Towards MSSM from F-theory

Kang-Sin Choi
Kyoto University

based on
0910.2571
1003.2126*
1007.3843
*with Tatsuo Kobayashi

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Origin of the Standard Model

The Standard Model summarizes the known facts around sub-TeV.

- A gauge theory with
- group $SU(3) \times SU(2) \times U(1)$
- fermions $l(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}, q(\mathbf{3},\mathbf{2})_{\frac{1}{6}}, u^{\mathrm{c}}(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}, d^{\mathrm{c}}(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}, e^{\mathrm{c}}(\mathbf{1},\mathbf{1})_{1}$
- $\times 3$ repetition
- boson $h(1,2)$
- renormalizable interactions

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- $\times 3$ repetition
- boson $h(1,2)$ why?
- renormalizable interactions why?

E_n Unification

Wisdom from Grand Unification: the structure of the SM is far from arbitrary

- $SU(5) = E_4$
  - The minimally unifying simple group.
  - $\overline{5}$, 10 harbors all quarks and leptons.
- $SO(10) = E_5$
  - $1_{\nu^c} + \overline{5} + 10 = 16$ all fermions in one single repr.
  - Includes R-handed neutrino
    A good explanation about small neutrino mass "See-saw mech".
- E_6
  - $16 + 10_h + 1_N = 27$ also unifies Higgs $10_h = 5_h + \overline{5}_h$.

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- $E_7,\ E_8$
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  - $133 + 56 + \overline{56} + 1 = 248$

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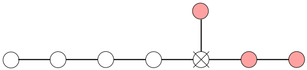
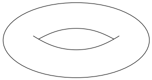
Unification along E_n series: unique position of the SM: $E_3 \times U(1)$ [Ramond] [Olive]



String theory

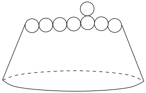
String theory may address such problems.

heterotic



one-loop consistency

F-theory



the shape of singularity

Predictions

- Gauge group $E_8 \times E_8$ or $SO(32)$
- Extra dimensions
- Spacetime supersymmetry
- $\vdots$

Symmetry breaking in extra dimension

Choice of non-simply connected internal space and background vector/scalar

A higher dim gauge fields $A_M = \underbrace{A_\mu}_{\text{4D gauge boson}} \oplus \underbrace{A_5 \oplus A_6}_{\text{4D scalars}} \oplus \cdots$

- Constant $\langle A_5 \rangle$ = conventional Higgs mech.: 4D vectorlike spectrum $R + \overline{R}$
- Position dependent $\langle A_5 \rangle = mx^6$: chiral: exclusively R or $\overline{R}$ [Landau]
- Generalized and classified by Chern characters
  F(monopole), $F \wedge F$(instanton), $F \wedge F \wedge F \ldots$

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Dirac equation in higher dim

$$(i\Gamma^\mu \partial_\mu + \Gamma^m (i\partial_m - A_m + \tfrac{1}{2}\omega_m))\psi = 0$$

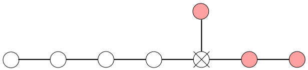
- Eigenvalue of $\Gamma^m (i\partial_m - \underbrace{A_m}_{\text{VEV}} + \underbrace{\tfrac{1}{2}\omega_m}_{\text{geometry}}) \equiv i \not{\nabla}$ looks like 4D mass
- # degenerate massles states counted by index theorem. Ex. 6D

$$n_R - n_{\overline{R}} = \text{index} i \not{\nabla} = \int_{6D} trF \wedge F \wedge F - \frac{1}{8} trF \wedge trR \wedge R$$

Gauge-Higgs unification: chiralty and repeted generations

From E_8 adjoint

Assuming E_8 (to be justified later)



Under $E_8 \to SU(3) \times SU(2) \times U(1)_Y \times SU(5)_\perp$,

$$\begin{aligned}\mathbf{248} \to \text{adjoints} &+ e^c(\mathbf{1},\mathbf{1},\mathbf{5})_1 + l(\mathbf{1},\mathbf{2},\mathbf{10})_{-1/2} \\ &+ q(\mathbf{3},\mathbf{2},\mathbf{5})_{1/6} + X(\mathbf{3},\mathbf{2},\mathbf{1})_{-5/6} \\ &+ d^c(\overline{\mathbf{3}},\mathbf{1},\mathbf{10})_{1/3} + u^c(\overline{\mathbf{3}},\mathbf{1},\mathbf{5})_{-2/3} + c.c.\end{aligned}$$

A desirable spectrum!

