On low-energy effective actions in three-dimensional supergauge models

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Reference

I.L. Buchbinder, N.G. Pletnev, I.B. Samsonov, JHEP 1004 (2010) 124
I.L. Buchbinder, N.G. Pletnev, I.B. Samsonov, to appear.

- Progress in studying M2 branes from the point of view of 3d supergauge theories
- Bagger-Lambert-Gustavsson (BLG) theory: $\mathcal{N}=8$ $d=3$ Chern-Simons matter theory with SU(2)×SU(2) gauge group
- Aharony-Bergman-Jafferis-Maldacena (ABJM) theory: $\mathcal{N}=6$ $d=3$ Chern-Simons matter theory with U(n)×U(n) gauge group
- Quantum aspects of multiple M2 branes?

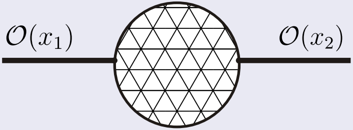
ABJM theory

O. Aharony, O. Bergman, D. Jafferis, J. Maldacena, JHEP 0810 (2008) 091
M. Benna, I. Klebanov, T. Klose, M. Smedback, JHEP 0809 (2008) 072

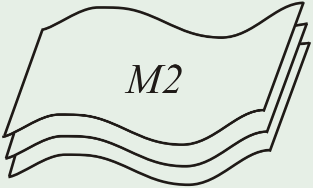
$$
\begin{aligned}
S_{ABJM} &= S_{CS} + S_{mat} + S_{pot} \\
S_{CS} &= \frac{k}{4\pi}\int d^3x d^4\theta \int_0^1 dt[V\bar{D}^\alpha(e^{tV}D_\alpha e^{-tV}) - \hat{V}\bar{D}^\alpha(e^{t\hat{V}}D_\alpha e^{-t\hat{V}})] \\
S_{mat} &= -\int d^3x d^4\theta \left[\bar{Q}_+^a e^V Q_{+a} e^{-\hat{V}} + \bar{Q}_-^a e^{\hat{V}} Q_{-a} e^V\right] \\
S_{pot} &= \int d^3x d^2\theta\, W(Q) + \int d^3x d^2\bar{\theta}\, \bar{W}(\bar{Q})
\end{aligned}
$$

- k is the Chern-Simons level
- V and $\hat{V}$ are two gauge superfields
- Q_{+a}, Q_{-a}, $a = 1, 2$ are chiral superfields
- Gauge group is U(n)×U(n), the supersymmetry is $\mathcal{N} = 6$
- When the gauge group is SU(2)×SU(2) the supersymmetry is raised up to $\mathcal{N} = 8$ (BLG theory)
- The model describes the stack of M2 branes on the $\mathbb{C}^4/\mathbb{Z}_k$ background

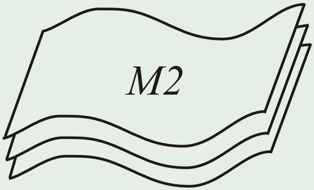
Correlation functions of composite operators $\langle\mathcal{O}_1\ldots\mathcal{O}_n\rangle$: scattering of M2 branes



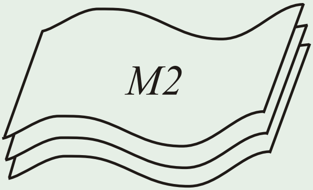
The effective action $\Gamma[\Phi]$: effective quantum dynamics of a stack of M2 branes



The effective action $\Gamma[\Phi]$: effective quantum dynamics of a stack of M2 branes



The effective action $\Gamma[\Phi]$: effective quantum dynamics of a stack of M2 branes



Alert

The M2 brane is a strongly coupled system. The perturbation theory may make no sense for the ABJM or BLG theory

Consider $\mathcal{N}=4$ $d=4$ SYM with gauge group SU(n)

