Heterotic Z7 Orbifolds in Blowup

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Based on work in progress with M.Blaszczyk , H.P.Nilles, F.Ruehle and M.Trapletti.

Motivation

- Heterotic orbifold compactifications of $E_8 \times E_8$ have proved to offer an interesting landscape region to search for nice phenomenology [Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz,Vaudrevange,Wingerter,06-08].
- Connect Orbifold compactifications of Heterotic $E_8 \times E_8$ with the Supergravity coupled to SYM limit of the theory on smooth CY with Abelian gauge fluxes[Nibbelink *et al*, 09]. See the talks by Stefan Groot Nibbelink and Michael Blaszczyk.
- Explore different compactifications to search for "MSSM"-like models, also with non-anomalous $U(1)$ on Blowup.

Introduction

- Here: compactifications of 10D Heterotic $E_8 \times E_8$ SUGRA SYM with abelian fluxes on smooth CY that are Blowups of the Orbifold $T^6/\mathbb{Z}_7$.
- Two toy models with the gauge group of the *SM*: One comming from an *SM*-like orbifold model, and the other being a blowup of an orbifold model with E_6-gauge symmetry.
- Field redefinitions [Nibbelink,Nilles,Trapletti,07] are performed to match the spectrum in both sides in Model 1.
- A classification of blowups from *SE* Orbifold is given. Values of $\sin^2\theta_W$ are explored in Model 2. Anomaly analysis of the spectrum is performed in Model 2.

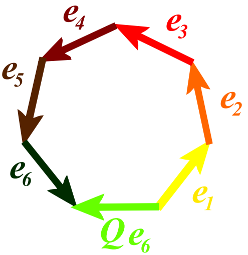
$T^6/\mathbb{Z}_7$ orbifold

The lattice is the non-factorizable root lattice of $SU(7)$, $\Gamma_{SU(7)}$ with vectors e_i, $i = 1...6$.

The $\mathbb{Z}_7$ twist action is given by

$$\theta : (z_1, z_2, z_3) \longrightarrow (e^{2\pi v_1} z_1, e^{2\pi i v_2} z_2, e^{2\pi i v_3} z_3), \quad v = (1/7, 2/7, 4/7),$$

where the $\mathcal{N} = 1$ condition $\sum_i v_i = 0 \mod 1$ is fulfilled.
Equivalent coordinates under space group are $z \equiv \theta^k z + n_\alpha e_\alpha$.
Lattice vectors fulfill $Qe_i = e_{i+1}, i = 1...5,\ Qe_6 = -\sum_i e_i$.



- $Qe_i = e_{i+1}$ implies $A_i \equiv A_{i+1}$ i.e. only one independent W.L.
- There are three independent twisted sectors $\theta, \theta^2, \theta^4$.
- There are 7 fixed points.

Heterotic $E_8 \times E_8$ String in $T^6/\mathbb{Z}_7$

Orbifolds invariant states are given by

$$|q_{sh}\rangle_R \otimes \tilde{\alpha}|p_{sh}\rangle_L \otimes \left(\sum_n e^{-2\pi i n\gamma} |h^n g h^{-n}\rangle \right)$$

The orbifold action reflects in the 16 gauge degrees of freedom by the action of local shifts V_g given by

$$V_g = kV + n_\alpha A_\alpha, \quad 7V \in \Lambda_{E_8\times E_8}, \quad 7A_a \in \Lambda_{E_8\times E_8}.$$

Modular invariance imposes $7V^2 = 7(A\cdot V) = 7A^2 = 0 \mod 2.$
Matter states can be cast as
Untwisted: $\forall_i p \cdot V = v_i \mod 1$. Twisted: $p_{sh}^2 = 10/7 - \widetilde{N}$
Gauge bosons with $p \in \Lambda_{E_8\times E_8}$ have to be shift and W.L. singlets $p \cdot V = p \cdot A = 0 \mod 1.$

Resolution of $\mathbb{C}^3/\mathbb{Z}_7$

Every fixed point in the orbifold is resolved by separate with the aim of "gluing" the pieces together [Lüst,Reffert,Scheideger,Stieberger, 06].