From $U(1)_Y \times SU(5)_\perp$ background instanton $\langle F \wedge F \rangle$

We have chiral solution to Dirac equation.

Monodromy

The fate of internal quantum numbers

$$\mathbf{248} \to \text{adjoints} + e^c(\mathbf{1},\mathbf{1},\mathbf{5})_1 + l(\mathbf{1},\mathbf{2},\mathbf{10})_{-1/2} + \ldots .$$

How many e^c, l in the low energy limit?
Instanton background reduces the field components under Weyl symmetry "monodromy."
In general $SU(5)_\perp$ has [Hayashi,Kawano,Tata,Tsuchya,Watari].

S_5 monodromy: 5! permuting all the weights of $\mathbf{5}_\perp$

Ex. $\mathbf{5}_\perp=\{t_1,t_2,t_3,t_4,t_5\}$ are globally 'connected' by S_5

- Just one kind of $e^c, \ldots$
- 3 generations from 3 zero modes of Dirac eq.

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$$\text{Ex.}\mathbf{10}_\perp = \left\{ \begin{array}{cccc} t_1+t_2 & t_1+t_3 & t_1+t_4 & t_1+t_5 \\ & t_2+t_3 & t_2+t_4 & t_2+t_5 \\ & & t_3+t_4 & t_3+t_5 \\ & & & t_4+t_5 \end{array} \right\} \quad {}_5C_2 = 10$$

Higgs doublets h_u, h_d are not yet distinguihsed.

$\mathbb{Z}_4$ Monodromy

To distinguish $SU(2)$ doublets l, h_u, h_d, we need more special monodromy
Mod out $SU(5)_\perp$ by $\mathbb{Z}_4$ monodromy. [Marsano, Saulina, Schaffer-Nameki]

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   the famous '$B-L$' symmetry $SU(5)_{GUT} \times U(1)_M \subset SO(10)_{GUT}$

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1. Singles out t_5:
   the famous '$B-L$' symmetry $SU(5)_{GUT} \times U(1)_M \subset SO(10)_{GUT}$
2. Now lepton is distinguished from Higgs
   - $\{t_1, t_2, t_3, t_4, t_5\} \to \{t_1, t_2, t_3, t_4\}, \{t_5\}$

$$q_\circ(\mathbf{3}, \mathbf{2}, \mathbf{5}) \to q(\mathbf{3}, \mathbf{2}, \mathbf{4})_{\frac{1}{6}} + q'(\mathbf{3}, \mathbf{2}, \mathbf{1})_{\frac{1}{6}}$$

   - $$\left\{ \begin{array}{cccc} t_1 + t_2 & t_1 + t_3 & t_1 + t_4 & t_1 + t_5 \\ & t_2 + t_3 & t_2 + t_4 & t_2 + t_5 \\ & & t_3 + t_4 & t_3 + t_5 \\ & & & t_4 + t_5 \end{array} \right\}$$

$$l_\circ(\mathbf{1}, \mathbf{2}, \mathbf{10})_{-\frac{1}{2}} \to l(\mathbf{1}, \mathbf{2}, \mathbf{4})_{-\frac{1}{2}, -3} + h_u(\mathbf{1}, \mathbf{2}, \mathbf{4})_{-\frac{1}{2}, 2} + h_d^c(\mathbf{1}, \mathbf{2}, \mathbf{2})_{-\frac{1}{2}, 2}$$

Yukawa coupling

Yukawa coupling originates from gauge invariant interaction

$$y_{abc} = \int \lambda_a \wedge A_b \wedge \lambda_c$$

Emerges at a triple intersection of branes

- $lh_d e^c : (t_j + t_k + t_6) + (t_l + t_m + t_6) + (t_l - t_6) = 0, \qquad \epsilon_{ijklm} \neq 0$
- $qh_u u^c : (t_i) + (-t_i - t_j - t_6) + (t_j + t_6) = 0, \qquad i \neq j, \ldots$