- Give a vev to a scalar $\langle\phi\rangle\neq 0$
- Spontaneous gauge symmetry breaking, say SU(n)$\to$U(1)$^{n-1}$
- The fields in the U(1)$^{n-1}$ are massless, others are massive
- Integrate out the massive fields (internal lines)
- Get the effective action for the massless ones

$$\Gamma_{bos}\sim\int d^4x\left(F^2+\frac{F^4}{\phi^4}+\frac{F^6}{\phi^8}+\ldots\right)$$

Consider the ABJM theory with gauge group U(n)×U(n)

- Give a vev to a scalar $v = \langle X^8 \rangle \neq 0$
- Spontaneous gauge symmetry breaking U(n)×U(n)→U(n)$_{\mathrm{diagonal}}$
- No separation of light and heavy fields
- Instead of this we get the quantum field theory for $\mathcal{N} = 8$ SYM (D2 brane) rather than for the M2 brane.

$$S_{ABJM} \to S_{\mathcal{N}=8\ SYM} + O(\frac{1}{v}).$$

We can study the effective action in the $\mathcal{N} = 8$ SYM model rather than in the ABJM theory.

Goals of our research

We study the effective actions in the following $d = 3$ models:

1. $\mathcal{N} = 2$ chiral superfield in the gauge superfield
2. $\mathcal{N} = 4$ hypermultiplet in external $\mathcal{N} = 4$ gauge superfield
3. $\mathcal{N} = 2$ SYM
4. $\mathcal{N} = 4$ SYM
5. $\mathcal{N} = 8$ SYM (D2 brane)

Classical action

$$S = -\int d^3x d^4\theta\, \bar{Q} e^{2V} Q$$

Effective action

$$\Gamma = \int d^3x d^4\theta\, \mathcal{L}(V, G, D_\alpha G, \bar{D}_\alpha G, D_\alpha \bar{D}_\beta G)$$

here $G = \frac{i}{2}\bar{D}^\alpha D_\alpha V$ is the abelian superfield strength

The effective action should be gauge and superconformal invariant. $\Rightarrow$

$$\begin{aligned}
\Gamma &= \Gamma_{\rm CS} + \Gamma_{\rm Maxwell} + \Gamma_{\rm higher} \\
\Gamma_{\rm CS} &\sim \int d^3x d^4\theta\, VG \sim \int d^3x\, \varepsilon_{mnp} A^m F^{np} \\
\Gamma_{\rm Maxwell} &\sim \int d^3x d^4\theta\, G \ln G \sim \int d^3x \frac{F^2}{\phi} \\
\Gamma_{\rm higher} &\sim \int d^3x \sum_{n=2}^{\infty} \frac{F^{2n}}{\phi^{4n-3}}
\end{aligned}$$

Direct quantum computations

$$\begin{aligned} \Gamma &= \frac{i}{2}\mathrm{tr}\ln\Box_+ \\ \Box_+ &= \nabla^m\nabla_m + G^2 + \frac{i}{2}D^\alpha W_\alpha + iW^\alpha\nabla_\alpha \end{aligned}$$

As a result we get

$$\begin{aligned} \Gamma_{\rm CS} &= \frac{1}{4\pi}\int d^3x d^4\theta\, V G \\ \Gamma_{\rm Maxwell} &= \frac{1}{4\pi}\int d^3x d^4\theta\, G\ln G \\ \Gamma_{\rm higher} &= \frac{1}{32\pi}\int d^3x d^4\theta\, G\frac{\Psi^2}{\Omega^2}\int_0^\infty \frac{dte^{it}}{\sqrt{i\pi t}}\left(\frac{\tanh(t\Omega)}{t\Omega}-1\right) \end{aligned}$$

where

$$\left.\begin{aligned} \Psi &= \tfrac{i}{G}\bar D^\alpha D_\alpha \ln G \\ \Omega^2 &= \tfrac{1}{8}(\tfrac{1}{G}\bar D^\alpha D_\alpha)^2\ln G \end{aligned}\right\}\mathcal{N}=2 \text{ superconformal invariants}$$

