities

So far, so good

What about more complete cosmologies

with conventional expansion in the end (inflationary or not)?

Early examples: either Big Rip singularity in future,
$\pi = \infty$, $H = \infty$ at $t < \infty$

Creminelli, Nicolis, Trincherini '2010

or gradient instability

Cai, Easson, Brandenberger '2012;
Koehn, Lehners, Ovrut '2014;
Pirtskhalava, Santoni, Trincherini, Uttayarat '2014;
Qiu, Wang '2015;
Kobayashi, Yamaguchi, Yokoyama '2015;
Sosnovikov '2015

Is instability generic
or just a drawback of models constructed so far?

Can one construct healthy bounce and/or Genesis
within the original theory?

No-go for Horndeski

To make long story short

Consider cubic theory

$$L = \frac{1}{2\kappa} R + F(\pi, X) - K(\pi, X)\Box\pi$$

Assume that there exists bounce or Genesis solution (spatially flat).

Calculate quadratic Lagrangian for salar perturbations (metric included)

$$L^{(2)} = A\dot{\chi}^2 - \frac{1}{a^2} B(\partial_i \chi)^2 + \ldots$$

No ghosts, gradient instabilities:

$$A > 0 \,, \qquad B > 0$$

$$\frac{B\dot{\pi}^2}{a} = \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 \, , \qquad \mathscr{R} = a^{-1}\left(K_X \dot{\pi}^3 - \frac{1}{\kappa} H\right)$$

$B > 0 \implies \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 > 0$. Integrate $\dot{\mathscr{R}}/\mathscr{R}^2 - \kappa a > 0$:

$$\frac{1}{\mathscr{R}(t_i)} - \frac{1}{\mathscr{R}(t_f)} > \kappa \int_{t_i}^{t_f} dt \; a(t) \, .$$

Bouncing scenario, Genesis: $\int_{-\infty}^{t_f} dt \; a(t) = \infty \, , \qquad \int_{t_i}^{\infty} dt \; a(t) = \infty \, .$

- Suppose $\mathscr{R}(t_i) > 0$. Then at $t > t_i$ one has $\mathscr{R}(t) > 0$ (since $\dot{\mathscr{R}} > 0$).

  $$\frac{1}{\mathscr{R}(t_f)} < \frac{1}{\mathscr{R}(t_i)} - \kappa \int_{t_i}^{t_f} dt \; a(t) \, .$$

  Right hand side changes sign at some $t_f \implies \mathscr{R}(t_f) = \infty$, singularity in future.

- Case $\mathscr{R}(t) < 0$: singularity in past. QED

- Similar argument forbids wormholes (in that case problem is with $A \Longleftrightarrow$ ghosts)

- Argument intact in presence of extra matter (obeying NEC) which interacts with Galileon only gravitationally:

$$\frac{B\dot{\pi}^2}{a} = \dot{\mathscr{R}} - \kappa a \mathscr{R}^2 - \frac{\rho_M + p_M}{2a} \; ,$$

even worse.

- Extends to general Horndeski theories with all four allowed terms present in Lagrangian (below)

Kobayashi '2016

- Extends to model with extra conventional scalar ϕ and

$$L = -\frac{1}{2\kappa}R + F(\pi, X, \phi, X_{\pi\phi}, X_\phi) + K(\pi, X, \phi)\Box\pi$$

where $X_{\pi\phi} = \nabla_\mu \pi \cdot \nabla^\mu \phi, \quad X_\phi = (\nabla\phi)^2$.

Kolevatov, Mironov '2016

Are there ways to repair?

Attitudes:

- Gradient instability would be cured by higher order terms in low energy effective action

  Pirtskhalava, Santoni, Trincherini, Uttayarat '2014;
  Koehn, Lehners, Ovrut '2016

  Take low UV cutoff and cook up short enough period of instability $\Longrightarrow$ instability does not have time to develop.

  Diffiult but possible at the expence of sort of "fine tuning"

- $\int_{-\infty}^{t} dt\ a(t) =$ finite $\Longrightarrow$ modified Genesis (not bounce).

  But past geodesic incompleteness. Time-like geodesics going backwards reach spatial infinity in finite proper time.

Problematic!

General Horndeski theory

$$L = F(\pi, X) - K(\pi, X)\Box\pi$$
$$+ G_4(\pi, X)R + G_{4,X}\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$
$$+ G_5 \cdot G^{\mu\nu}\nabla_\mu\nabla_\nu\pi - \frac{1}{6}G_{5,X}\left[(\Box\pi)^3 - 3\Box\pi\cdot(\nabla_\mu\nabla_\nu\pi)^2 + 2(\nabla_\mu\nabla_\nu\pi)^3\right]$$

- Modified gravity (scalar-tensor). Second order field eqs (!)
- Again instability of Genesis and bounce.

