SUSY conference 2010

Minimal Supersymmetric SU(5) and Gauge Coupling Unification at Three Loops

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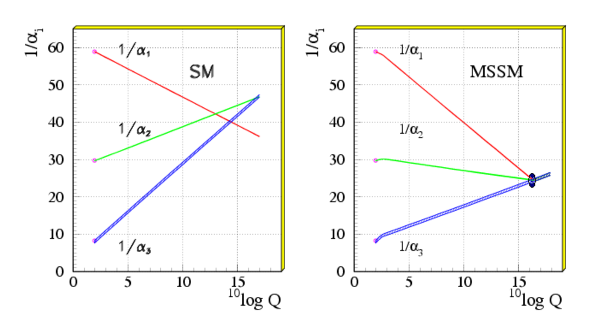
Overview

- Review of minimal SUSY SU(5) and motivation
- Setup and analysis
- Results
- Conclusions

Running of the gauge couplings



[1]

[1][Amaldi, de Boer, Fürstenau, 1991]

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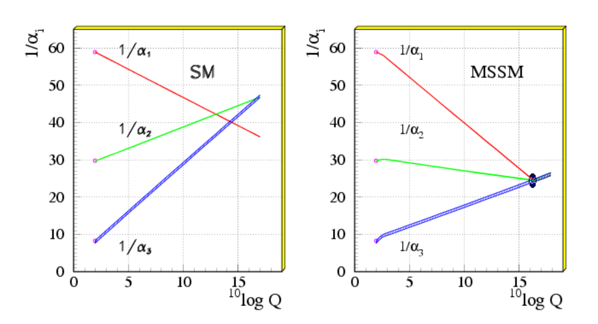

[1]

Minimal SUSY SU(5) is the simplest SUSY GUT!
Field content: ϕ_i $(\overline{\mathbf{5}})$, Ψ_i $(\mathbf{10})$, Σ $(\mathbf{24})$, H $(\mathbf{5})$, $\overline{H}$ $(\overline{\mathbf{5}})$

$$\begin{aligned}\mathcal{W} &= M_1 \mathrm{Tr}(\Sigma^2) + \lambda_1 \mathrm{Tr}(\Sigma^3) + \lambda_2 \overline{H}\Sigma H + M_2 \overline{H} H \\ &\quad + \sqrt{2} Y_d^{ij} \Psi_i \phi_j \overline{H} + \frac{1}{4} Y_u^{ij} \Psi_i \Psi_j H\end{aligned}$$

[1] [Amaldi, de Boer, Fürstenau, 1991]

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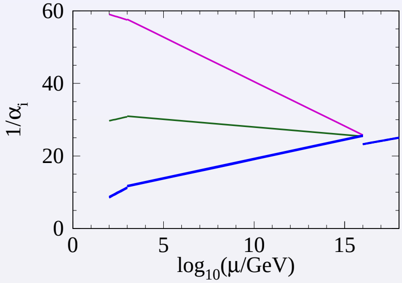

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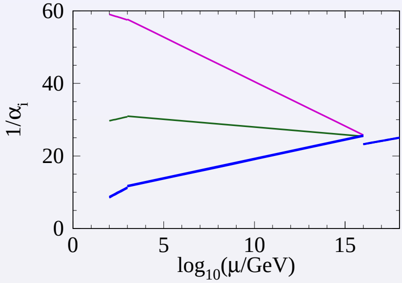

$$\mathcal{L}_{\text{GUT}} = \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{heavy}}$$

$$\Rightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MSSM}} + \mathcal{O}\left(\frac{1}{M_{\text{GUT}}}\right) \quad +$$

parameter redefinitions

$$\alpha_i(\mu_{\text{GUT}}) = \zeta_{\alpha_i}(\mu_{\text{GUT}}) \; \alpha(\mu_{\text{GUT}})$$

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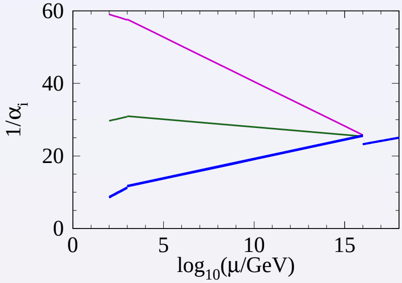