Classical action

$$S=-\int d^3xd^4\theta(\bar{Q}_+e^{2V}Q_+ + \bar{Q}_-e^{-2V}Q_-) - \left(\int d^3xd^2\theta\, Q_+\Phi Q_- + c.c.\right)$$

here (Q_+,Q_-) $-$ hypermultiplet $\qquad (V,\Phi)$ $-$ $\mathcal{N}=4$ vector multiplet

Effective action

$$\begin{aligned}\Gamma &= \frac{1}{2\pi}\int d^3xd^4\theta\Big[-\sqrt{G^2+\bar{\Phi}\Phi}+G\ln(G+\sqrt{G^2+\bar{\Phi}\Phi})\\ &+\frac{1}{8}\frac{\mathbf{\Phi}^2}{\mathbf{\Omega}^2}\sqrt{G^2+\bar{\Phi}\Phi}\int_0^\infty\frac{dte^{it}}{\sqrt{i\pi t}}\left(\frac{\tanh(t\mathbf{\Omega})}{t\mathbf{\Omega}}-1\right)\Big]\end{aligned}$$

here

$$\begin{aligned}\mathbf{\Psi} &= \frac{i}{G}\bar{D}^\alpha D_\alpha\ln(G+\sqrt{G^2+\bar{\Phi}\Phi})\,,\\ \mathbf{\Omega}^2 &= \frac{1}{8}\frac{1}{\sqrt{G^2+\bar{\Phi}\Phi}}\bar{D}^\alpha D_\alpha\frac{1}{G}\bar{D}^\beta D_\beta\ln(G+\sqrt{G^2+\bar{\Phi}\Phi})\,.\end{aligned}$$

Classical action

$$S = -\int d^3x d^4\theta (\bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_-) - \left(\int d^3x d^2\theta \, Q_+ \Phi Q_- + c.c. \right)$$

here (Q_+, Q_-) − hypermultiplet (V, Φ) − $\mathcal{N}=4$ vector multiplet

Effective action

$$\begin{aligned} \Gamma &= \frac{1}{2\pi} \int d^3x d^4\theta \Big[-\sqrt{G^2 + \bar{\Phi}\Phi} + G \ln(G + \sqrt{G^2 + \bar{\Phi}\Phi}) \\ &\quad + \frac{1}{8} \frac{\mathbf{\Phi}^2}{\mathbf{\Omega}^2} \sqrt{G^2 + \bar{\Phi}\Phi} \int_0^\infty \frac{dt e^{it}}{\sqrt{i\pi t}} \left(\frac{\tanh(t\mathbf{\Omega})}{t\mathbf{\Omega}} - 1 \right) \Big] \end{aligned}$$

here

$$\begin{aligned} \mathbf{\Psi} &= \frac{i}{G} \bar{D}^\alpha D_\alpha \ln(G + \sqrt{G^2 + \bar{\Phi}\Phi}) \,, \\ \mathbf{\Omega}^2 &= \frac{1}{8} \frac{1}{\sqrt{G^2 + \bar{\Phi}\Phi}} \bar{D}^\alpha D_\alpha \frac{1}{G} \bar{D}^\beta D_\beta \ln(G + \sqrt{G^2 + \bar{\Phi}\Phi}) \,. \end{aligned}$$

Classical action

$$S_{\rm GW} = \int d^3x d^4\theta [V\hat{G} + \bar{Q}_+ e^{2V} Q_+ + \bar{Q}_- e^{-2V} Q_-]$$

$(V, \hat{V})$ – two U(1) gauge superfields with superfield strengths $(G, \hat{G})$

Equation of motion for V:

$$\hat{G} + 2\bar{Q}_+ e^{2V} Q_+ - 2\bar{Q}_- e^{-2V} Q_- = 0 \quad \Rightarrow \quad V = V(\hat{G}, Q_+, Q_-)$$

