

# SHOWER MONTE CARLO + NLO: POWHEG

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Loops and Legs in Quantum Field Theory

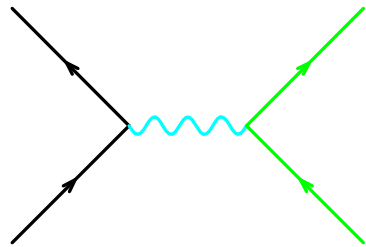
Wörlitz, 26 April 2010

- Shower Monte Carlo + NLO
- The POWHEG method
- The POWHEG BOX
- A few POWHEG results
- Conclusions

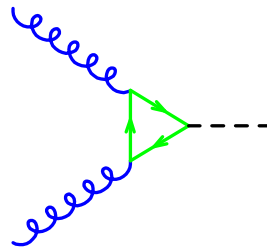


# High energy collisions

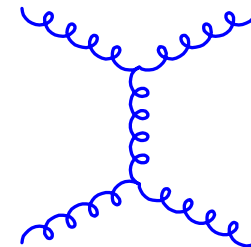
High-energy particle physics deals with the scattering and the production of elementary constituents



$$e^+e^- \rightarrow q\bar{q}$$



$$gg \rightarrow H$$

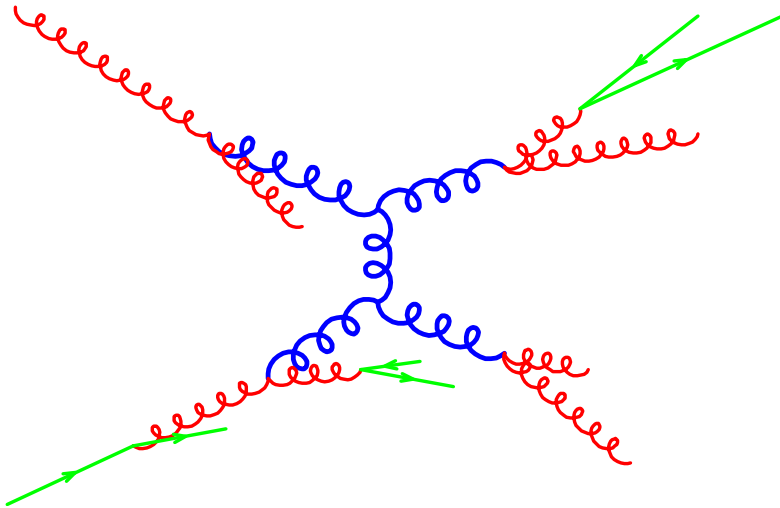


$$gg \rightarrow gg$$

Ideally, one needs elementary constituents as projectiles and targets, (i.e. a collider for leptons, gluons and quarks) and a final-state detector of leptons, gluons and quarks. **Not obvious** for quarks and gluons:

- at **short distance**, due to asymptotic freedom, quarks and gluons behave as free particles
- at **long distance**, infrared slavery: very strong interactions hide the simplicity of the description of the constituents.

## Dominant corrections



**Collinear-splitting** processes in the initial and final state (always with **transverse momenta**  $> \Lambda_{\text{QCD}}$ ) are **strongly enhanced**. This is due to the fact that, in perturbation theory, the **denominators** in the propagators are **small**.

- The algorithms that evaluate (and **resum**) all these enhanced contributions are called **shower algorithms**.
- Shower algorithms give a description of a hard collision up to **distances of order**  $1/\Lambda_{\text{QCD}}$ .
- At larger distances, perturbation theory breaks down and we need to resort to **non-perturbative methods** (i.e. lattice calculations). However, these methods can be applied only to simple systems. The only viable alternative is to use **models of hadron formation**.

## Shower Monte Carlo

- In high-energy collider physics not many questions can be answered without a Shower Monte Carlo (SMC).
- The name **shower** comes from the fact that we **dress** a **hard event** with **QCD radiation**.
- In general, the accuracy of SMC programs is **Leading Order + Leading Log**. They resum the largest logarithmic terms coming from the **collinear (and soft) regions**.
- Events are then characterized by a **small number** of **high- $p_T$** , **well-separated**, final-state partons (the ones described by the **tree-level** Born amplitude) + many **collinear partons**, whose collinear divergences have been correctly resummed.

# NLO

LO matrix elements are (often, but not always) good for shapes. Uncertain absolute normalization

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + b_0 \alpha_s(\mu_0) \log(\mu^2/\mu_0^2)}, \quad b_0 = \frac{11C_A - 4n_f T_F}{12\pi} \Big|_{n_f=5} \approx 0.6$$

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu) [1 - b_0 \alpha_s(\mu) \log(4)]^n \approx \alpha_s^n(\mu) (1 - n\alpha_s(\mu))$$

For  $\mu = 100 \text{ GeV}$ ,  $\alpha_s = 0.12$ , normalization uncertainty:

$W + 1J$	$W + 2J$	$W + 3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

To improve on this, we need to go to NLO

- Positive experience with NLO calculations at LEP, HERA and Tevatron
- NLO results are cumbersome to compute: typically made up of an  $n$ -body (Born + virtual + soft and collinear remnants) and  $(n + 1)$ -body (real emission) terms, both divergent (finite only when summed up).

## NLO vs Shower Monte Carlo

### NLO

- ✓ accurate shapes at high  $p_T$
- ✓ normalization accurate at NLO order
- ✓ reduced dependence on renormalization and factorization scales
- ✗ wrong shapes at small  $p_T$
- ✗ description only at the parton level

### SMC (LO + shower)

- ✗ bad description at high  $p_T$
- ✗ normalization accurate only at LO
- ✓ correct Sudakov suppression at small  $p_T$
- ✓ simulate events at the hadron level

It is natural to try to merge the two approaches, keeping the good features of both

- MC@NLO [Frixione and Webber, 2001]
- POWHEG [Nason, 2004]

## POsitive-Weight Hardest Emission Generator

- ✓ POWHEG is a method for interfacing NLO calculations with parton shower programs [Nason, hep-ph/0409146]
- ✓ it generates the hardest emission first, with NLO accuracy, producing events with positive weights. The acronym comes from this feature
- ✓ The rest of the shower is performed by the (usual) LO Shower Monte Carlo programs, such as PYTHIA, HERWIG ...  
It is then possible to compare the different outputs
- ✓ As far as the hardest emission is concerned, POWHEG guarantees
  - NLO accuracy on integrated quantities
  - collinear, double-log (soft-collinear), large- $N_c$ -soft single-log of the Sudakov (in fact, corrections that exponentiates are obviously OK)
- ✓ The subsequent (less hard) emissions have the accuracy of the Shower Monte Carlo program one is using.

## Existing implementations

Up to now, the following processes have been implemented using the POWHEG method:

- $pp \rightarrow ZZ$  [Nason and Ridolfi, hep-ph/0606275]
- $e^+e^- \rightarrow \text{hadrons}$  [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]  
 $e^+e^- \rightarrow t\bar{t}$  with top decay [Latunde-Dada, arXiv:0806.4560]
- $pp \rightarrow Q\bar{Q}$  ( $c\bar{c}$ ,  $b\bar{b}$ ,  $t\bar{t}$ ) with **spin correlations** [Frixione, Nason and Ridolfi, arXiv:0707.3088]
- $pp \rightarrow W/Z$  with **spin correlations** [Alioli, Nason, Oleari and Re, arXiv:0805.4802; Hamilton, Richardson and Tully, arXiv:0806.0290]
- $pp \rightarrow H$  [Alioli, Nason, Oleari and Re, arXiv:0812.0578; Hamilton, Richardson and Tully, arXiv:0903.4345]
- $pp \rightarrow H + W/Z$  [Hamilton, Richardson and Tully, arXiv:0903.4345]



## Existing implementations

- [single-top production](#), in the  $s$  and  $t$  channel, with top decay [Alioli, Nason, Oleari and Re, arXiv:0907.4076]
- [Higgs boson production in vector boson fusion](#) [Nason and Oleari, arXiv:0911.5299] in the POWHEG BOX

All POWHEG implementations for hadronic colliders have been interfaced to both [PYTHIA](#) and [HERWIG](#).

## To appear very soon

- $pp \rightarrow Z + 1 \text{ jet}$  [Alioli, Nason, Oleari and Re] in the POWHEG BOX
- $pp \rightarrow VV$  [Hamilton]

## The POWHEG BOX



## The POWHEG BOX

The POWHEG BOX is a public-available computer framework, presented in [Alioli, Nason, Oleari and Re, arXiv: 1002.2581], that implements in practice the theoretical construction of the POWHEG formalism, for generic NLO processes, according to the general formulation of POWHEG given in [Frixione, Nason and Oleari, arXiv:0709.2092]

More precisely, the user should only supply:

- ✓ the lists of the Born and real processes (i.e.  $sc \rightarrow gud \iff [3, 4, 0, 2, 1]$ )
- ✓ the Born phase space
- ✓ the Born squared amplitudes, the color-correlated and spin-correlated amplitudes, for all partonic subprocesses  
All these amplitudes are common ingredients of a NLO calculation
- ✓ the real squared amplitude for all the relevant real-emission subprocesses
- ✓ the finite part of the virtual corrections, computed in conventional dimensional regularization or in dimensional reduction
- ✓ the Born color structures in the limit of large number of colors.

All the rest will be done **automatically!**

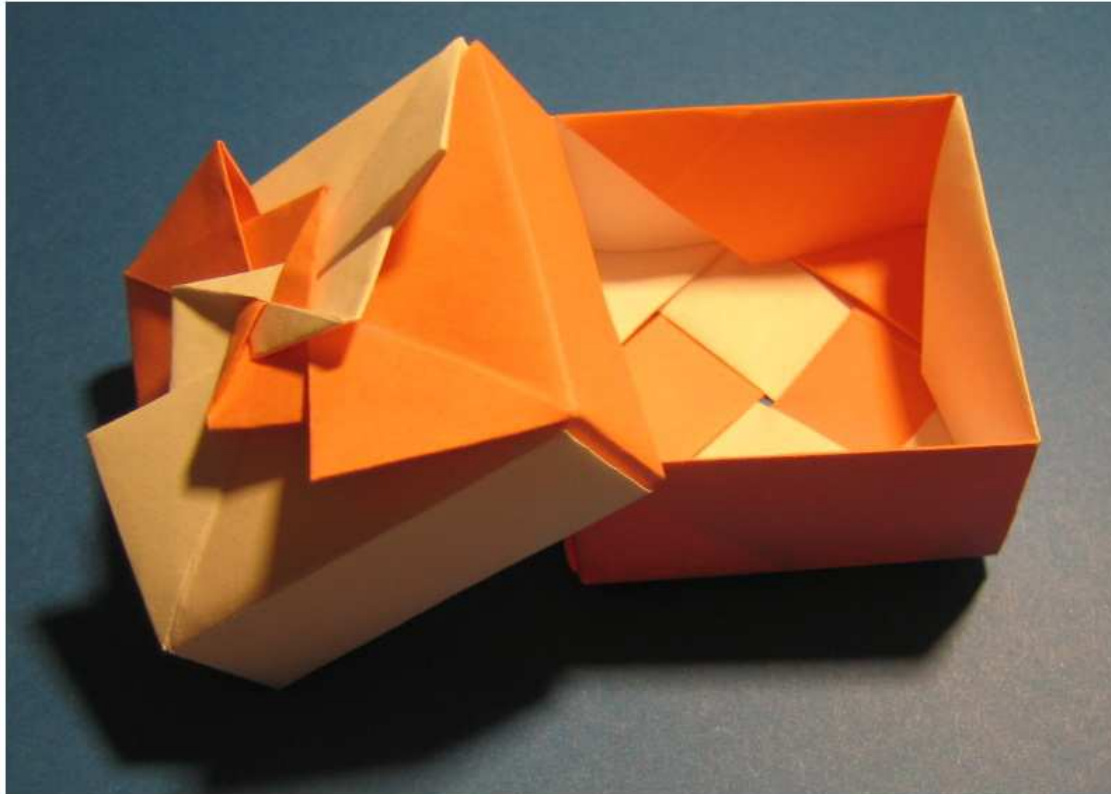
## The POWHEG BOX

The user **should not worry** about

- ✓ the **phase space** for initial-state radiation and final-state radiation (i.e. the phase space for real emission)
- ✓ the **combinatorics**, the identification of all **singular regions** in the real amplitude  $R$ , the **soft** and **collinear limits**, the calculation of **all the counterterms**
- ✓ the calculation of the differential NLO cross section  
Spinoff: **NLO** results using the **FKS subtraction scheme**
- ✓ the calculation of the upper bounds for the generation of radiation (for an efficient generation of the Sudakov-suppressed events)
- ✓ the **generation of radiation**
- ✓ writing the event into the Les Houches interface (to communicate with the LO Shower Monte Carlo programs)

The user has **only to know** in which format to supply the ingredients listed before.

## The POWHEG BOX



No need to open the BOX!

## The POWHEG BOX How-To

- parameter (`nlegborn=5`) [ $pp \rightarrow (Z \rightarrow e^+ e^-) + j$ ] in included file `pwhg_flst.h`  
`flst_nborn` and `flst_nreal`
- `flst_born(k=1..nlegborn, j=1..flst_nborn)`: flavour of the  $k$ -th leg of the  $j$ -th Born graph  
`flst_real(k=1..nlegreal, j=1..flst_nreal)`: flavour of the  $k$ -th leg of the  $j$ -th real graph.  
It is required that legs in the Born and real processes have to be ordered as follows:
  - leg 1, incoming parton with positive rapidity
  - leg 2, incoming parton with negative rapidity
  - from leg 3 onward, final state particles, in the order: colorless particles first, massive coloured particles, massless coloured particles.

The flavour is taken incoming for the two incoming particles and outgoing for the outgoing particles. The flavour index is assigned according to PDG conventions, except for gluons, where 0 is used instead of 21.

Example:  $pp \rightarrow (Z \rightarrow e^+ e^-) + 2j$ , the string `[1,0,-11,11,1,0]` labels the process  $dg \rightarrow e^+ e^- dg$

- `init_couplings`

- `born_phsp(xborn)` for Born phase space  
`xborn(1..ndim)` array of random numbers  $\text{ndim}=(\text{nlegborn}-2)*3-4+2-1$ 
  - the Born Jacobian `kn_jacborn`, Born momenta in the laboratory frame `kn_pborn(0:3,1..nlegborn)`, Born momenta in the partonic CM frame `kn_cmpborn(0:3,1..nlegborn)` and Bjorken  $x$  (`kn_xb1` and `kn_xb2`).
- `set_ren_fac_scales(mur,muf)`
- `setborn(p,bflav,born,bornjk,bmunu)`
  - the momenta `p(0:3,1..nlegborn)`
  - the flavour string `bflav(1..nlegborn)`
  - `bornjk(1..nlegborn,1..nlegborn)`
  - the Born helicity-correlated squared amplitudes `bmunu(0:3,0:3,j=1..nlegborn)`
- `setvirtual(p,vflav,virtual)` returns finite part of the interference  $2 \operatorname{Re}(M_B \times M_V)$ , after factorizing out ( $d = 4 - 2\epsilon$ )

$$\mathcal{N} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left( \frac{\mu^2}{Q^2} \right)^\epsilon \frac{\alpha_s}{2\pi}$$

- `setreal(p,rflav,amp2)`
  - the momenta `p(0:3,1..nlegreal)`
  - the flavour string `rflav(1..nlegreal)`
  - `amp2`: spin and color summed and averaged real squared amplitudes

## The POWHEG BOX today

The POWHEG BOX is a package **in evolution**.

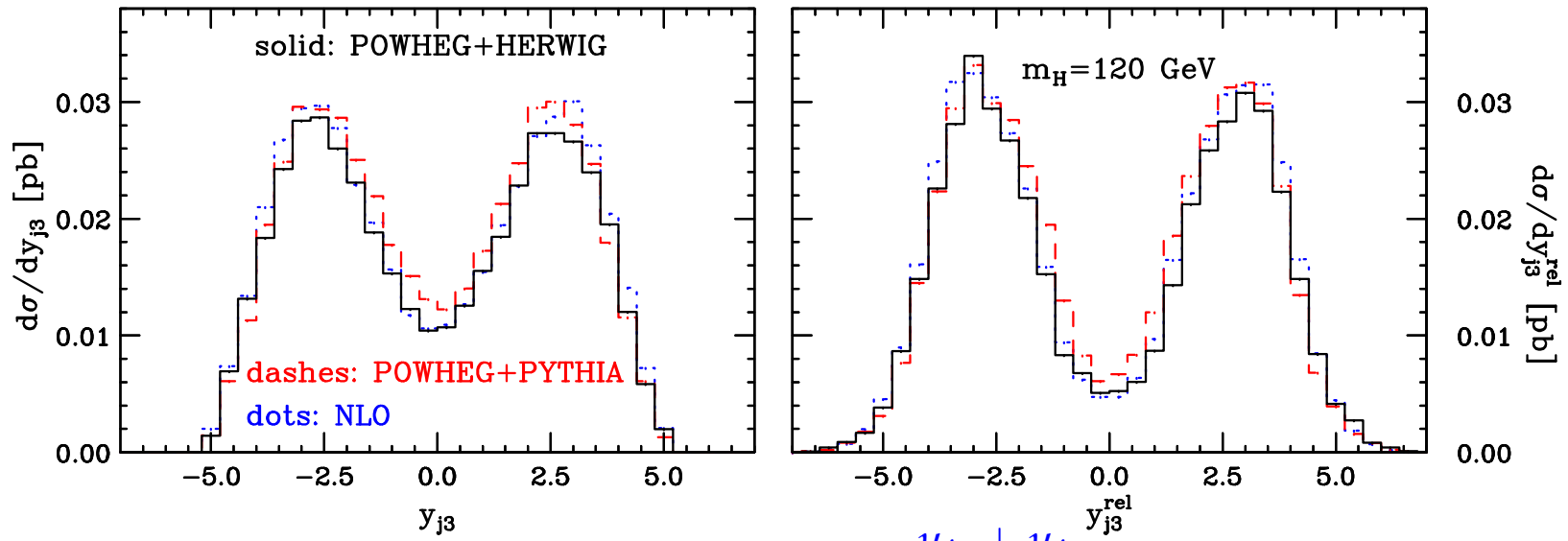
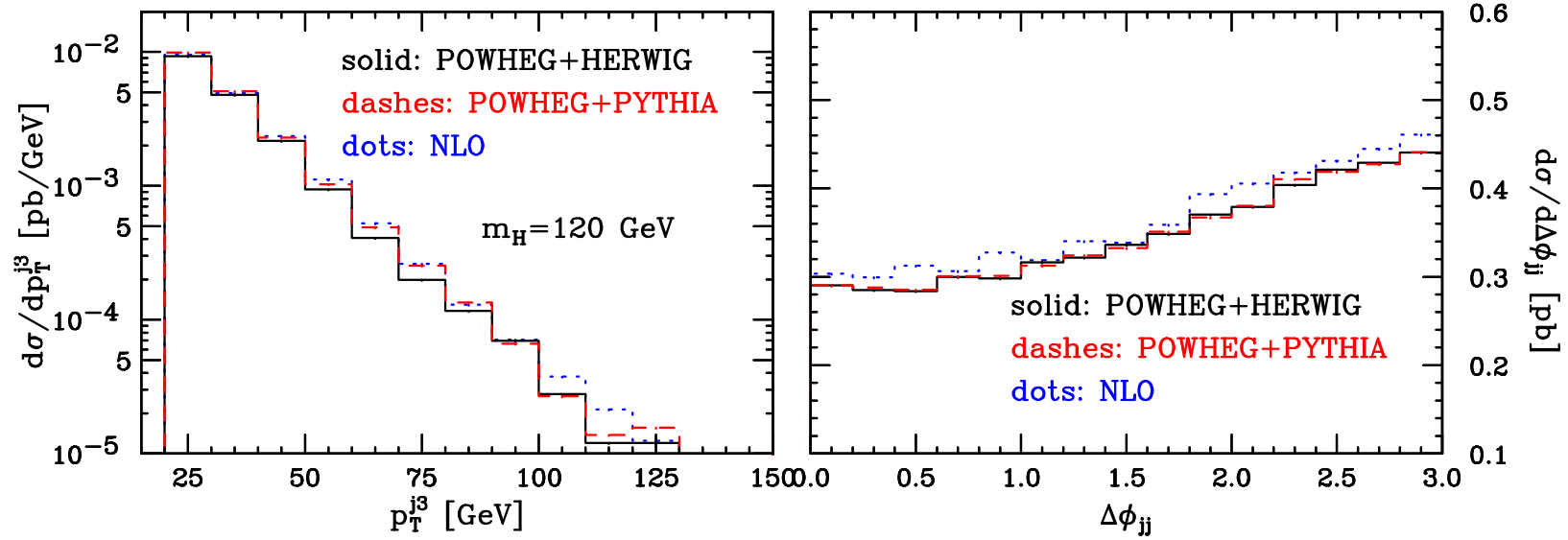
As new processes are implemented in the BOX, **new problems** will probably need to be solved and the code **will change** accordingly.

Right now, in the code, you can find

- **W** production:  $pp(\bar{p}) \rightarrow W \rightarrow l\nu_l$   $\Leftarrow$  **Born zero diagrams**
- **Z** production:  $pp(\bar{p}) \rightarrow Z \rightarrow l^-l^+$
- **Higgs production** in gluon fusion  $\Leftarrow$  **tuning of the real cross section**
- **Higgs production** in VBF  $\Leftarrow$  **tagging parton lines**
- **Z + 1 jet**:  $pp(\bar{p}) \rightarrow Z + 1 \text{ jet} \rightarrow l^-l^+ + 1 \text{ jet}$   $\Leftarrow$  **divergent Born**

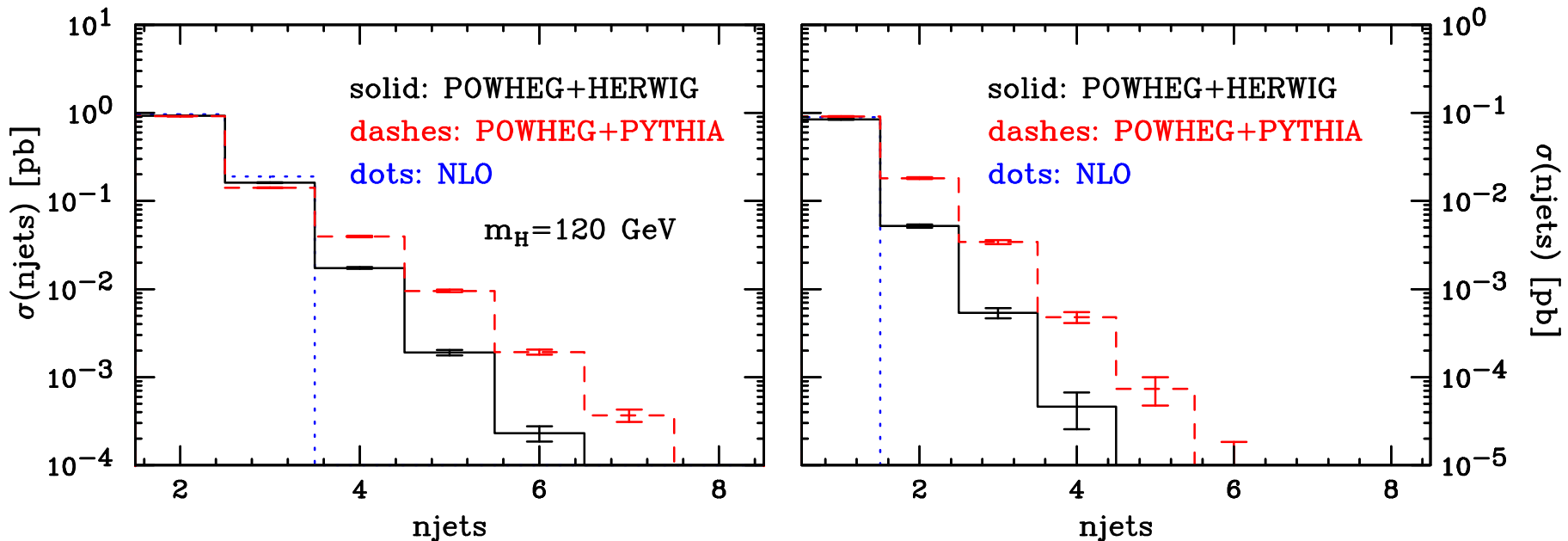


# Higgs boson in VBF



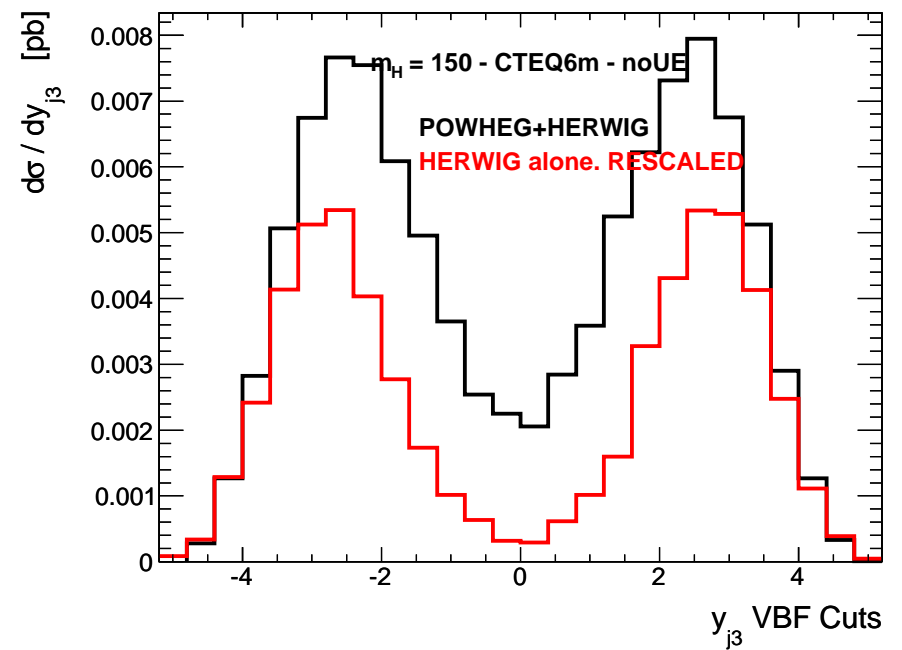
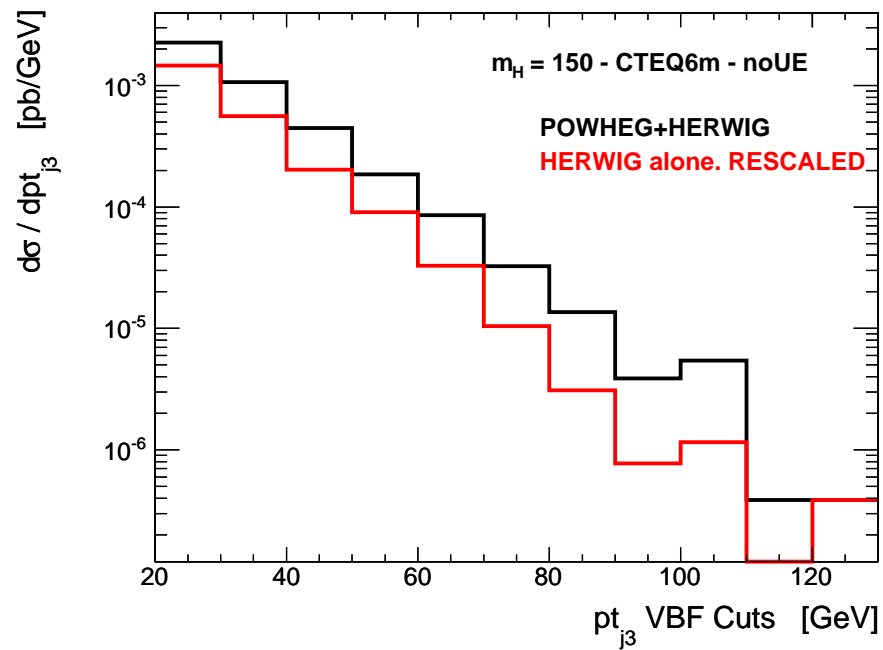
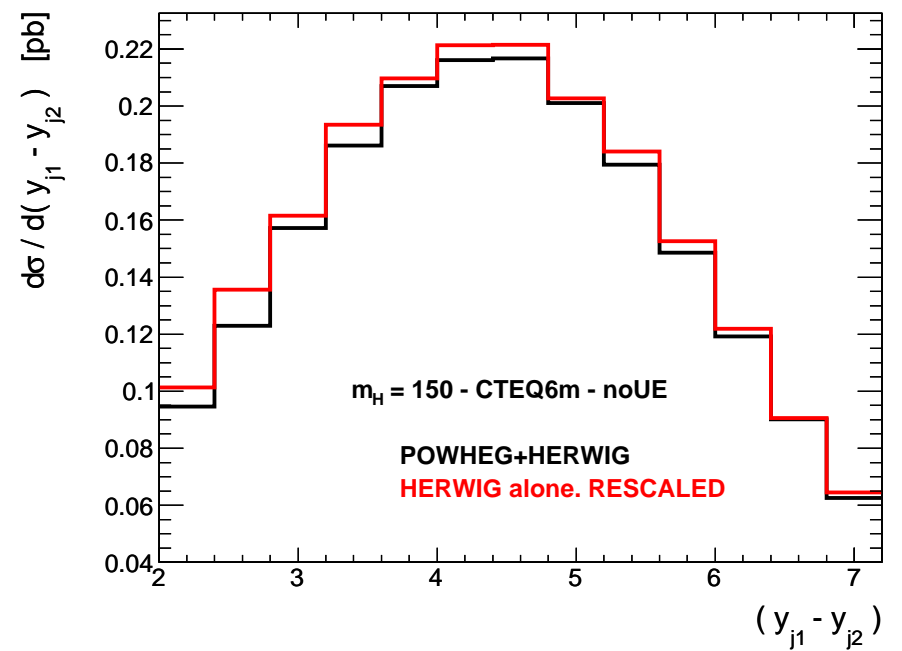
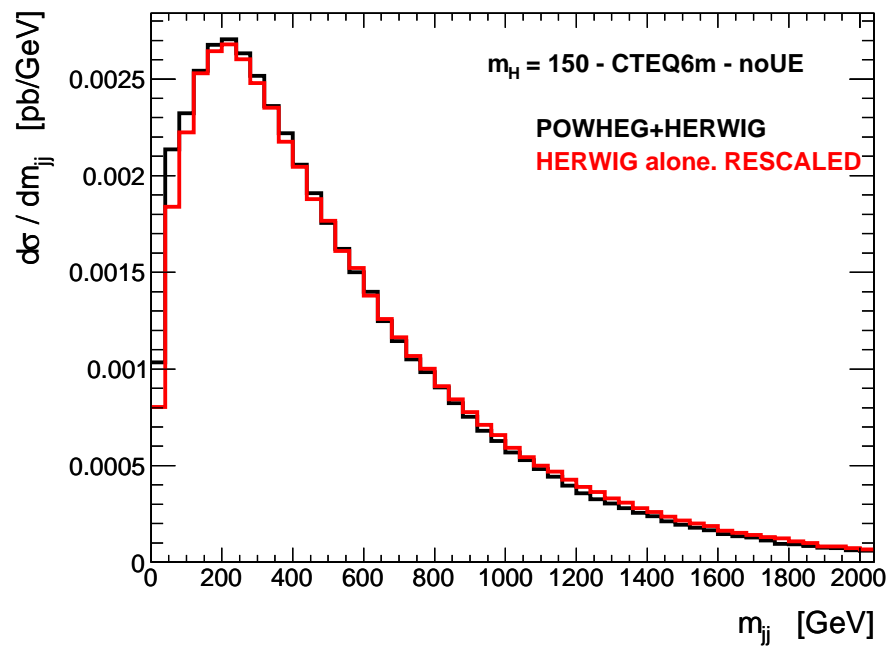
$$y_{j3}^{\text{rel}} = y_{j3} - \frac{y_{j1} + y_{j2}}{2}$$