Kobayashi '2016; Ijjas, Steinhardt '2016

Choose unitary gauge $\delta\pi = 0$.

$$ds^2 = N^2 dt^2 - a^2 e^{2\zeta}(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}k_{kj})(N^i dt + dx^i)(N^j dt + dx^j)$$

Dynamical variables in scalar sector: transverse traceless h_{ij} and ζ.

$$L_\zeta = A_\zeta \dot{\zeta}^2 - a^{-2} B_\zeta (\partial_i \zeta)^2 \,, \quad L_h = A_h \dot{h_{ij}}^2 - a^{-2} B_h (\partial_k h_{ij})^2$$

Key relation

$$\frac{d}{dt}\left(\frac{a(t) A_h^2(t)}{\Theta(t)}\right) = -a(t)(B_\zeta + B_h)$$

where $\Theta(t) = -2HG_4 + \dot{\pi} X K_X + \dots$, a complicated expression involving backrgound $\pi(t)$ and $H(t)$. Same story:

$$\frac{a(t_f) A_h^2(t_f)}{\Theta(t_f)} - \frac{a(t_i) A_h^2(t_i)}{\Theta(t_i)} = - \int_{t_i}^{t_f} dt \; a(t)(B_\zeta + B_h)$$

Impossible for $B_\zeta > 0$, $B_h > 0$, finite A_h, Θ and

$$\int_{-\infty}^{t_f} dt \; a(t)(B_\zeta + B_h) = \infty \,, \quad \int_{t_i}^{+\infty} dt \; a(t)(B_\zeta + B_h) = \infty \,.$$

$$\frac{a(t) A_h^2(t)}{\Theta(t)} = \infty \quad \text{at some time } \; t$$

Yet another approach

Another modified Genesis and bounce

Wetterich' 2015; Kobayashi '2016; Ijjas, Steinhardt '2016

$$\frac{a(t_i)A_h^2(t_i)}{\Theta(t_i)} = \frac{a(t_f)A_h^2(t_f)}{\Theta(t_f)} - \int_{t_i}^{t_f} dt\ a(t)(B_\zeta + B_h)$$

Arrange for convergent integral

$$\int_{-\infty}^{t_f} dt\ a(t)(B_\zeta + B_h) < \infty$$

No-go theorem does not work.

But gravity unconventional as $t \to -\infty$: $B_\zeta, B_T \to 0$.

In explicit examples so far

$A_h, A_\zeta \to 0$ as $t \to -\infty$. Effective Planck mass vanishes as $t \to -\infty$.

Strong coupling?

Beyond Horndeski theories

Zumalacárregui, Gacia-Bellido' 2014

Gleyzes, Langlois, Piazza, Vernizzi' 2014

- Give up requirement of second order field equations
- Require that there remains one scalar degree of freedom + tensor

  Allowed terms

$$G_4(\pi, X)R + F_4(\pi, X)\left[(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2\right]$$

  F_4 and G_4 no longer related.

- Way to understand: disformal transformation

$$g_{\mu\nu} \to \Omega(\pi, X)g_{\mu\nu} + \Lambda(\pi, X)\partial_\mu\pi\partial_\nu\pi$$

  Horndeski $\to$ beyond Horndeski

  NB: This is formal trick. Ω, Λ may be singular

Now

$$a(t)(B_\zeta + B_h) = -\frac{d}{dt}\left[\frac{aA_h(A_h-\Delta)}{\Theta}\right]$$

$(A_h - \Delta)$ can cross zero without singularity.

No-go theorem no longer holds

Effective field theory: Cai et.al.' 2016, Creminelli et.al.'2016

Covariant formalism: Kolevatov et.al.' 2017, Cai, Piao' 2017

NB: $\Theta = 0$ not a problem, gauge artifact

Ijjas'2017;