Dual form of the GW action

$$\tilde{S}_{\rm GW} = \int d^3x d^4\theta [-\sqrt{\hat{G}^2 + \bar{\Phi}\Phi} + \hat{G}\ln(\hat{G} + \sqrt{\hat{G}^2 + \bar{\Phi}\Phi})]$$

where $\Phi = Q_+ Q_-$

The dual action of the Gaiotto-Witten theory is induced as the leading quantum contribution to the hypermultiplet effective action

$\mathcal{N}=2$ SYM

$$S=\frac{1}{g^2}\mathrm{tr}\int d^3xd^4\theta\, G^2\,, \qquad [g]=\frac{1}{2}$$

One-loop effective action is scale invariant

$$\Gamma^{(1)}=-\frac{3}{2\pi}\int d^3xd^4\theta[G\ln G+\ldots]\sim\int d^3x\frac{F^2}{\phi}+\ldots$$

$\mathcal{N}=4$ SYM

$$S=\frac{1}{g^2}\mathrm{tr}\int d^3xd^4\theta[G^2-\frac{1}{2}e^{-2V}\bar\Phi e^{2V}\Phi]$$

$$\Gamma^{(1)}=-\frac{1}{\pi}\int d^3xd^4\theta[-\sqrt{G^2+\bar\Phi\Phi}+G\ln(G+\sqrt{G^2+\bar\Phi\Phi})+\ldots]\sim\int d^3x\frac{F^2}{\phi}+\ldots$$

Dual action of the Gaiotto-Witten model is induced as the low-energy contribution in the $\mathcal{N}=4$ SYM

The stack of M2 branes can be considered as an effective theory for the stack of D2 branes at the strong coupling

$$\begin{aligned} \mathcal{N}=4 \text{ SYM} &\approx \text{D2 brane} \\ \mathcal{N}=4 \text{ Gaiotto-Witten theory} &\approx \text{M2 brane} \end{aligned}$$

⇒ The Gaiotto-Witten theory should appear in some limit form the effective action of $\mathcal{N}=4$ SYM

Problem

Is it possible to extend this correspondence to the case of the ABJM/BLG theory and $\mathcal{N}=8$ SYM?

Abelian ABJM theory

$$S_{\rm ABJM} = -\int d^3x d^4\theta [V\hat{G} + \bar{Q}^a_+ e^{2V} Q_{+a} + \bar{Q}^a_- e^{-2V} Q_{-a}]$$

here (Q^a_+, Q^a_-), – two hypermultiplets, $a = 1, 2$
Dual form of the ABJM action:

$$\tilde{S}_{\rm ABJM} = \int d^3x d^4\theta [\hat{G}\ln\hat{G} + O(Q)] \sim \int d^3x\, F^2/\phi$$

$\mathcal{N} = 8$ SYM

$$S = \frac{1}{g^2}{\rm tr}\int d^3x d^4\theta [G^2 - \frac{1}{2}e^{-2V}\bar{\Phi}^i e^{2V}\Phi_i] + \frac{1}{12g^2}\left({\rm tr}\int d^3x d^2\theta\, \varepsilon^{ijk}\Phi_i[\Phi_j, \Phi_k] + c.c.\right)$$

Here Φ_i – three chiral superfields, $i = 1, 2, 3$.
Effective action

$$\Gamma \sim \int d^3x d^4\theta \frac{W^2\bar{W}^2}{(G^2 + \bar{\Phi}^i\Phi_i)^{5/2}} + \ldots \sim \int d^3x \frac{F^4}{\phi^5} + \ldots.$$

Conclusions

1. The abelian Gaiotto-Witten model appears as the part in the effective action in $\mathcal{N} = 4$ SYM
2. The ABJM action cannot appear as the part of the effective action in $\mathcal{N} = 8$ SYM
3. Can the ABJM theory appear as a part of the low-energy effective action in $\mathcal{N} = 2$ quiver gauge theory with four chirals (D2 brane probing conifold singularoty)? - work in progress