# Higgs boson in VBF

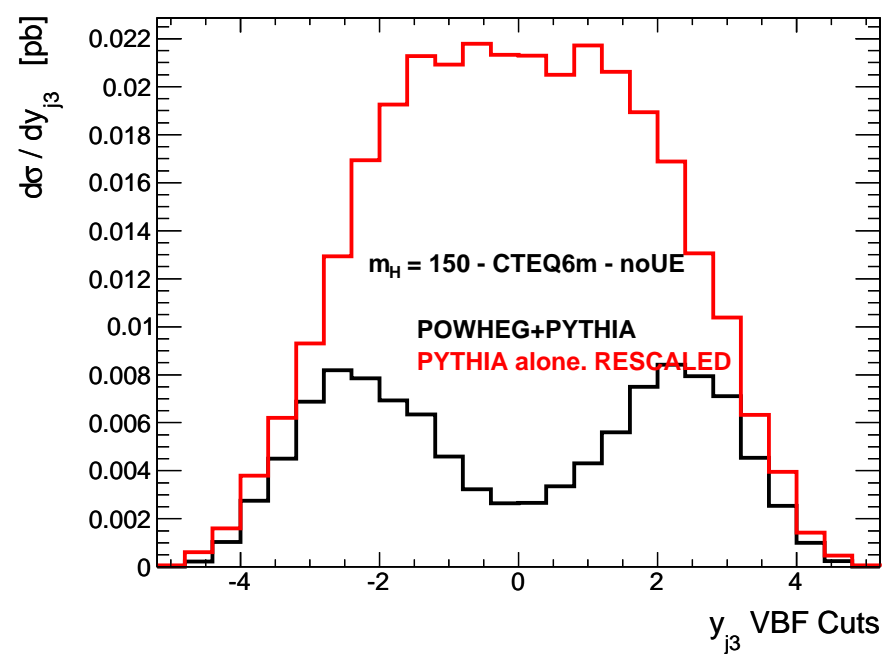
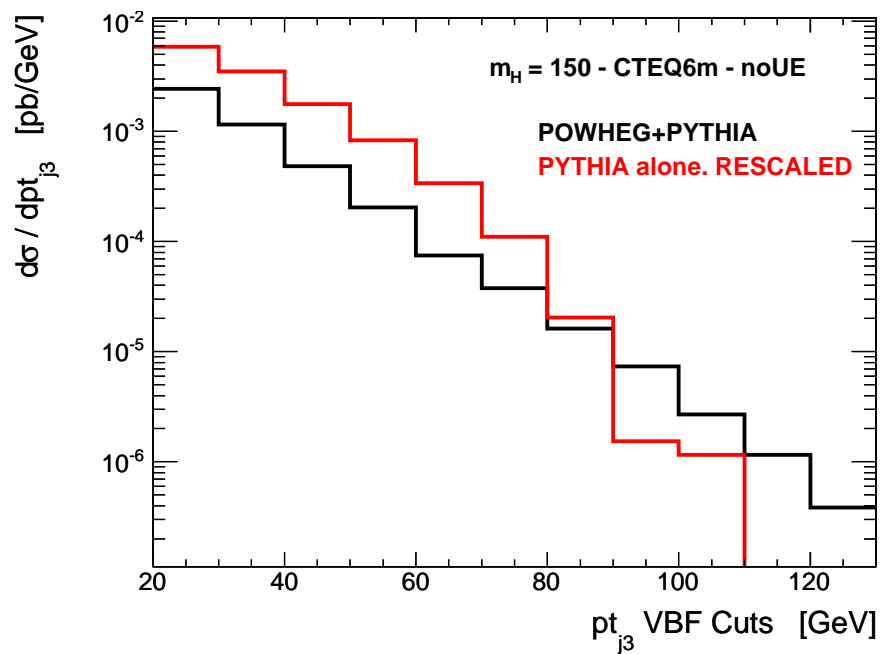
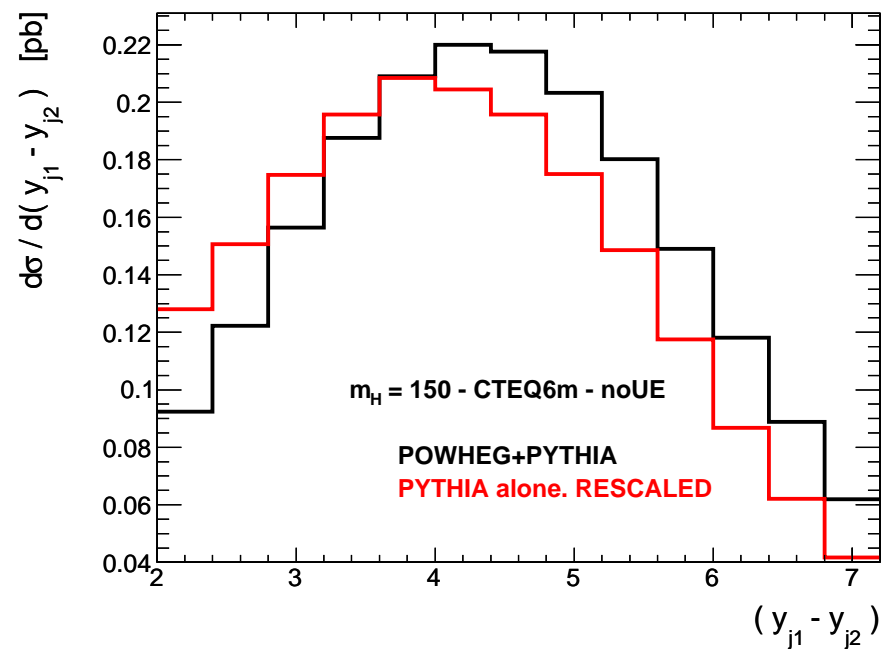
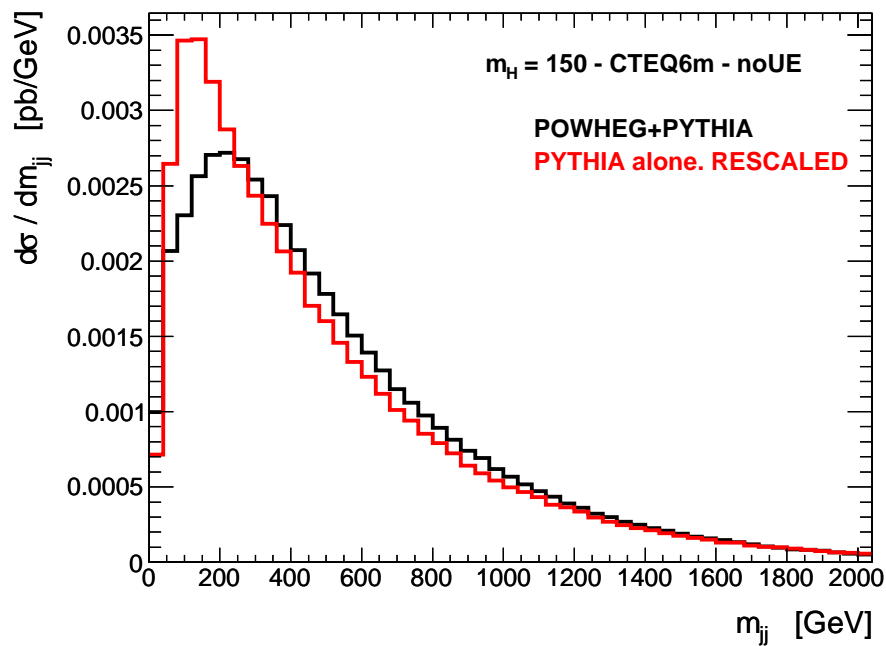


$$\begin{aligned}
 & p_{Tj} > 20 \text{ GeV}, & |y_j| < 5 \\
 & p_T^{\text{tag}} > 30 \text{ GeV}, & |y_{j_1} - y_{j_2}| > 4.2, & y_{j_1} \cdot y_{j_2} < 0, & m_{jj} > 600 \text{ GeV}
 \end{aligned}$$

$$\text{veto jet: } \min(y_{j_1}, y_{j_2}) < y_j < \max(y_{j_1}, y_{j_2})$$

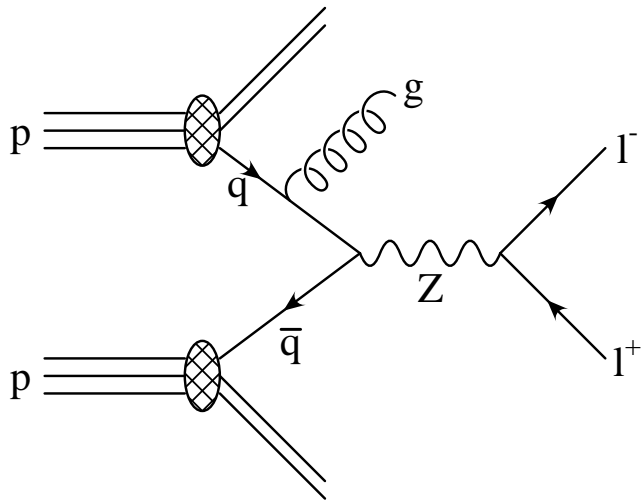


Neither HW alone nor PY alone are expected to describe correctly the 3rd jet. HW results **rescaled** to the NLO total Xsec



Harder PY jets “mimic” tagging jets. Wrong shapes for  $m_{jj}$  and  $(y_{j1} - y_{j2})$ . Same for  $y_{j3}$ . PY results **rescaled** to the NLO total Xsec

## Z + 1 jet



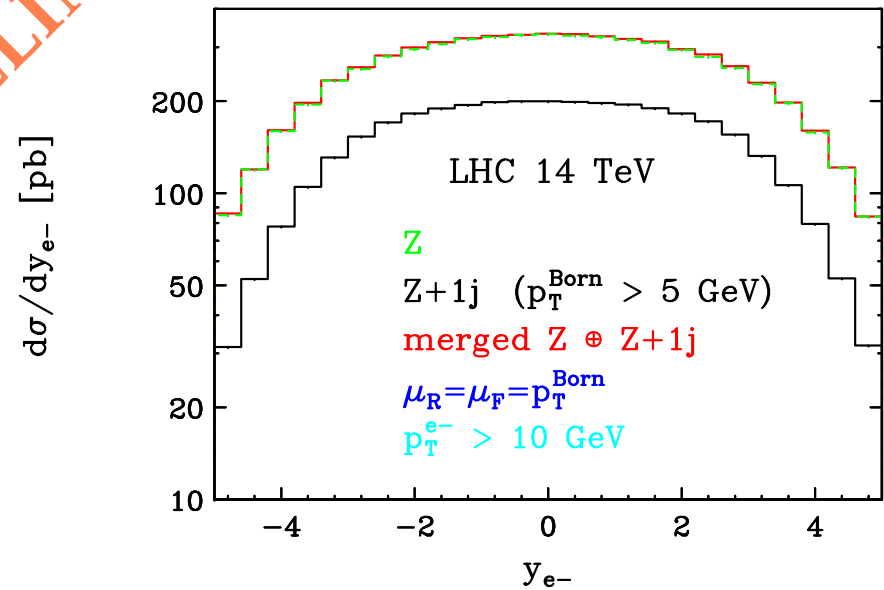
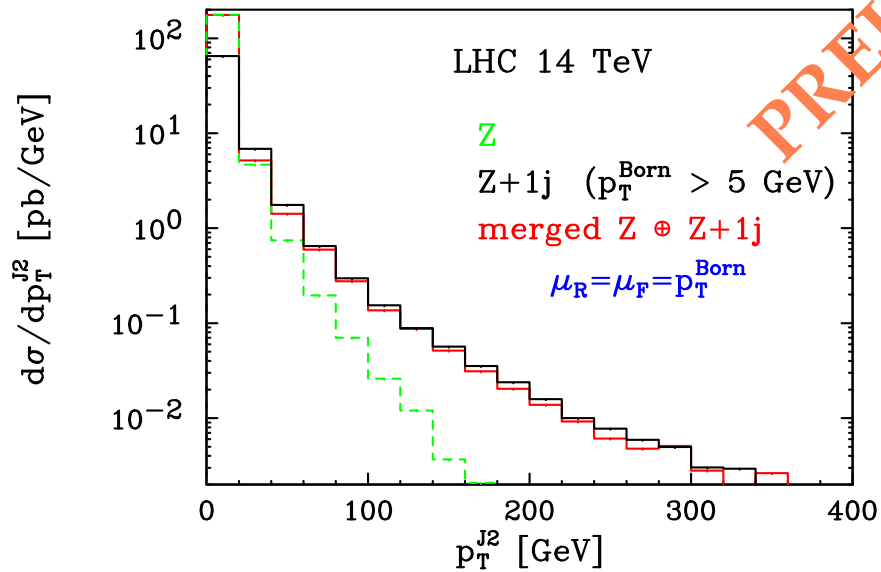
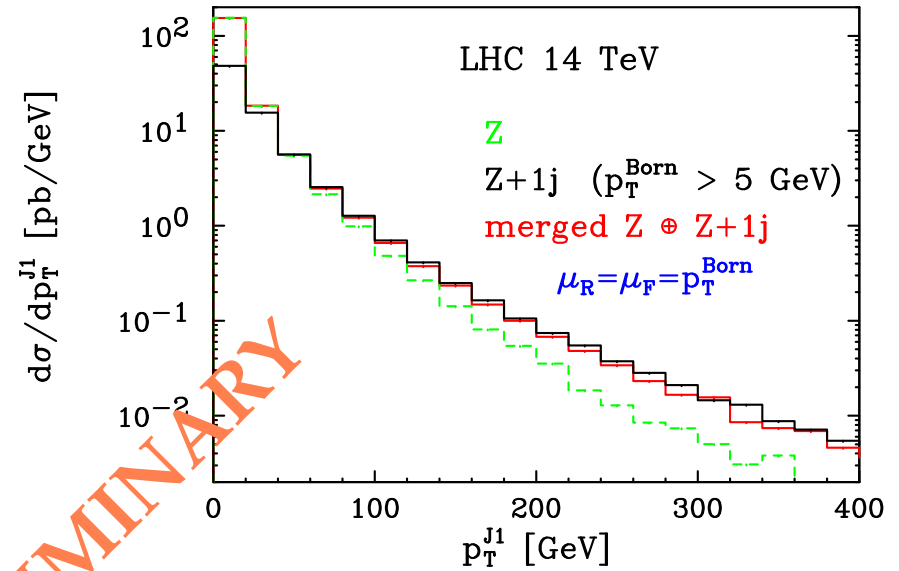
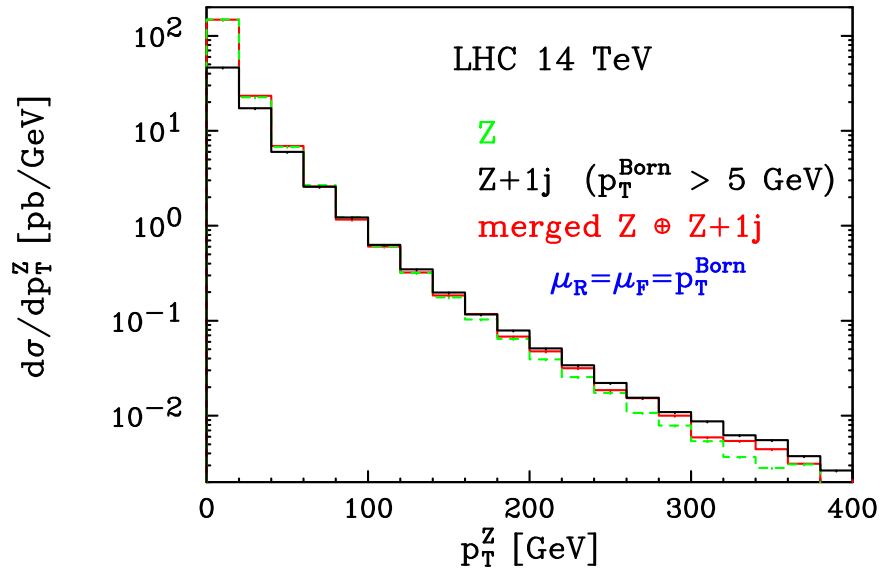
New problem to solve: the Born contributions are divergent.

POWHEG starts from a Born diagram and attaches radiation.

Simplest solution: introduce a cutoff. Generate events starting from partonic Born events with  $p_T^{\text{Born}} > p_T^{\text{min}}$

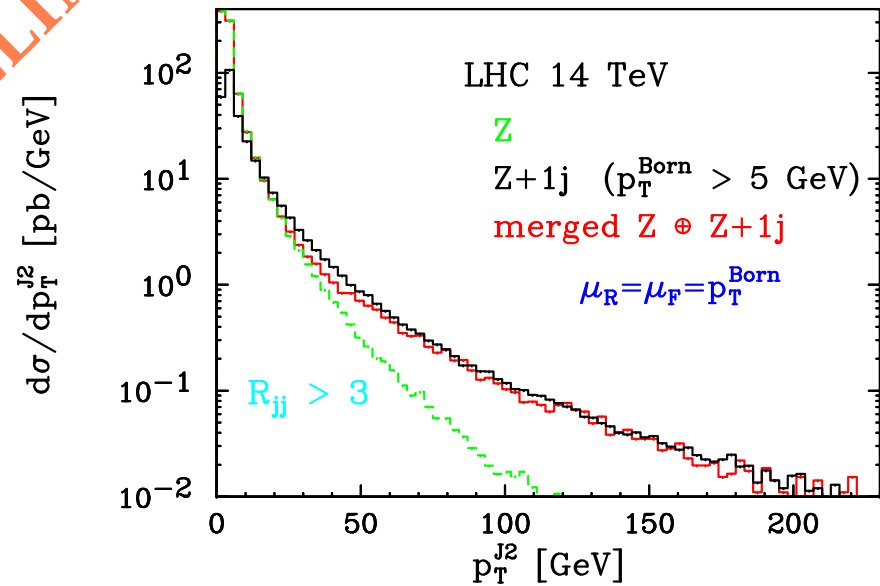
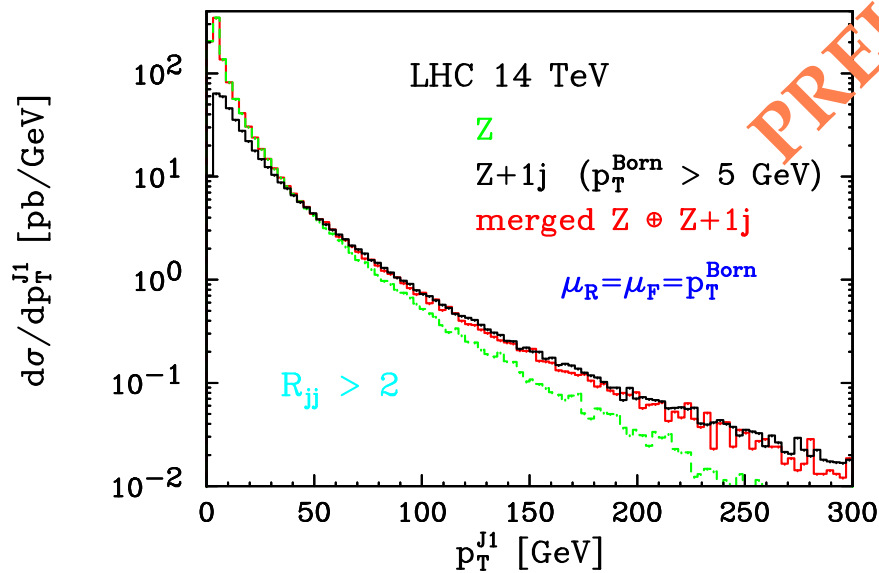
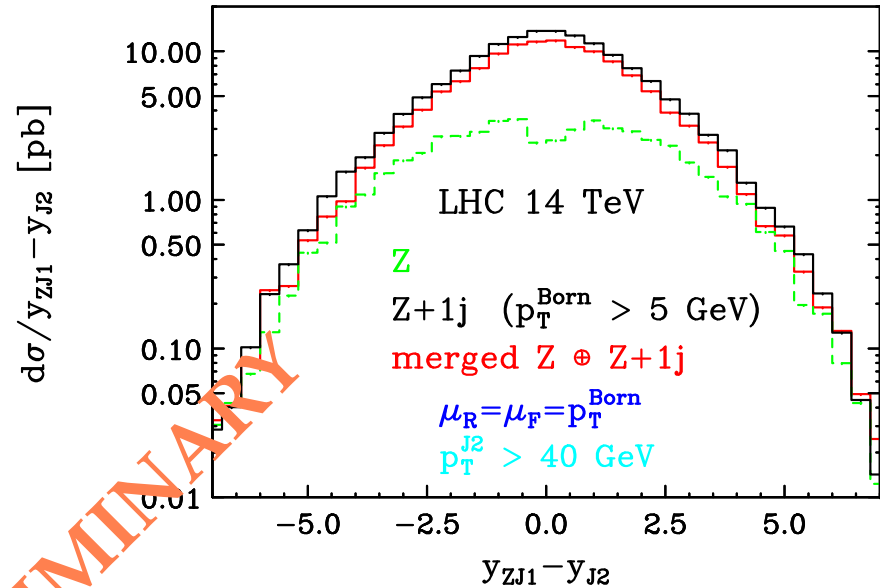
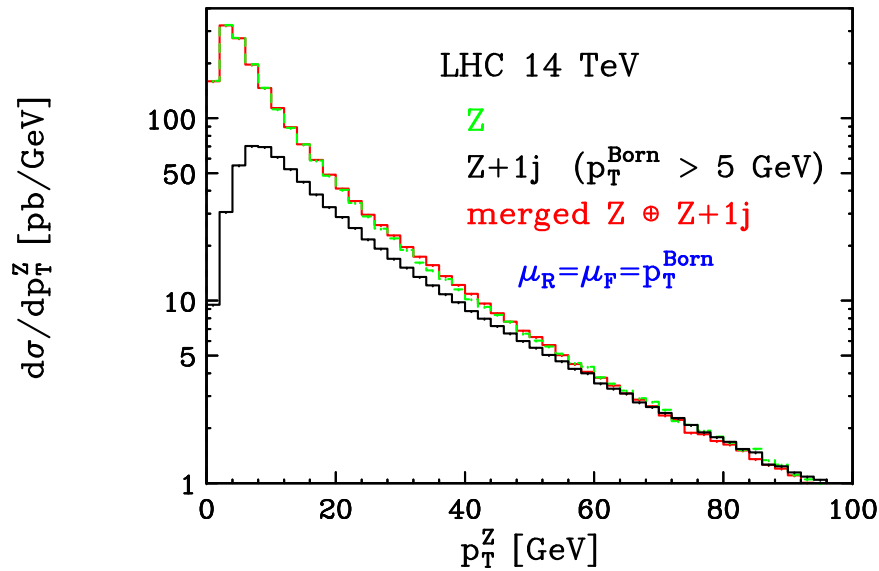
- Study the effect of the cutoff at the partonic Born level on showered events
- Find a way to merge consistently NLO Z and Z + 1 jet events.

# Preliminary merged Z and Z + 1 jet events



All results showered by **HERWIG**

# Preliminary merged Z and Z + 1 jet events



All results showered by **HERWIG**

## Conclusions

- ✓ It is **relatively easy** to add new processes in the POWHEG BOX.
- ✓ No need to know how it works but only how to “**communicate**” with it.
- ✓ Please, feel free to get in touch with us if you want to insert **new NLO calculations** into the POWHEG BOX.

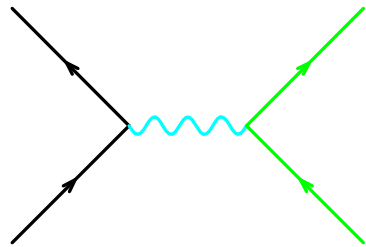




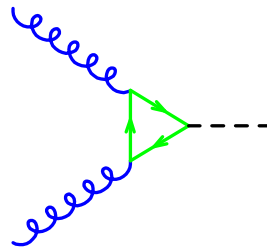
**Backup slides**

## High energy collisions

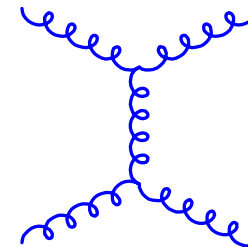
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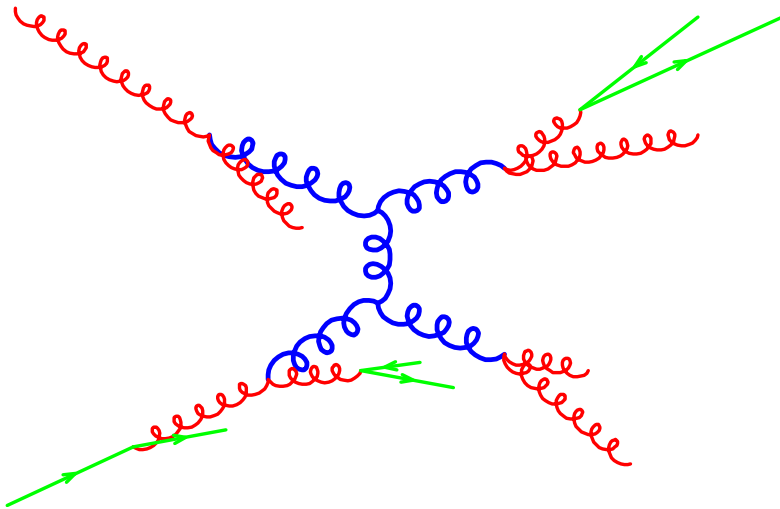


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## Hadronic final states

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS	V-X	V-Y	V-Z	V-C*T
30	NU_E	12	1	28	23	0	0	64.30	25.12	-1194.4	1196.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	E+	-11	1	29	23	0	0	-22.36	6.19	-234.2	235.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
230	PI0	111	1	155	24	0	0	0.31	0.38	0.9	1.0	0.13	4.209E-11	6.148E-11	-3.341E-11	5.192E-10
231	RHO+	213	197	155	24	317	318	-0.06	0.07	0.1	0.8	0.77	4.183E-11	6.130E-11	-3.365E-11	5.189E-10
232	P	2212	1	156	24	0	0	0.40	0.78	1.0	1.6	0.94	4.156E-11	6.029E-11	-4.205E-11	5.250E-10
233	NBAR	-2112	1	156	24	0	0	-0.13	-0.35	-0.9	1.3	0.94	4.168E-11	6.021E-11	-4.217E-11	5.249E-10
234	PI-	-211	1	157	9	0	0	0.14	0.34	286.9	286.9	0.14	4.660E-13	8.237E-12	1.748E-09	1.749E-09
235	PI+	211	1	157	9	0	0	-0.14	-0.34	624.5	624.5	0.14	4.056E-13	8.532E-12	2.462E-09	2.462E-09
236	P	2212	1	158	9	0	0	-1.23	-0.26	0.9	1.8	0.94	-4.815E-11	1.893E-11	7.520E-12	3.252E-10
237	DLTABR--	-2224	197	158	9	319	320	0.94	0.35	1.6	2.2	1.23	-4.817E-11	1.900E-11	7.482E-12	3.252E-10
238	PI0	111	1	159	9	0	0	0.74	-0.31	-27.9	27.9	0.13	-1.889E-10	9.893E-11	-2.123E-09	2.157E-09
239	RHO0	113	197	159	9	321	322	0.73	-0.88	-19.5	19.5	0.77	-1.888E-10	9.859E-11	-2.129E-09	2.163E-09
240	K+	321	1	160	9	0	0	0.58	0.02	-11.0	11.0	0.49	-1.890E-10	9.873E-11	-2.135E-09	2.169E-09
241	KL_1-	-10323	197	160	9	323	324	1.23	-1.50	-50.2	50.2	1.57	-1.890E-10	9.879E-11	-2.132E-09	2.166E-09
242	K-	-321	1	161	24	0	0	0.01	0.22	1.3	1.4	0.49	4.250E-11	6.333E-11	-2.746E-11	5.211E-10
243	PI0	111	1	161	24	0	0	0.31	0.38	0.2	0.6	0.13	4.301E-11	6.282E-11	-2.751E-11	5.210E-10

High-energy experimental physicists feed this kind of output through their detector-simulation software, and use it to determine **efficiencies** for signal detection, and perform **background estimates**.

**Analysis strategies** are set up using **these simulated data**.

## A word of warning

“The Monte Carlo simulation has become the major mean of visualization of not only detector performance but also of physics phenomena. So far so good. But it **often happens** that the **physics simulations** provided by the Monte Carlo generators **carry the authority of data itself**. They look like data and feel like data, and if one is not careful they are **accepted** as if they were **data**.”

J.D. Bjorken

Talk given at the 75th anniversary celebration of the Max-Planck Institute of Physics, Munich, Germany, December 10th, 1992, as quoted in Beam Line, Winter 1992, Vol. 22, No. 4. Reference taken from Sjöstrand.

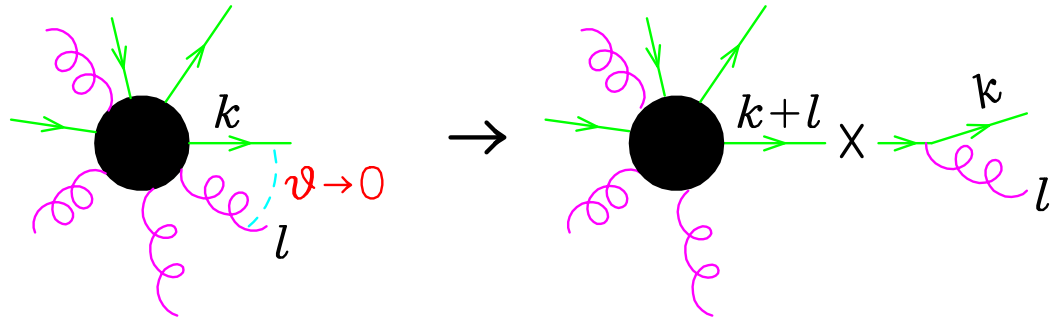
## Summarizing

- In high-energy collider physics not many questions can be answered without a Shower Monte Carlo (SMC).
- The name **shower** comes from the fact that we **dress** a **hard event** with **QCD radiation**.
- After a latency period, many physicists are now looking at shower Monte Carlo models again, under different perspective: Catani, Krauss, Kühn & Webber; Mangano, Moretti, Piccinini, Pittau, Polosa & Treccani; Frixione & Webber; Kramer, Mrenna, Nagy & Soper; Giele, Kosower & Skands; Bauer & Schwartz; Schumann & Krauss; Dinsdale, Ternick & Weinzierl; ...
- **Shower algorithms** summarize most of our knowledge in perturbative QCD: **infrared cancellations**, **Altarelli-Parisi** equations, **soft coherence**, **Sudakov form factors**. All have a simple interpretation in terms of shower algorithms.

# Shower basics: collinear factorization

QCD emissions are **enhanced** near the **collinear limit**

Cross sections factorize near collinear limit



$$d\Phi_{n+1} = d\Phi_n d\Phi_r \quad d\Phi_r \doteq dt dz d\varphi$$

$$|M_{n+1}|^2 d\Phi_{n+1} \implies |M_n|^2 d\Phi_n \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi} \left\{ \begin{array}{l} \frac{dt}{t} \approx \frac{d\theta}{\theta} \quad \text{collinear singularity} \\ \frac{dz}{1-z} \approx \frac{dE_g}{E_g} \quad \text{soft singularity} \end{array} \right.$$

$$t : (k+l)^2, p_T^2, E^2\theta^2 \dots$$

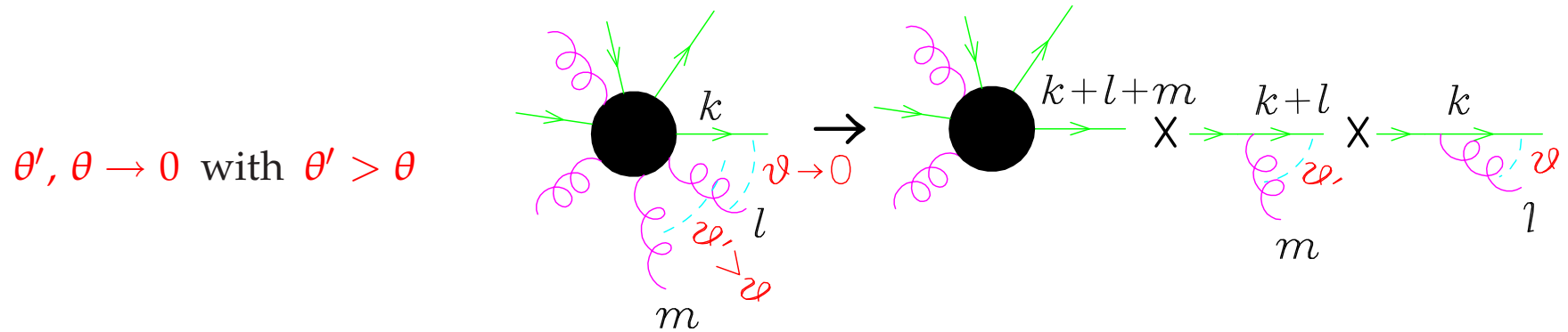
$$z = k^0 / (k^0 + l^0) : \text{energy (or } p_{\parallel} \text{ or } p^+) \text{ fraction of quark}$$

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z} : \text{Altarelli-Parisi splitting function}$$

(ignore  $z \rightarrow 1$  IR divergence for now)

## Shower basics: collinear factorization

If another gluon becomes collinear, **iterate** the previous formula



$$\begin{aligned}
 |M_{n+1}|^2 d\Phi_{n+1} &\implies |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z') dz' \frac{d\varphi'}{2\pi} \\
 &\times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi} \theta(t' - t)
 \end{aligned}$$

Collinear partons can be described by a factorized integral ordered in  $t$ .



## Collinear factorization: multiple emissions

For  $n$  collinear emissions, the cross section goes as

$$\begin{aligned}\sigma &\approx \sigma_0 \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \frac{dt_2}{t_2} \dots \frac{dt_n}{t_n} \theta(Q^2 > t_1 > t_2 > \dots > t_n > t_0) \\ &= \sigma_0 \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \int_{t_0}^{t_1} \frac{dt_2}{t_2} \dots \int_{t_0}^{t_{n-1}} \frac{dt_n}{t_n} \approx \sigma_0 \alpha_s^n \frac{1}{n!} \left( \log \frac{Q^2}{t_0} \right)^n\end{aligned}$$

- $Q^2$  is an upper cutoff for the ordering variable  $t$
- $t_0 \approx \Lambda^2 \approx \Lambda_{\text{QCD}}^2$  is an **infrared cutoff** (quark mass, confinement scale)
- Due to the log dependence, we call it **leading-log approximation**.
- According to the Kinoshita-Lee-Nauenberg theorem, the **virtual corrections**, order by order, contribute with a comparable term, with **opposite sign**.
- The virtual leading-log contribution should be included in order to get sensible results!

## Simple probabilistic interpretation of “not-resolved” corrections

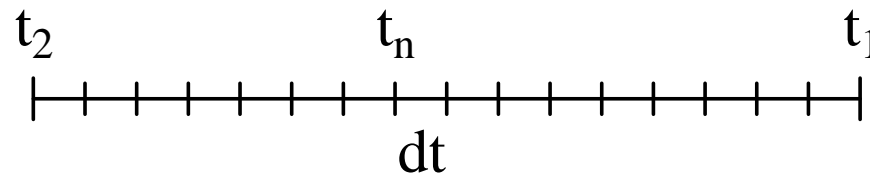
- probability of emission in the interval  $dt$ , at order  $\alpha_s$  (multiple emissions are of higher orders in  $\alpha_s$ )

$$dP_{\text{emis}}(t + dt, t) = \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz P_{i,jk}(z)$$

- probability of no emission in the interval  $dt$

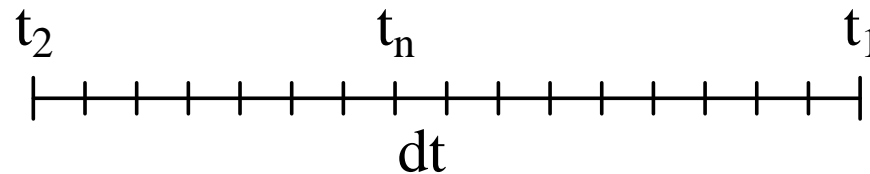
$$dP_{\text{no emis}}(t + dt, t) = 1 - dP_{\text{emis}}(t + dt, t) = 1 - \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz P_{i,jk}(z)$$

The “no emission” probability contains, through the **1**, all the **virtual corrections** (in the collinear approximation, that is at the leading-log level).