$lh_d e^c$ $qh_u u^c$ $qh_d d^c$ $lh_u \nu^c$	MSSM superpotential
$\nu_M \nu_M^c$	
$Xh_u d^c$ $q' h_d^c D^c$ $q' h_u D'$	nonvanishing but X and q' are absent

B and/or L violating couplings absent

$$lh_u, \; lle^c, \; lqd^c, \; u^c d^c d^c$$

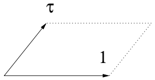
Superpotential is renormalizable MSSM superpotential without R-parity violating terms→would be broken cf.[Ambroso, Ovrut]

$$W = W_{MSSM}(\mu = 0) + m_h h_u h_d + m_D D D'$$

F-theory

Type IIB string theory 10D $\overset{SL(2,\mathbb{Z})}{=}$ F-theory on a torus 12D

- $\tau = C_0 + ie^{-\phi}$ $\qquad e^{-\langle\phi\rangle} = g^{-1}$.
- The symmetry of a torus with comp struct τ



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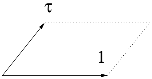

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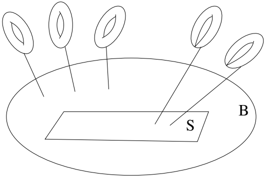
We compactify F-theory on a Calabi–Yau fourfold: 4D $\mathcal{N} = 1$ SUSY

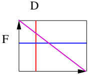
- 1 of 4 $\dim_{\mathbb{C}}$ is torus — nontrivial product on 3 $\dim_{\mathbb{C}}$base

elliptic (torus) fibered: shape of the torus varies, as we move around B



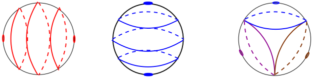
Singular torus locus S supports 8D worldvol gauge theory.
cf. CY condition: $x \sim -4c_1(B)$, $\quad y \sim -6c_1(B)$, $\quad y^2 = x^3 + fx + g$

Gauge theory from geometry

Gauge coupling $\tau = C_0 + ig^{-1}$ = CS of an extra torus 

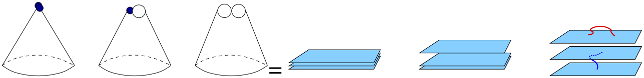
$(C_0 = 0)$

- Cartan subalgebra $A_\mu = \sum G_{\mu np} \wedge \omega_{np}$
- $W^\pm$ components M2 brane in F-theory wrapping on S^2



D-brane F-string str. junction

- Shrinking sphere = singularity: massless string btwn the coincident branes



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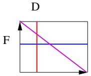
Gauge coupling $\tau = C_0 + ig^{-1}$ = CS of an extra torus 

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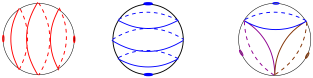

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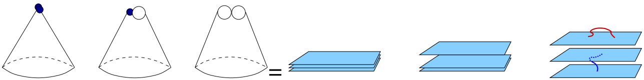
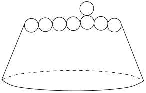
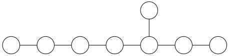


The connectedness of Lie algebra = geometry

ex. E_8   $y^2 = x^3 + z^5$

Classified and tabulated: Just refer to tables.

The shape of singularities

Singularities are classified: They are all special cases of the elliptic equation $y^2 + a_1 xy + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$ describing the torus.

