

# SUSY-Yukawa Sum Rule at the LHC

David Curtin

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In Collaboration with Maxim Perelstein, Monika Blanke



Cornell Institute for High Energy Phenomenology

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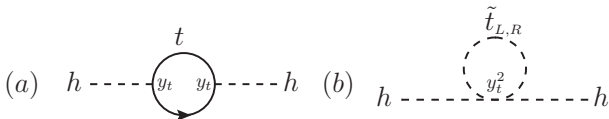
Monday August 23, 2010

1. Introducing the SUSY-Yukawa Sum Rule
2. How can we use it at the LHC?
3. A Simple LHC Case Study

# Introducing the SUSY-Yukawa Sum Rule

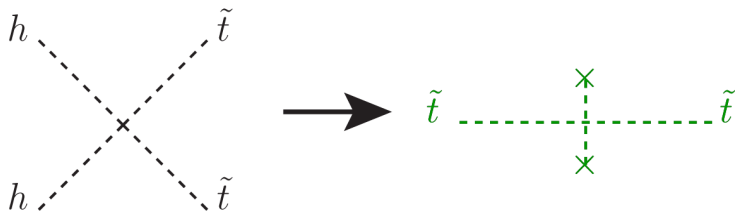
# Introduction

- **Hierarchy problem:** In the SM, Higgs mass receives quadratically divergent corrections, most importantly from the top quark
- In **SUSY**, top contribution cancelled by stop



- This relies on both **particle content** and **coupling relations**. **We want to test the coupling relations.**

# How to probe the Quartic Higgs Coupling?



$$\begin{aligned}
 M_{\tilde{t}_i \tilde{t}_j}^2 &= \begin{bmatrix} M_L^2 + \hat{m}_t^2 + g_{UL} \hat{m}_Z^2 c_{2\beta} & m_t (A_t + \mu \cot \beta) \\ m_t (A_t + \mu \cot \beta) & M_T^2 + \hat{m}_t^2 + g_{UR} \hat{m}_Z^2 c_{2\beta} \end{bmatrix} \\
 &= \begin{bmatrix} m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 & c_t s_t (m_{t1}^2 - m_{t2}^2) \\ c_t s_t (m_{t1}^2 - m_{t2}^2) & m_{t1}^2 s_t^2 + m_{t2}^2 c_t^2 \end{bmatrix}
 \end{aligned}$$

Extract this contribution to diagonal sfermion mass terms!

# SUSY-Yukawa Sum Rule

Consider stop/sbottom  $LL$  mass terms at tree level:

$$M_{\tilde{t}_L \tilde{t}_L}^2 = M_L^2 + \hat{m}_t^2 + g_{uL} \hat{m}_Z^2 \cos 2\beta = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 \quad (1)$$

$$M_{\tilde{b}_L \tilde{b}_L}^2 = M_L^2 + \hat{m}_b^2 + g_{bL} \hat{m}_Z^2 \cos 2\beta = m_{b1}^2 c_b^2 + m_{b2}^2 s_b^2 \quad (2)$$

Soft masses Higgs Quartic Coupling D-term contributions measurable

(1) – (2) eliminates the soft mass:

$$\hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta$$

We call this the **SUSY-Yukawa Sum Rule**: It has its origins in the same coupling relations that cancel higgs mass corrections.

**Testing this sum rule at a collider would constitute a highly nontrivial check on SUSY.**

# How to test the sum rule?

## SUSY-Yukawa Sum Rule:

$$\hat{m}_t^2 - \hat{m}_b^2 = \mathbf{m}_{t1}^2 \mathbf{c}_t^2 + \mathbf{m}_{t2}^2 \mathbf{s}_t^2 - \mathbf{m}_{b1}^2 \mathbf{c}_b^2 - \mathbf{m}_{b2}^2 \mathbf{s}_b^2 - \hat{m}_Z^2 \cos^2 \theta_W \cos 2\beta$$

Define an observable  $\Upsilon$  for which the sum rule gives a definite prediction:

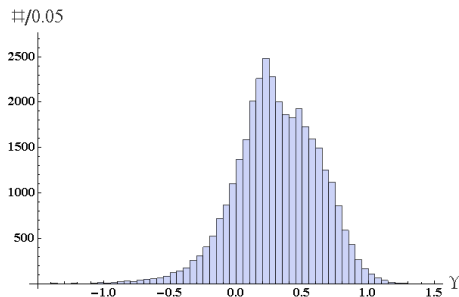
$$\Upsilon \equiv \frac{1}{v^2} \left( m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 \right)$$

## Tree-Level Prediction for $\Upsilon$ from SUSY-Yukawa Sum Rule

$$\begin{aligned} \Upsilon_{\text{SUSY}}^{\text{tree}} &= \frac{1}{v^2} \left( \hat{m}_t^2 - \hat{m}_b^2 + m_Z^2 \cos^2 \theta_W \cos 2\beta \right) \\ &= \begin{cases} 0.39 & \text{for } \tan \beta = 1 \\ \mathbf{0.28} & \text{for } \tan \beta \rightarrow \infty \text{ (converges quickly for } \tan \beta \gtrsim 5) \end{cases} \end{aligned}$$

# Radiative Corrections

- **Radiative Corrections** wash out SUSY tree-level prediction for  $\Upsilon$
- MSSM Parameter Scan with `SuSpect` for  $M_{\text{SUSY}} < 2 \text{ TeV}$ :



- $\Upsilon_{\text{SUSY}}^{\text{tree}} \approx 0.3 \longrightarrow |\Upsilon_{\text{SUSY}}| \lesssim 1$   
(For comparison, the 'generic' perturbative theory prediction is  $|\Upsilon| \lesssim 16\pi^2$ .)
- Sidenote: R.C. can be constrained by measuring **additional parameters**.



- Introduce the SUSY-Yukawa Sum Rule,

$$\hat{m}_t^2 - \hat{m}_b^2 = m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta$$

which relies on the same coupling relations that cancel contributions from stop & top loops to the higgs coupling.

- Introduce **new observable** which can be measured at a collider:

$$\Upsilon \equiv \frac{1}{v^2} \left( m_{t1}^2 c_t^2 + m_{t2}^2 s_t^2 - m_{b1}^2 c_b^2 - m_{b2}^2 s_b^2 \right)$$

- SUSY-Yukawa Sum Rule  $\Rightarrow |\Upsilon| \lesssim 1$
- Measuring  $\Upsilon$  constitutes a **powerful nontrivial check** that SUSY is the solution to the hierarchy problem.

How can we use the  
SUSY-Yukawa Sum Rule  
at the LHC?

# SUSY-Yukawa Sum Rule at the LHC

- To measure every ingredient of  $\Upsilon$  (especially  $\theta_b$ ) we probably need a lepton collider.
- What good is the sum rule at the LHC?  $\Rightarrow$  Use SUSY-prediction for  $\Upsilon$  to constrain 'unmeasurable' parameters!
- Which parts can we measure? Often  $m_{t1}$ ,  $m_{b1}$  are the lightest two masses.

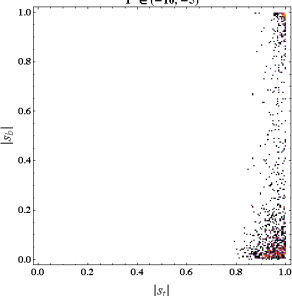
$$\Upsilon = \underbrace{\frac{1}{\sqrt{2}} (m_{t1}^2 - m_{b1}^2)}_{\Upsilon'} + \underbrace{\frac{s_t^2}{\sqrt{2}} (m_{t2}^2 - m_{t1}^2)}_{\Delta\Upsilon_t} - \underbrace{\frac{s_b^2}{\sqrt{2}} (m_{b2}^2 - m_{b1}^2)}_{\Delta\Upsilon_b}$$

We can try to measure  $\Upsilon'$  at the LHC.