## Simple probabilistic interpretation of “not-resolved” corrections

- divide a finite interval  $[t_2, t_1]$  in  $N$  small intervals  $dt = (t_1 - t_2)/N$ .



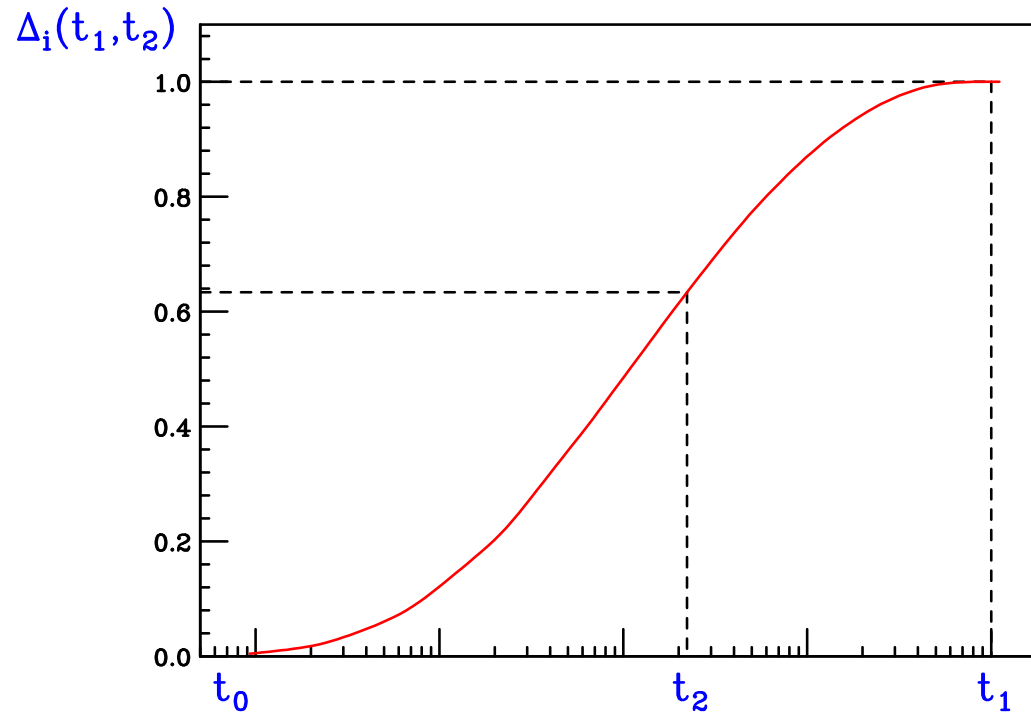
The probability of **not emitting** radiation between the two ordering scales  $t_1$  and  $t_2$  is given by the product

$$\begin{aligned} P_{\text{no emis}}(t_1, t_2) &= \lim_{N \rightarrow \infty} \prod_{n=1}^N \left[ 1 - \frac{dt}{t_n} \frac{\alpha_s(t_n)}{2\pi} \int dz P_{i,jk}(z) \right] \\ &= \exp \left\{ - \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz P_{i,jk}(z) \right\} \\ &\equiv \Delta(t_1, t_2) \end{aligned}$$

- The weight  $\Delta(t_1, t_2)$  is called **Sudakov form factor**. It resums all the **dominant virtual corrections** to the tree graph (in the collinear approximation).

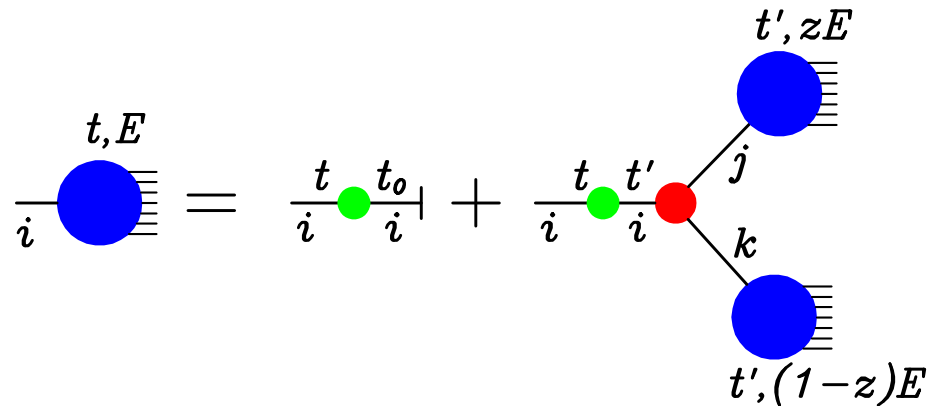
## Sudakov form factors

$$\Delta_i(t_1, t_2) = \exp \left\{ - \sum_{jk} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz P_{i,jk}(z) \right\}$$



Notice that, when  $t_2 \ll t_1$ ,  $\Delta \rightarrow 0$ , i.e. the probability that a hard parton turns into a narrow jet, or that it does not radiate at all, is small (it is **Sudakov suppressed**)

## Final recipe



$$\mathcal{S}_i(t, E) = \Delta_i(t, t_0) \mathbb{1} + \sum_{(jk)} \int_{t_0}^t \frac{\alpha_s(t')}{2\pi} \frac{dt'}{t'} \int dz \int \frac{d\varphi}{2\pi} \Delta_i(t, t') P_{i,jk}(z) \mathcal{S}_j(t', zE) \mathcal{S}_k(t', (1-z)E)$$

- consider all **tree graphs**.
- assign values to the radiation variables  $\Phi_r$  ( $t$ ,  $z$  and  $\varphi$ ) to **each vertex**.
- at each vertex,  $i \rightarrow jk$ , include a factor

$$\frac{dt}{t} dz \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{d\varphi}{2\pi}$$

- include a factor  $\Delta_i(t_1, t_2)$  to each internal parton  $i$ , from hardness  $t_1$  to hardness  $t_2$ .
- include a factor  $\Delta_i(t, t_0)$  on final lines ( $t_0 = \mathbf{IR\ cutoff}$ )

## Accuracy: soft divergences and double-log regions

$z \rightarrow 1$  ( $z \rightarrow 0$ ) region problematic. In fact, for  $z \rightarrow 1$ ,  $P_{qq}, P_{gg} \div 1/(1-z)$

The **choice** of the **ordering variable**  $t$  makes a **difference**

virtuality:	$t \equiv E^2 z(1-z)$	$\overbrace{\theta^2}^{2(1-\cos\theta)}$	
$p_T^2$ :	$t \equiv E^2 z^2(1-z)^2$		
angle:	$t \equiv E^2 \theta^2$		

$$\text{virtuality: } z(1-z) > t/E^2 \implies \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{1}{4} \log^2 \frac{t}{E^2}$$

$$p_T^2: z^2(1-z)^2 > t/E^2 \implies \int \frac{dt}{t} \int_{t/E^2}^{1-t/E^2} \frac{dz}{1-z} \approx \frac{1}{2} \log^2 \frac{t}{E^2}$$

$$\text{angle: } \implies \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \log \Lambda$$

Sizable difference in double-log structure!

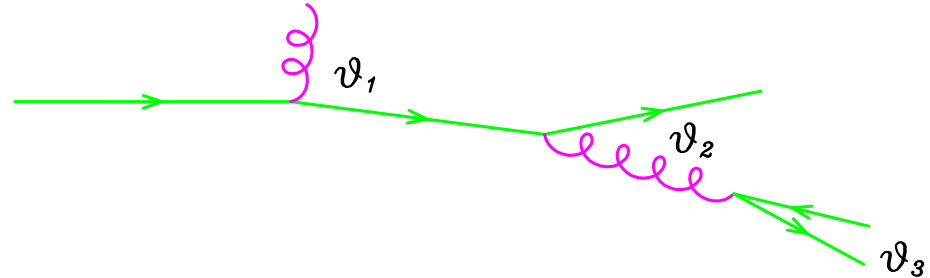
# Angular ordering and color coherence

Mueller (1981) showed that **angular ordering** is the correct choice

$$\frac{d\theta}{\theta} \frac{\alpha_s(p_T^2)}{2\pi} P(z) dz$$

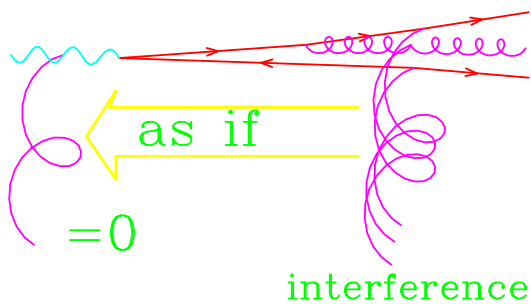
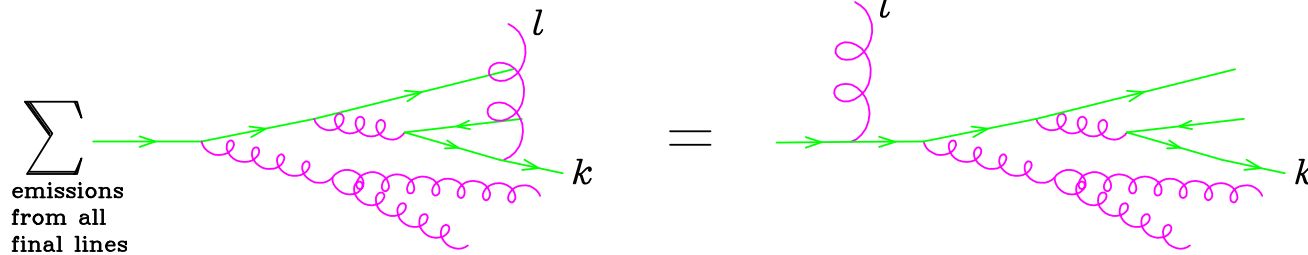
$$\theta_1 > \theta_2 > \theta_3 \dots$$

$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$  for a correct treatment of charge renormalization in **soft region**

Soft gluons emitted at **large angles** from final-state partons add **coherently**



- angular ordering accounts for soft gluon interference.
- intensity for **photon** jets = 0
- intensity for gluon jets =  $C_A$  instead of  $2C_F + C_A$

## New developments

- Interfacing **Matrix Elements** (ME) generators with **Parton Showers**
  - **CKKW** matching [Catani, Krauss, Küen, Webber]
  - **MLM** matching [Mangano]
- Interfacing **NLO** calculations with **Parton Showers**
  - **MC@NLO** [Frixione, Webber]
  - **POWHEG** [Nason]

Several other approaches have appeared

- $e^+e^- \rightarrow 3$  **partons** [Kramer, Mrenna, Soper]
- Shower by **antenna factorization** [Giele, Kosower, Skands]
- Shower by **Catani-Seymour dipole factorization** [Schumann, Krauss]
- Shower with **quantum interference** [Nagy, Soper]
- Shower by **Soft Collinear Effective Theory** [Bauer, Schwartz]
- Shower from the **dipole formalism** [Dinsdale, Ternick, Weinzierl]

Up to now, **complete results** for hadron colliders **only** from **MC@NLO** and **POWHEG**.



## NLO + Parton Shower

LO-ME good for **shapes**. Uncertain absolute normalization

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu) (1 - b_0 \alpha_s(\mu) \log(4))^n \approx \alpha_s^n(\mu) (1 - n \alpha_s(\mu))$$

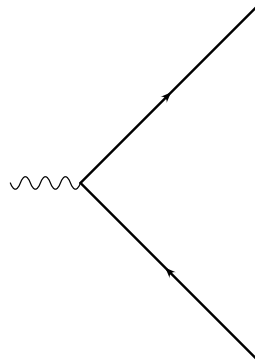
For  $\mu = 100 \text{ GeV}$ ,  $\alpha_s = 0.12$ , normalization uncertainty:

$W + 1J$	$W + 2J$	$W + 3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

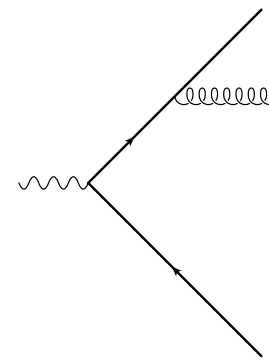
To improve on this, we need **to go to NLO**

The main problem in **merging** a **NLO** result and a **Parton Shower** is **not to double-count** radiation: the shower might produce some radiation **already present** at the NLO level (both at the **virtual** and at the **real** level).

**LO:**



**NLO:**



## NLO vs Shower Monte Carlo

### NLO

- ✓ accurate shapes at high  $p_T$
- ✓ normalization accurate at NLO order
- ✓ reduced dependence on renormalization and factorization scales
- ✗ wrong shapes at small  $p_T$
- ✗ description only at the parton level

### SMC (LO + shower)

- ✗ bad description at high  $p_T$
- ✗ normalization accurate only at LO
- ✓ correct Sudakov suppression at small  $p_T$
- ✓ simulate events at the hadron level

It is natural to try to merge the two approaches, keeping the good features of both

MC@NLO [Frixione and Webber, 2001] and POWHEG [Nason, 2004] do this in a consistent way

## POsitive-Weight Hardest Emission Generator

- ✓ it generates events with **positive weights**. **NO negative weights** to handle
- ✓ it is **independent** from **parton-shower** programs. Can be interfaced with **PYTHIA, HERWIG, SHERPA...**

It is then possible to **compare** the **different outputs**

- ✓ **No need to implement new interfaces**

Two possible ways to interface to shower Monte Carlo programs

1. **Les Houches Event** format. The event is written on a file that is subsequently showered by HERWIG, PYTHIA...
2. **on the fly**. We provide UPINIT and UPEVNT directly running in HERWIG and PYTHIA

## Existing implementations

The POWHEG method has already been **successfully** used in

- $pp \rightarrow ZZ$  [Nason and Ridolfi, hep-ph/0606275]
- $e^+e^- \rightarrow \text{hadrons}$  [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]  
 $e^+e^- \rightarrow t\bar{t}$  with top decay [Latunde-Dada, arXiv:0806.4560]
- $pp \rightarrow Q\bar{Q}$  ( $c\bar{c}$ ,  $b\bar{b}$ ,  $t\bar{t}$ ) with **spin correlations** [Frixione, Nason and Ridolfi, arXiv:0707.3088].
- $pp \rightarrow W/Z$  with **spin correlations** [Alioli, Nason, C.O. and Re, arXiv:0805.4802; Hamilton, Richardson and Tully, arXiv:0806.0290].
- $pp \rightarrow H$  [Alioli, Nason, C.O. and Re, arXiv:0812.0578; Hamilton, Richardson and Tully, arXiv:0903.4345]
- $pp \rightarrow H + W/Z$  [Hamilton, Richardson and Tully, arXiv:0903.4345]

**All POWHEG** implementations for hadronic colliders have been interfaced to both **PYTHIA** and **HERWIG**.

## To appear very soon

- single-top production [Alioli, Nason, C.O. and Re]
- $pp \rightarrow W/Z + 1 \text{ jet}$  [Alioli, Nason, C.O. and Re]
- $pp \rightarrow VV$  [Hamilton and Nason]

We are working now on a **general framework** for the implementation of **any NLO process** into the POWHEG formalism.

Given the Born, real and virtual amplitudes, combine them **automatically** to produce POWHEG events.

### Truncated shower

- in an approximate form, truncated shower has been studied in  $e^+e^- \rightarrow \text{hadrons}$  [Latunde-Dada Gieseke and Webber, hep-ph/0612281]
- included in the **HERWIG++** framework [Bähr, Gieseke, Gigg, Grellscheid, Hamilton, Plätzer, Richardson, Seymour and Tully, arXiv:0812.0529] and in all the HRT's papers

# POWHEG

$$d\sigma_{\text{NLO}} = d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] d\Phi_r \right\}$$

$$d\Phi_{n+1} = d\Phi_n d\Phi_r \quad d\Phi_r \doteq dt dz d\varphi$$

$$V(\Phi_n) = V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \quad \Leftarrow \text{finite}$$

$$d\sigma_{\text{SMC}} = B(\Phi_n) d\Phi_n \left\{ \Delta_{t_0} + \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \Delta_t d\Phi_r \right\}$$

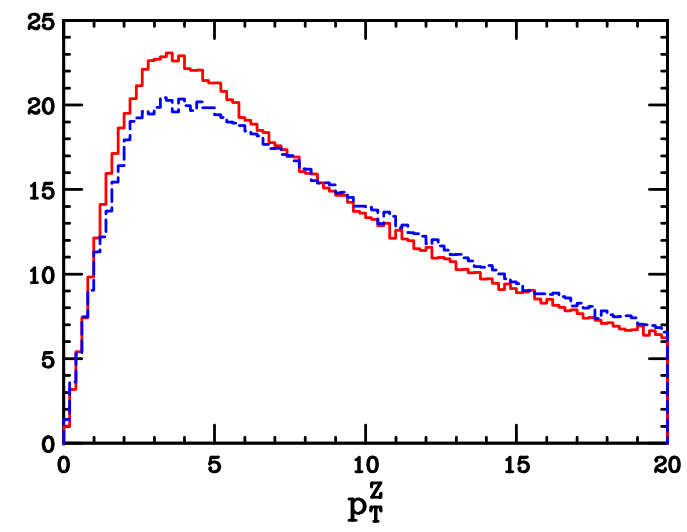
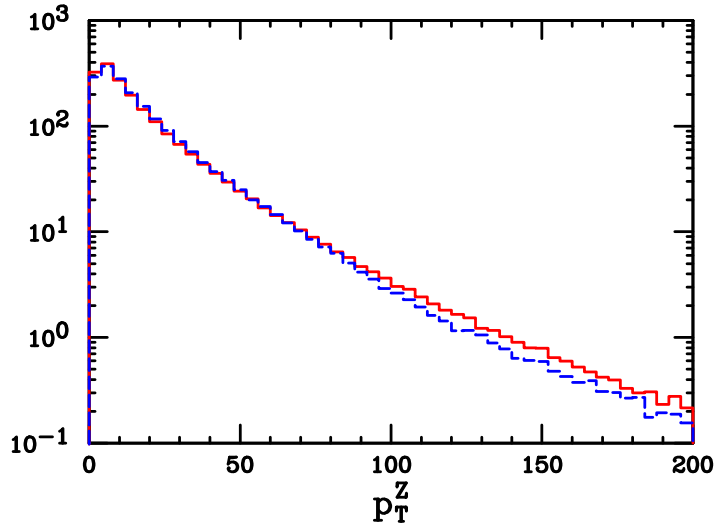
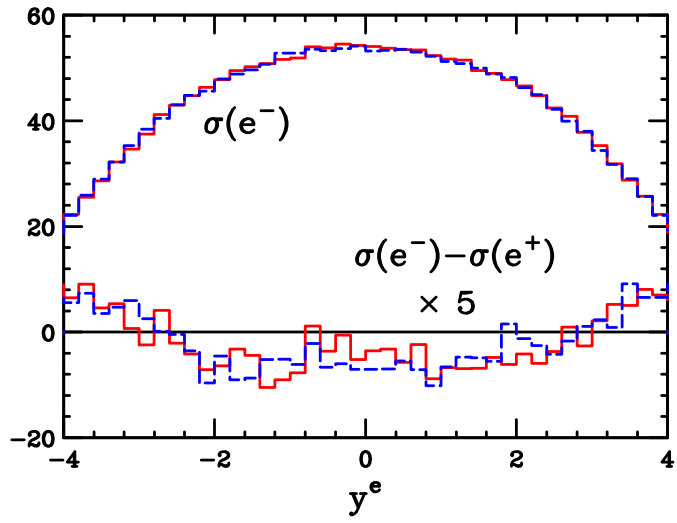
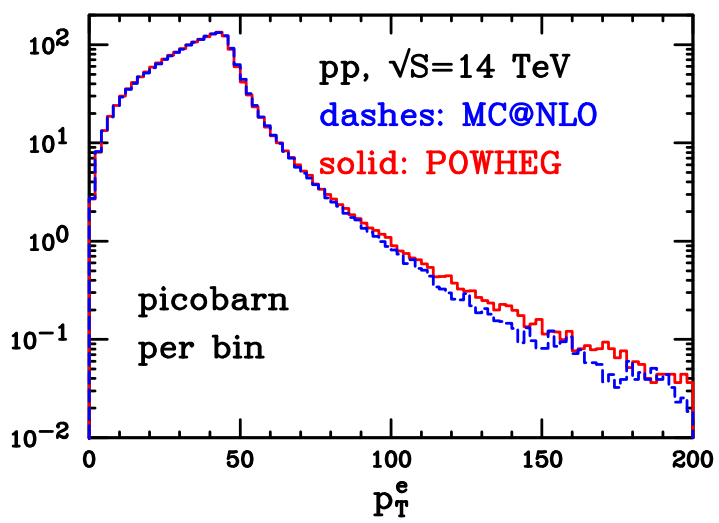
$$\Delta_t = \exp \left[ - \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right] \quad \text{SMC Sudakov form factor}$$

$$d\sigma_{\text{POWHEG}} = \bar{B}(\Phi_n) d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \Delta(\Phi_n, p_T) d\Phi_r \right\}$$

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

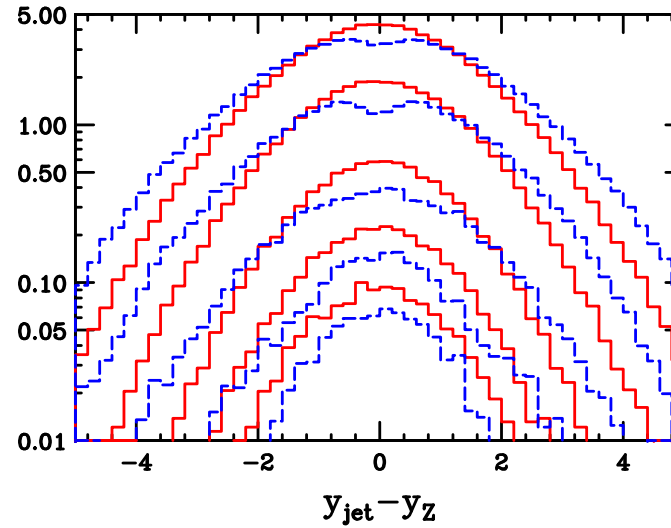
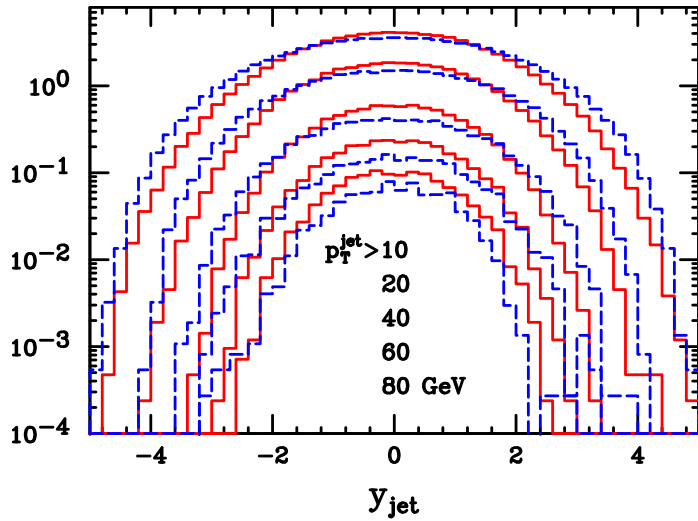
$$\Delta(\Phi_n, p_T) = \exp \left[ - \int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(k_T(\Phi_n, \Phi'_r) - p_T) \right] \quad \text{POWHEG Sudakov}$$

# Z production: POWHEG + HERWIG vs MC@NLO

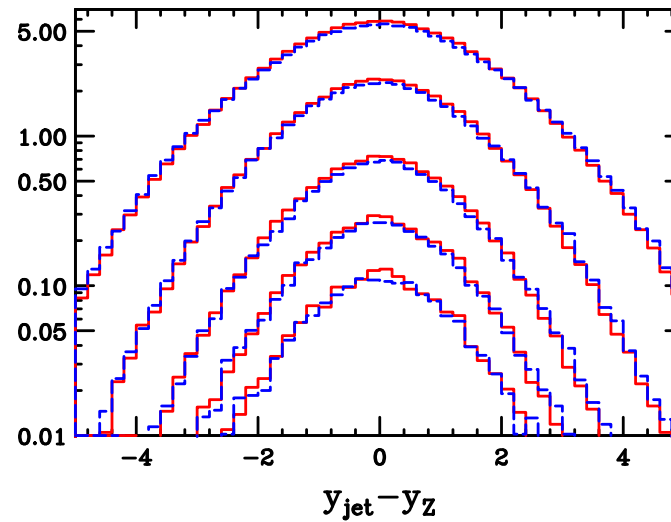
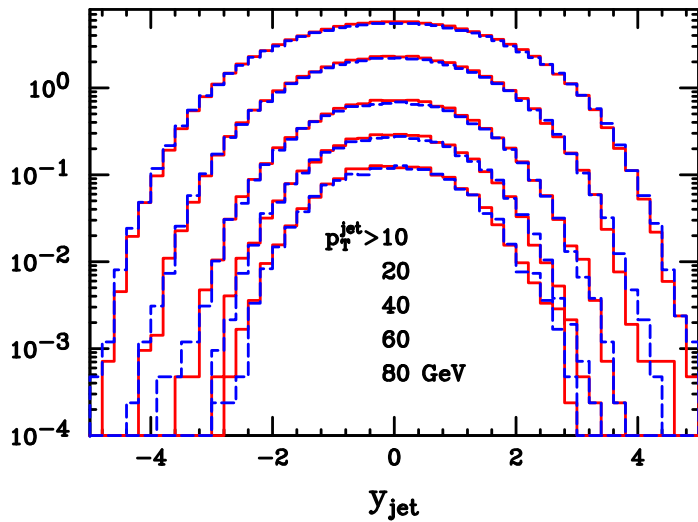


Small differences in the high- and low- $p_T$  regions.

# Rapidity distribution of hardest jet at Tevatron



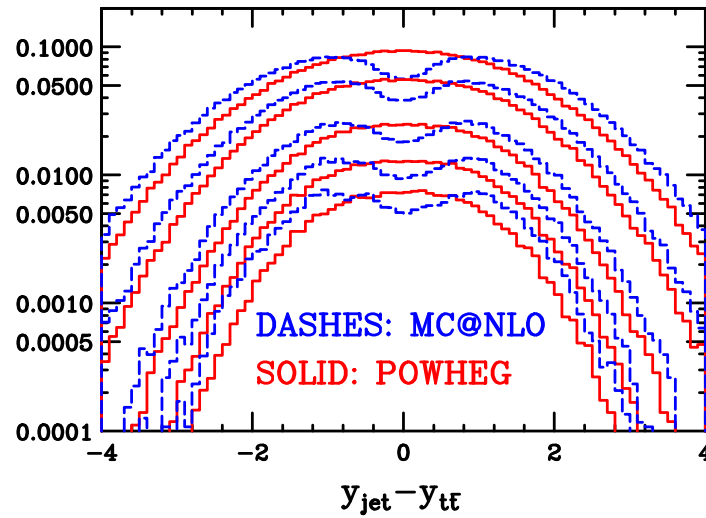
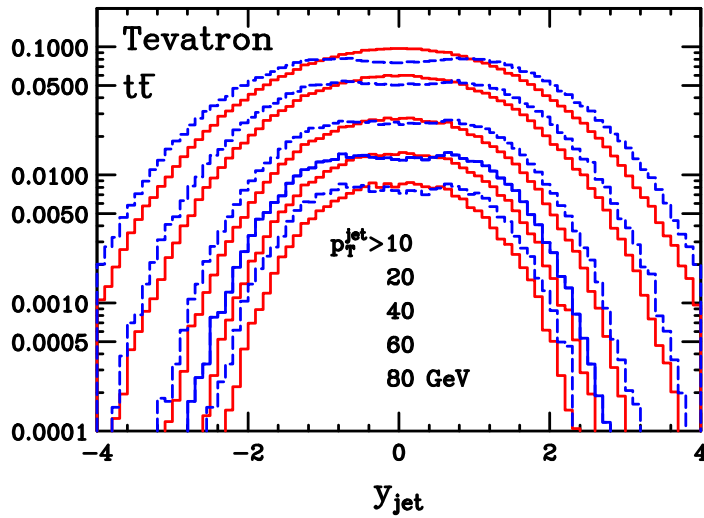
POWHEG+HERWIG  
MC@NLO



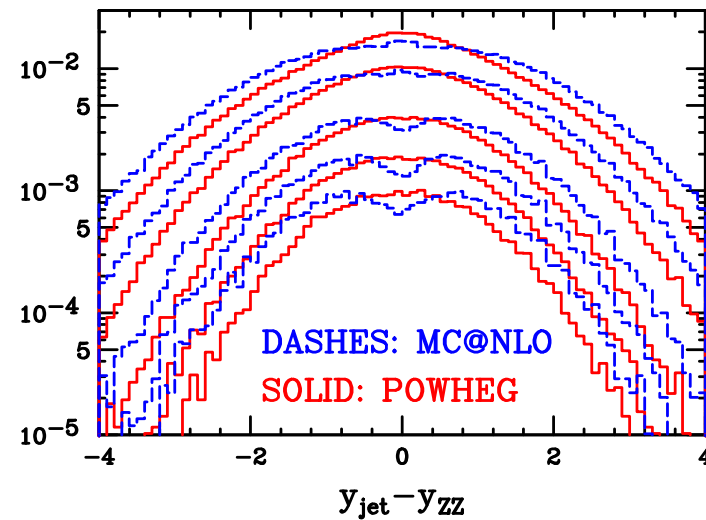
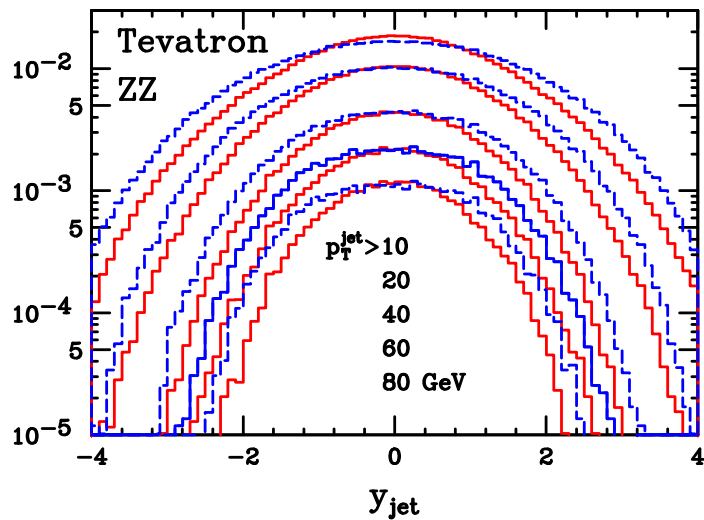
POWHEG+PYTHIA  
PYTHIA



# Rapidity distribution of hardest jet at Tevatron

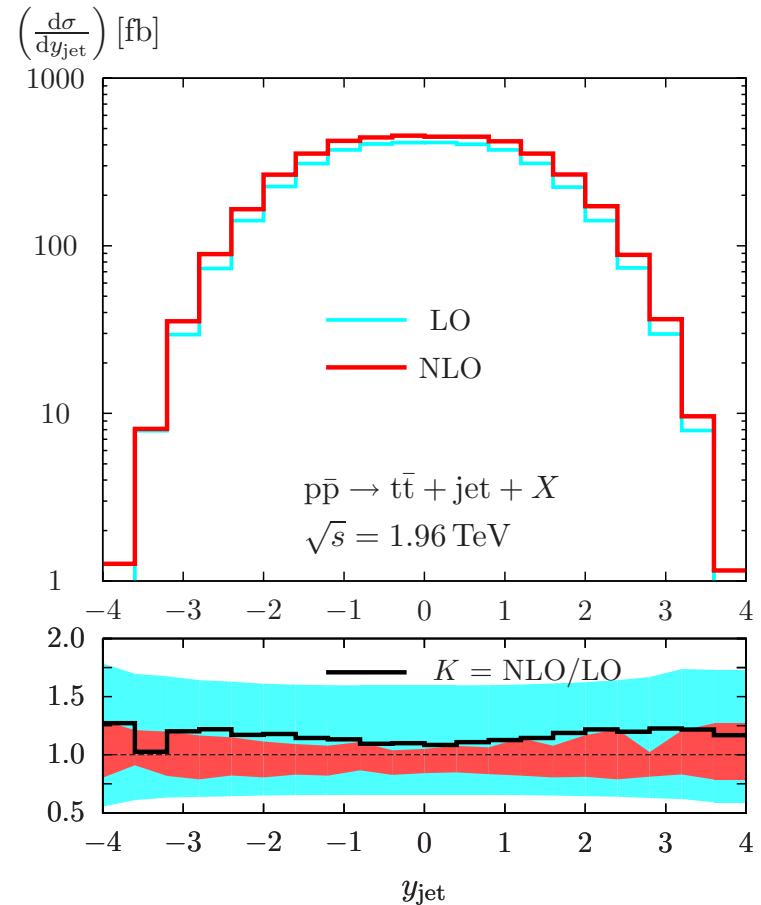
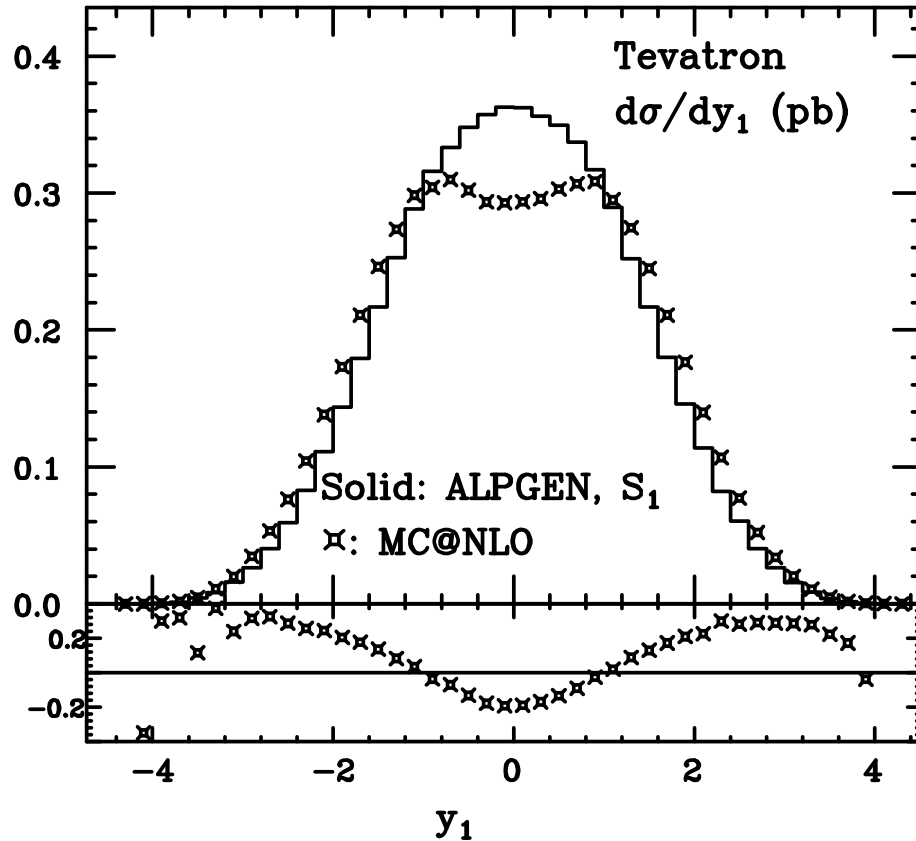