Ordinary Divisors D_i

θ-invariant monomials $u_j = z_1^{(v_1)_j} z_2^{(v_2)_j} z_3^{(v_3)_j}$ give the condition $v_1\phi_1 + v_2\phi_2 + v_3\phi_3 = 0 \mod 1$ on the vectors. Every v_i is associated with a codimension 1 hypersurface D_i.

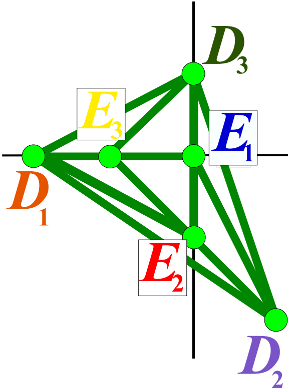
Exceptional Divisors E_k

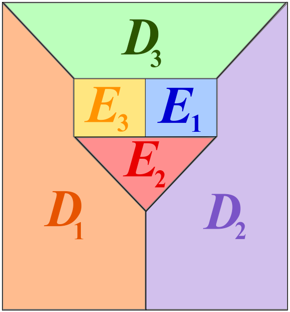
For every twist $\theta^k : (z_1, z_2, z_3) \to (e^{2\pi i g_1} z_1, e^{2\pi i g_2} z_2, e^{2\pi i g_3} z_3)$, a divisor E_r will be placed at $\omega_r = g_1 v_1 + g_2 v_2 + g_3 v_3$.

Here

$$\begin{aligned} v_1 &= (-2,0,1), \quad v_2 = (1,-2,1), \quad v_3 = (0,1,1), \\ \omega_1 &= (0,0,1), \quad \omega_2 = (0,-1,1), \quad \omega_3 = (-1,0,1) \end{aligned}$$

Resolution of $\mathbb{C}^3/\mathbb{Z}_7$





Triple intersections

They are 1 if divisors form corners of a basic triangle, and zero if not. i.e. $E_1 E_2 E_3 = D_1 E_2 E_3 = 1$ and $E_1 D_1 D_2 = E_3 E_1 D_2 = 0$.

Resolution of $\mathbb{C}^3/\mathbb{Z}_7$

Coordinates y_r are associated to every E_r. Invariant monomials are read from the diagram

$$u_j = \prod_{i=1}^{3}\prod_{r=1}^{3} z_i^{(v_i)_j}(y_r)^{(\omega_r)_j} \Rightarrow \sum_i (v_i)_j D_i + \sum_r (\omega_r)_j E_r \sim 0.$$

Via Poincaré duality cycles can be associated with forms. The equivalence relations of the cycles is given as an equivalences of the dual forms.

Triple intersections of distinct divisors and equivalences lead to the non-zero values $E_r^2 = 8, E_3^2 E_2 = E_2^2 E_1 = E_1^2 E_3 = -2$.

Resolution of $\mathbb{T}^6/\mathbb{Z}_7$

One has to "glue" together the local resolutions in the 7 fixed points. The $\#$ of divisors: $7 \times (3+3)+3 = 45$.

Inherited divisors R_i

Information from the 6-dim torus is included by means of the Poincaré duals of the (1,1) orbifold invariant forms.

New equivalence relations: Ex. $R_1 \sim 7D_{1,a} + (E_{1,a} + 2E_{2,a} + 4E_{3,a})$.

An auxiliary polyhedra [Lüst,Reffert,Scheideger,Stieberger, 06] constructed with D_J, E_r and R_i leads to the intersection numbers

$$\forall_{r_1 r_2} R_i E_{r_1} E_{r_2} \sim 0.$$

Resolution of $\mathbb{T}^6/\mathbb{Z}_7$

The total Chern class of the resolution can be computed using the dual forms [Nibbelink *et al*, 09], one gets here

$$c_1(X) \;=\; 0, \;\; c_2(X)E_r = -4, \;\; c_3(X) = 48$$

The gauge field strength $F_{10D} = F_{4D} + \mathcal{F}$ and the Ricci tensor $R_{10D} = R_{4D} + \mathcal{R}$. $\mathcal{F}$ and $\mathcal{R}$ are the gauge flux and the Ricci-tensor on the internal space.