# What does $\Upsilon'$ tell us?

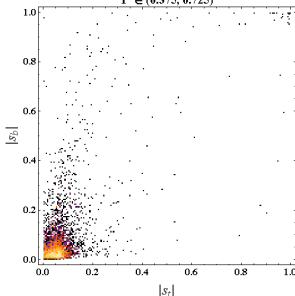
$$\Upsilon = \Upsilon' + \frac{s_t^2}{v^2} (m_{t2}^2 - m_{t1}^2) - \frac{s_b^2}{v^2} (m_{b2}^2 - m_{b1}^2)$$

$\Upsilon' \in (-10, -5)$



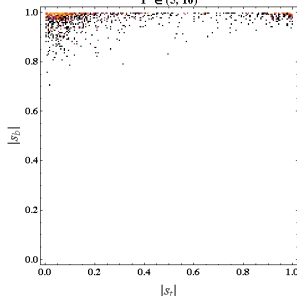
$\Upsilon' \ll 0 \Rightarrow \text{RH } \tilde{t}_1$

$\Upsilon' \in (0.375, 0.725)$



$\Upsilon' \sim 0 \Rightarrow \text{LH } \tilde{t}_1, \tilde{b}_1$

$\Upsilon' \in (5, 10)$



$\Upsilon' \gg 0 \Rightarrow \text{RH } \tilde{b}_1$

**Even a rough measurement of  $\Upsilon'$  gives strong constraints on the stop and/or sbottom mixing angles!**

# A Simple LHC Case Study

# Measuring $\Upsilon'$ at the LHC

- Want to demonstrate that the **SUSY-Yukawa Sum Rule** can be used to **measure stop & sbottom mixing angles at the LHC**.
- Choose a particular MSSM **Benchmark Point** with light  $\tilde{t}_1, \tilde{b}_1$  and small mixing:

## Parameters:

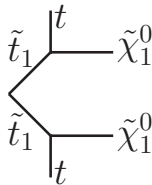
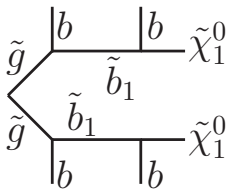
$\tan \beta$	$M_1$	$M_2$	$M_3$	$\mu$	$M_A$	$M_{Q3L}$	$M_{tR}$	$A_t$
10	100	450	450	400	600	310.6	778.1	392.6

## Spectrum: (GeV)

$m_{t1}$	$m_{t2}$	$s_t$	$m_{b1}$	$m_{b2}$	$s_b$	$m_{\tilde{g}}$	$m_{\tilde{\chi}_1^0}$
371	800	-0.095	341	1000	-0.011	525	98

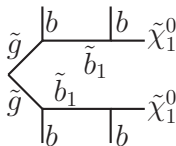
# Outline of Measurement

- **We will measure**  $\gamma' = \frac{1}{v^2}(m_{t1}^2 - m_{b1}^2)$
- Parton-level Analysis with gaussian momentum smearing. (More realistic analysis in progress.)
- **Gluino pair production**  $\implies m_{b1}$  (Bonus:  $m_{\tilde{g}}$  and  $m_{\tilde{\chi}_1^0}$ )
- **Stop pair production**  $\implies m_{t1}$



# (I) Gluino Pair Production

- Analyze the process<sup>1</sup>  
 $\tilde{g}\tilde{g} \rightarrow 2\tilde{b}_1 + 2b \rightarrow 4b + 2\tilde{\chi}_1^0$ .
- $\sigma_{\tilde{g}\tilde{g}} \approx 11.6 \text{ pb} @ \sqrt{s} = 14 \text{ TeV}$ .



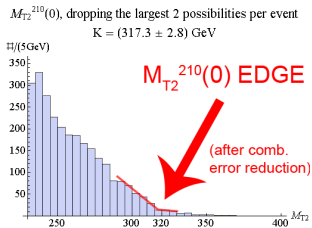
- Impose basic  $p_T$ , MET-cuts and require **4  $b$ -tags**.
- No SUSY-BG. SM-BG suppressed by  $b$ -tag requirement.
- Use  $\mathcal{L} = 10 \text{ fb}^{-1}$ . After cuts we are left with 4800 signal events.
- Even with parton-level pure signal, full mass extraction is challenging!

<sup>1</sup>MadGraph/Madevent & BRIDGE



# Edge Extraction & Mass Measurement

- To measure masses at hadron colliders with **invisible massive particles in the final state**, we go **Edge Hunting!**
- Distributions of  $M_{T2}$ -**subsystem-variables**<sup>2</sup> and  $M_{bb}$  show **edges** which tell us mass combinations.
- Big Problem: **Combinatorial Error** (especially for  $M_{T2}$ 's).