POWHEG+HERWIG  
 MC@NLO



POWHEG+HERWIG  
 MC@NLO

# ALPGEN and NLO vs MC@NLO: $t\bar{t} + 1$ jet

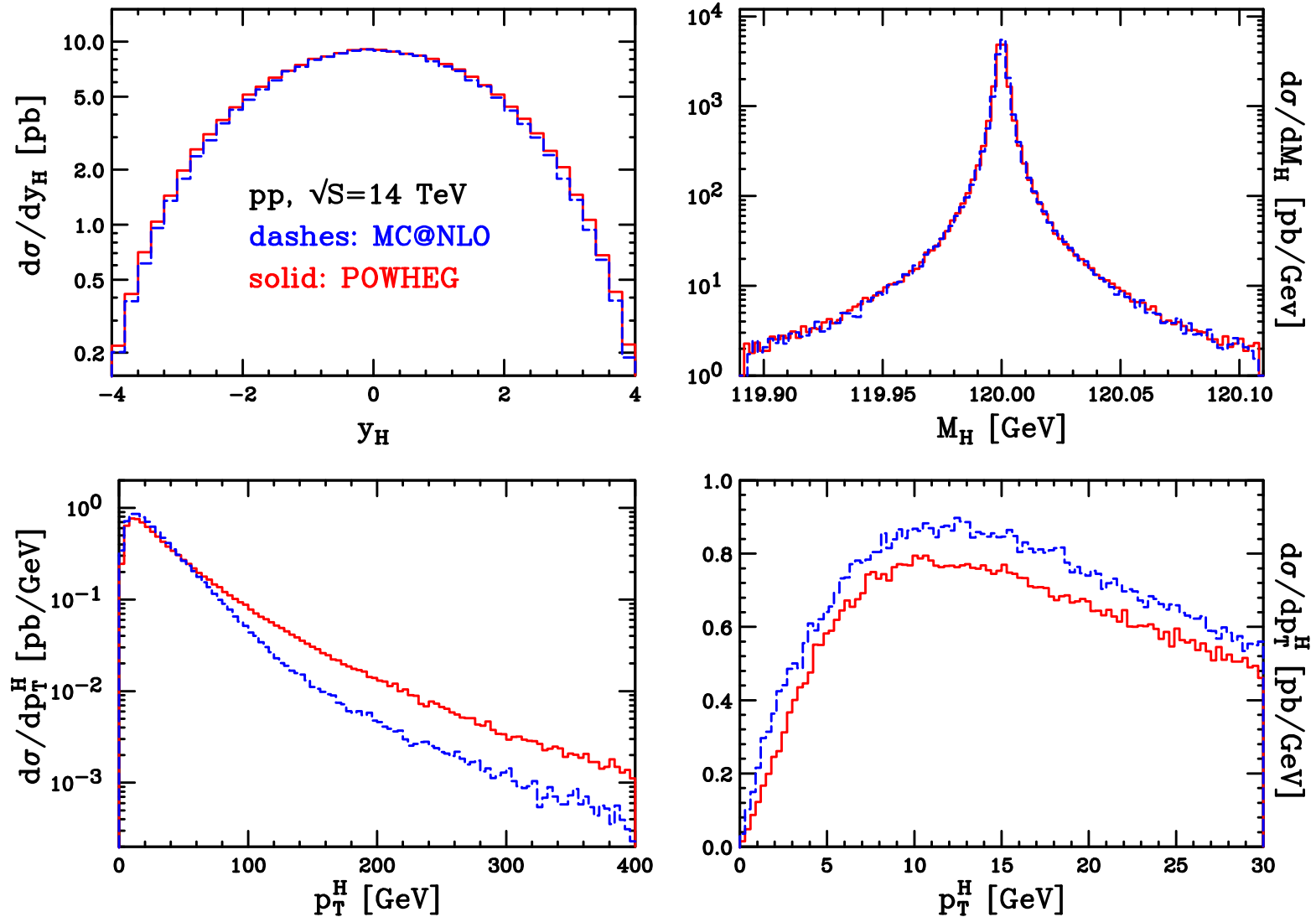


Rapidity  $y_1$  of the leading jet (highest  $p_T$ ).

POWHEG's distribution as in ALPGEN: **no dip** present. The size of discrepancy can be attributed to different treatment of higher-order terms.

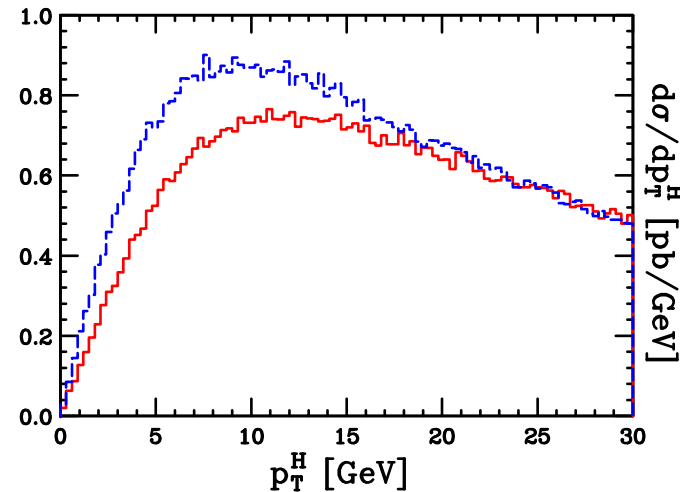
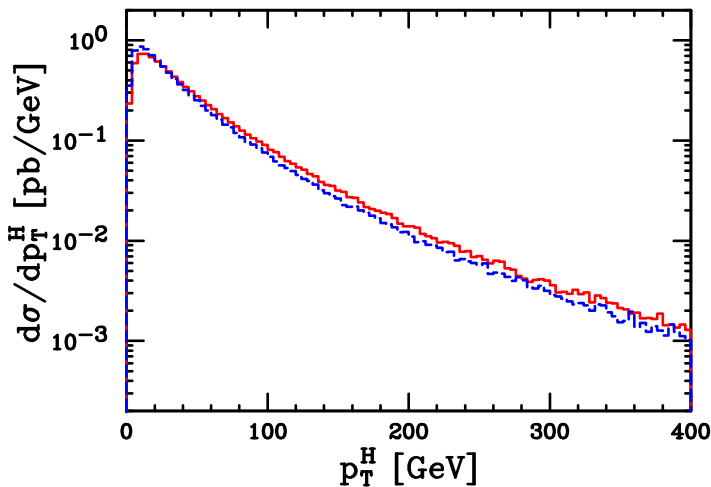
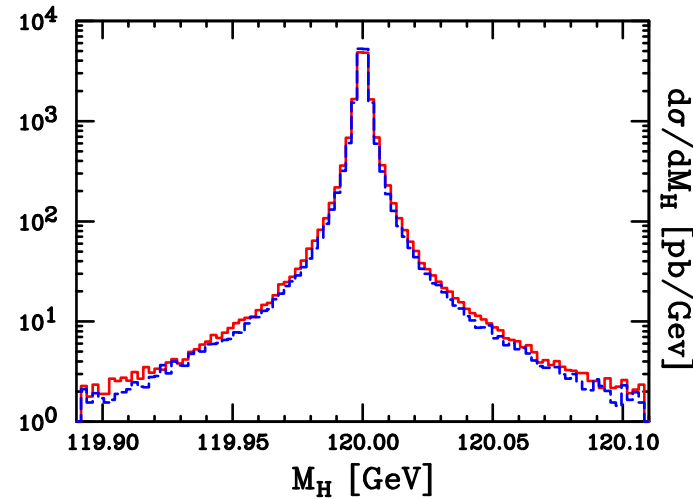
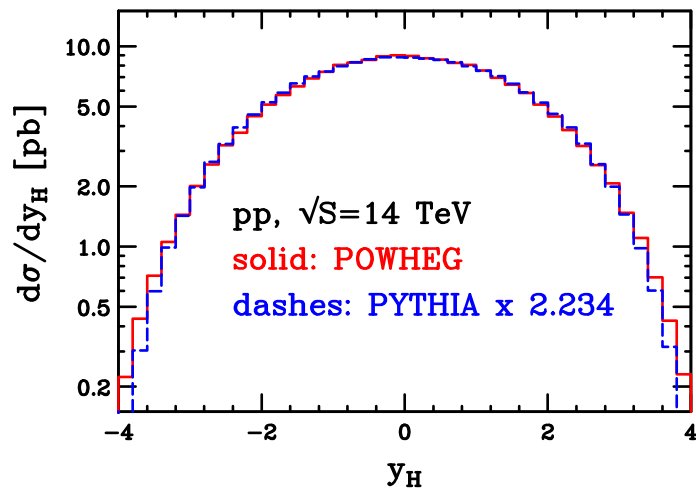
$pp \rightarrow t\bar{t} + \text{jet}$  at **NLO** [Dittmaier, Uwer and Weinzierl, arXiv:0810.0452] shows **no dip** too.

# Higgs boson production: POWHEG + HERWIG vs MC@NLO



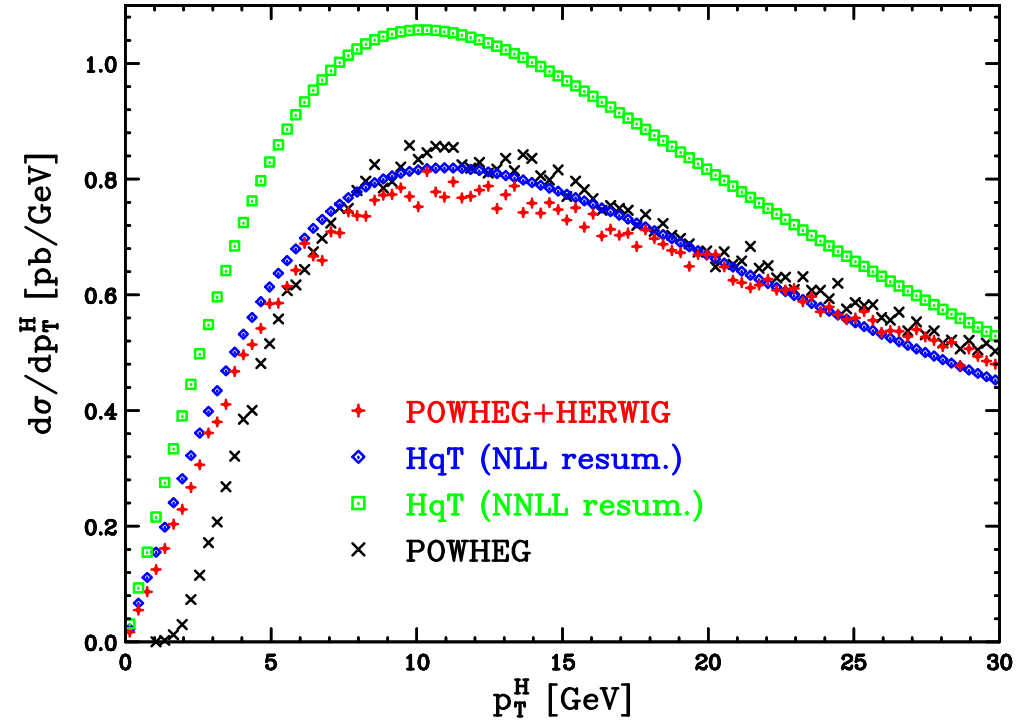
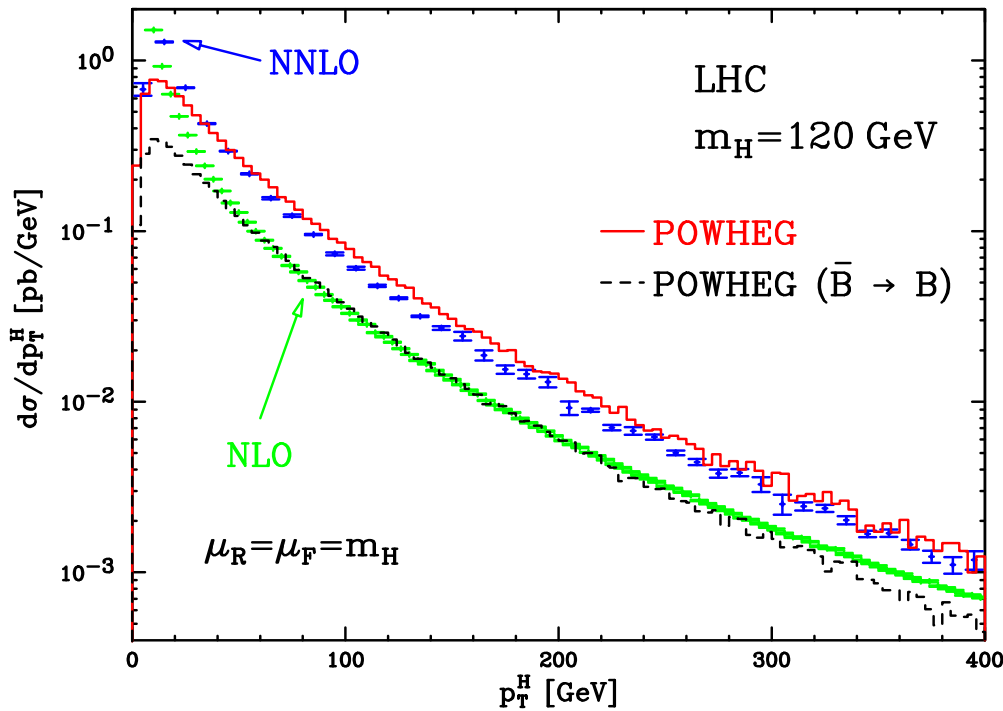
Differences in the **high-** and **low-** $p_T$  regions. More in the next slides.

# Higgs boson production: POWHEG + PYTHIA vs PYTHIA



- A shower Monte Carlo is accurate in the radiation of the **hardest jet only** in the **collinear regions**.
- **Only** because the generation of radiation in **vector-boson** and **Higgs boson** production in PYTHIA is **very similar** to the POWHEG one, we can make comparisons of  $p_T^{\text{jet}}$  and  $p_T^H$  distributions.

# Higgs boson production at the LHC



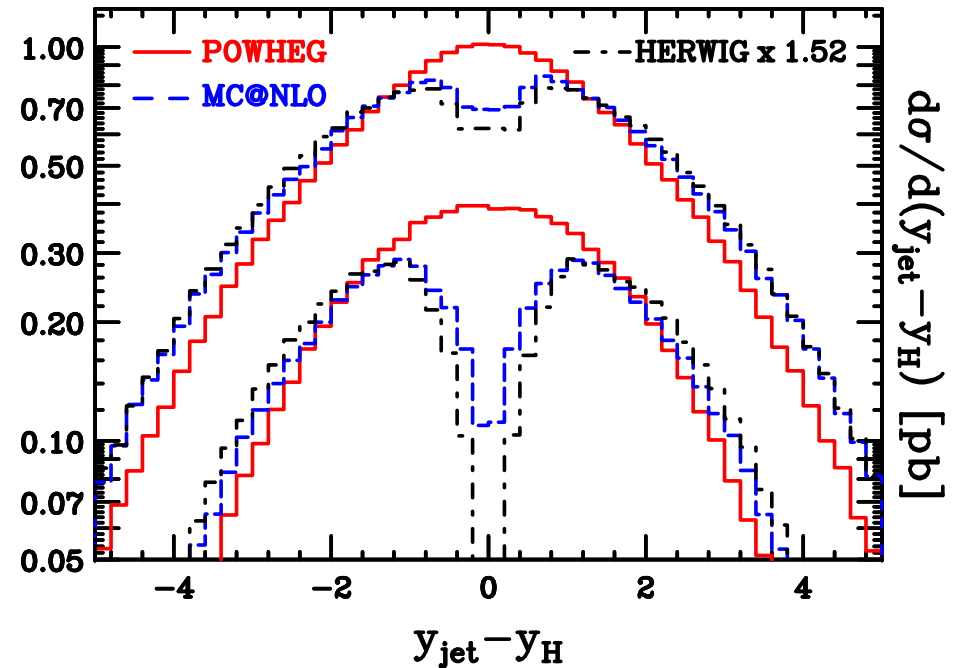
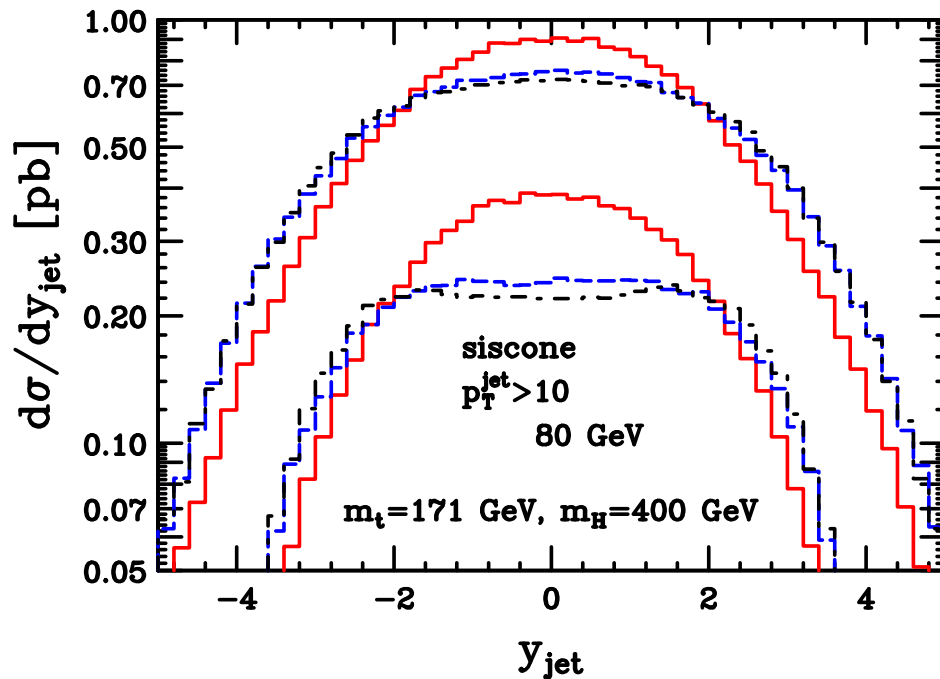
$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$d\sigma = \bar{B}(\Phi_n) d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \Delta(\Phi_n, p_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

$$d\sigma_{\text{rad}} \approx \frac{\bar{B}(\Phi_n)}{B(\Phi_n)} R(\Phi_{n+1}) d\Phi_{n+1} = \left\{ 1 + \mathcal{O}(\alpha_s) \right\} R(\Phi_{n+1}) d\Phi_{n+1}$$

Better agreement with NNLO in this way

# Higgs boson production at the LHC



- Dip **inherited** from the **deeper dip** of **HERWIG**. **MC@NLO** fills partially the dip.
- It gets **worse** for **large**  $p_T^{\text{jet}}$
- Why **MC@NLO** has a **dip** in the hardest jet rapidity?
- Why **POWHEG** has **no dip**? Is that because of the hardest  $p_T$  spectrum?

## Hard $p_T$ spectrum in POWHEG

We have enough flexibility to get rid of higher cross section at high  $p_T$ , if we want. Go back to the POWHEG cross section

$$d\sigma = \bar{B}(\Phi_n) \left\{ \Delta(p_T^{\min}) + \Delta(p_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} d\Phi_n$$

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

$$\Delta(p_T) = \exp \left[ - \int d\Phi_r' \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(p_T' - p_T) \right]$$

Break  $R = R_s + R_f$ , with  $R_f$  finite in collinear and soft limit. Define

$$d\sigma' = \bar{B}_s(\Phi_n) \left\{ \Delta_s(p_T^{\min}) + \Delta_s(p_T) \frac{R_s(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} d\Phi_n + R_f(\Phi_{n+1}) d\Phi_{n+1}$$

$$\bar{B}_s(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R_s(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$$

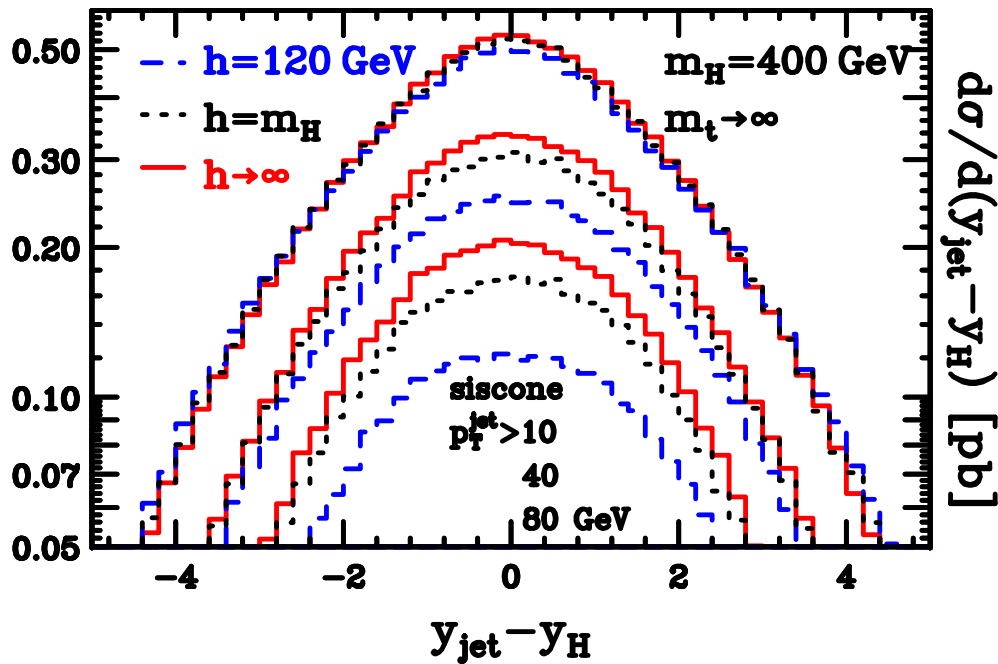
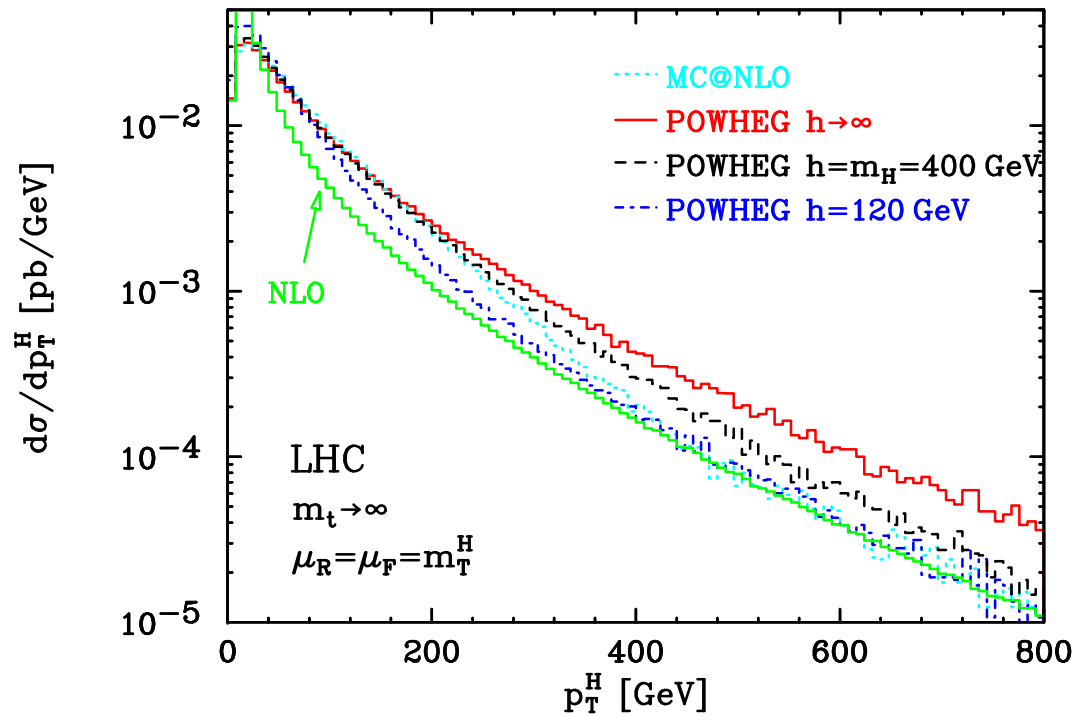
$$\Delta_s(p_T) = \exp \left[ - \int d\Phi_r' \frac{R_s(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(p_T' - p_T) \right]$$

Easy to prove that  $d\sigma'$  is equivalent to  $d\sigma$ . In other words, the part of the real cross section that is treated with the shower technique can be varied.

$$R_s = \frac{h^2}{p_T^2 + h^2} R$$

$$R_f = \frac{p_T^2}{p_T^2 + h^2} R$$

agrees with **NLO** at high  $p_T$



No new features appear in **all** the other **distributions**

High  $p_T$  cross section and dips are **unrelated issues**



## Why is there a dip in MC@NLO?

Write the MC@NLO hardest jet cross section in the POWHEG language. Hardest emission can be written as [Nason 2004]

$$d\sigma = \underbrace{\bar{B}_{\text{HW}} d\Phi_n}_{\text{S event}} \underbrace{\left[ \Delta_{\text{HW}}(p_T^{\min}) + \Delta_{\text{HW}}(p_T) \frac{R_{\text{HW}}(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right]}_{\text{HERWIG event}} + \underbrace{\left[ R(\Phi_{n+1}) - R_{\text{HW}}(\Phi_{n+1}) \right] d\Phi_{n+1}}_{\text{H event}}$$

$$\bar{B}_{\text{HW}}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[ R_{\text{HW}}(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right] d\Phi_r$$

$$\Delta_{\text{HW}}(p_T) = \exp \left[ - \int d\Phi_r' \frac{R_{\text{HW}}(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(p_T' - p_T) \right]$$

Like POWHEG with  $\begin{cases} R_s = R_{\text{HW}} \\ R_f = R - R_{\text{HW}} \end{cases} \iff \text{can be negative}$

This formula illustrates why MC@NLO and POWHEG are **equivalent at NLO!**

But differences can arise at **NNLO...**

At high  $p_T$  the cross section goes as

$$\begin{aligned}
 d\sigma &\approx \left[ \frac{\bar{B}_{\text{HW}}(\Phi_n)}{B(\Phi_n)} R_{\text{HW}}(\Phi_{n+1}) + R(\Phi_{n+1}) - R_{\text{HW}}(\Phi_{n+1}) \right] d\Phi_{n+1} \\
 &= \underbrace{R(\Phi_{n+1})}_{\text{no dip}} d\Phi_{n+1} + \underbrace{\left( \frac{\bar{B}_{\text{HW}}(\Phi_n)}{B(\Phi_n)} - 1 \right)}_{\mathcal{O}(\alpha_s) \text{ but large for Higgs}} \underbrace{R_{\text{HW}}(\Phi_{n+1})}_{\text{pure HERWIG dip}} d\Phi_{n+1}
 \end{aligned}$$

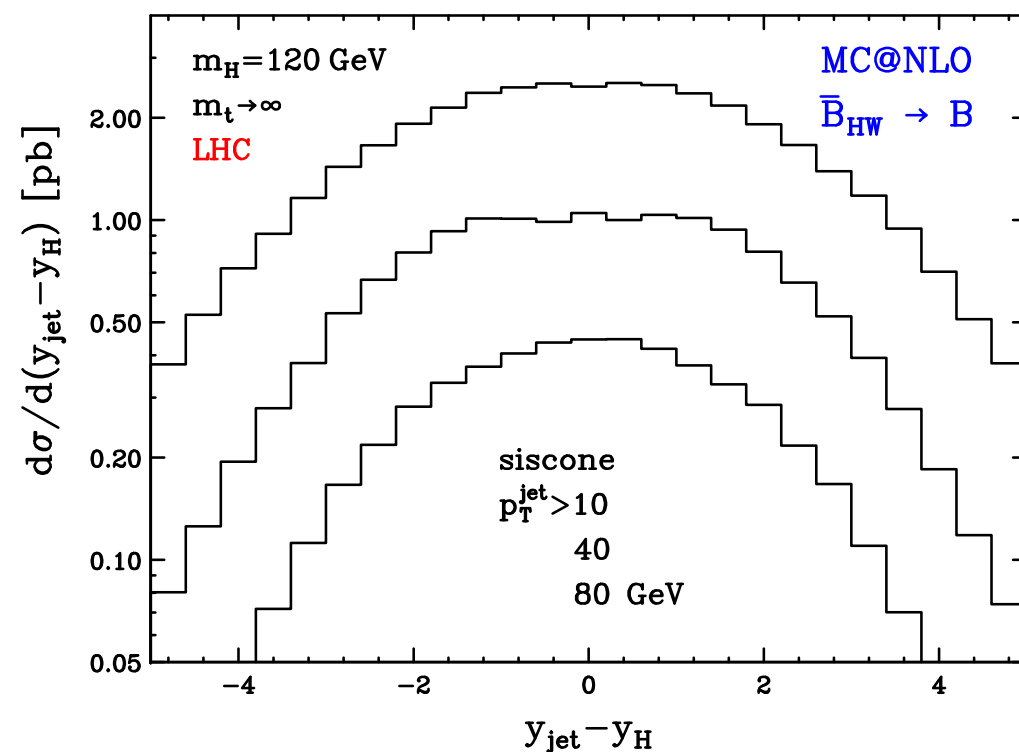
So: a **contribution** with a **dip** is added to the exact NLO result.

The contribution is  $\mathcal{O}(\alpha_s R)$ , i.e. **NNLO**

Can we test this hypothesis?

**Replace**  $\bar{B}_{\text{HW}} \rightarrow B$  in MC@NLO.

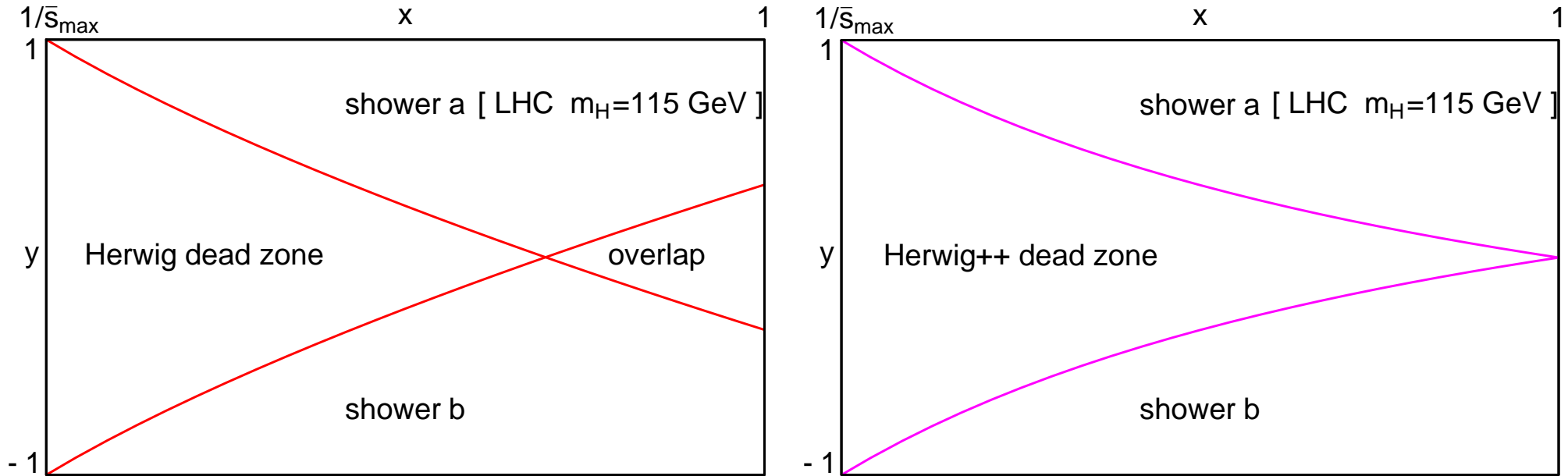
The dip should disappear...



