Matching of singular and resolved Orbifolds

Patrick K. S. Vaudrevange
LMU München

August 23, 2010

Based on:
M. Blaszczyk, S. Groot Nibbelink, F. Ruehle, M. Trapletti and P. V.: arXiv:1007.0203

M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trapletti and P. V.: arXiv:0911.4905,

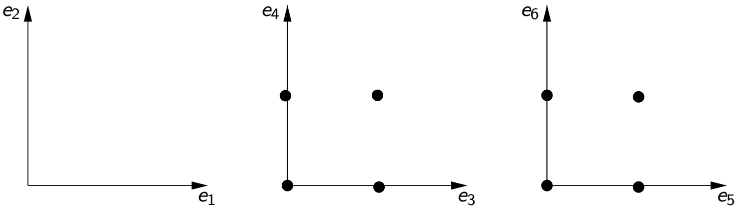
Phys.Lett.B683:340-348,2010

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold		Resolution
shifts and Wilson lines	$\Leftrightarrow$	flux quantization condition
gauge momentum p_{sh} of blow-up mode	$\Leftrightarrow$	local U(1) gauge background $V_r, r = 1, \ldots, 48$
massless blow-up mode $p_{sh}^2 = \frac{3}{2}$	$\Rightarrow$	Bianchi identity $\sum_{\text{sector}} V_r^2 = 24$
(Abelian) D-flatness	$\approx$	Donaldson-Uhlenbeck-Yau $\sum_r \text{vol}(E_r)\, V_r = \xi_{1L}$
VEV of blow-up mode	$\Leftrightarrow$	Kähler modulus b_r
Gauge group after VEVs	$\Leftrightarrow$	Abelian gauge flux commutant
(redefined) twisted matter	$\approx$	Matter from $E_8 \times E_8$ gauginos
speculative		
Ratio of VEVs at intersection of 3 fixed points	$\overset{?}{\Leftrightarrow}$	Triangulation of singularity
F-flatness $F = 0$	$\overset{?}{\Leftrightarrow}$	Bianchi identity + triangulation

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold geometry

- Torus T^6 divided by $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{1\!\!1, \theta_1, \theta_2, \theta_1\theta_2\}$
- Space group $= \{$torus translations e_p, rotations $\theta_i\}$
- Geometry:

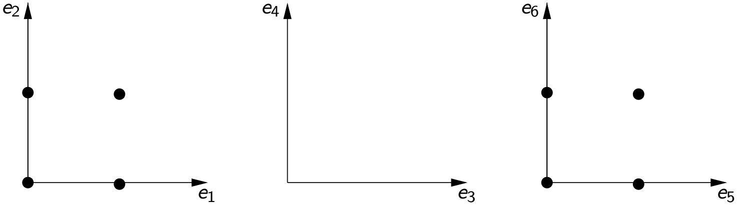
fixed points of θ_1 in T^6 torus:



$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold geometry

- Torus T^6 divided by $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{\mathbb{1}, \theta_1, \theta_2, \theta_1\theta_2\}$
- Space group $= \{$torus translations e_p, rotations $\theta_i\}$
- Geometry:

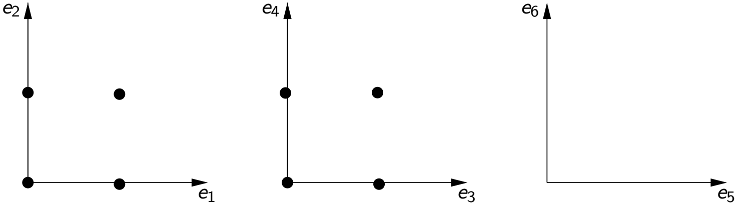
fixed points of θ_2 in T^6 torus:



$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold geometry

- Torus T^6 divided by $\mathbb{Z}_2 \times \mathbb{Z}_2 = \{1\!\!1, \theta_1, \theta_2, \theta_1\theta_2\}$
- Space group $= \{$torus translations e_p, rotations $\theta_i\}$
- Geometry:

fixed points of $\theta_1\theta_2$ in T^6 torus:



$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold spectrum

$\Rightarrow$ 16+16+16=48 fixed points

- Later: freely acting $\mathbb{Z}_2$ with Wilson line
  $\Rightarrow$ 24 fixed points and GUT breaking
- Embed space group into gauge d.o.f.
  as shifts V_i and Wilson lines W_p, i.e.

$$(\theta_1^k \theta_2^l, n_p e_p) \hookrightarrow V_{loc} = kV_1 + lV_2 + n_p W_p$$

- Compute massless spectrum:
  - Untwisted string: gauge group from $E_8 \times E_8$ and matter (vector-like for $\mathbb{Z}_2 \times \mathbb{Z}_2$)
  - Twisted matter: localized at fixed point

$$p_{sh}^2 = \frac{3}{2} \quad \text{with} \quad p_{sh} = p + V_{loc} \quad \text{no oscillator}$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold MSSM