We are able to successfully measure  $M_{bb}$ ,  $M_{T2}^{210}(0)$  and  $M_{T2}^{220}(0)$  edges

⇒

mass	th.	68 % c.l.
$m_{b1}$	341	(316, 356)
$m_{\tilde{g}}$	525	(508, 552)
$m_{\tilde{\chi}_1^0}$	98	(45*, 115)

<sup>2</sup>Barr, Lester, Stephens, 2003; Cho, Choi, Kim, Park 2008; Burns, Kang, Matchev, Park 2009

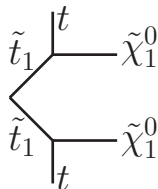
\* LEP bound

## (II) Stop Pair Production

- Analyze the process  $\tilde{t}_1 \tilde{t}_1^* \rightarrow t\bar{t} + 2\tilde{\chi}_1^0$ .

- $\sigma_{\tilde{t}_1 \tilde{t}_1^*} \approx 2 \text{ pb} @ \sqrt{s} = 14 \text{ TeV}$ .

- Impose standard cuts & use hadronic tops<sup>3</sup>.



- Use  $\mathcal{L} = 100 \text{ fb}^{-1}$ . After cuts: 1481 signal and 105 BG events.

- Easy to extract  $M_{T2}^{\text{max}}$  edge  $\implies$  Gives  $m_{t\tilde{t}_1}(m_{\tilde{\chi}_1^0})$

- Combine with (I)  $\implies$

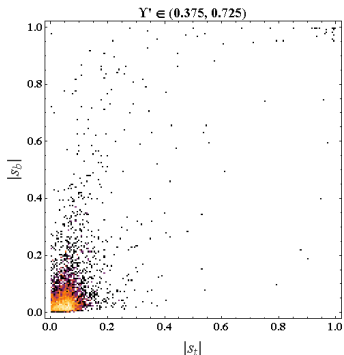
	th.	68 % c.l.
$m_{t\tilde{t}_1}$	371	(356, 414)

<sup>3</sup>Meade, Reece 2006

# $\Upsilon'$ Measurement and SUSY-prediction for $\Upsilon$

Putting all these measurements together, we get

	th.	meas.
$\Upsilon'$	0.350	$0.525^{+0.20}_{-0.15}$
$\Upsilon$	0.423	—



Unless there is a strong accidental cancellation, such a small  $\Upsilon'$  measurement implies that **both stop and sbottom mixing angles are small,  $\lesssim 0.2$ .**

Compare to actual values:

$$s_t = -0.095$$

$$s_b = -0.011$$

# Conclusions

# Summary & Conclusions

- Confirmation of the **SUSY-Yukawa Sum Rule**

$$\hat{m}_t^2 - \hat{m}_b^2 = m_{t_1}^2 c_t^2 + m_{t_2}^2 s_t^2 - m_{b_1}^2 c_b^2 - m_{b_2}^2 s_b^2 - \hat{m}_Z^2 \cos^2 \theta_w \cos 2\beta$$

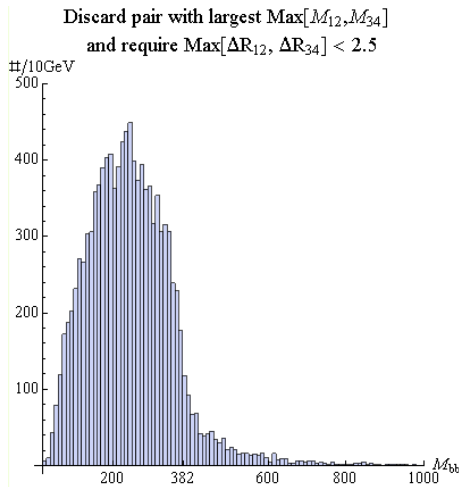
(probably at a **lepton collider**) would be strong support for TeV-scale SUSY as the solution for hierarchy problem.