**No visible dip** is present.

# HERWIG and HERWIG++ studies

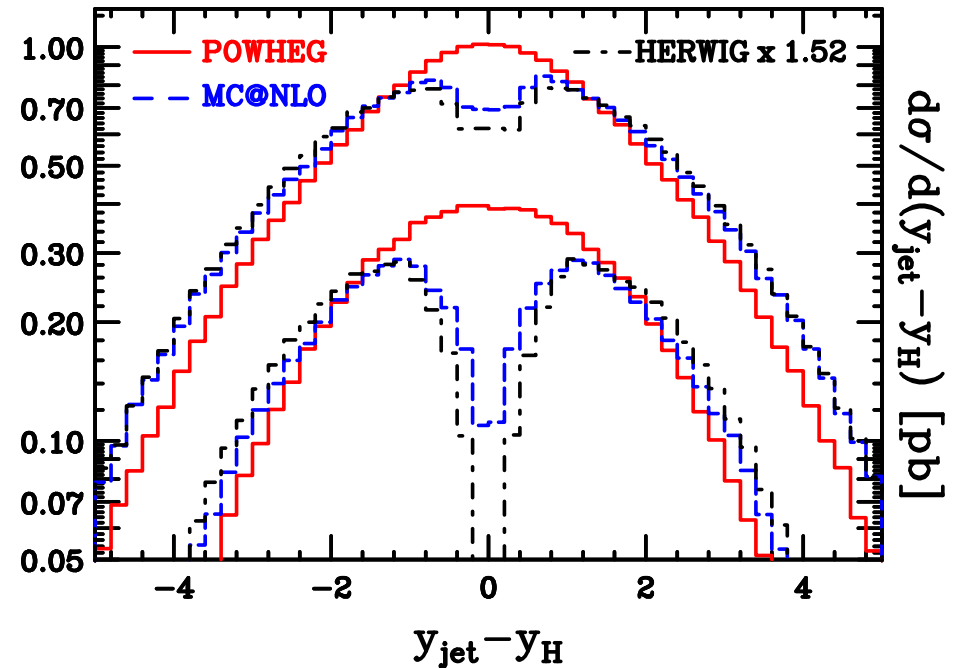
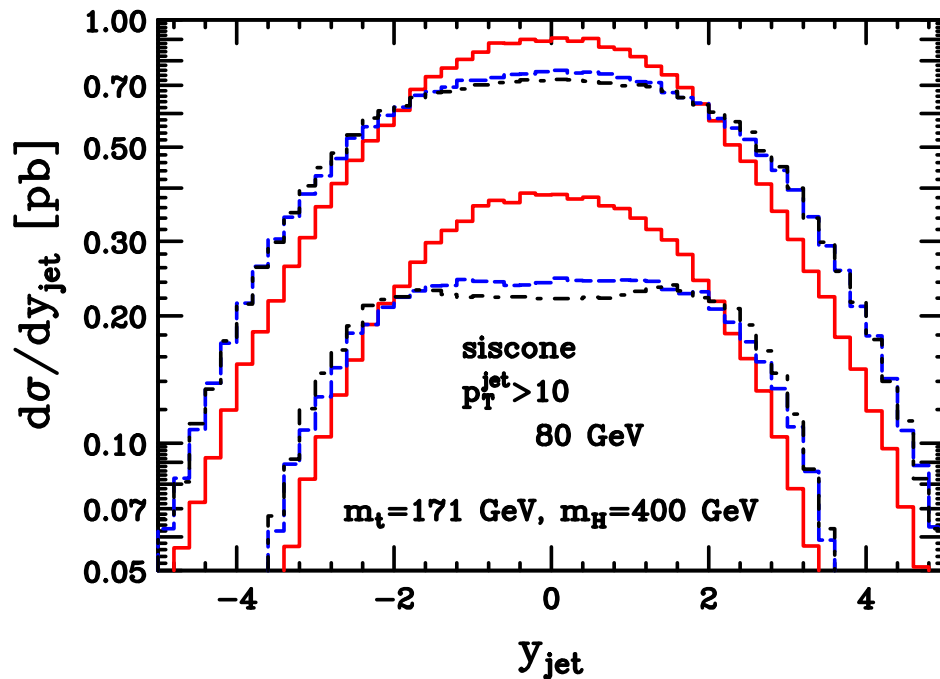
[Hamilton, Richardson and Tully] arXiv:0903.4345



Both HERWIG and HERWIG++ have a **dead radiation region** corresponding to **central rapidity** and **high energy**.

**Dip in central region** in HERWIG can be attributed to the **dead zone**.

# Higgs boson production at the LHC

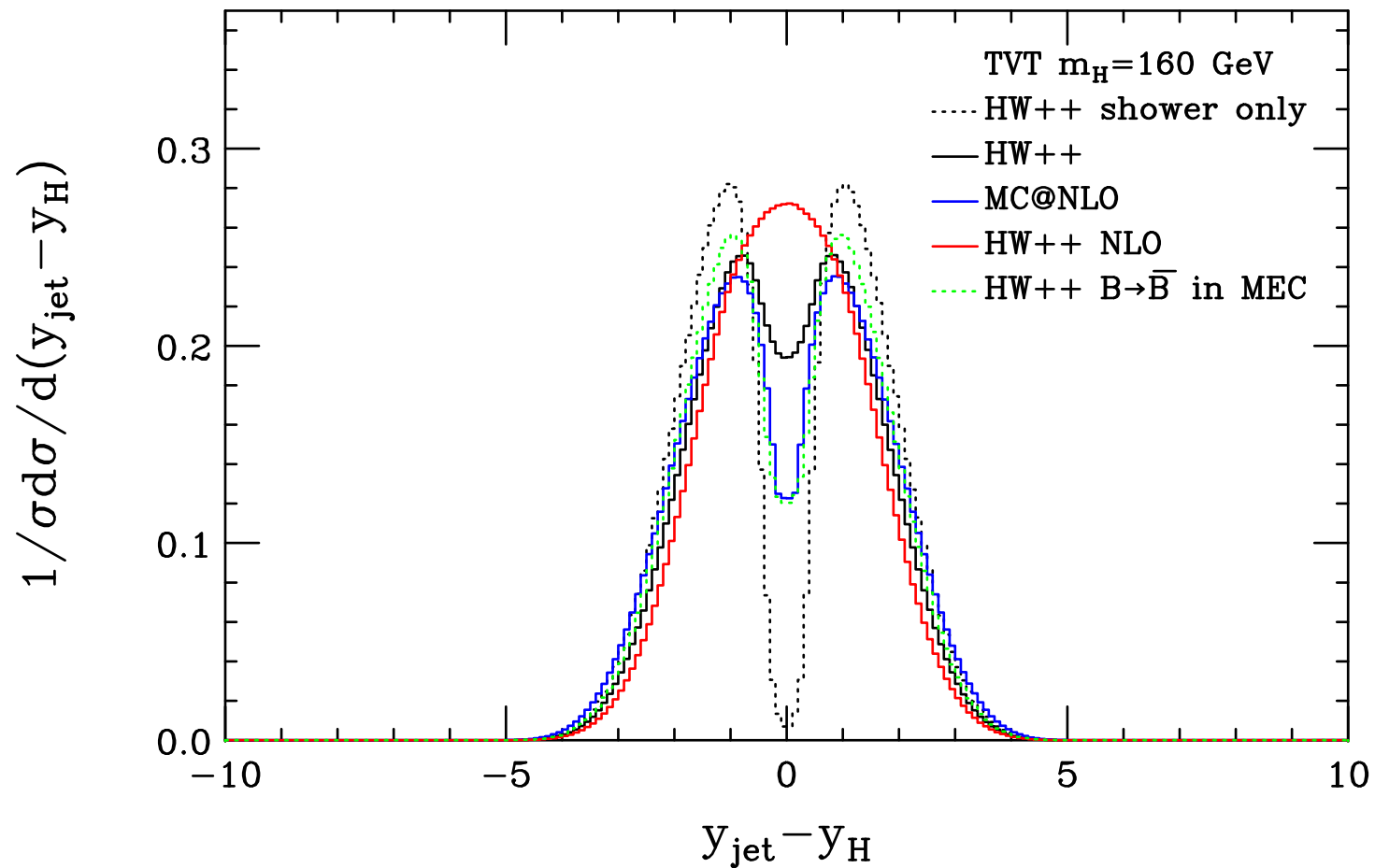


- Why **MC@NLO** has a **dip** in the hardest jet rapidity?  
**ANSWER**: because it is very **sensitive** to the **dead zone** in the HERWIG phase space
- Why **POWHEG** has **no dip**? Is that because of the hardest  $p_T$  spectrum?  
**ANSWER**: NO, it does **not depend** on the hardest  $p_T$  spectrum. POWHEG generate by **itself** the **hardest radiation**.

# HERWIG and HERWIG++ studies

[Hamilton, Richardson and Tully] arXiv:0903.4345

Hardest jet rapidity – Higgs rapidity ( $p_T > 40$  GeV)

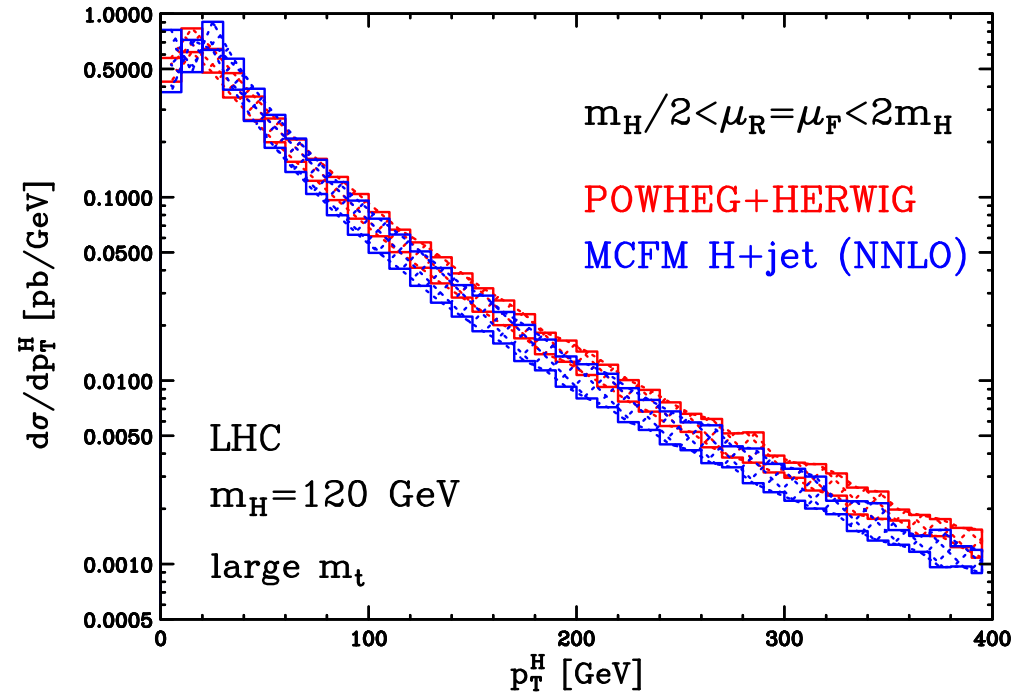
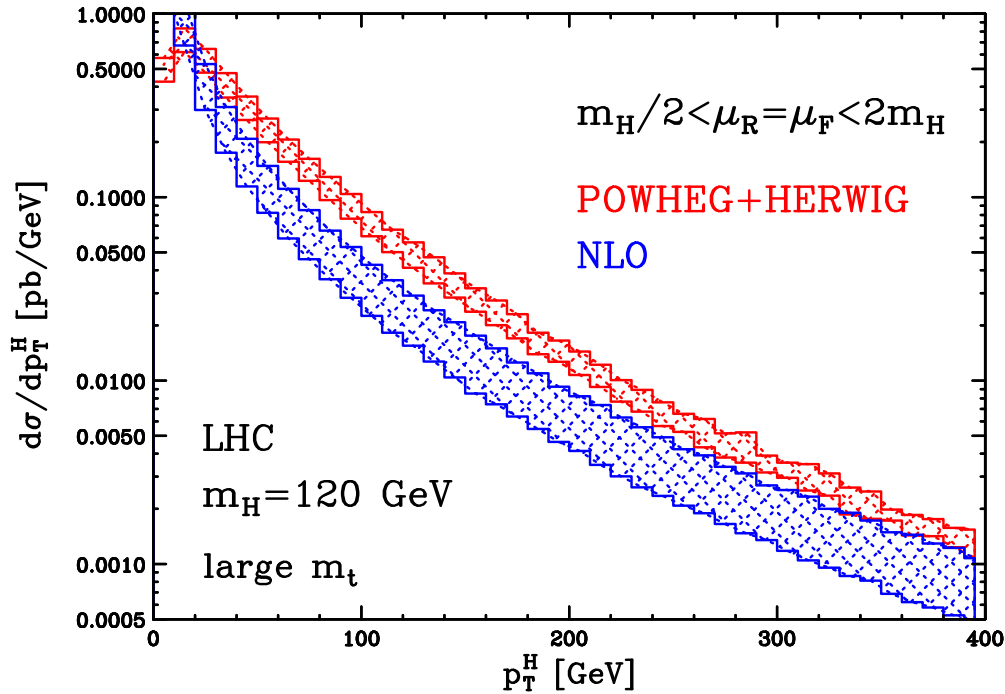


## Summary of MC@NLO and POWHEG comparisons

- Fairly good agreement on most distributions
- Areas of disagreement can be tracked back to **NNLO terms**, arising mostly because of the use of an **NLO inclusive** cross section (the  $\bar{B}$  function) to shower out the hardest radiation.
- In POWHEG, since the **hardest radiation** is generated by **POWHEG itself**, one has **high flexibility** in tuning the magnitude of these NNLO terms.
- For MC@NLO, these NNLO terms can generate **unphysical behavior** in **physical distributions**, reflecting the **dead zones** structure of the underlying shower Monte Carlo.

Since MC@NLO uses the underlying Monte Carlo to generate the hardest emission, to remedy to these problems one has to intervene on the Monte Carlo itself

# Higgs boson production at the LHC



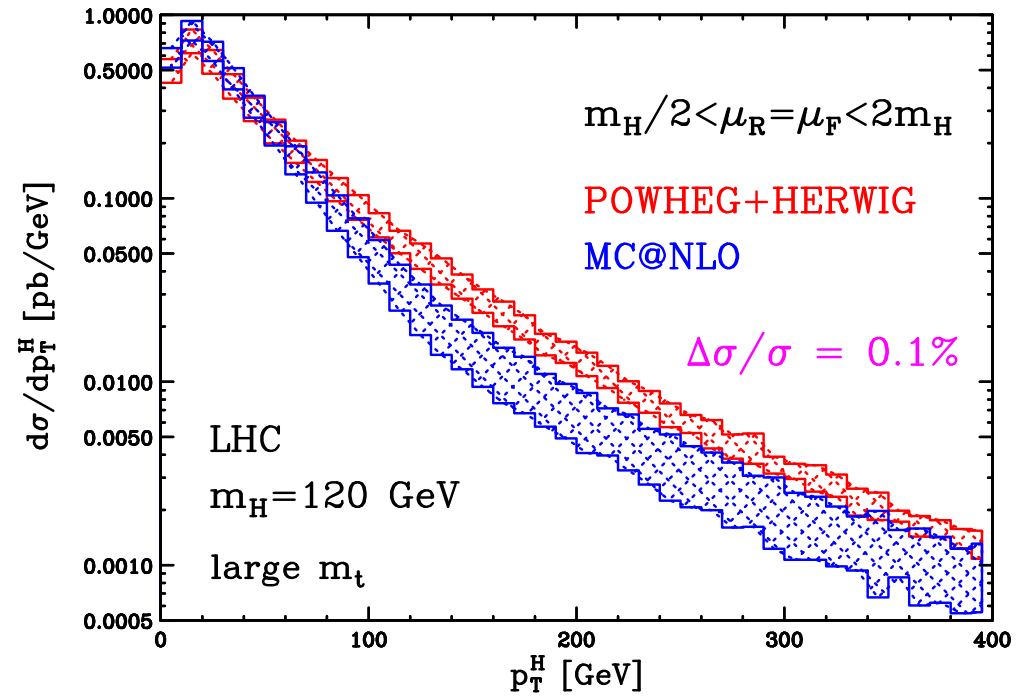
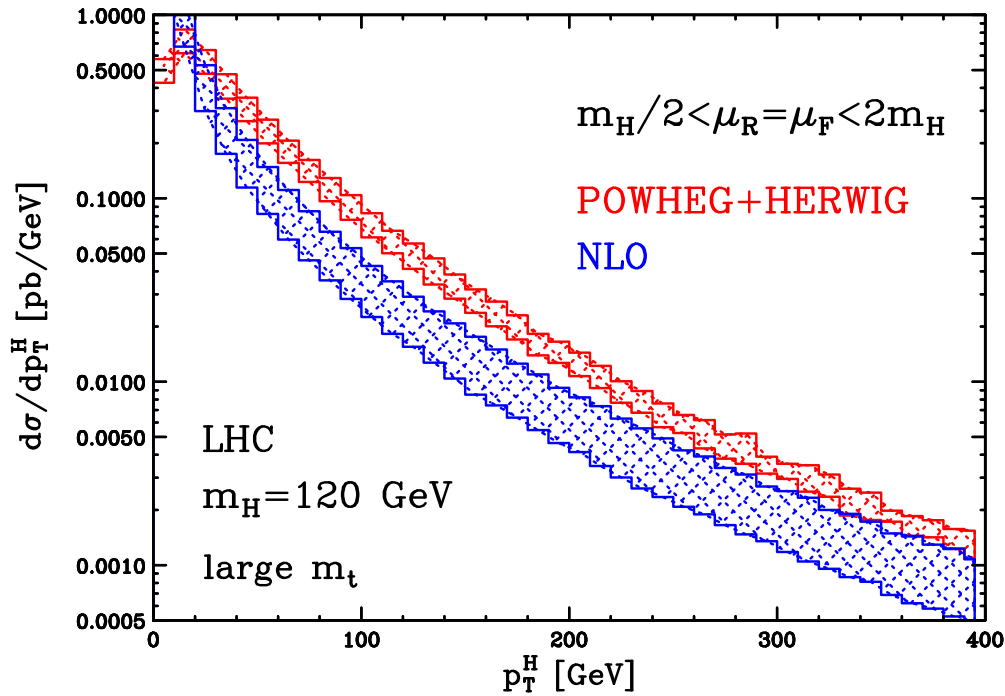
The NLO result is in reality a **LO** one  $\implies$  it depends upon  $\alpha_s^3(\mu_R)$

$$\bar{B}(\Phi_n, \mu_R) = B(\Phi_n) + V(\Phi_n, \alpha_s(\mu_R)) + \int d\Phi_r [R(\Phi_n, \Phi_r, \alpha_s(\mu_R)) - C(\Phi_n, \Phi_r)]$$

$$d\sigma = \bar{B}(\Phi_n, \mu_R) d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \Delta(\Phi_n, p_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

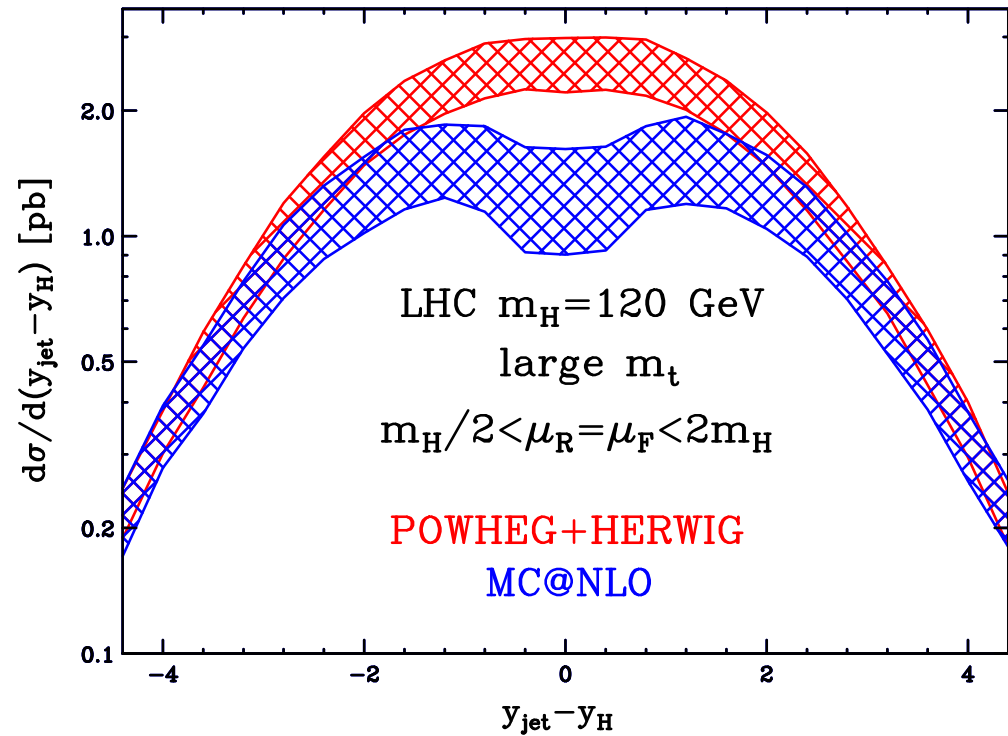
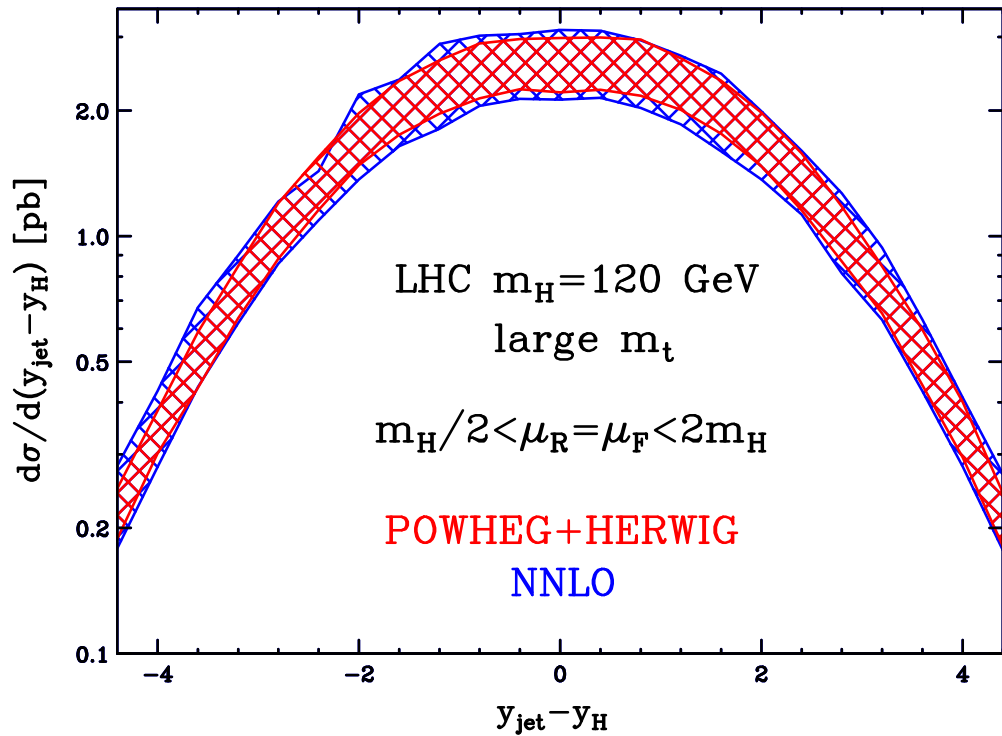
$$\Delta(\Phi_n, p_T) = \exp \left[ - \int d\Phi'_r \frac{R(\Phi_n, \Phi'_r, \alpha_s(k_T))}{B(\Phi_n)} \theta(k_T(\Phi_n, \Phi'_r) - p_T) \right]$$

# Higgs boson production at the LHC



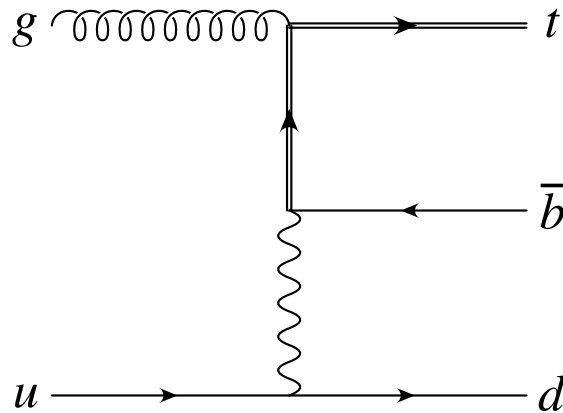
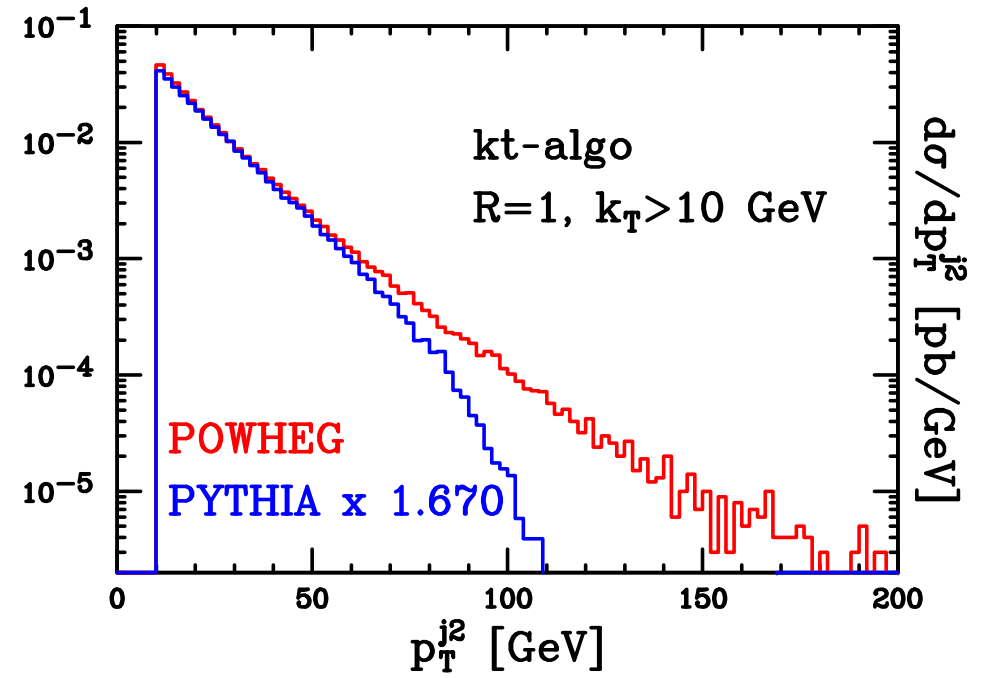
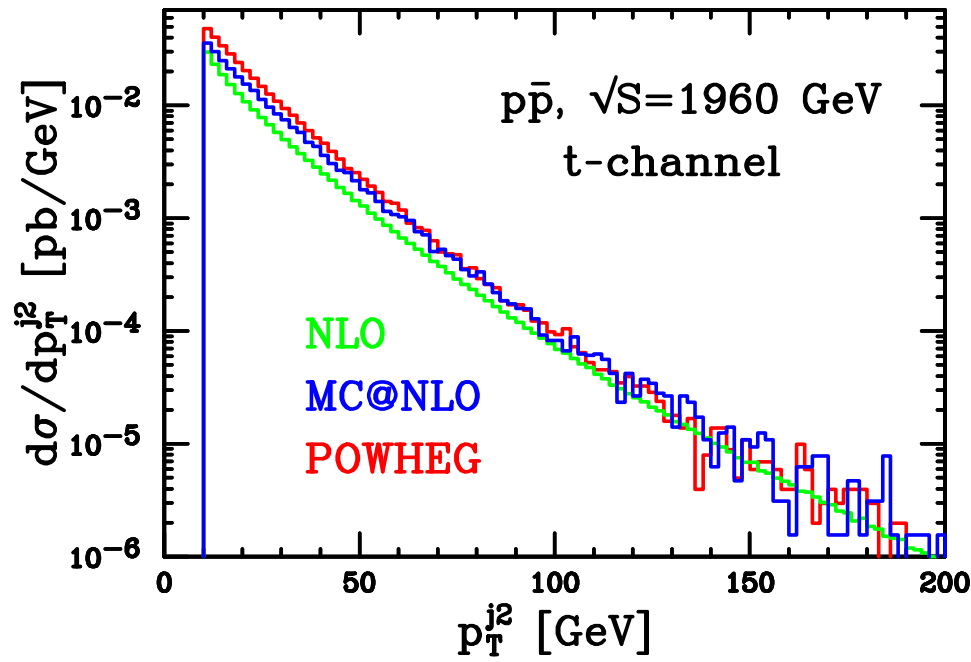


# Higgs boson production at the LHC

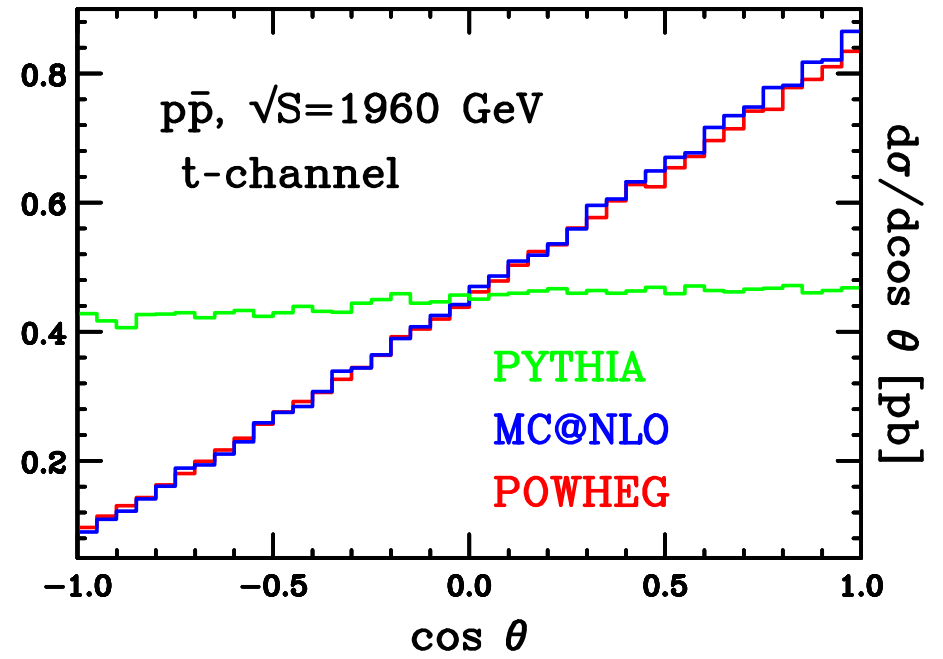
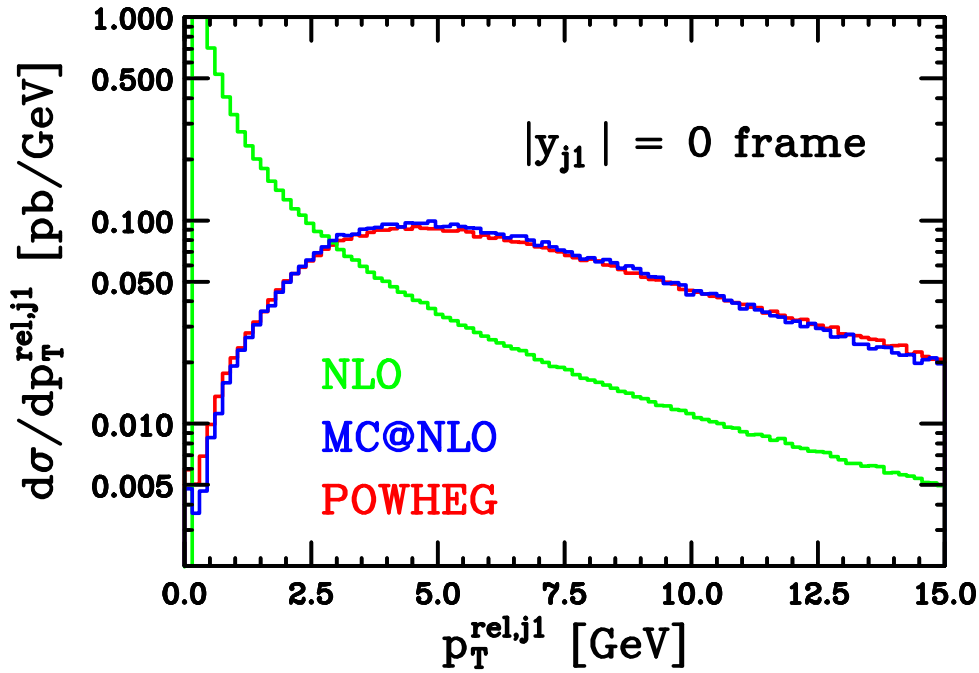


NNLO result obtained with HNNLO by Catani & Grazzini

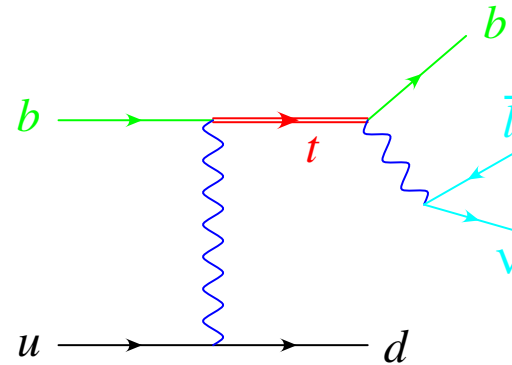
# Single-top production + spin correlations