Mironov, V.R., Volkova' 2018

Bounce: proof of principle

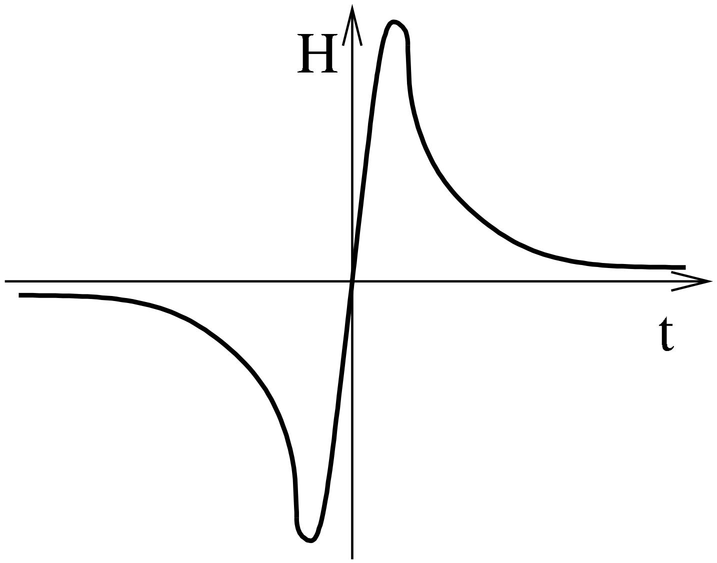
"Inverse method"

Term by Ijjas, Steinhardt '2016

- Choose background $\pi(t) = t$, no loss of generality

  Then $X = (\partial\pi)^2 = 1$. Field equations and stability conditions involve $f_0(t) = F(\pi(t))$, $f_1(t) = F_X(\pi(t))$, etc., all at $X = 1$.

- Choose your favorite $H(t)$ such that $H(t) \to \frac{1}{3t}$ as $|t| \to \infty$
GR + Galileon = conventional massless scalar.