Bianchi Identities(BI)

$$\int_S \mathrm{d}H \;\sim\; \int_S \left(\mathrm{tr}\mathcal{R}^2 - \mathrm{tr}\mathcal{F}^2\right) = 0$$
$$S \;:\; \text{Compact hypersurface.}$$

Heterotic SUGRA on the resolution

Abelian gauge fluxes $\frac{\mathcal{F}}{2\pi} = E_r V_r^I H_I$ are considered[Nibbelink *et al*,08]. Here $r = (k, a)$, denotes sector $k = 1, 2, 4$ and fixed point $a = 1...7$.

Relation btw. V_g and V_r is obtained integrating the flux over non-compact curves. For example $\int_{D_{2a}D_{3a}} \frac{\mathcal{F}}{2\pi} = V_{1,a}^I H_I \equiv V_{\theta, l_a}$.

Considering the anomaly polynomial of the gaugino in 10D is obtained the multiplicity operator N, which gives he number of times a given state appears on the resolution $N|\omega\rangle = N_\omega|\omega\rangle$.

The spectrum is free on Non-abelian anomalies. The 4D the anomaly polynomial $I_6 = \int_X I_{12}$ has a common factor depending on $\mathrm{tr}[\mathcal{F}' F']$ which allows to determine the # of anomalous $U(1)$s.

Model 1.

The shift and wilson line are [Casas *et al*, 90]

$$V = \frac{1}{7}\{0,0,1,1,1,2,2,1,1,1,0^6\} + \{0^8,1,0,1,0^5\},$$
$$A = \frac{1}{7}\{1^5,3,-2,2,3,-3,-4,0^5\}.$$

The orbifold gauge group $\mathcal{G}$ is given by

$$\mathcal{G} = [SU(3) \times SU(2) \times U(1)^5] \times [SO(10) \times U(1)^3]'.$$

The V_r solution of BI leaves the $\mathcal{G}$ invariant.
All U(1)s are anomalous on Blowup.
To test the match [Nibbelink *et al*, 09, Nibbelink,Nilles,Trapletti,07] the following identifications are relevant

$$p_{sh}^{\mathcal{S}} = V_r, \quad V_g \equiv V_r, \quad \Psi_{orb} = \mathcal{S}\Psi_{Bup}.$$

The fields $\mathcal{S}$ with $p_{sh}^{\mathcal{S}}$ should take VEVs leading the Blowup process.

Model 1. Matching

Spectrum

Space	$(\mathbf{3},\mathbf{2},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})$	$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{10})$
BlowUp	3	5	11	13	1
Orbifold	3	12	18	21	1

Mismatchs

State	Blowup	Orbifold
$(\mathbf{3},\mathbf{2},\mathbf{1})$	None	None
$(\mathbf{3},\mathbf{1},\mathbf{1})$	2	9
$(\bar{\mathbf{3}},\mathbf{1},\mathbf{1})$	3	10
$(\mathbf{1},\mathbf{2},\mathbf{1})$	2	10
$(\mathbf{1},\mathbf{1},\mathbf{10})$	None	None

- "Higgs" masses for orbifold states, due to VEV of Blowup modes.
- Blowup states with no counterpart are due to differences in multiplicities.

Classification of Blowups.

The orbifold model is the one of the Standard embedding.

$\mathcal{SSS}$	$E_6 \to E_6$
U	$3 \times (27) + 3 \times (1)$
T	$7 \times [3 \times (27) + 3 \times (1)]$
$\mathcal{SS}f$	$E_6 \to SO(10)$
U	$3 \times (16) + 3 \times (10) + 6 \times (1)$
T	$7 \times [3 \times (16) + 1 \times (10) + 12 \times (1)]$
$\mathcal{S}ff$	$E_6 \to SU(5)$
U	$3 \times (16) + 3 \times (10) + 6 \times (1)$
T	$7 \times [3 \times (16) + 1 \times (10) + 12 \times (1)]$
fff	$E_6 \to SU(3) \times SU(2)$
U	$3 \times (3,2) + 3 \times (3,1) + 9 \times (\bar{3},1) + 9(1,2) + 12(1,1)$
T	$7 \times [3 \times (3,2) + 6 \times (\bar{3},1) + 1 \times (1,2) + 24 \times (1,1)]$

$\mathcal{S}$: Singlets under non-abelian sector

f : Fields in the non-abelian sector.