# Single-top production + spin correlations

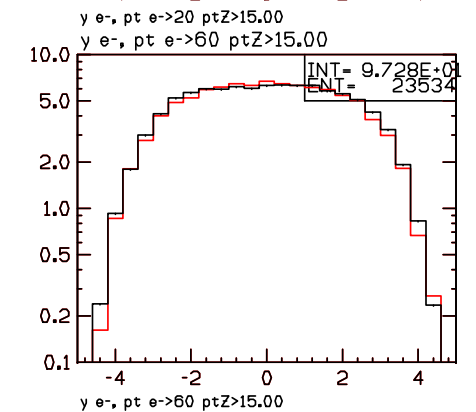
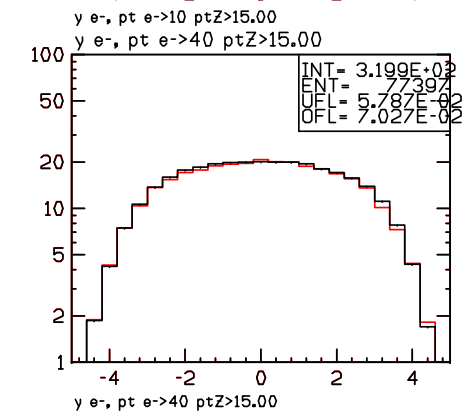
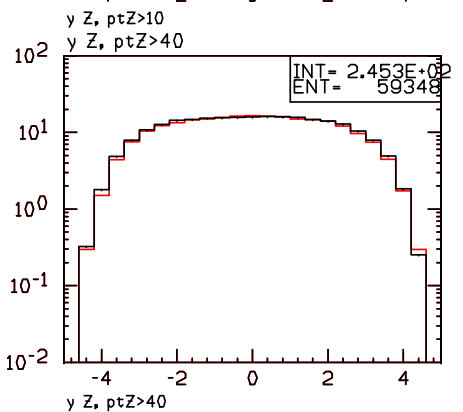
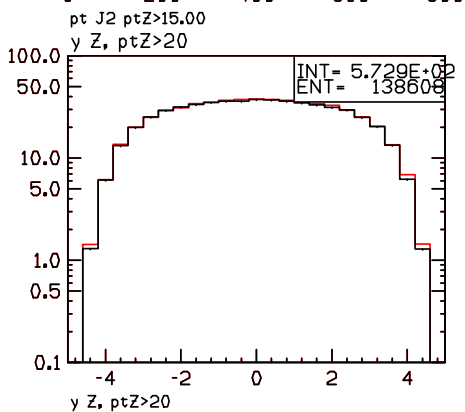
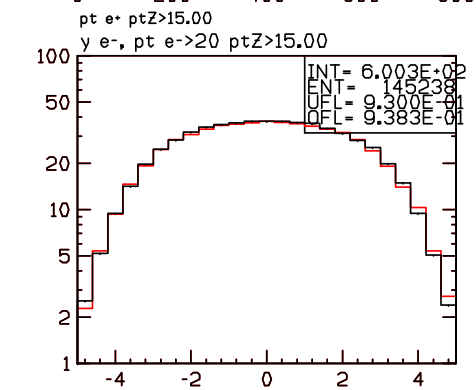
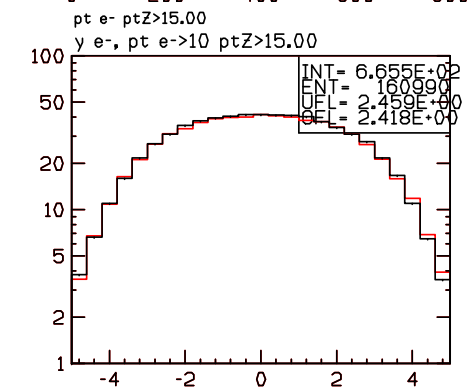
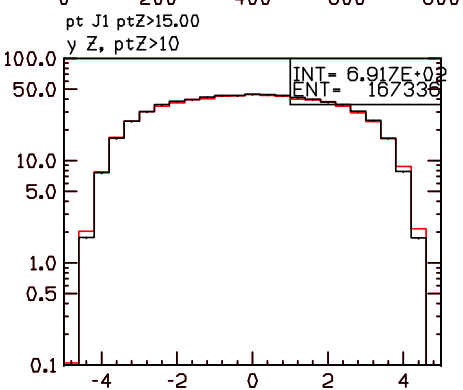
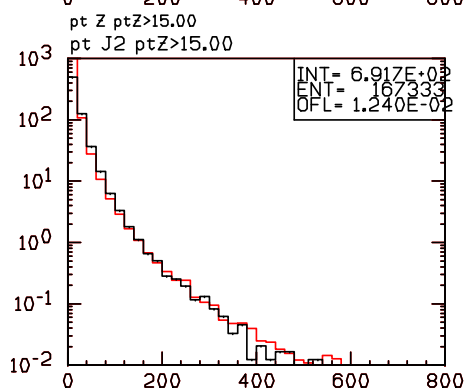
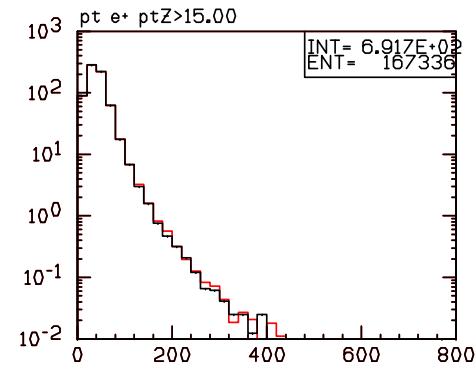
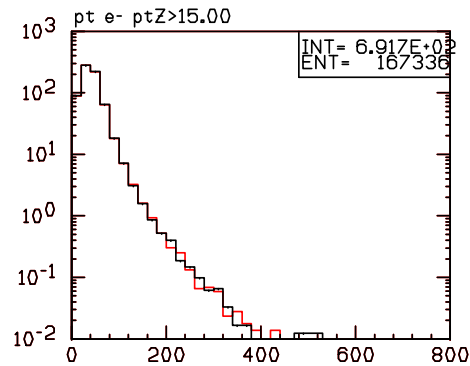
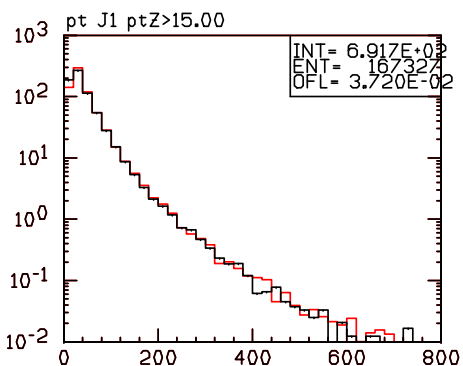
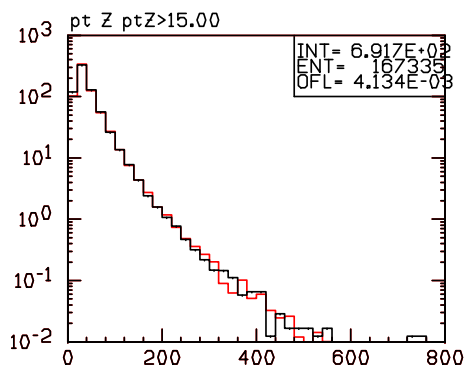


$$p_T^{\text{rel},j_1} = \sum_{i \in j_1} \frac{|\vec{k}^i \times \vec{p}^{j_1}|}{|\vec{p}^{j_1}|}$$

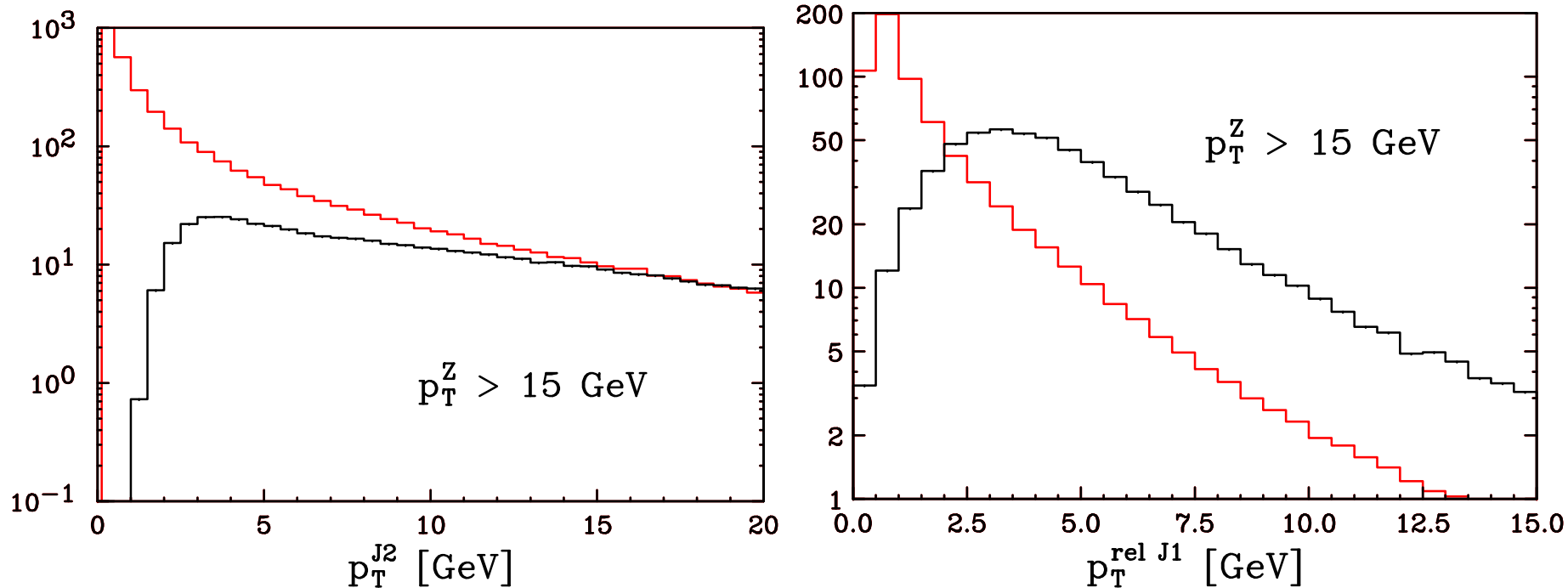


$\theta$  = angle between the **charged lepton**  $\bar{l}$  and the **hardest jet** ( $d$  quark), in the top rest frame

# Z + 1 jet: POWHEG+HERWIG with NLO



## Z + 1 jet: POWHEG+HERWIG with NLO



Distributions **sensitive to more than two jets** show a **noticeable difference**

All others in agreement with NLO

First process not present in MC@NLO

## The POWHEG BOX

Automatic implementation of POWHEG for generic NLO processes

More precisely, the **user** must supply

- ✓ the **Born phase space**
- ✓ the **lists of Born and real processes** (i.e.  $u\bar{u} \rightarrow e^+e^-gg$ )
- ✓ the **Born squared amplitudes**  $\mathcal{B} = |\mathcal{M}|^2$ ,  $\mathcal{B}_{ij}$  and  $\mathcal{B}_{j,\mu\nu}$ , for all relevant partonic processes.  $\mathcal{B}_{ij}$  is the color-ordered Born amplitude squared,  $\mathcal{B}_{j,\mu\nu}$  is the spin-correlated amplitude, where  $j$  runs over all external gluons in the amplitude. All these amplitudes are **common ingredients** of a NLO calculation.
- ✓ The **real squared amplitude**.
- ✓ The finite part of the **virtual amplitude**.

**All the rest** will be done **automatically!** The user should not worry about the subtraction scheme we use and all the other details.

## The POWHEG BOX

Use the **FKS** (Frixione-Kunszt-Signer) subtraction scheme according to the general formulation of POWHEG given in [Frixione, Nason and C.O., 2007] (FNO), **hiding all FKS implementation details**.

In other words, the user **needs not to know it!**

It includes:

- ✓ the **phase space** for ISR and FSR, according to FNO.
- ✓ the **combinatorics**, the calculation of all **singular regions** in the real amplitude  $R$ , the soft and collinear limit
- ✓ the calculation of  $\bar{B}$  (spinoff: **NLO** results using the **FKS subtraction scheme**)
- ✓ the calculation of the upper bounds for the generation of radiation
- ✓ the **generation of radiation**
- ✓ writing the event into the Les Houches interface

**More testing needed.** Further problems solved while we implement new processes.

## The POWHEG BOX How-To

- parameter (`nlegborn=5`) [ $pp \rightarrow (Z \rightarrow e^+ e^-) + j$ ] in included file `pwhg_flst.h`  
`flst_nborn` and `flst_nreal`
- `flst_born(k=1..nlegborn, j=1..flst_nborn)`: flavour of the  $k$ -th leg of the  $j$ -th Born graph  
`flst_real(k=1..nlegreal, j=1..flst_nreal)`: flavour of the  $k$ -th leg of the  $j$ -th real graph.  
It is required that legs in the Born and real processes have to be ordered as follows:
  - leg 1, incoming parton with positive rapidity
  - leg 2, incoming parton with negative rapidity
  - from leg 3 onward, final state particles, in the order: colorless particles first, massive coloured particles, massless coloured particles.

The flavour is taken incoming for the two incoming particles and outgoing for the outgoing particles. The flavour index is assigned according to PDG conventions, except for gluons, where 0 is used instead of 21.

Example:  $pp \rightarrow (Z \rightarrow e^+ e^-) + 2j$ , the string `[1,0,-11,11,1,0]` labels the process  $dg \rightarrow e^+ e^- dg$

- `init_couplings`



- `Born_phsp(xborn)` for Born phase space  
`xborn(1..ndim)` array of random numbers  $\text{ndim}=(\text{nlegborn}-2)*3-4+2-1$ 
  - the Born Jacobian `kn_jacborn`, Born momenta in the laboratory frame `kn_pborn(0:3,1..nlegborn)`, Born momenta in the partonic CM frame `kn_cmpborn(0:3,1..nlegborn)` and Bjorken  $x$  (`kn_xb1` and `kn_xb2`).
- `set_ren_fac_scales(mur,muf)`
- `setborn(p,bflav,born,bornjk,bmunu)`
  - the momenta `p(0:3,1..nlegborn)`
  - the flavour string `bflav(1..nlegborn)`
  - `bornjk(1..nlegborn,1..nlegborn)`
  - the Born helicity-correlated squared amplitudes `bmunu(0:3,0:3,j=1..nlegborn)`
- `setvirtual(p,vflav,virtual)` returns finite part of the interference  $2 \operatorname{Re}(M_B \times M_V)$ , after factorizing out ( $d = 4 - 2\epsilon$ )

$$\mathcal{N} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left( \frac{\mu^2}{Q^2} \right)^\epsilon \frac{\alpha_s}{2\pi}$$

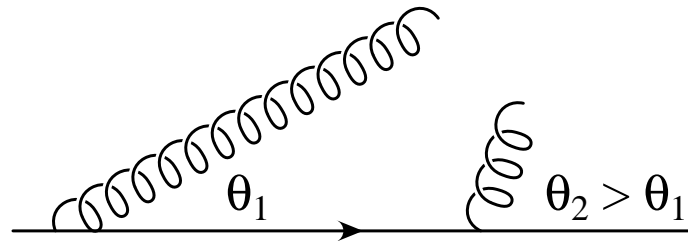
- `real_ampsq(p,rflav,amp2)`
  - the momenta `p(0:3,1..nlegreal)`
  - the flavour string `rflav(1..nlegreal)`
  - `amp2`: spin and color summed and averaged real squared amplitudes

## Conclusions

- ✓ NLO accuracy with Shower Monte Carlo has become a reality in recent years.
- ✓ The POWHEG method is progressing, with new processes being included
- ✓ Progress in understanding agreement and differences between MC@NLO and POWHEG
- ✓ A path to full automation of POWHEG implementations of arbitrary NLO calculation is open: the POWHEG BOX
- ✓ Many interesting problems remain to be addressed, and the NLO+Shower community is steadily growing

## Backup slides

## POWHEG: truncated shower

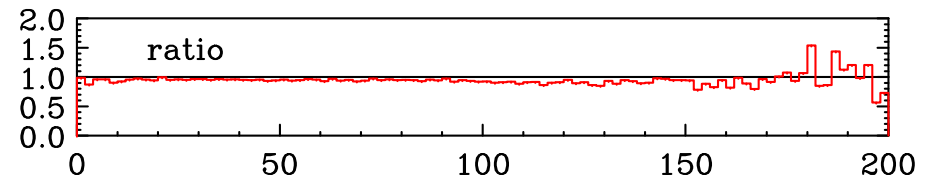
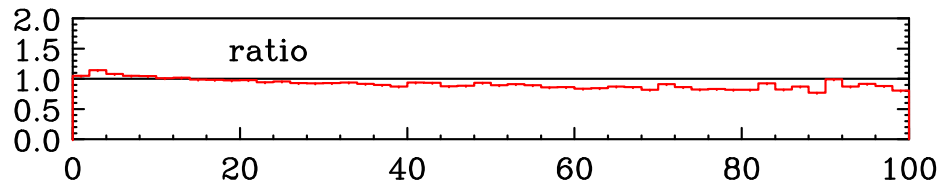
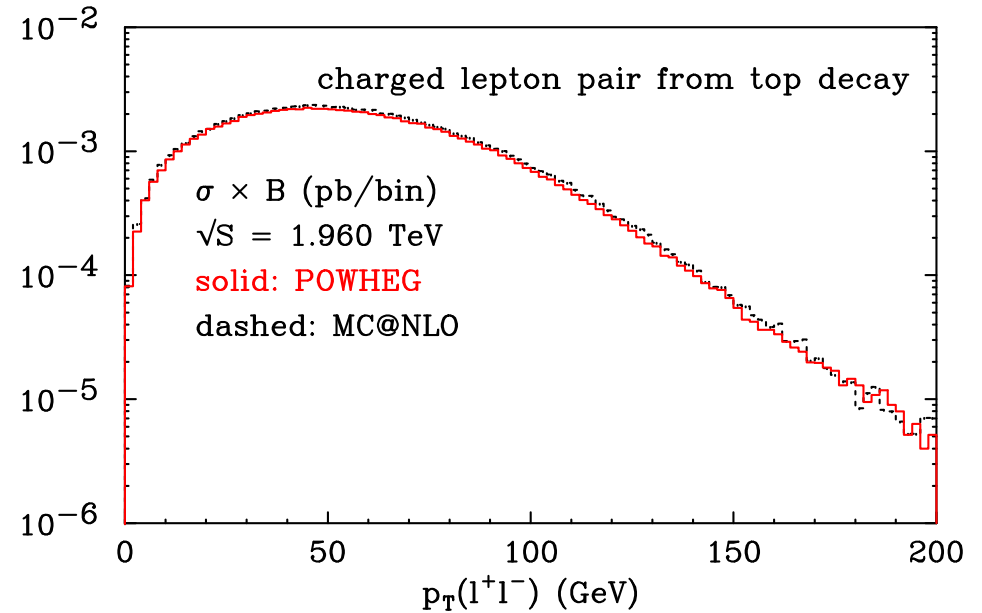
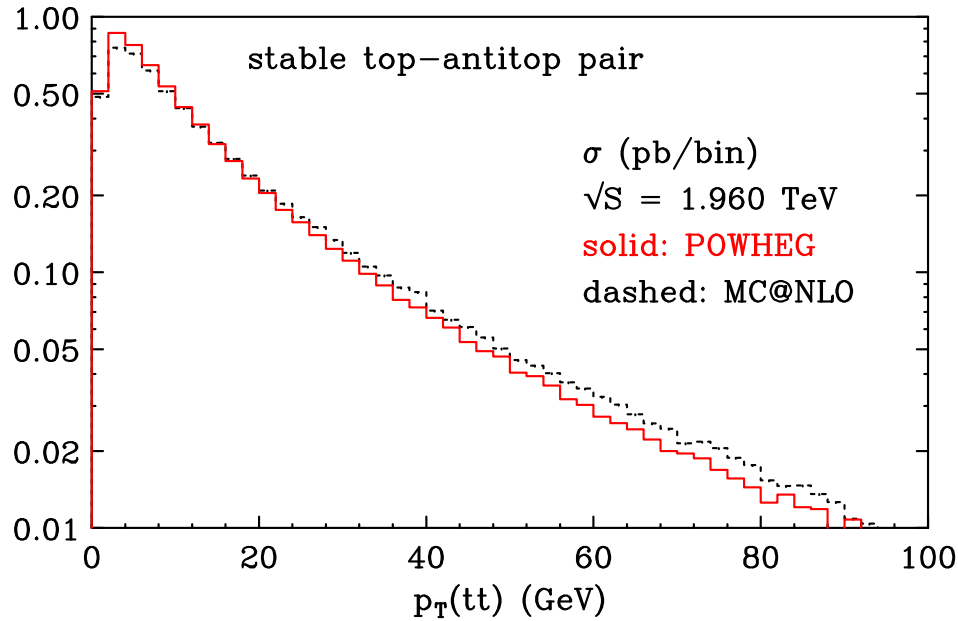


- if the shower is **ordered in  $p_T$**  (for example PYTHIA), nothing else needs to be done
- if the shower is **ordered in angle** (for example HERWIG), we need to generate correctly soft radiation at large angle.
  - pair up the partons that are nearest in  $p_T$
  - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (**truncated shower**)
  - generate all subsequent **vetoed showers**

This is a problem that affects **all the angular-ordered** shower Monte Carlo programs when the shower is initiated by a relatively complex matrix element.

**Truncated shower** implemented only in HERWIG++

# $t\bar{t}$ production



Good agreement for all observables considered. There are **sizable differences** that can be ascribed to different treatment of higher terms. But more investigation needed (different **scale choices**, **no truncated shower**, different **hard/soft radiation emission**,...).

## ALPGEN vs MC@NLO: $t\bar{t} + 1$ jet

ALPGEN can generate samples of  $t\bar{t} + n$  jets. Can be compared to NLO + Parton Shower [Mangano, Moretti, Piccinini & Treccani, hep-ph/0611129]

- ✓ **advantage**: better high jet multiplicity (exact Matrix Element)
- ✗ **disadvantage**: worse normalization (no NLO)

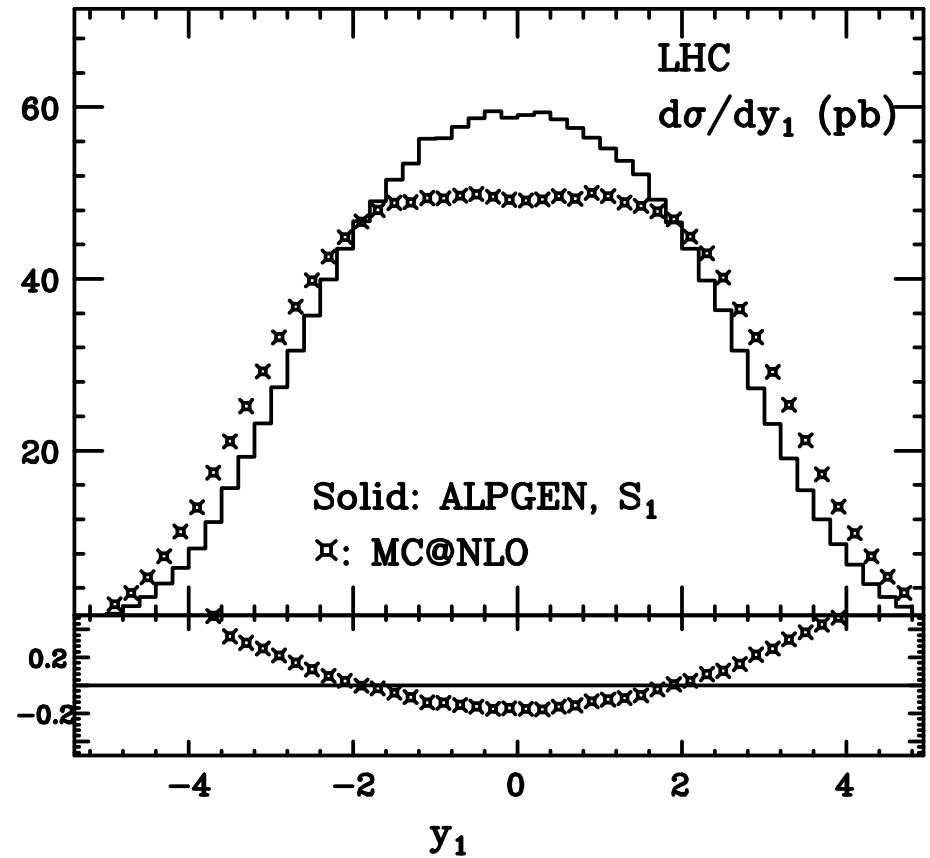
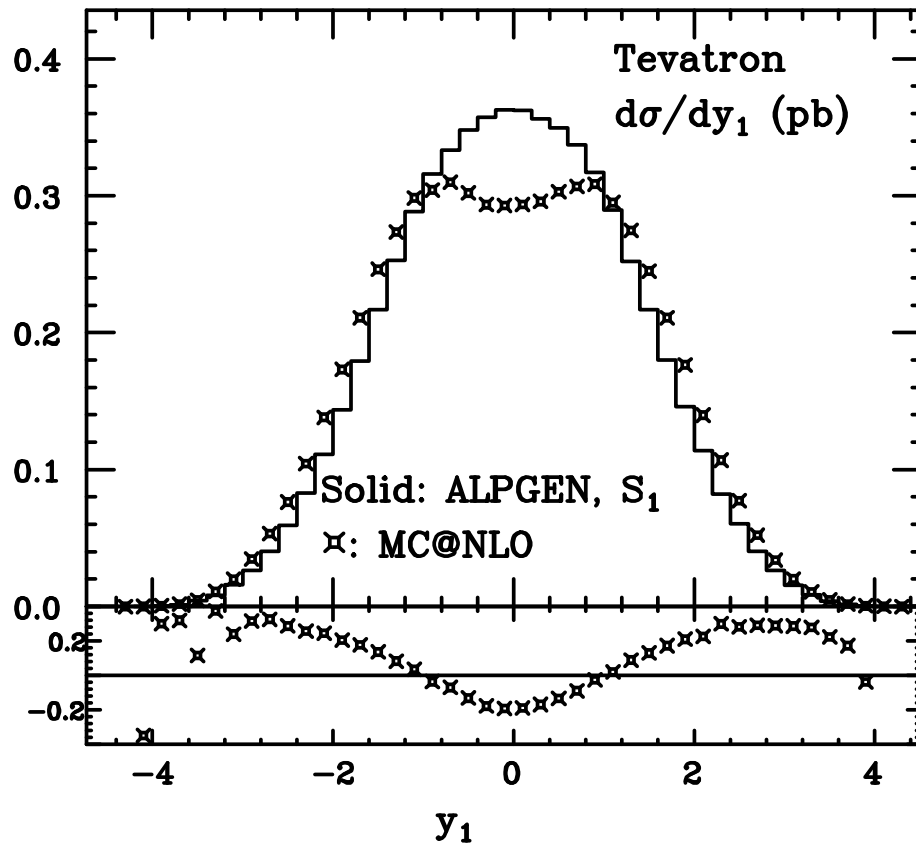
### ALPGEN

- Generation:  $P_{\min}^T = 30$  GeV,  $\Delta R = 0.7$
- Matching:  $E_{\min}^T = 30$  GeV,  $\Delta R = 0.7$

### Jet definitions

- **Tevatron**:  $E_{\min}^T = 15$  GeV,  $\Delta R = 0.4$ ,  $K$  factor = 1.45
- **LHC**:  $E_{\min}^T = 20$  GeV,  $\Delta R = 0.5$ ,  $K$  factor = 1.57

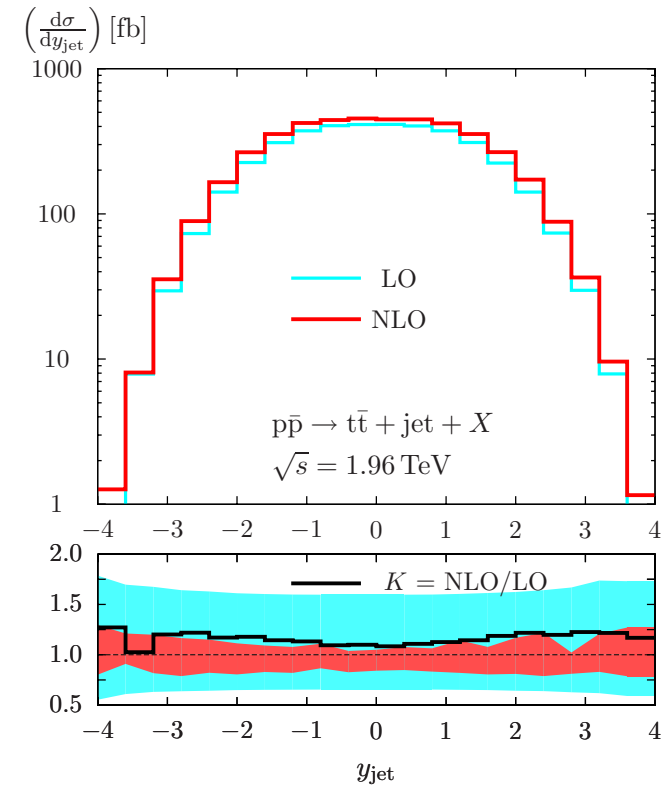
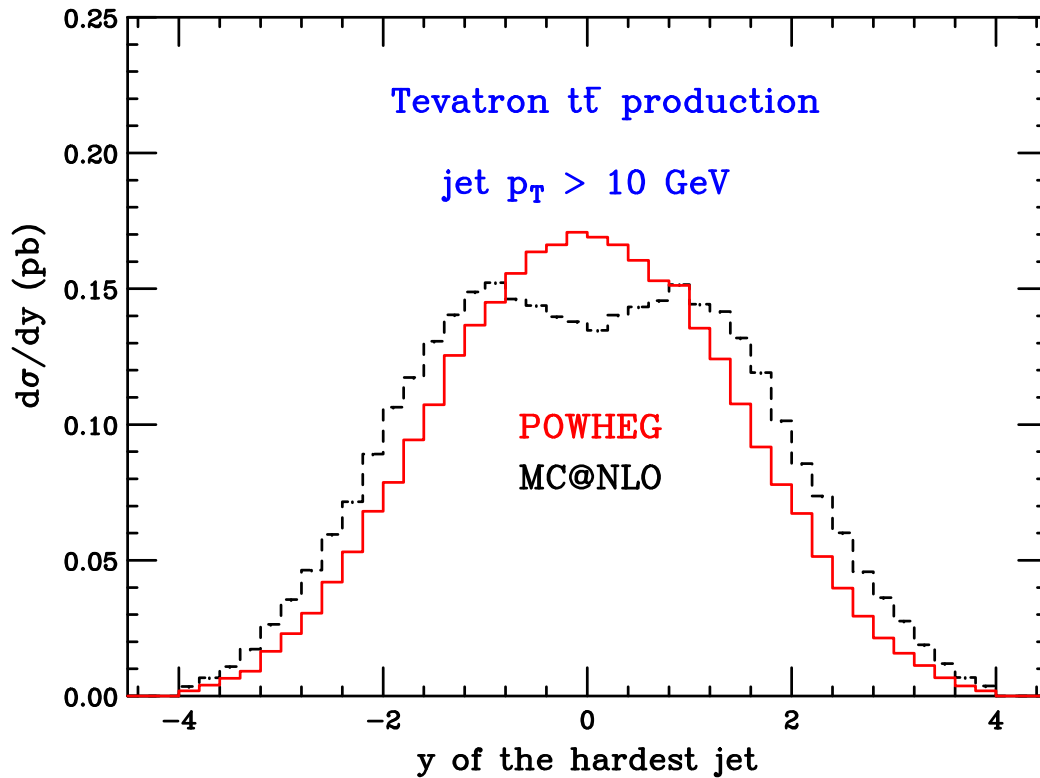
# ALPGEN vs MC@NLO: $t\bar{t} + 1$ jet



Rapidity  $y_1$  of the leading jet (highest  $p_T$ ).

Different shapes both at Tevatron and at the LHC

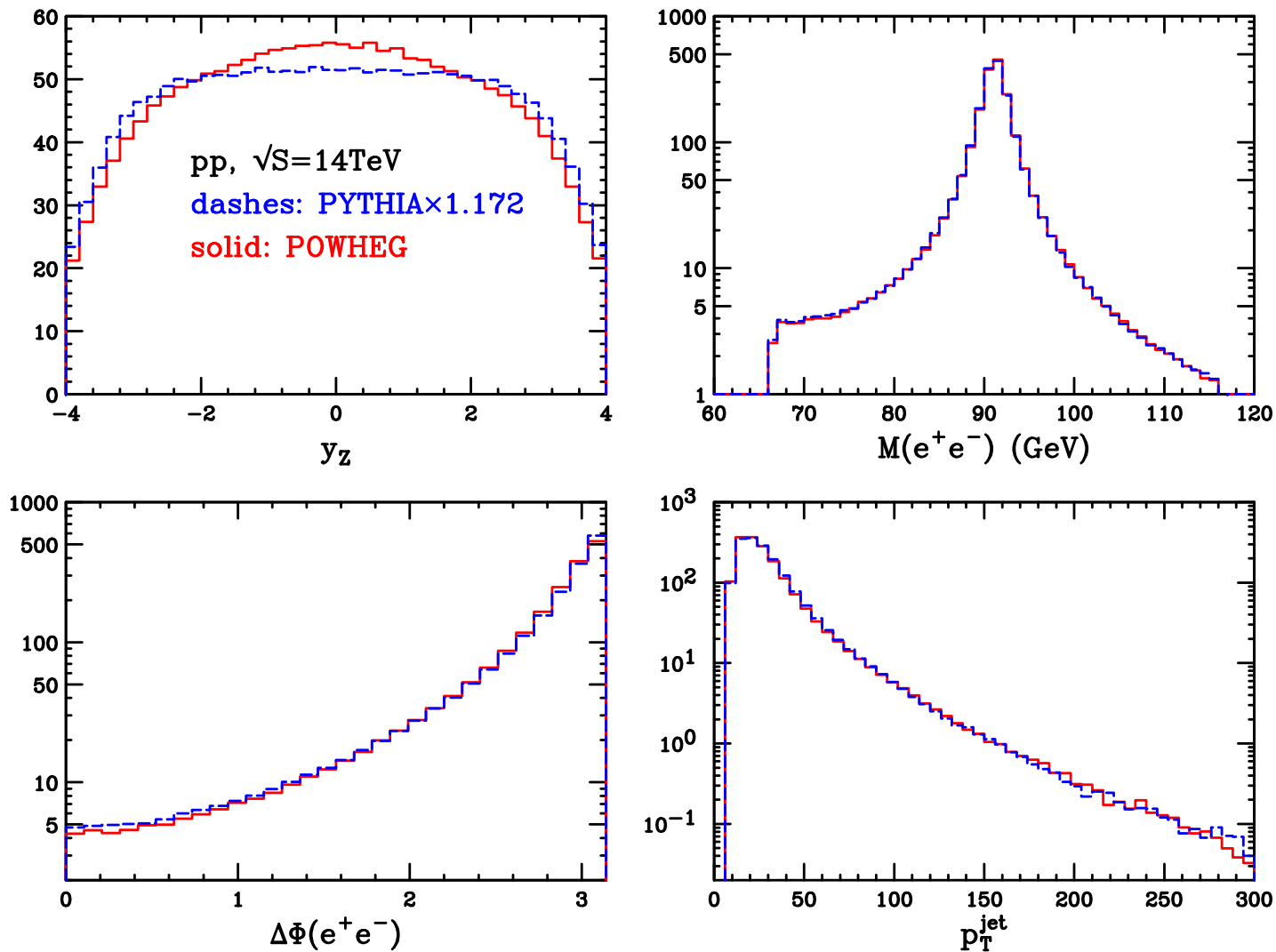
# POWHEG: rapidity of the leading jet



POWHEG's distribution as in ALPGEN: **no dip** present. The size of discrepancy can be attributed to different treatment of higher-order terms. Is this "feature" really there? The new  $pp \rightarrow t\bar{t} + \text{jet}$  at NLO [Dittmaier, Uwer, Weinzierl, hep-ph/0703120] shows **no dip** too.