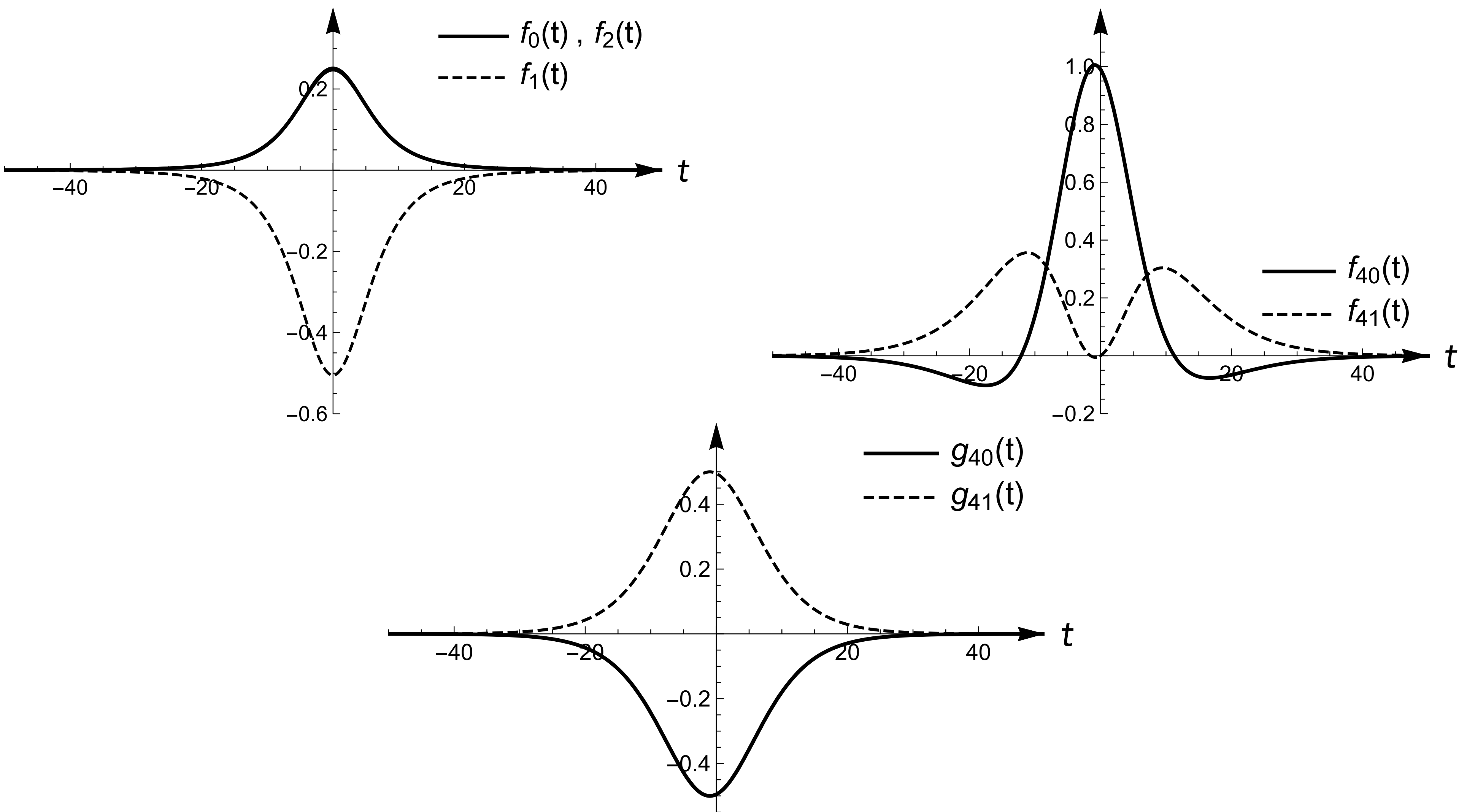


- Asymptotics of Lagrangian functions as $|t| \to \infty$:

$$F(t) = \frac{1}{t^2}, \quad F_X(t) = \frac{1}{t^2} \Longrightarrow F = \frac{(\partial \pi)^2}{\pi^2} = (\partial \log \pi)^2$$

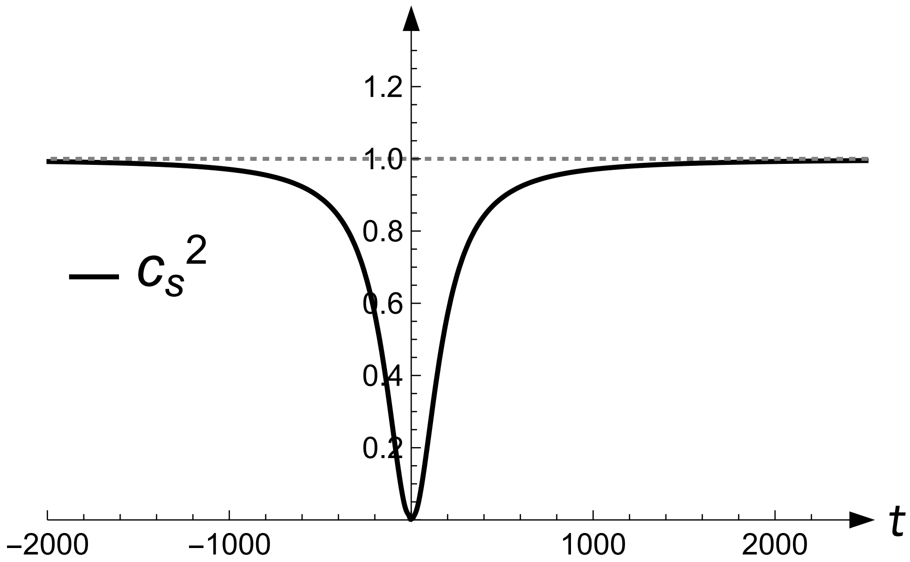
$$G_4 = \frac{M_{Pl}^2}{16\pi}, \qquad K = F_4 = 0$$

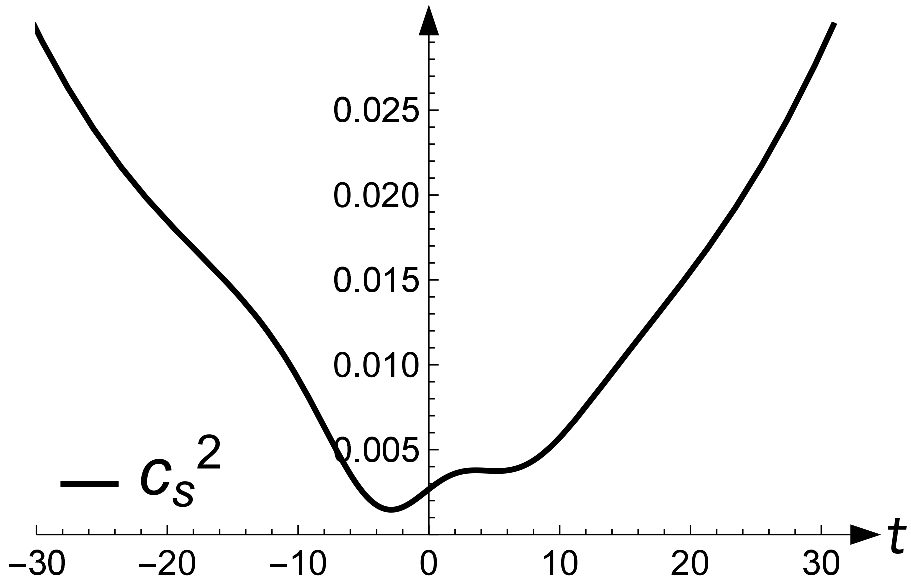
- Cook up Lagrangian functions in such a way that
  - Field equation are satisfied
  - Stability conditions are satisfied at all times



No kidding: speed of gravity waves is always 1.