type	group	a_1	a_2	a_3	a_4	a_6	Δ	f	g
I_0	smooth	0	0	0	0	0	0	0	0
I_1	$U(1)$	0	0	1	1	1	1	0	0
I_{2k-1}^{ns}	unconven.	0	0	k	k	$2k-1$	$2k-1$	0	0
I_{2k-1}^{s}	$SU(2k-1)$	0	1	$k-1$	k	$2k-1$	$2k+1$	0	0
I_{2k}^{ns}	$Sp(k)$	0	0	k	k	$2k$	$2k$	0	0
I_{2k}^{s}	$SU(2k)$	0	1	k	k	$2k$	$2k$	0	0
II	—	1	1	1	1	1	2	1	1
III	$SU(2)$	1	1	1	1	2	3	1	1
IV^{ns}	unconven.	1	1	1	2	2	4	1	1
IV^{s}	$SU(3)$	1	1	1	2	3	4	1	1
$I_0^{*\,ns}$	G_2	1	1	2	2	3	6	2	3
$I_0^{*\,ss}$	$SO(7)$	1	1	2	2	4	6	2	3
$I_0^{*\,s}$	$SO(8)^*$	1	1	2	2	4	6	2	3
$I_{2k-3}^{*\,ns}$	$SO(4k+1)$	1	1	k	$k+1$	$2k$	$2k+3$	2	3
$I_{2k-3}^{*\,s}$	$SO(4k+2)$	1	1	k	$k+1$	$2k+1$	$2k+3$	2	3
$I_{2k-2}^{*\,ns}$	$SO(4k+3)$	1	1	$k+1$	$k+1$	$2k+1$	$2k+4$	2	3
$I_{2k-2}^{*\,s}$	$SO(4k+4)^*$	1	1	$k+1$	$k+1$	$2k+1$	$2k+4$	2	3
$IV^{*\,ns}$	F_4	1	2	2	3	4	8	3	4
$IV^{*\,s}$	E_6	1	2	2	3	5	8	3	4
III^*	E_7	1	2	3	3	5	9	3	5
II^*	E_8	1	2	3	4	5	10	3	5
non-min	—	1	2	3	4	6	12	4	6

Modding out by monodromy: we can get all the possible Lie algebra [Tate]

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$IV^{*\,s}$	E_6	1	2	2	3	5	8	3	4
III^*	E_7	1	2	3	3	5	9	3	5
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non-min	—	1	2	3	4	6	12	4	6

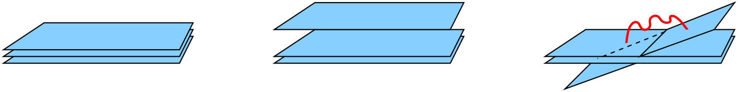
Modding out by monodromy: we can get all the possible Lie algebra [Tate]

We have 8D gauge symmetry of the same name of singularity

Matter and localization

Matter comes from the ‘off-diagonal’ component of gaugino
Ex. From $U(m+n) \to U(m) \times U(n)$

$$(\mathbf{m}+\mathbf{n})^2 \to (\mathbf{m}^2, \mathbf{1}) + (\mathbf{1}, \mathbf{n}^2) + (\mathbf{m}, \mathbf{n}) + (\overline{\mathbf{m}}, \overline{\mathbf{n}})$$



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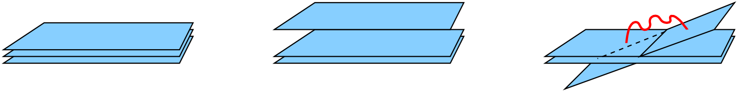

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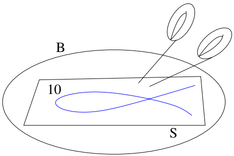
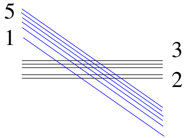
Focusing on one gauge brane, matters localized around 'matter' curves on S



Ex. From $E_6 \to SO(10) \times U(1)$, $\mathbf{78} \to \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1}$, spinorial $\mathbf{16}$ obtainable.
For this reason, in $SU(5)$, $\epsilon_{\alpha\beta\gamma\delta\varepsilon}\mathbf{10}^{\alpha\beta}\mathbf{10}^{\gamma\delta}\mathbf{5}^{\varepsilon}$ allowed (forbidden in perturbative D-branes)

Spectral cover

The background instanton/Higgs bundle of $U(1)_Y \times SU(5)_\perp$ is described by spectral covers 'flavor branes.'
Describing a SUSY bundle for elliptic fibered space [Friedman, Morgan, Witten]



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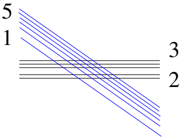

- A brany description: t_i positions reflecting monodromy

$$0 = \underbrace{(a_0 s + a_1)}_{U(1)_Y}\underbrace{(b_0 s^5 + b_1 s^4 + b_2 s^3 + b_3 s^2 + b_4 s + b_5)}_{SU(5)_\perp}$$
$$= a_0 b_0 (s - t_Y)(s - t_1)(s - t_2)(s - t_3)(s - t_4)(s - t_5) \qquad \mathrm{mod}\ S_5$$