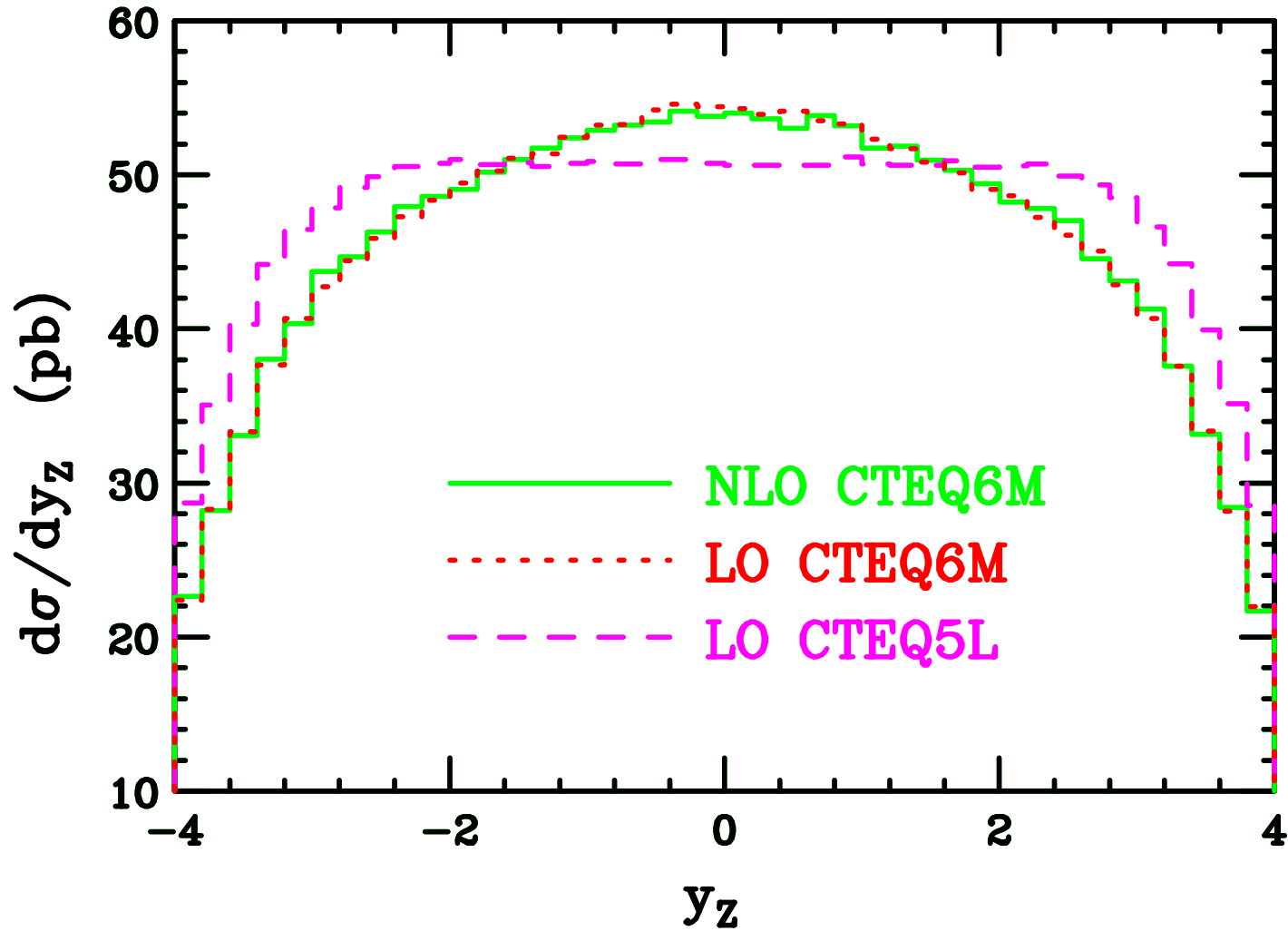
## Z production: POWHEG + PYTHIA vs PYTHIA



For vector-boson production, PYTHIA generates radiation in a way similar to POWHEG.

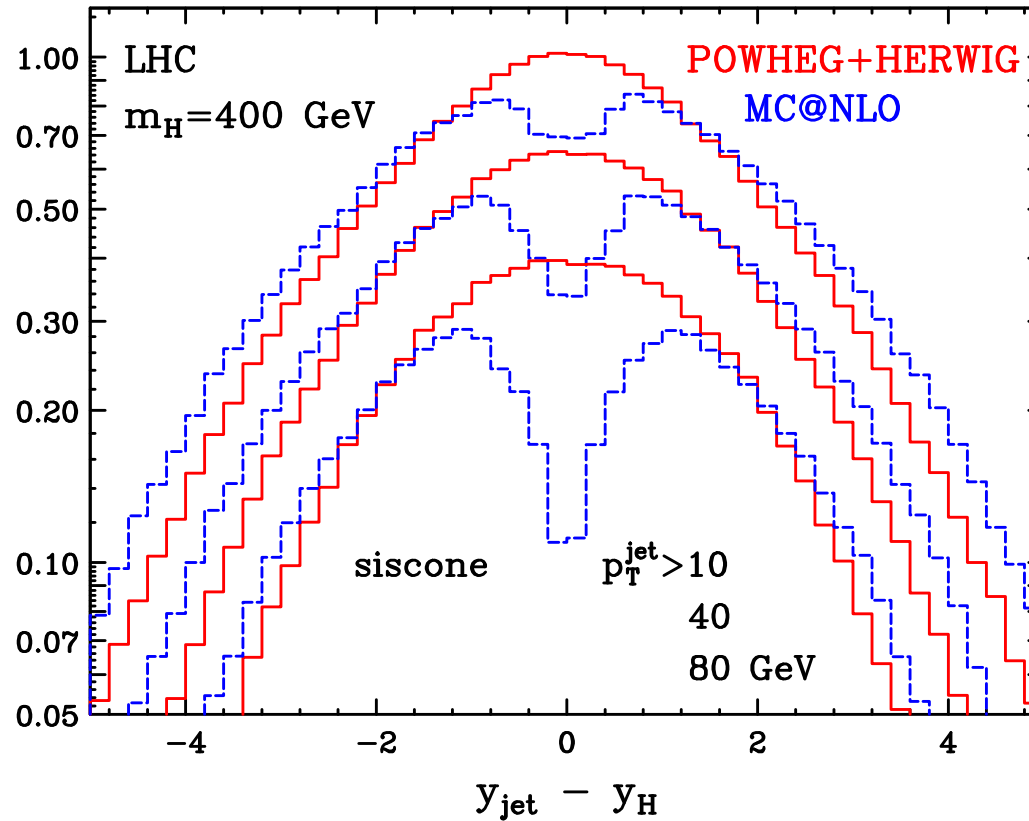
Differences ascribed to the use of LO parton densities.

## Z production: POWHEG + PYTHIA vs PYTHIA



Plots normalized to the NLO total cross section.

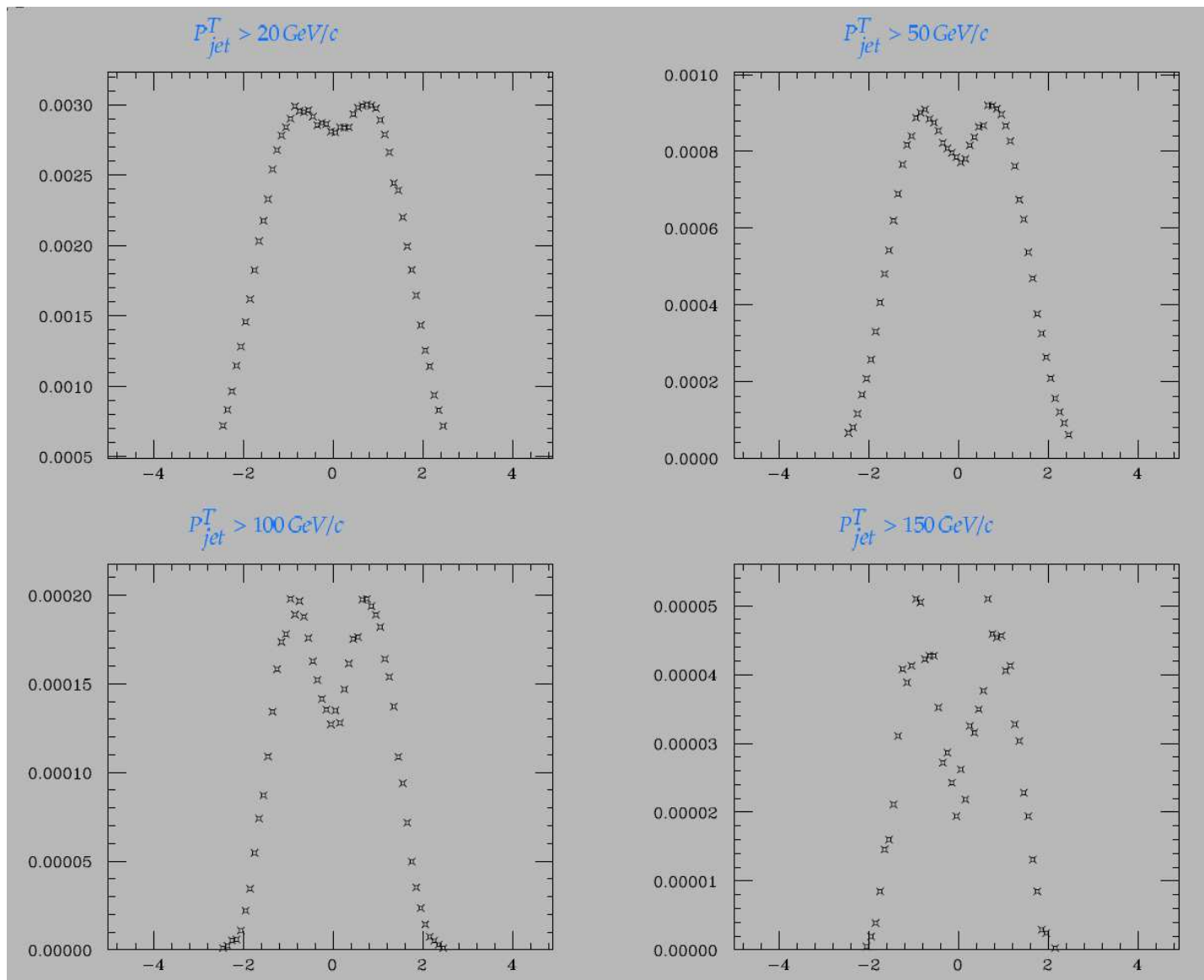
# Higgs boson production at the LHC



POWHEG + HERWIG vs MC@NLO

$m_H = 400$  GeV

# HERWIG: rapidity of the leading jet



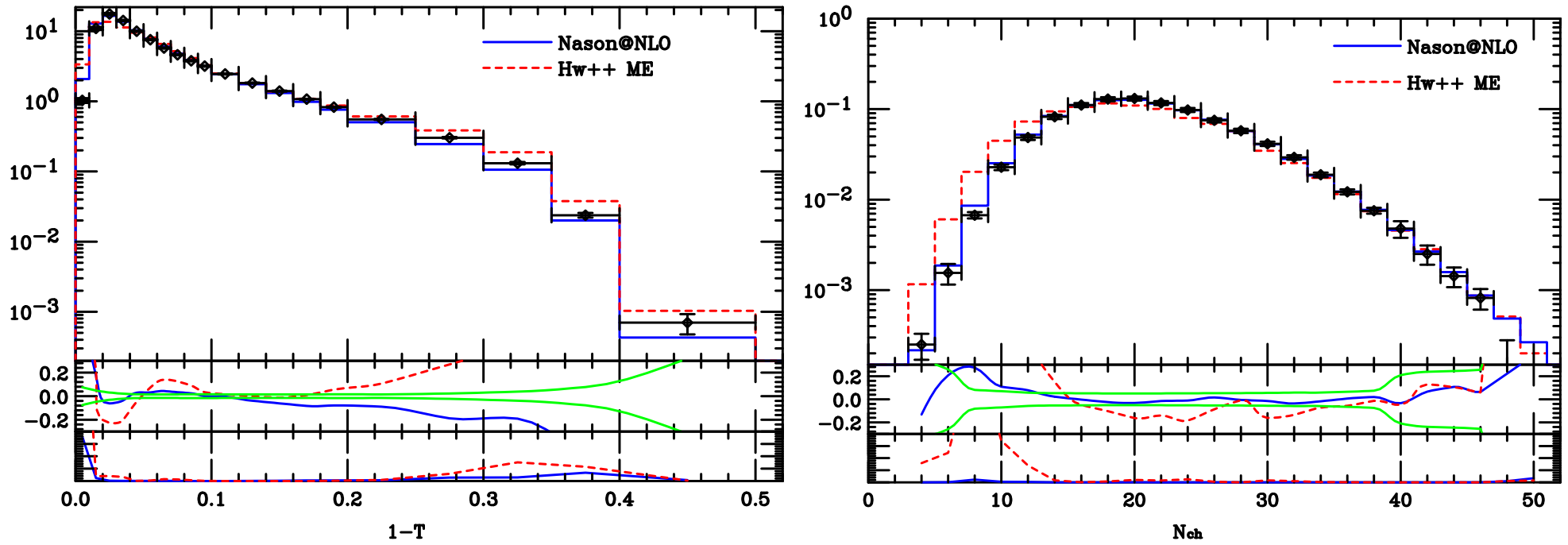
# POsitive-Weight Hardest Emission Generator

[Nason, hep-ph/0409146]

is a **method**, **NOT** (only) a set of programs

- ✓ generates events with **positive weights**
- ✓ can be interfaced to **any** Parton Shower Monte Carlo, **if** the **vetoed shower** is implemented, according to the **Les Houches Interface**.  
It is **independent** from **parton-shower** programs. POWHEG can be interfaced with both **PYTHIA** and **HERWIG**, or with your favorite showering program,
- ✓ As far as the **hardest emission** is concerned, POWHEG guarantees:
  - **NLO accuracy** of (integrated) shape variables
  - **Collinear, double-log, soft (large- $N_c$  limit) accuracy** of the Sudakov (in fact, corrections that exponentiates are obviously OK)
- ✓ As far as **subsequent (less hard) emissions**, the output has the accuracy of the SMC one is using.

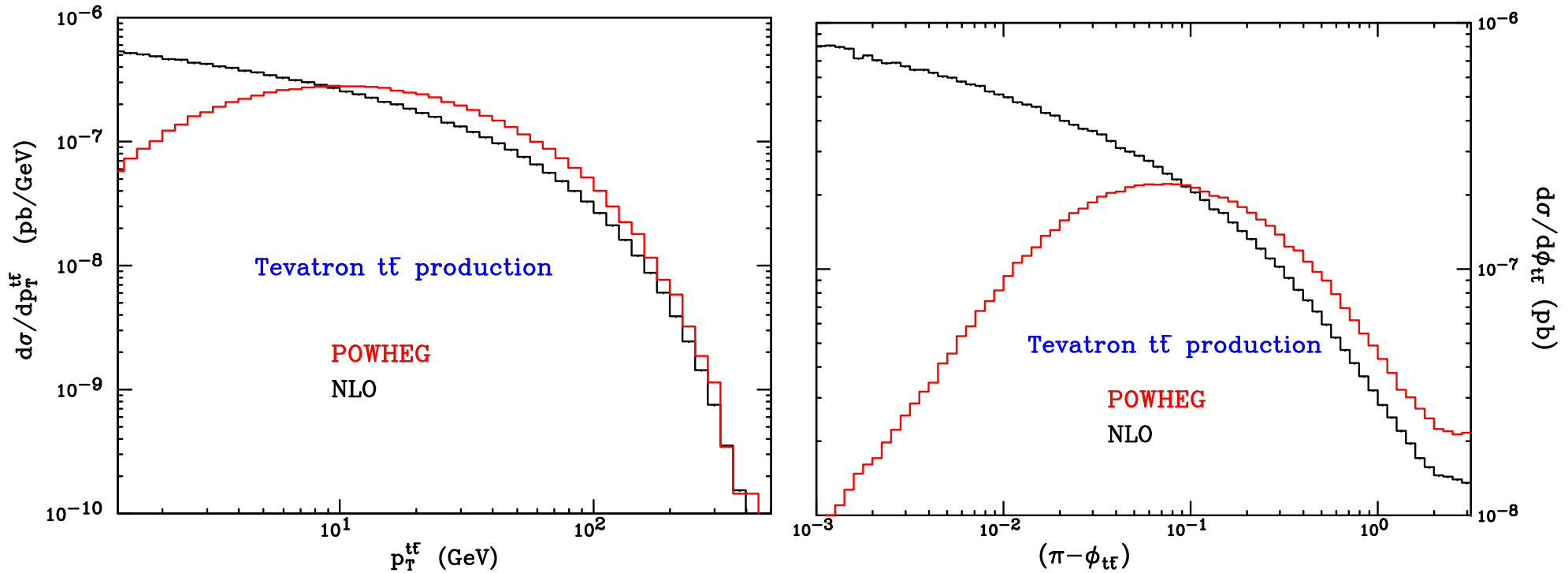
# $e^+e^- \rightarrow \text{hadrons}$



[Latunde-Dada, Gieseke and Webber, hep-ph/0612281]

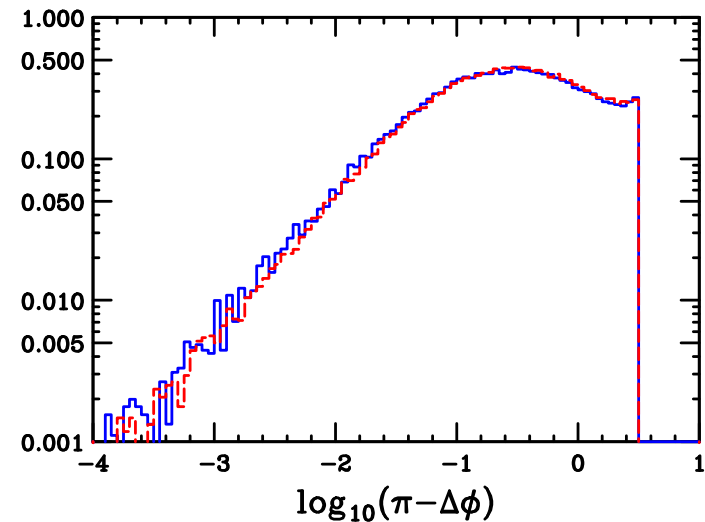
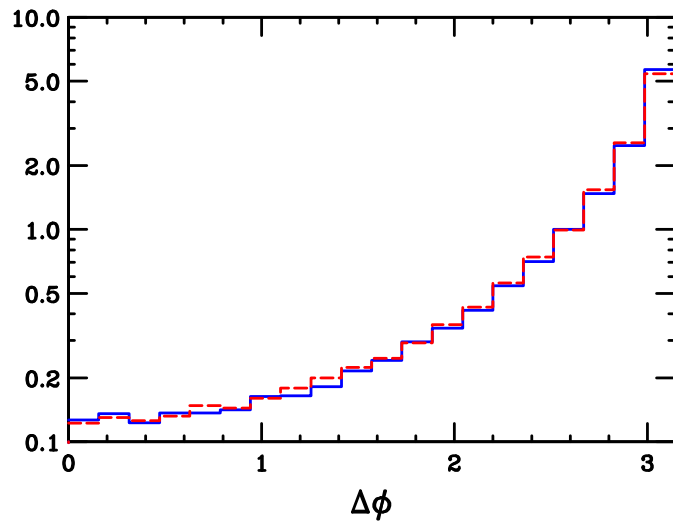
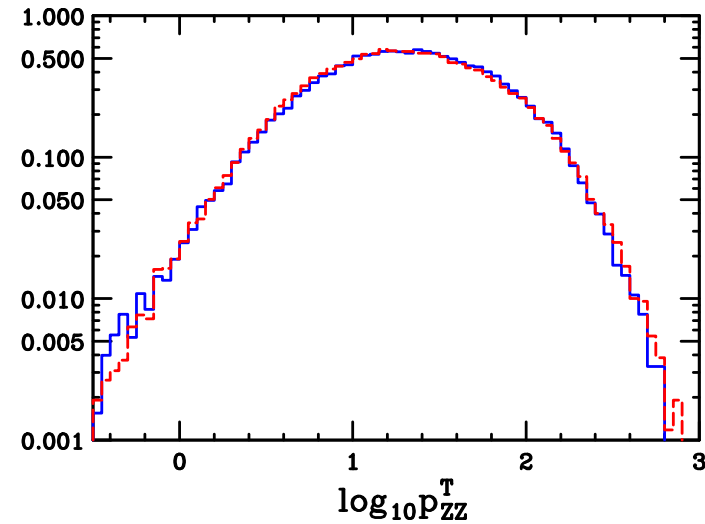
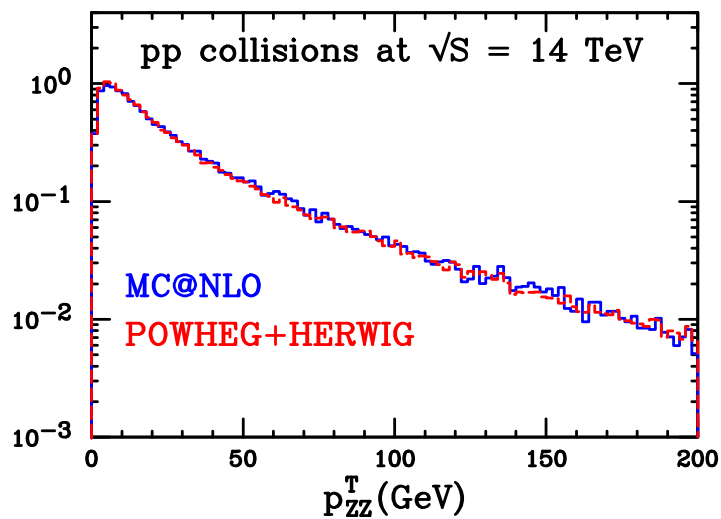
Fit to  $e^+e^-$  data: **better agreement** than in the standard matrix-element correction approach.

## $t\bar{t}$ production: POWHEG vs. NLO



- when  $p_T^{t\bar{t}} \rightarrow 0$ , POWHEG treats correctly the resummation of soft/collinear radiation
- when  $p_T^{t\bar{t}}$  becomes **large**, POWHEG approaches the NLO result
- when  $\Phi_{t\bar{t}} \rightarrow 0$ , the emitted **radiation** becomes **hard** and POWHEG goes to the NLO result.

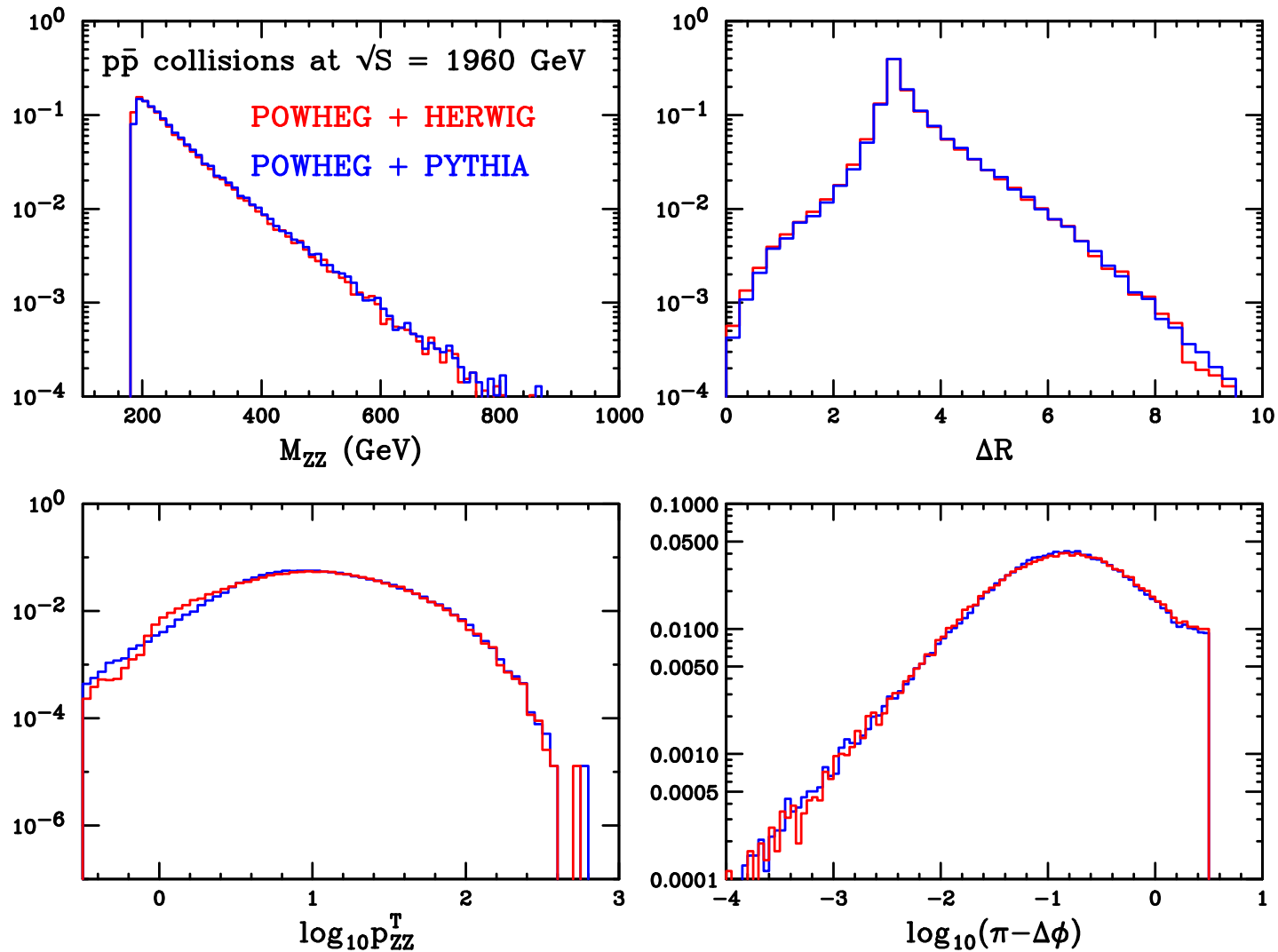
# ZZ production: POWHEG + HERWIG vs MC@NLO



No significant difference with MC@NLO [Nason and Ridolfi, hep-ph/0606275]



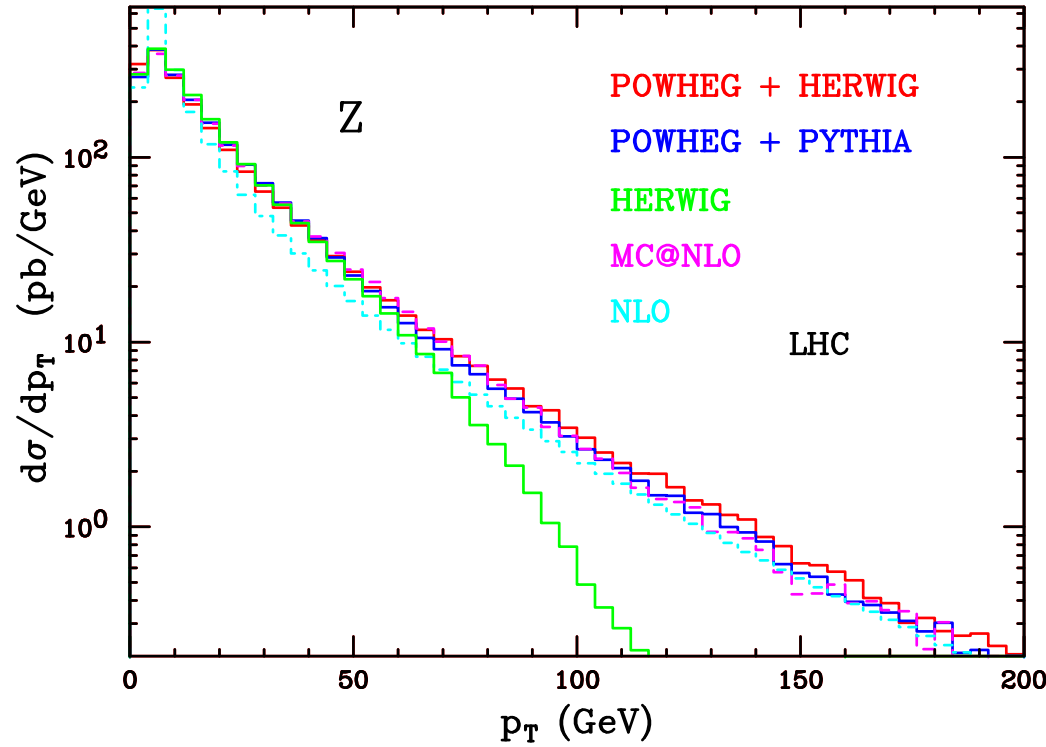
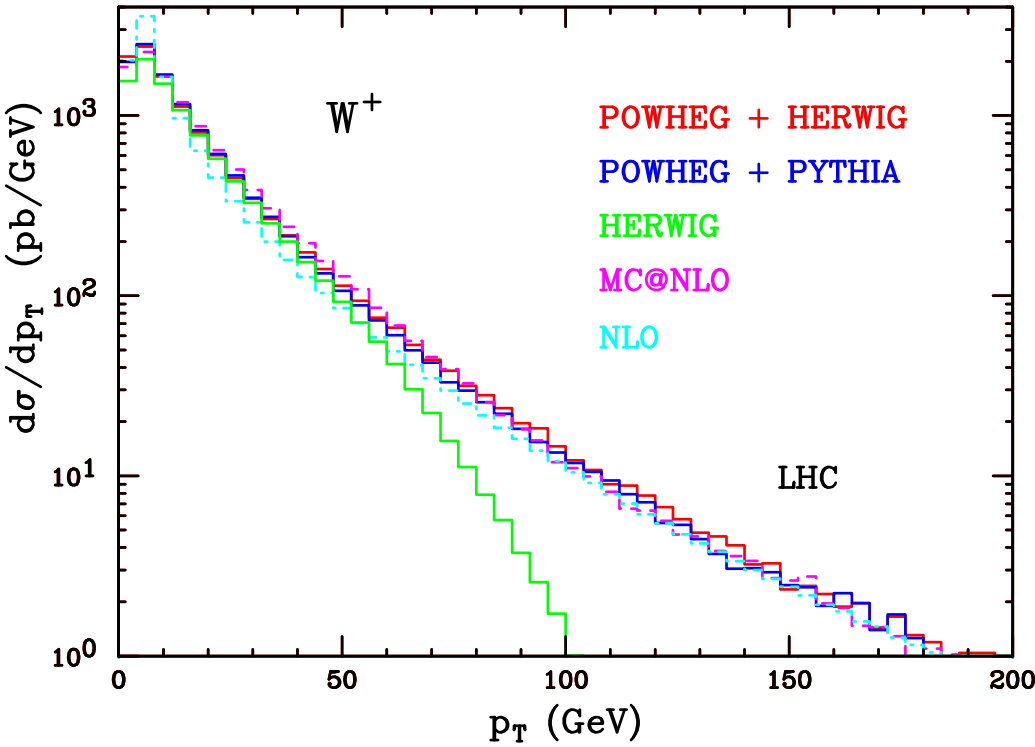
# POWHEG + HERWIG vs POWHEG + PYTHIA



**Agreement** between POWHEG + HERWIG and POWHEG + PYTHIA

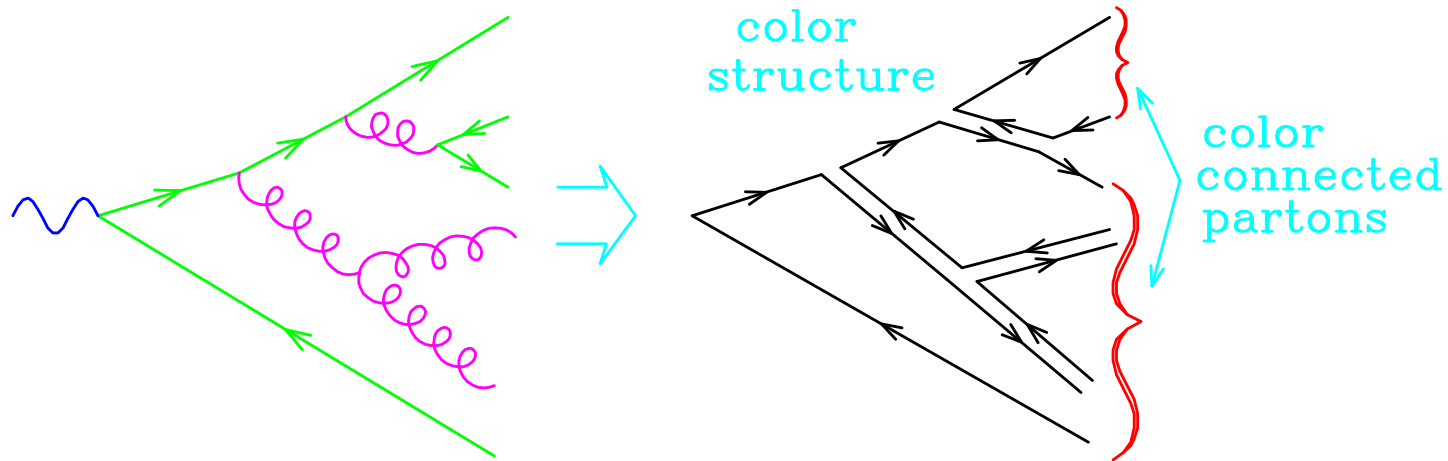
[Nason and Ridolfi, hep-ph/0606275]

# W/Z production



## Color and hadronization

Shower Monte Carlo programs assign **color labels** to partons. Only color connections are recorded (in **large  $N_c$  limit**). The initial color is assigned according to hard cross section.



Color assignments are used in the **hadronization model**.