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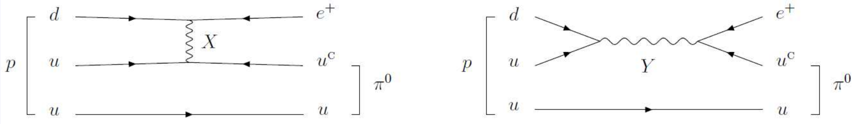
$\alpha_i(\mu_{\text{GUT}}) = \zeta_{\alpha_i}(\mu_{\text{GUT}})\ \alpha(\mu_{\text{GUT}})$

$$4\pi\left(-\frac{1}{\alpha_1(\mu)} + 3\,\frac{1}{\alpha_2(\mu)} - 2\,\frac{1}{\alpha_3(\mu)}\right) = -\frac{12}{5}\ln\left(\frac{\mu^2}{M_{H_c}^2}\right),$$

$$4\pi\left(5\,\frac{1}{\alpha_1(\mu)} - 3\,\frac{1}{\alpha_2(\mu)} - 2\,\frac{1}{\alpha_3(\mu)}\right) = -24\ln\left(\frac{\mu^3}{M_X^2 M_\Sigma}\right) \Rightarrow M_G^3 := M_X^2 M_\Sigma$$

Proton decay

Dimension-6 decay ($\propto 1/M_X^4$):



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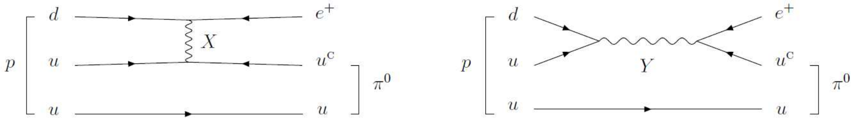

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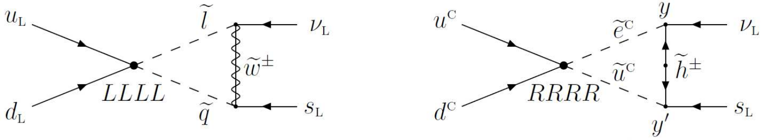

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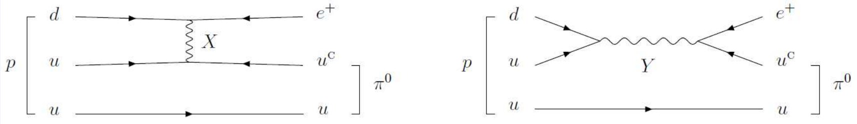

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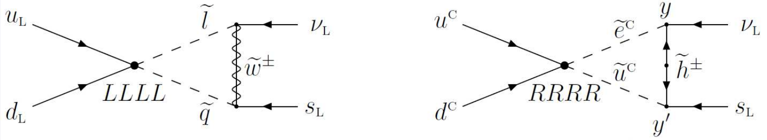

- Constraint on M_{H_c} lead [Goto, Nihei, 1999] and [Murayama, Pierce, 2002] to the exclusion of minimal SUSY SU(5).

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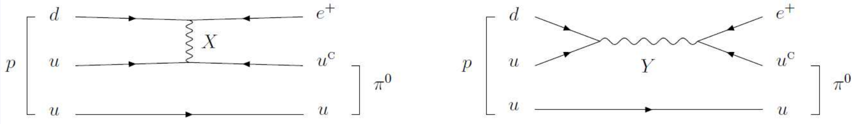

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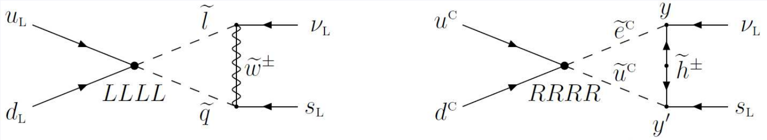

- Constraint on M_{H_c} lead [Goto, Nihei, 1999] and [Murayama, Pierce, 2002] to the exclusion of minimal SUSY SU(5).
- Later careful analyses[2] showed that those constraints were too strong and that minimal SUSY SU(5) is still perfectly viable.