- Reflecting S_5 monodromy
  b_m/b_0 elementary symmetric polynomials of deg. m of $t_1, \ldots t_5$.
- $a_0 = 1$ by anomaly cancellation
  The only combinations $b_i + b_{i-1} a_1$
- Other monodromies, e.g $\mathbb{Z}_4$ by further tuning b_m
- $U(1)_Y$: the trace part of $SU(5)_\perp$

$$b_1 = -a_1 b_0, \quad a_1 \sim -c_1(S)$$

The $SU(3) \times SU(2) \times U(1)_Y$ singularity

Symmetry breaking described by the spectral cover,

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contains also the info. about the unbroken part.

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$$\begin{aligned}
\mathsf{a}_1 &= -(b_5 + b_4 a_1) + O(z) \\
\mathsf{a}_2 &= (b_4 + b_3 a_1) z + O(z^2) \\
\mathsf{a}_3 &= -(b_3 + b_2 a_1)(a_1 b_5 + z) z + O(z^3) \\
\mathsf{a}_4 &= (b_2 + b_1 a_1)(a_1 b_5 + z) z^2 + O(z^4) \\
\mathsf{a}_6 &= b_0 (a_1 b_5 + z)^2 z^3 + O(z^6)
\end{aligned}$$

$$\Delta = (b_5 + a_1 b_4)^3 P_X^2 P_{q_\circ}^2 P_{d^c_\circ} P_{u^c_\circ} z^3 + P_{q_\circ} P_X Q_4 z^4 + O(z^5).$$

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- On $S = \{\Delta \simeq 0\}$ we have 8D worldvolume gauge theory of $SU(3) \times SU(2) \times U(1)_Y$
- From tables, generically $SU(3)$ at $z = 0$. (some simplification)
  Very specially tuned so the actual symmetry is larger.
- Change of coordinate $z \leftrightarrow z + a_1 b_5$, the singularity looks like generically $SU(2)$ tuned upto $O(z^5)$.

The $SU(3) \times SU(2) \times U(1)_Y$ singularity

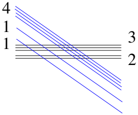
- $a_1 = 0 : SU(5)$ enhancement.
- In the weakly coupling limit $f, g \to \infty$, with f^3/g^2 finite,

$$P_{q_\circ} \simeq P_{u^c_\circ} \simeq P_{e^c_\circ} \simeq b_5, \quad P_{l_\circ} \simeq P_{d^c_\circ} \simeq R_{\mathbf{5}},$$
$$\Delta \simeq b_5^4 R_{\mathbf{5}} (b_5 a_1 + z)^2 z^3,$$

- Well known reduction of $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$
  - $U(1)_Y$: relative center-of-mass
  - Linearly equivalent $z \sim z - a_1 b_5 \sim -t$ cf. parallel separation
  - Parallel but intersecting $\sigma \cap \sigma \neq 0 : (\mathbf{3}, \mathbf{2})$ localized on the intersection.

Spectrum

4+1+1 factorization $C_4 \cup C_1 \cup C_1'$



matter	matter curve	living on	M
X	$t_6 \to 0$	C_1	0
q	$t_i \to 0$	C_4	1
q'	$t_5 \to 0$	C_1'	-4
d^c	$t_i + t_5 \to 0$	$C_4 \cap \tau C_1'$	-3
D^c	$t_i + t_{i+2} \to 0$	$C_4 \cap \tau C_4$	2
D'	$t_i + t_{i+1} \to 0$	$C_4 \cap \tau C_4$	2
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h_u	$t_i + t_{i+1} + t_6 \to 0$	$C_4 \cap \tau C_4$	2
h_d^c	$t_i + t_{i+2} + t_6 \to 0$	$C_4 \cap \tau C_4$	2
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e'	$t_5 - t_6 \to 0$	$C_1 \cap C_1'$	-4

The simplest flux on 4 part only
$\gamma_4 = (4 - p_4^*(\eta - c_1))(C_4 \cap \sigma)$,
$\gamma_1 = 0, \gamma_1' = 0$.