Model 2. $\mathcal{G}_{SM}$ in Blow up

Orbifold

Gauge group	$[E_6 \times U(1)^2] \times [E_8]'$
Untwisted matter	$3 \times (27) + 3 \times (1)$
$\frac{1}{7}$ Twisted matter	$3 \times (27) + 24 \times (1)$

The orbifold shift is the standard embeddig one

$$V = \left\{1/7, 2/7, 4/7, 0^{13}\right\}$$

Choosing a given V_r solution of the BI is posible to break the gauge group down to the $\mathcal{G}_{SM}$.

Blowup

Gauge group:$[SU(3) \times SU(2) \times U(1)^5] \times [E_8]'$

Matter	$(\mathbf{3}, \mathbf{2})$	$(\mathbf{3}, \mathbf{1})$	$(\bar{3}, 1)$	$(\mathbf{1}, \mathbf{2})$	$(\mathbf{1}, \mathbf{1})$
Untwisted	3	3	9	9	12
$\frac{1}{7}$Twisted	3	0	6	1	24

Model 2. Hypercharge

There are 2 $U(1)_{non-anom}$ and 3 $U(1)_{anom}$ in the Blow up.

Is possible to define an Hypercharge embedded in $SU(5)$ in the form

$$E_6 \to SO(10) \to SU(5) \to SU(3) \times SU(2)$$

i.e.

$$T_Y^{anom} = \left\{-5/6, 1/6, 1/6, -1/6, -1/6, -1/6, 0^{10}\right\}$$

This gives a correct value for $\sin^2(\theta_W) = \frac{3}{8}$.

Exploring for Hypercharge definitions liying inside the non-anomalous U(1)s there are different choices, one could be

$$T_Y^{non-anom} = \left\{2/3, 1/6, -5/6, -1/6, -1/6, 1/3, 0^{10}\right\}$$

which gives $\sin^2(\theta_W) = \frac{3}{11}$. Is not possible to have $T_Y^2 = 5/6$.

Model 2. Anomalies

$$2\pi I_6 = A(F'_I F'_j F'_K) + B(F'_I)\mathrm{TR}R^2 + C(F'_I)\mathrm{tr}F'^2 + ('\rightarrow'').$$

A: pure U(1)

$$\begin{aligned} A &= 1/(147\pi^2)(-860\widetilde{F}[3]^3 + 96\widetilde{F}[3]^2(-27\widetilde{F}[4] + 4\widetilde{F}[5]) + \\ &+ 96\widetilde{F}[3](-31\widetilde{F}[4]^2 + 11\widetilde{F}[4]\widetilde{F}[5] + \widetilde{F}[5]^2) - \\ &- 384(\widetilde{F}[4]^3 + 5\widetilde{F}[4]^2\widetilde{F}[5] - 8\widetilde{F}[4]\widetilde{F}[5]^2 + \widetilde{F}[5]^3)) \end{aligned}$$

B: U(1)-Gravitational

$B = -7/(2\pi^2)\widetilde{F}[3]$, checked with the triangle anomaly of 4D th.

C: U(1)-Non Abelian

$$C = 49/(48\pi^2)\widetilde{F}[3]$$

Conclusions & Outlook

- Resolution models of $\mathbb{Z}_7$ orbifold were obtained with "MSSM"-like particle spectrum. Blowing up with singlets from an Orbifold-$\mathcal{G}_{SM}$ or blowing up with charged fields which break the sym. down to $\mathcal{G}_{SM}$.
- Model 2 possses U(1) non-anomalous directions in Blow-up, into which is possible to define a "Hypercharge generator". Next: Search for more realistic Blowup model with similar features.
- In Model 1, field redefinitions which connect Blowups and orbifolds states work to some extent. Further understanding of the connection btw. both schemes is needed. See talks by Stefan Groot Nibbelink and Michael Blaszczyk
- Anomaly cancellation mechanism in the present models have to be further studied.

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Thanks for your attention!