[2] [Bajc, Fileviez Perez, Senjanovic, 2002], [Emmanuel-Costa, Wiesenfeldt, 2003]

Setup and analysis

- Goal: Predict M_{H_c} from the knowledge of α_s, α and $\sin\theta_W$ at M_Z.

[3] B.C. Allanach, 2001

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	run	dec	run	dec	run
	$M_Z \to \mu_{SUSY}$	$\alpha_i \to \alpha_i^{(MSSM)}$	$\mu_{SUSY} \to \mu_{GUT}$	$\alpha_i^{(MSSM)} \to \alpha^{(GUT)}$	$\mu_{GUT} \to M_{pl}$
EW	2(3)	1(2)	3	1(2)	3
QCD	3	2	3		

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	run $M_Z \to \mu_{\text{SUSY}}$	dec $\alpha_i \to \alpha_i^{(\text{MSSM})}$	run $\mu_{\text{SUSY}} \to \mu_{\text{GUT}}$	dec $\alpha_i^{(\text{MSSM})} \to \alpha^{(\text{GUT})}$	run $\mu_{\text{GUT}} \to M_{\text{pl}}$
EW	2(3)	1(2)	3	1(2)	3
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- Use SOFTSUSY[3] for the generation of the sparticle spectrum.
- Proton decay yields a lower bound on M_{H_c}.

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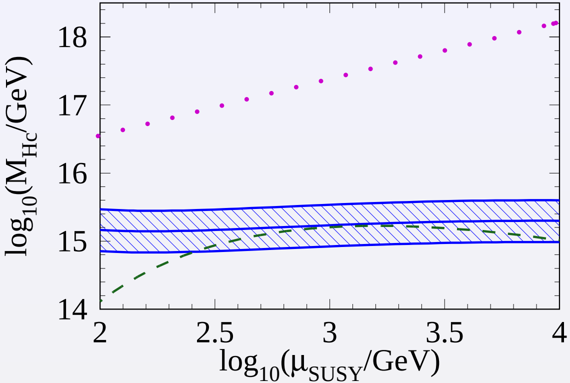
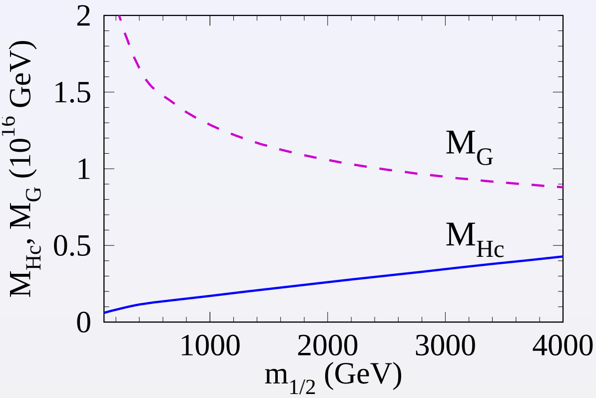
Dependence on the decoupling scale



mSUGRA parameters:

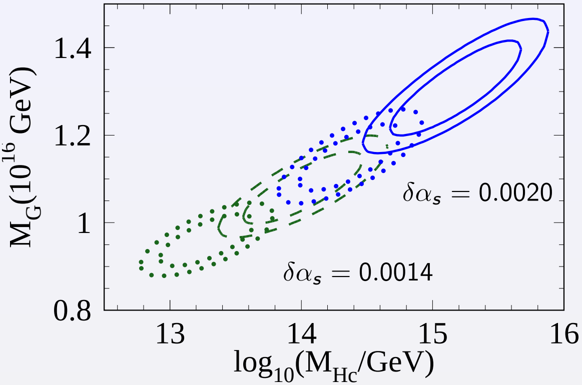
$$m_0 = m_{1/2} = -A_0 = 1000 \text{ GeV},$$
$$\tan\beta = 3, \qquad \mu > 0$$

Dependence on the SUSY spectrum



$$\mu_{\text{SUSY}} = 1000 \text{ GeV},$$
$$\mu_{\text{GUT}} = 10^{16} \text{ GeV},$$