- Shifts and Wilson lines

$$
\begin{aligned}
V_1 &= \left(\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{3}{4}, -\tfrac{3}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right) \left(1, 0, 0, 0, 0, 0, 0, 0\right) \\
V_2 &= \left(\tfrac{3}{4}, \tfrac{1}{4}, -\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{3}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right) \left(1, 0, 0, 0, 0, 0, 0, 0\right) \\
W_2 = W_4 = W_6 &= \left(-\tfrac{5}{4}, \tfrac{3}{4}, -\tfrac{3}{4}, \tfrac{9}{4}, -\tfrac{7}{4}, -\tfrac{3}{4}, \tfrac{5}{4}, -\tfrac{3}{4}\right) \left(-\tfrac{1}{4}, \tfrac{11}{4}, \tfrac{3}{4}, -\tfrac{3}{4}, -\tfrac{7}{4}, -\tfrac{3}{4}, \tfrac{5}{4}, \tfrac{3}{4}\right) \\
W_3 &= \left(-1, -1, 0, -2, 0, -2, 2, -3\right) \left(-\tfrac{7}{4}, -\tfrac{1}{4}, \tfrac{3}{4}, -\tfrac{1}{4}, -\tfrac{5}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{5}{4}\right) \\
W_5 &= \left(\tfrac{1}{4}, \tfrac{9}{4}, -\tfrac{13}{4}, \tfrac{11}{4}, \tfrac{3}{4}, \tfrac{11}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}\right) \left(\tfrac{3}{4}, \tfrac{1}{4}, -\tfrac{11}{4}, -\tfrac{3}{4}, -\tfrac{3}{4}, -\tfrac{1}{4}, -\tfrac{5}{4}, \tfrac{3}{4}\right)
\end{aligned}
$$

- 6 generations $\mathbf{10} + \overline{\mathbf{5}}$ of SU(5)

M. Blaszczyk, S. Groot Nibbelink, F. Ruehle, M. Trapletti and P. V.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold MSSM

- Freely acting $\mathbb{Z}_2 \Rightarrow$ non-local GUT breaking to SM group
- 3 generations of quarks and leptons plus vector-like exotics
- Supersymmetric configurations: $D = F = 0$
  c.f. talk by Michael Ratz

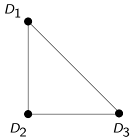
  R. Kappl, B. Petersen, M. Ratz, R. Schieren and P. V. in preparation

- Full blow-up:
  one blow-up mode (with p_{sh}) per fixed point gets VEV

Resolved orbifold

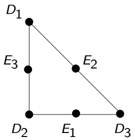
Resolution of singularity $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ using toric geometry

- local $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ singularity



with ordinary divisors D

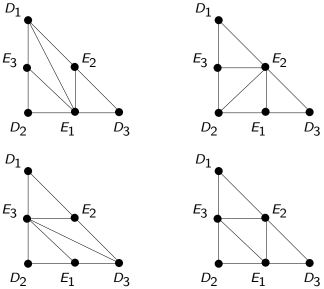
- add exceptional divisors E



Resolved orbifold

Resolution of singularity $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ using toric geometry

- triangulation



- different triangulations $\equiv$ different intersection numbers of divisors

MSSM on resolved orbifold

- Heterotic on resolved $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with U(1) gauge fluxes

$$\mathcal{F} = 2\pi V_r^I E_r H_I \quad \text{sum } r = 1, \ldots, 48,\ H_I \text{ Cartan}$$

- Quantization condition

$$\int_C \mathcal{F} = 2\pi \Lambda_I H_I \quad \text{for curve } C \text{ and } \Lambda \in \mathrm{E}_8 \times \mathrm{E}_8 \text{ lattice}$$

$\Rightarrow V_r \sim$ 8 indep. vectors $\Leftrightarrow$ shifts and Wilson lines

- $V_r = p_{sh}$ of blow-up mode
- Bianchi identity $\int_S dH = 0 \Rightarrow \sum V_r^2 = 24$ solved by $p_{sh}^2 = 3/2$
- Kähler form $J = a_i R_i - b_r E_r$

$$\text{VEV of blow-up mode } v_r^2 \sim \text{vol}(E_r) \sim a_i b_r + b_s^2 - b_t^2$$

- Donaldson-Uhlenbeck-Yau

$$\sum \text{vol}(E_r)\, V_r = \xi_{1L} \quad \Leftrightarrow \quad D = \sum v_r^2 p_{sh,r} = 0$$

MSSM on resolved orbifold

- Specify model by 48 vectors V_r
- Result: 6 generations of SU(5)
- Add freely acting involution with Wilson line
  $\Rightarrow$ GUT breaking and 3 generations of quarks and leptons
- Important: modular invariance condition from string partition function!

$\mathbb{Z}_2 \times \mathbb{Z}_2$ Orbifold		Resolution
shifts and Wilson lines	$\Leftrightarrow$	flux quantization condition
gauge momentum p_{sh} of blow-up mode	$\Leftrightarrow$	local U(1) gauge background $V_r, r = 1, \ldots, 48$
massless blow-up mode $p_{sh}^2 = \frac{3}{2}$	$\Rightarrow$	Bianchi identity $\sum_{\text{sector}} V_r^2 = 24$
(Abelian) D-flatness	$\approx$	Donaldson-Uhlenbeck-Yau $\sum_r \text{vol}(E_r)\, V_r = \xi_{1L}$
VEV of blow-up mode	$\Leftrightarrow$	Kähler modulus b_r
Gauge group after VEVs	$\Leftrightarrow$	Abelian gauge flux commutant
(redefined) twisted matter	$\approx$	Matter from $E_8 \times E_8$ gauginos
speculative		
Ratio of VEVs at intersection of 3 fixed points	$\overset{?}{\Leftrightarrow}$	Triangulation of singularity
F-flatness $F = 0$	$\overset{?}{\Leftrightarrow}$	Bianchi identity + triangulation