Speed of scalar perturbation $0 < c_s^2 \leq 1$





Completely stable bounce

Similar construction for Genesis.

Other issues

- Transition to hot epoch. Not a problem, similar to k-inflation.

  Armendariz-Picon, Damour, Mukhanov' 99

- Generation of density perturbations. Need a separate mechanism to generate nearly flat power spectrum.

  To name a few:

  - Matter bounce

    Finelli, Brandenberger' 2001
    Wands' 98

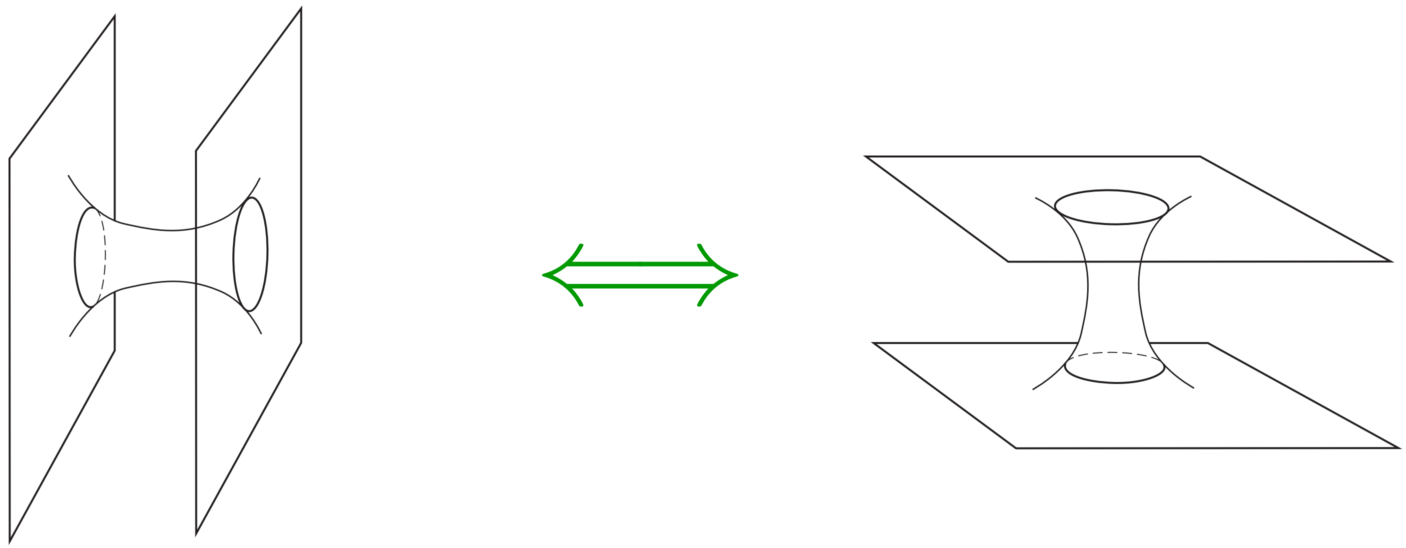
  - Conformal mechanism

    V R' 2009
    Creminelli, Nicolis, Trincherini' 2010
    Hinterbichler, Khouri' 2011, ...

- Tensor perturbations (gravity waves) are absent

What about wormholes?

Static wormhole $\Longleftrightarrow$ Bouncing Universe



No-go for Horndeski: no stable, static, spherically symmetric wormholes: always ghosts.

V.R. '2016

Evseev, Melichev' 2018

Theorem does not hold beyond Horndeski

Mironov, V.R., Volkova '2018

Franciolini, Hui, Penco, Santoni, Trincherini' 2018

Work in progress

Instead of conclusion

- Constructing bouncing or Genesis cosmology is a non-trivial task. Even harder than originally thought.
- Exotic fields are needed. It is “beyond Horndeski” that does the job.
  - UV completion not known (and may not exist)
- Fully consistent bouncing and Genesis cosmologies possible at classical field theory level
- Wormholes, creation of a universe in lab: open issues.
  - NB: wormhole $\Longleftrightarrow$ time machine

Morris, Thorne, Yurtsever’ 1988

- Ahead: more to understand