- At the **LHC**, the sum rule provides powerful **constraints on stop and sbottom mixing angles** (hard to come by otherwise) using only a **mass measurement** .
- We developed **new techniques for reducing  $M_{T2}$ -combinatorial background**, allowing us to measure  $\tilde{t}_1, \tilde{b}_1, \tilde{g}, \tilde{\chi}_1^0$  masses at our benchmark point.

# Backup Slides

# Glino Pair Production: Kinematic Edge

- $M_{bb}^{\max} = \sqrt{\frac{(m_{\tilde{g}}^2 - m_{b_1}^2)(m_{b_1}^2 - m_{\tilde{\chi}_1^0}^2)}{m_{b_1}^2}}$
- With known decay chain assignments get  $(M_{b_1 b_2}, M_{b_3 b_4})$  for each event, plot  $M_{bb}$ -distribution  $\Rightarrow$  edge at 382 GeV.
- Main problem: **Combinatorial Background!**
- Can reduce CB with  $\Delta R$  cuts and dropping largest  $M_{bb}$ 's per event.

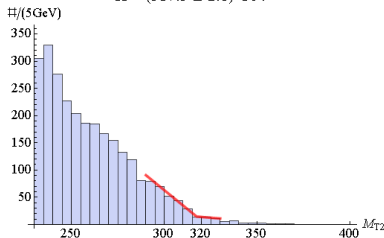
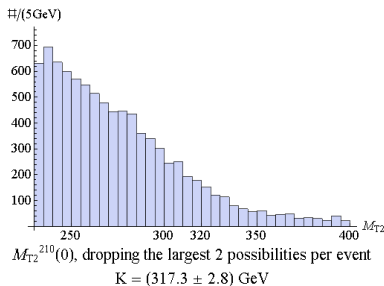


$$M_{bb\text{meas}}^{\max} = 395 \pm 15 \text{ GeV}$$

# Glino Pair Production: $M_{T2}$ -subsystem Edges

- The distributions of  $M_{T2}$  subsystem variables<sup>4</sup> also have edges we can measure. Look at  $M_{T2}^{210}(0)$ .
- **Combinatorial Background** is more dangerous.
  - To calculate  $M_{T2}^{210}$ , have to divide  $4b$  into an **upstream** and **downstream** pair: **6 possibilities**.
  - The  $M_{T2}$ -distribution for **wrong** pairings is **more featured** than  $M_{bb}$ .
- One way to reduce CB:  
**Drop largest 2  $M_{T2}^{210}$ 's per event** →

$M_{T2}^{210}(0)$  without combinatorial error reduction



<sup>4</sup>Barr, Lester, Stephens, 2003; Cho, Choi, Kim, Park 2008; Burns, Kang, Matchev, Park 2009



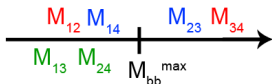
# Glino Pair Production: $M_{T2}$ -subsystem Edges

Another way to reduce CB:

Use Kinematic Edge Measurement!

Possible  $M_{bb}$  pairs:  $(M_{12}, M_{34})$ ,  $(M_{13}, M_{24})$ ,  $(M_{14}, M_{23})$

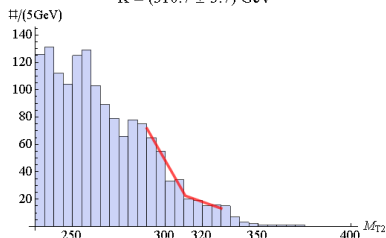
For ~30% of events, situation like:



Can deduce correct decay chain assignment!

$M_{T2}^{210}(0)$  with known decay chain assignment

$K = (310.7 \pm 3.7) \text{ GeV}$



For edge measurement, require two methods to agree!

edge	th.	measurement
$M_{bb}$	382	$395 \pm 15$
$M_{T2}^{210}(0)$	321	$314 \pm 13 \text{ GeV}$
$M_{T2}^{220}(0)$	507	$492 \pm 14 \text{ GeV}$



mass	th.	68 % c.l.
$m_{b1}$	341	(316, 356)
$m_{\tilde{g}}$	525	(508, 552)
$m_{\tilde{\chi}_1^0}$	98	(45, 115)

(Imposed  $m_{\tilde{\chi}_1^0} > 45 \text{ GeV}$  bound from LEP measurement of invisible  $Z$  decay width.)