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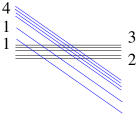

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For trivial $a_0 = 1$,

- Matter curves for q, u^c, e^c ($\in \mathbf{10}$) are linearly equivalent (but actual curves are different) $t_i \sim t_i + t_6 \sim t_i - t_6$
- Spectrum obeys a unification relation

$$16: \quad n_q = n_{d^c} = n_{u^c} = n_l = n_{e^c} = n_{\nu^c} = -\lambda\eta \cdot (\eta - 4c_1)$$

$$10: \quad n_{h_d} = n_{h_u} = n_D = n_{D'} = -2\lambda \cdot (\eta - 4c_1)$$

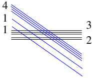
Choosing the base manifold S such that $-\lambda\eta \cdot (\eta - 4c_1) = 3$ e.g. [Donagi, Wijnholt], [Blumenhagen, Grimm, Weigand]

3 generations of quarks and leptons
plus 6 vectorlike Higgs doublets and tiplets

- no other exotics. 2-3 splitting problem

$\mathbb{Z}_4$ monodromy

Monodromy is encoded in the spectral cover

1+1+4 structure 

$$(U + a_1 V)(d_0 U + d_1 V)(e_0 U^4 + e_1 U^3 V + e_2 U^2 V^2 + e_3 U V^3 + e_4 V^4) = 0.$$

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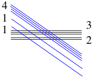
1+1+4 structure 

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$\mathbb{Z}_4$ monodromy needs just one tuning [KSC, Kobayashi]

$$e_2 = e_2' + e_2''$$

$$e_2'/e_0 \sim t_1 t_2 + t_2 t_3 + t_3 t_4 + t_4 t_1, \quad e_2''/e_0 \sim t_1 t_3 + t_2 t_4.$$

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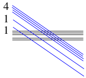
1+1+4 structure 

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Down type quarks are decomposed into colored Higgses

$$S_4 \text{ orbit } \prod_{i<j}^{4} (t_i + t_j) \sim e_0^{-3}(-e_0 e_3^2 + e_1 e_2 e_3 - e_1^2 e_4),$$

Two $\mathbb{Z}_4$ orbits give matter curves

$$h_u^c : (t_1 + t_3)(t_2 + t_4) \sim e_2'/e_0,$$

$$h_d : (t_1 + t_2)(t_2 + t_3)(t_3 + t_4)(t_4 + t_1) \sim (e_2''^2 + e_1 e_3 - 4 e_0 e_4)/e_0^2.$$

Actual matter homologies

$$
\begin{aligned}
X &: C_X \cap \sigma = -c_1 \cap \sigma \\
q &: C_q \cap \sigma = \sigma \cap (\eta - 4c_1 - x) \\
q' &: C_{q'} \cap \sigma = \sigma \cap (-c_1 + x) \\
d^c, l &: (C_q - V) \cap C_{q'} = \sigma \cap (\eta - 4c_1 + 2x) + (\eta - c_1 - x) \cap x \\
D^c, h_d^c &: \tfrac{1}{2}(C_q - 2U) \cap (C_q - 4V) = \sigma \cap (\eta - 4c_1 - x) + \tfrac{1}{2}(\eta - x) \cap (\eta - 4c_1 - x) \\
D', h_u &: \tfrac{1}{2} C_q \cap (U + V) = \sigma \cap (\eta - 2c_1 - x) + \tfrac{1}{2}(\eta - x) \cap c_1 \\
u^c, e^c &: C_q \cap C_X = \sigma \cap (\eta - 4c_1 - x) \\
u', e' &: C_{q'} \cap C_X = \sigma \cap (-c_1 + x)
\end{aligned}
$$

Conclusion

$SU(3) \times SU(2) \times U(1)_Y$ model

- Grand Unification is a compelling structure.
- Unique direction $E_3 \times U(1)_Y$ is the commutant to $SU(5)_\perp \times U(1)_Y$ in E_8
- $\mathbb{Z}_4$ monodromy: R-parity distinguishing Higgses from leptons

String theory/F-theory well-realizes unification with more degrees of freedom.
Construction of gauge singularity, spectral covers in F-theory

- Renormalizable MSSM superpotential
- Three generations of quarks and leptons, vectorlike Higgs doublets and triplets mu-term and triplet masses, but without mixing of quarks.
- Partial flux on $SU(4)$ part gives rise to $SU(5), SO(10)$ GUT structure