Impact of the experimental uncertainties of $\alpha_i(M_Z)$



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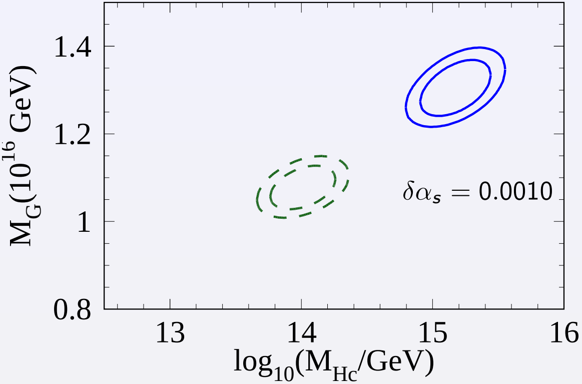

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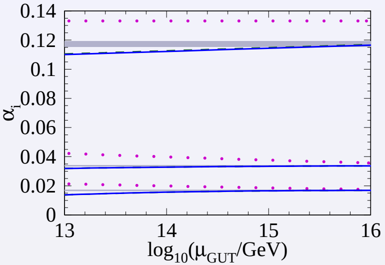
Top-down approach



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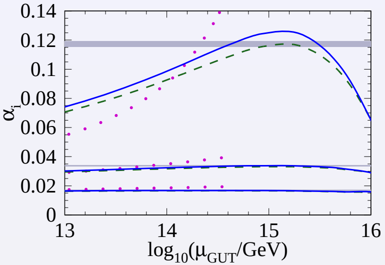

(b) Missing Doublet Model

SUSY spectrum: SPS1a

$\mu_{SUSY} = 500$ GeV

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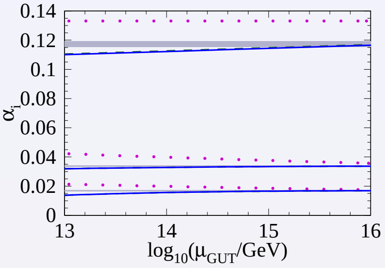

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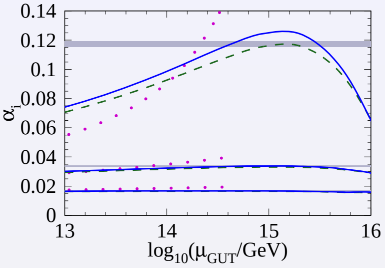

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Thanks for listening!

Backup slides

Grand Unified Theories (GUTs)

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$\begin{pmatrix} d_1^c & d_2^c & d_3^c \end{pmatrix}_L$,	$\bar{\mathbf{5}} = \begin{pmatrix} d_1^c & d_2^c & d_3^c & e & -\nu_e \end{pmatrix}_L$,
$\begin{pmatrix} u_1^c & u_2^c & u_3^c \end{pmatrix}_L$,	
$\begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix}_L$,	$\mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$
$\begin{pmatrix} \nu_e & e \end{pmatrix}_L$, e_L^c	

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$\begin{pmatrix} d_1^c & d_2^c & d_3^c \end{pmatrix}_L$, $\begin{pmatrix} u_1^c & u_2^c & u_3^c \end{pmatrix}_L$, $\begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix}_L$, $\begin{pmatrix} \nu_e & e \end{pmatrix}_L$, e_L^c	$\overline{\mathbf{5}} = \begin{pmatrix} d_1^c & d_2^c & d_3^c & e & -\nu_e \end{pmatrix}_L$, $\mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$
arbitrary normalization of hypercharge Y	$Y = \frac{1}{2\sqrt{15}}\ \mathrm{diag}(-2, -2, -2,\ 3,\ 3)$

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$$\begin{pmatrix} \nu_e & e \end{pmatrix}_L ,\ e_L^c$$

GUTs:

$$\overline{\mathbf{5}} = \left(\begin{array}{ccc|c} d_1^c & d_2^c & d_3^c & e \ -\nu_e \end{array} \right)_L ,$$

$$\mathbf{10} = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ \hline u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{array} \right)_L$$

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