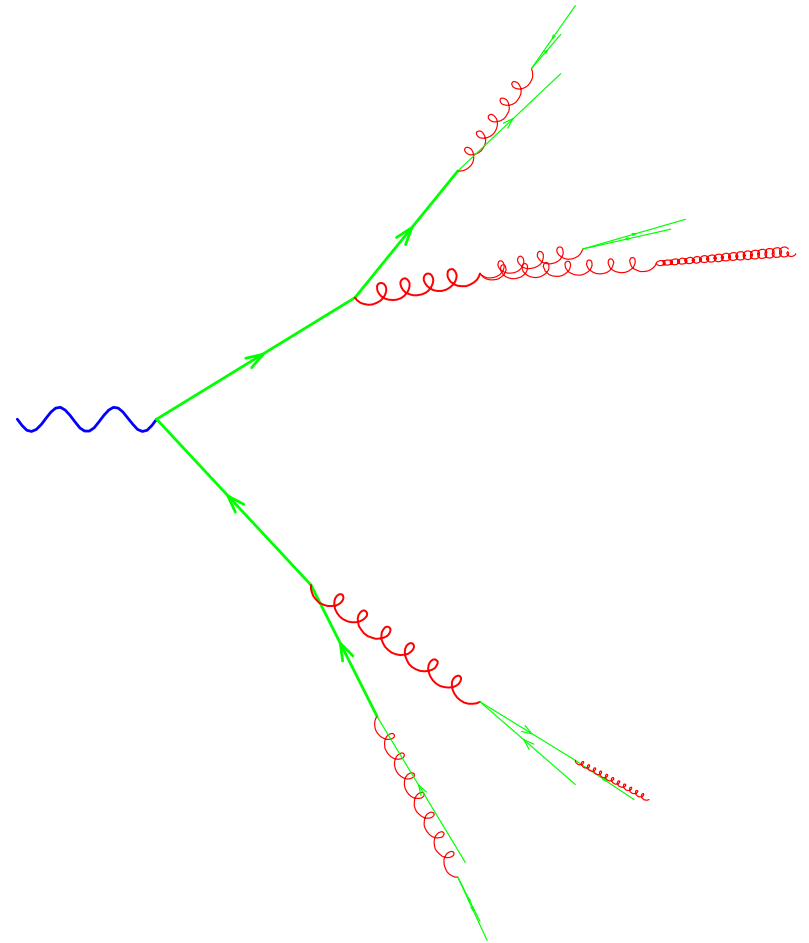
Most popular models: **Lund string model**, **cluster model**.

In all models, color singlet structures are formed out of color connected partons, and are decayed into hadrons, preserving energy and momentum.

## Typical dominant configuration at very high $Q^2$

$\gamma^* \rightarrow$  hadrons

- Besides  $q \rightarrow qg$ , also  $g \rightarrow gg, g \rightarrow q\bar{q}$  come into play.
- In the **typical configurations**, intermediate angles are of order of geometric average of upstream and downstream angles.
- Each angle is  $\mathcal{O}(\alpha_s)$  **smaller** than its upstream angle, and  $\mathcal{O}(\alpha_s)$  **bigger** than its downstream angle.
- As relative **momenta** become **smaller**,  $\alpha_s$  becomes **bigger**, and this picture breaks down.



## First branching

The probability of the **first branching** is independent of subsequent branchings because of Kinoshita-Lee-Nauenberg cancellation. It is given by

$$dP_{\text{first}} = \Delta_i(t, t') \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} P_{i,jk}(z) dz \frac{d\varphi}{2\pi}$$

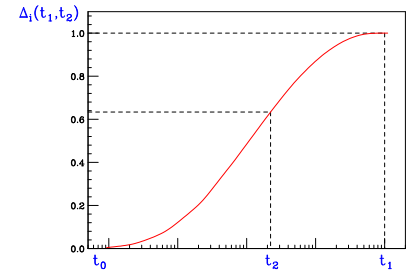
Upon integrating in  $z$  and  $\varphi$ , and summing over  $jk$ , we have

$$dP_{\text{first}} = \Delta_i(t, t') \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int \sum_{(jk)} P_{i,jk}(z) dz \frac{d\varphi}{2\pi} = d\Delta_i(t, t')$$

i.e. the distribution is **uniform** in the **Sudakov form factor**.

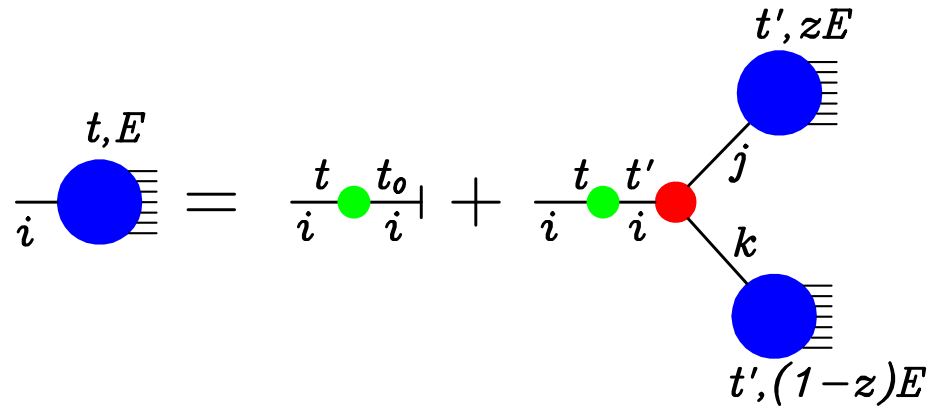
The integral over the whole  $t'$  range, from the minimum value  $t_0$  (**IR cutoff**) up to  $t$ , is given by

$$\int_{t_0}^t dP_{\text{first}} = \int_{t_0}^t d\Delta_i(t, t') = \Delta_i(t, t) - \Delta_i(t, t_0) = 1 - 0 = 1$$



as it should be for a **correct probabilistic interpretation**.

## Final recipe I

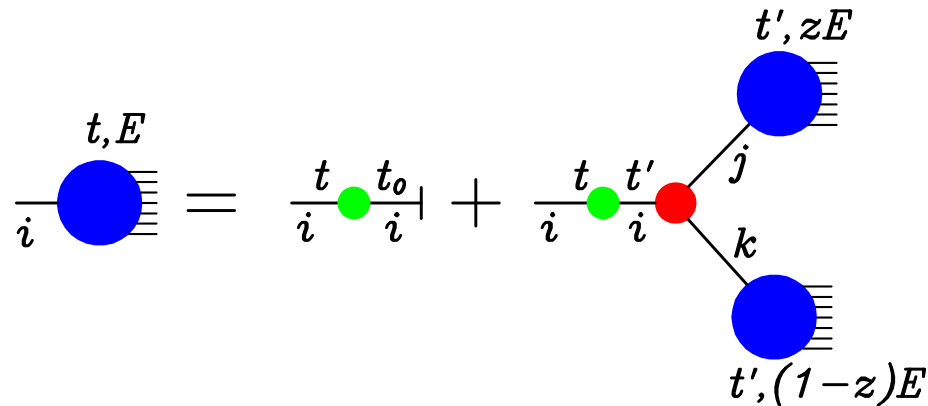


$$\mathcal{S}_i(t, E) = \Delta_i(t, t_0) \mathbb{1} + \sum_{(jk)} \int_{t_0}^t \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int dz \int \frac{d\varphi}{2\pi} \Delta_i(t, t') P_{i,jk}(z) \mathcal{S}_j(t', zE) \mathcal{S}_k(t', (1-z)E)$$

- consider all **tree graphs**.
- assign values to the radiation variables  $\Phi_r(t, z \text{ and } \varphi)$  to **each vertex**.
- at each vertex,  $i \rightarrow jk$ , include a factor

$$\frac{dt}{t} dz \frac{\alpha_S(t)}{2\pi} P_{i,jk}(z) \frac{d\varphi}{2\pi}$$

## Final recipe II



- include a factor  $\Delta_i(t_1, t_2)$  to each internal parton  $i$ , from hardness  $t_1$  to hardness  $t_2$ .

$$\Delta_i(t_1, t_2) = \exp \left[ - \sum_{(jk)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz P_{i,jk}(z) \int \frac{d\varphi}{2\pi} \right]$$

The weights  $\Delta_i(t_1, t_2)$  are called **Sudakov form factors**. They resum all the **dominant virtual corrections** to the tree graph (in the collinear approximation). Notice also that the inclusion of real and virtual corrections gives a net result of 1 (cancellation of collinear singularities in inclusive quantities).

- include a factor  $\Delta_i(t, t_0)$  on final lines ( $t_0 = \text{IR cutoff}$ )

## Actual implementation of the shower algorithm

We **start** from a given value of the ordering variable  $t$ . We want to generate the value  $t'$  for the **next emission**, according to the probability

$$dP_{\text{first}} = \Delta_i(t, t') \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int \sum_{(jk)} P_{i,jk}(z) dz \frac{d\varphi}{2\pi} = d\Delta_i(t, t')$$

Since this is an **exact differential** form, we proceed as in the case we want to generate a random variable  $x$  according to a distribution function  $f(x)$ , whose **indefinite** integral is known, starting from a uniform random variable  $r$

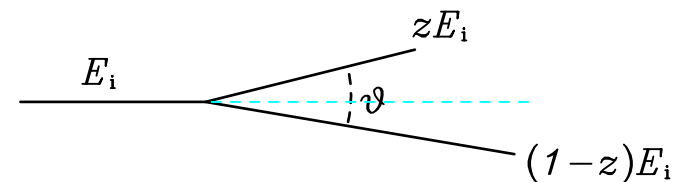
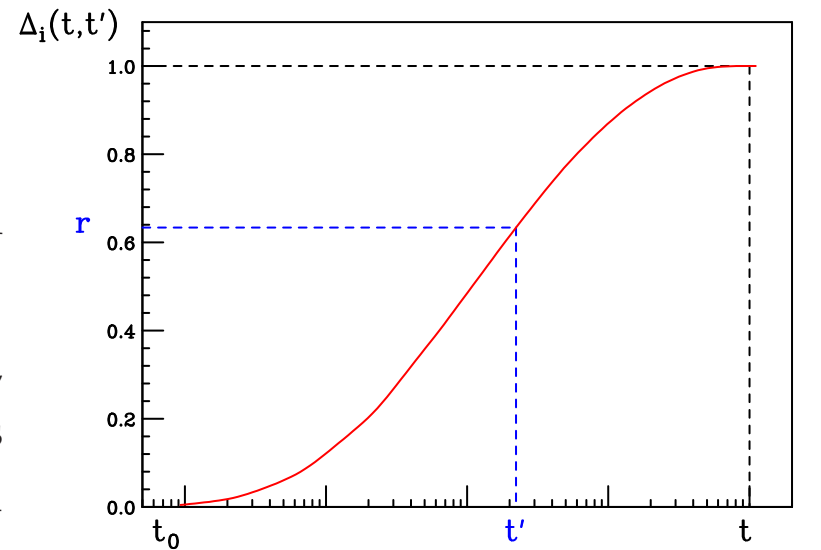
$$dP = f(X) dX = 1 dR \quad \text{where} \quad f(X) dX = dF(X)$$

$$\int_{x_{\min}}^x f(X) dX = F(x) = \int_0^r 1 dR = r \quad \implies \quad x = F^{-1}(r)$$



## Actual implementation of the shower algorithm

- ✓ generate a **hard process** configuration with a probability proportional to its parton-level cross section. Parton densities are evaluated at the typical “high” scale  $Q$  of the process
- ✓ for each **final-state colored parton**, generate a shower
  - set  $t = Q^2$
  - generate a uniform random number  $0 < r < 1$
  - solve the equation  $\Delta_i(t, t') = r$  for  $t'$
  - if  $t' < t_0$  stop here (final state line). Begin hadronization
  - if  $t' > t_0$ , generate  $z, jk$  with probability  $P_{i,jk}(z)$ , and  $0 < \varphi < 2\pi$  uniformly. Assign energies  $E_j = zE_i$  and  $E_k = (1 - z)E_i$  to partons  $j$  and  $k$ . The angle  $\theta$  between their momenta is fixed by  $t'$  and with  $\varphi$  their direction is completely specified
  - restart shower from each of the two branched parton  $j$  and  $k$ , setting the ordering parameter  $t = t'$ .



## Shower algorithm

- ✓ for each **initial-state colored parton**, generate a shower in a similar way, but using a “trick”: the **backward evolution** (Sjöstrand)

$$\frac{f_i^h(t', x) \Delta(t, t')}{f_i^h(t, x)} = r$$

where  $f_i^h$  is the parton density for the colliding hadron  $h$ , where parton  $i$  carries a momentum fraction  $x = E_i/E_h$

Some **momentum reshuffling** is needed in order to preserve local (at each vertex) and global momentum conservation

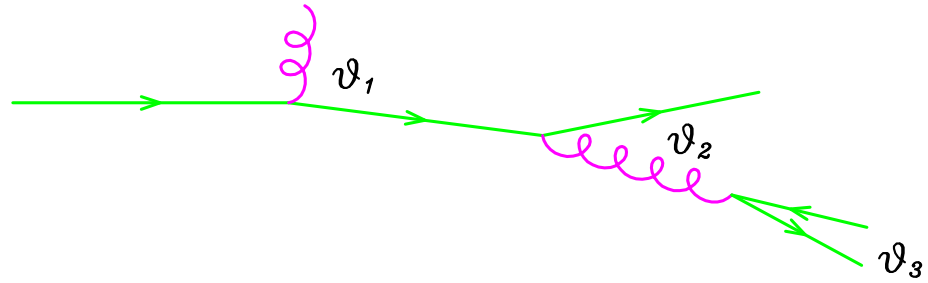
# Angular ordering

Mueller (1981) showed that **angular ordering** is the correct choice

$$\frac{d\theta}{\theta} \frac{\alpha_s(p_T^2)}{2\pi} P(z) dz$$

$$\theta_1 > \theta_2 > \theta_3 \dots$$

$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$  for a correct treatment of charge renormalization in **soft region** ( $p_T^2$  equals to the maximum virtuality of the gluon line).

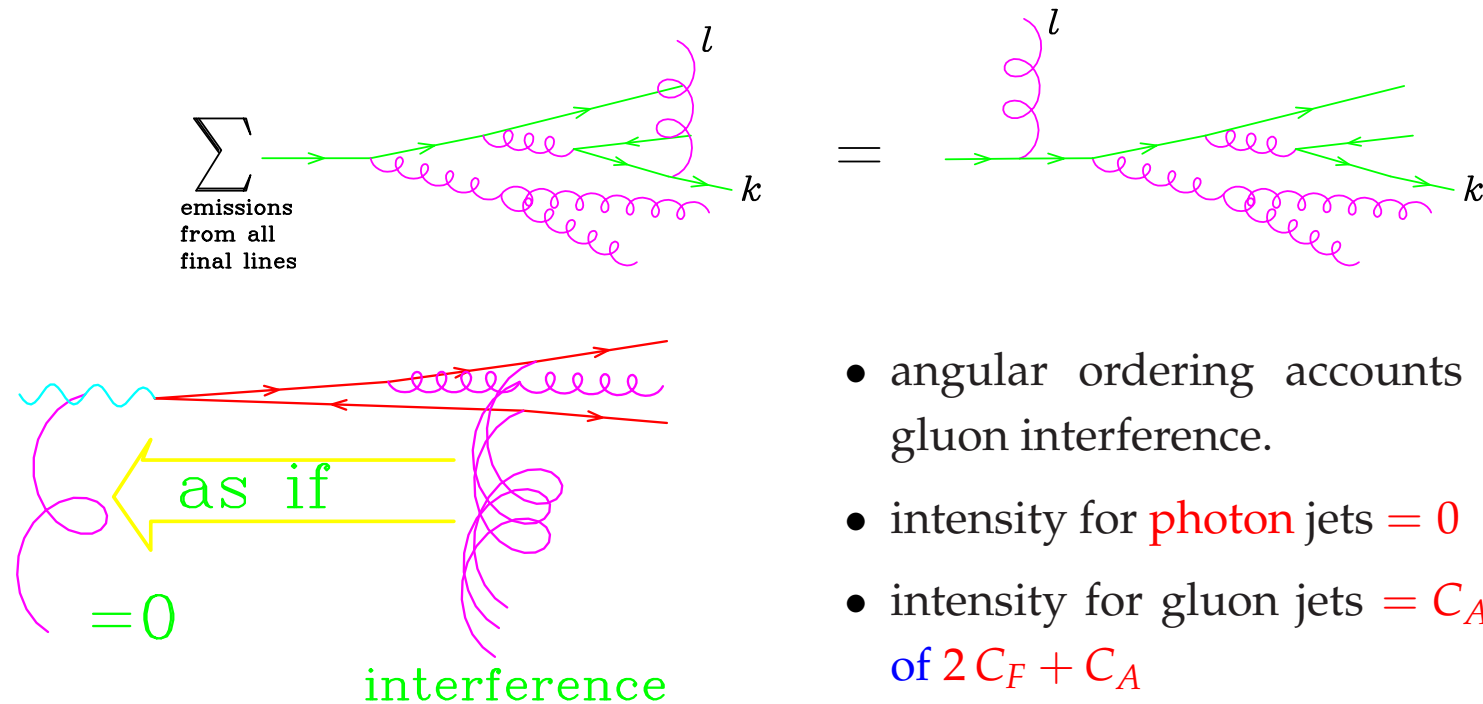
$$\Delta_i(t, t') = \exp \left[ - \int_{t'}^t \frac{dt}{t} \int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz \frac{\alpha_s(p_T^2)}{2\pi} \sum_{(jk)} P_{i,jk}(z) \right]$$

$$\approx \exp \left\{ - \frac{c_i}{4\pi b_0} \left[ \log \frac{t}{\Lambda^2} \log \frac{\log \frac{t}{\Lambda^2}}{\log \frac{t_0}{\Lambda^2}} - \log \frac{t}{t_0} \right]_{t'}^t \right\} \quad (c_q = C_F, c_g = 2C_A)$$

Sudakov dumping stronger than any power of  $t$ .

# Color coherence

Soft gluons emitted at **large angles** from final-state partons add **coherently**



- angular ordering accounts for soft gluon interference.
- intensity for **photon** jets = 0
- intensity for gluon jets =  $C_A$  instead of  $2C_F + C_A$

In angular-ordered shower Monte Carlo, **large-angle soft emission** is generated **first**.

**Hardest emission**, i.e. highest  $p_T = E z(1 - z) \theta$ , in general, **happens later**.

## Some available codes

- **COJETS** Odorico (1984)
- **ISAJET** Paige+Protopopescu (1986)
- **FIELDJET** Field (1986)
- **JETSET** Sjöstrand (1986)
- **PYTHIA** Bengtsson+Sjöstrand (1987), Sjöstrand+Skands (2004)
- **HERWIG** Marchesini+Webber (1988),  
Marchesini+Webber+Abbiendi+Knowles+Seymour+Stanco (1992)
- **ARIADNE** Lönnblad (1992)
- **SHERPA** Gleisberg+Höche+Krauss+Schälicke+Schumann+Winter (2004)

## Available accuracy

	collinear	soft-collinear	soft large- $N_c$	soft
PYTHIA	leading	partial	no	no
HERWIG	leading	leading	no	no
ARIADNE	partial	partial	leading	no
PYTHIA6.4	partial	partial	leading	no
SHERPA	leading	partial	no	no

One can realistically aim at

leading collinear, leading double log, leading soft in large- $N_c$  limit

Soft effects for finite  $N_c$  require matrix exponentiation in the Sudakov form factor.

## New developments

- Interfacing **Matrix Elements** (ME) generators with **Parton Showers** : **CKKW** matching [Catani, Krauss, Küen, Webber], **MLM** matching [Mangano]
- Interfacing **NLO** calculations with **Parton Showers**: **MC@NLO** [Frixione, Webber], **POWHEG** [Nason]

Several other approaches have appeared

- $e^+e^- \rightarrow 3$  **partons** [Kramer, Mrenna, Soper]
- Shower by **antenna factorization** [Giele, Kosower, Skands]
- Shower by **Catani-Seymour dipole factorization** [Schumann, Krauss]
- Shower with **quantum interference** [Nagy, Soper]
- Shower by **Soft Collinear Effective Theory** [Bauer, Schwartz]
- Shower from the **dipole formalism** [Dinsdale, Ternick, Weinzierl]

Up to now, complete results for hadron colliders only from MC@NLO and POWHEG.

## NLO + Parton Shower

LO-ME good for **shapes**. Uncertain absolute normalization

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu) (1 - b_0 \alpha_s(\mu) \log(4))^n \approx \alpha_s^n(\mu) (1 - n \alpha_s(\mu))$$

For  $\mu = 100 \text{ GeV}$ ,  $\alpha_s = 0.12$ , normalization uncertainty:

$W + 1J$	$W + 2J$	$W + 3J$
$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

To improve on this, we need to go to NLO

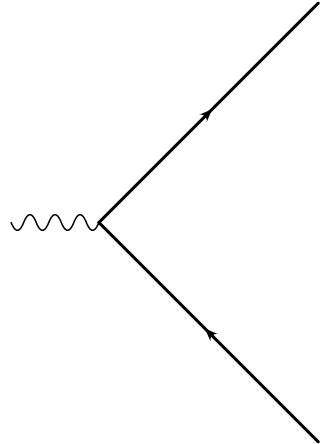
- **Positive experience** with **NLO calculations** at LEP, HERA and Tevatron
- NLO results are cumbersome to compute: typically made up of an  $n$ -body (Born + virtual + soft and collinear remnants) and  $(n + 1)$ -body (real emission) terms, both divergent (finite only when summed up).
- **Merging NLO with shower** is a natural extension of present approaches.



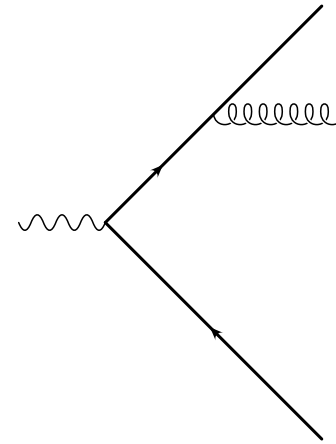
## NLO + Parton Shower

The main problem in **merging** a **NLO** result and a **Parton Shower** is **not to double-count** radiation: the shower might produce some radiation **already present** at the NLO level.

**LO:**



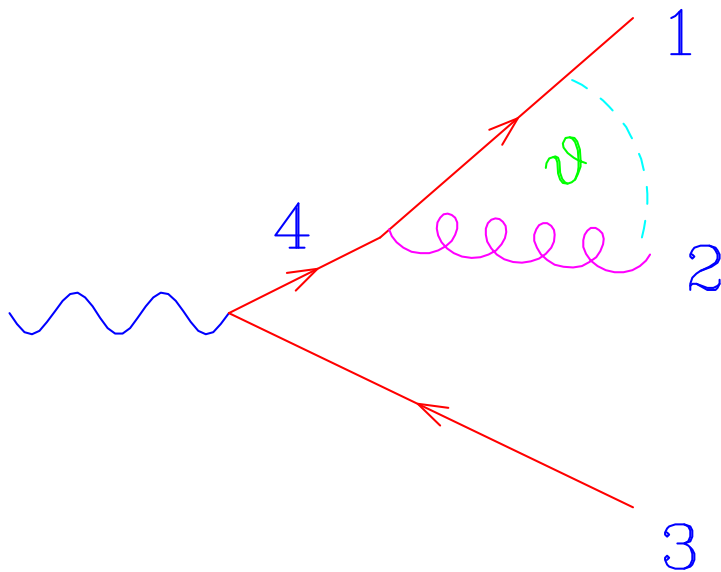
**NLO:**



## POWHEG: how it works

1. **POWHEG**, **PO**sitive **W**eight **H**ardest **E**mission **G**enerator, [Nason, hep-ph/0409146], generates **first** a partonic event with just **one single emission**, at **NLO level**, and with the **correct probability** in order not to have double-counting coming from (subsequent) radiation. The  $p_T$  of the produced radiation works as an **upper cutoff** for the  $p_T$ 's of the entire subsequent shower.
2. The event is written on a file using the standard **Les Houches Interface** and is processed by the Parton Shower program (HERWIG, PYTHIA...), that showers the event, but with a  $p_T$  less than the  $p_T$  generated by POWHEG ( **$p_T$  veto**).
  - if the shower is **ordered in  $p_T$**  (for example PYTHIA), nothing else needs to be done
  - if the shower is **ordered in angle** (for example HERWIG), we need to generate correctly soft radiation at large angle.
    - pair up the partons that are nearest in  $p_T$
    - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (**truncated shower**)
    - generate all subsequent **vetoed showers**

## Example of truncated shower: $e^+e^-$



- nearby partons: 1 and 2
- truncated shower: 1 and 2 pair, from  $\theta$  up to a maximum angle. The truncated shower reintroduces coherent soft radiation from 1 and 2 at angles larger than  $\theta$  (angular-ordered shower Monte Carlo programs generate those earlier).
- 1 and 2 shower from  $\theta$  to cutoff
- 3 showers from maximum to cutoff

Truncated showers not yet implemented.

No evidence of effects from their absence in  $ZZ$  and  $e^+e^-$  production. Might be some effects in heavy-quark production.

## NLO calculations

We can always parametrize the  $(n + 1)$ -body phase space  $\Phi_{n+1}$  in terms of the Born phase space  $\Phi_n$  and three radiation variables  $\Phi_r$ :  $\Phi_{n+1} = \{\Phi_n, \Phi_r\}$

$$\langle O \rangle = \int O d\sigma = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V_b(\Phi_n)] + \int d\Phi_n d\Phi_r O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r)$$

where  $V_b$  is the (divergent) virtual differential cross section. The virtual and real-radiation integrals are separate divergent. Their sum is finite (for any infra-red safe observable).

A typical subtraction method re-organize the integrals in the form

$$\begin{aligned} \langle O \rangle &= \int d\Phi_n O(\Phi_n) \left[ B(\Phi_n) + V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \right] \\ &+ \underbrace{\int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)]}_{\text{finite}} \end{aligned}$$

Defining

$$V(\Phi_n) = V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \quad \Leftarrow \text{finite}$$

we have

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] + \int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)]$$

## NLO in SMC

Shower Monte Carlo (SMC) cross section for first emission ( $d\Phi_r = dt dz d\varphi$ )

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int_{t_0} \frac{dt}{t} dz d\varphi O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \right\}$$

with

$$\Delta_t = \exp \left[ - \int_t \frac{dt'}{t'} dz' d\varphi' \frac{\alpha_s}{2\pi} P(z') \right]$$

The expansion at order  $\alpha_s$  gives the  $\text{NLO}_{\text{SMC}}$

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) + \int_{t_0} \frac{dt}{t} dz d\varphi [O(\Phi_n, \Phi_r) - O(\Phi_n)] \frac{\alpha_s}{2\pi} P(z) \right\}$$

This is the **inexact** NLO correction implemented by the SMC

How do we reach exact NLO accuracy?

## Towards NLO accuracy

$$\begin{aligned}
 \langle O \rangle &= \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] \\
 &+ \int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)] \\
 &= \int d\Phi_n O(\Phi_n) \left\{ B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] \right\} \\
 &+ \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)]
 \end{aligned}$$

Define

$$\begin{aligned}
 \bar{B}(\Phi_n) &= B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] \\
 \langle O \rangle &= \int d\Phi_n O(\Phi_n) \bar{B}(\Phi_n) + \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)]
 \end{aligned}$$

In NLO<sub>SMC</sub>, it was

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) B(\Phi_n) + \int d\Phi_n d\Phi_r B(\Phi_n) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} [O(\Phi_n, \Phi_r) - O(\Phi_n)]$$

## POWHEG

$$\text{NLO}_{\text{SMC}} \leftrightarrow \text{NLO} : \quad B(\Phi_n) \leftrightarrow \bar{B}(\Phi_n) \quad B(\Phi_n) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \leftrightarrow R(\Phi_n, \Phi_r)$$

All-order emission probability in SMC

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int_{t_0} d\Phi_r O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \right\}$$

with

$$\Delta_t = \exp \left[ - \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right]$$

All order emission probability in POWHEG

$$\langle O \rangle = \int d\Phi_n \bar{B}(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int d\Phi_r O(\Phi_n, \Phi_r) \Delta_t \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \right\}$$

$$\Delta_t = \exp \left[ - \int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(t' - t) \right]$$

with  $t = k_T(\Phi_n, \Phi_r)$  and  $\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$

**POSITIVE** if  $\bar{B}$  is positive (i.e. NLO < LO).

## Accuracy of the Sudakov form factor

POWHEG's Sudakov form factor has the form (with  $c \approx 1$ )

$$\Delta_t = \exp \left[ - \int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(c k_T^2)}{\pi} \left\{ A \log \frac{E^2}{k_T^2} + B \right\} \right]$$

The next-to-leading log (NLL) Sudakov form factor has the form

$$\Delta_t^{\text{NLL}} = \exp \left[ - \int_t^{Q^2} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(k_T^2)}{\pi} \left\{ \left( A_1 + A_2 \frac{\alpha_s(k_T^2)}{\pi} \right) \log \frac{E^2}{k_T^2} + B \right\} \right]$$

provided the color structure of the process is sufficiently simple ( $\leq 3$  colored legs). Can use this to fix  $c$  in POWHEG's Sudakov form factor as suggested in Catani, Webber, Marchesini, (1991). HERWIG uses this.

For colored legs  $\geq 4$ , exponentiation only holds at leading-log (LL) or LL + NLL in the large- $N_c$  limit (i.e. planar color structure of Feynman diagrams)

POWHEG's Sudakov form factor is **always LL accurate**. NLL accurate for  $\leq 3$  colored legs, NLL accurate in leading  $N_c$  in all cases.



## Mathematical tricks

- ✓ To **generate** the underlying **Born variables** ( $\Phi_n$ ), distributed according to  $\overline{B}(\Phi_n)$ , one uses programs like BASES/SPRING or MINT, that, after a **single integration**, can generate points distributed according to the **integrand function**.
- ✓ Use the **veto technique** and the **highest- $p_T$  bid** procedure, to generate the **radiation variables**, distributed according to  $d\Delta_i(t, t')$ .

These tricks are well known to Monte Carlo experts.

We have collected a few of them in the appendixes of our paper [Frixione, Nason and C.O., arXiv:0709.2092 [hep-ph]].

## POsitive-Weight Hardest Emission Generator

- ✓ it is **independent** from **parton-shower** programs. POWHEG can be interfaced with both **PYTHIA** and **HERWIG**, or with your favorite showering program, **if** the **vetoed shower** is implemented, according to the **Les Houches Interface**.
- ✓ it can use **existing NLO results**
- ✓ it generates events with **positive weights**
- ✓ As far as the **hardest emission** is concerned, POWHEG guarantees:
  - **NLO accuracy** on **integrated quantities**
  - **collinear, double-log (soft-collinear), large- $N_c$ -soft single-log** of the Sudakov (in fact, corrections that exponentiates are obviously OK)
- ✓ As far as **subsequent** (less hard) **emissions**, the output has the accuracy of the SMC one is using.
- ✗ **no truncated shower** implemented up to now. But this is a problem that affects all the angular-ordered SMC when the shower is initiated by a relatively complex matrix element.

## Existing implementations

The POWHEG method has already been **successfully** used in

- $pp \rightarrow ZZ$  [Nason and Ridolfi, hep-ph/0606275]
- $e^+e^- \rightarrow \text{hadrons}$  [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]  
 $e^+e^- \rightarrow t\bar{t}$  with top decay [Latunde-Dada, arXiv:0806.4560]
- $pp \rightarrow Q\bar{Q}$  ( $c\bar{c}$ ,  $b\bar{b}$ ,  $t\bar{t}$ ) with **spin correlations** [Frixione, Nason and Ridolfi, arXiv:0707.3088].
- $pp \rightarrow W/Z$  with **spin correlations** [Alioli, Nason, C.O. and Re, arXiv:0805.4802; Hamilton, Richardson and Tully, arXiv:0806.0290].
- $pp \rightarrow H$  [Alioli, Nason, C.O. and Re, **arXiv:0812.0578**]

**All POWHEG** implementations for hadronic colliders have been interfaced to both **PYTHIA** and **HERWIG**.

## To appear soon

- $pp \rightarrow H$  [Hamilton, Richardson and Tully, HERWIG++ group]
- single-top production [Alioli, Nason, C.O. and Re]
- $pp \rightarrow W/Z + 1 \text{ jet}$  [Alioli, Nason, C.O. and Re]

We are working now on a **general framework** for the implementation of **any NLO process** into the POWHEG formalism.

Given the Born, real and virtual amplitudes, combine them **automatically** to produce POWHEG events.

**Truncated showers** have been studied in  $e^+e^- \rightarrow \text{hadrons}$  [Latunde-Dada, Gieseke and Webber] and are included in the **HERWIG++** framework [Bähr, Gieseke, Gigg, Grellscheid, Hamilton, Plätzer, Richardson, Seymour and Tully, [arXiv:0812.0529](#)]

## From NLO to POWHEG

POWHEG is a **method**, **NOT** (only) a set of programs!

POWHEG is fully general and can be applied to **any NLO subtraction framework**.

We have provided any user with **all the formulae and ingredients** to implement an **existing NLO** calculation in the **POWHEG formalism** [Frixione, Nason and C.O., arXiv:0709.2092 [hep-ph]].

We have looked in detail at POWHEG in two subtraction schemes:

- the **Frixione, Kunszt** and **Signer** scheme
- the **Catani** and **Seymour** scheme.

We have discussed, in a pedagogical way, two examples:

- $e^+e^- \rightarrow q\bar{q}$
- $q\bar{q} \rightarrow V$

The fortran implementation of the POWHEG code for these two processes (and **all the others**) can be found at

<http://moby.mib.infn.it/~nason/POWHEG>

## Conclusions

- ✓ POWHEG is a **viable method** for interfacing NLO and Shower Monte Carlo programs
- ✓ It is **easy to implement** and does **not require** new NLO computations
- ✓ **No commitment** to a **specific Shower Monte Carlo** implementation is required
- ✓ It outputs **positive, unweighted events**, as in traditional Shower Monte Carlo programs
- ✓ **Several processes** already **available**. More to come
- ✓ Anybody can work on it!

POWHEG is a **method**, **not** (only) a set of programs!

- ✓ We have collected and published material to make it easy for others to implement POWHEG with their NLO calculations

<http://moby.mib.infn.it/~nason/POWHEG>

- ✓ A **general framework** for implementing arbitrary processes is **work in progress**