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Relativistic plasmas for TeV acceleration

Dortmund, 2010

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Outline

Two pathways to TeV energies using plasmas:

- **Laser-driven acceleration in the bubble regime**
- **Proton beam-driven plasma wake fields**

Basic Plasma Parameters

- Plasma (Langmuir) frequency $$\omega_p^2 = \frac{4\pi n_p e^2}{m\langle\gamma\rangle}$$
- Plasma wave number $$k_p = \frac{\omega_p}{c}$$
- Plasma wavelength $$\lambda_p = \frac{2\pi}{k_p},$$

$$\lambda_p = 1\mu\text{m for } n_p = 10^{21}\text{cm}^{-3}$$

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Important laser parameters

Dimensionless laser amplitude $$a = \frac{eA}{mc^2}$$

relativistic when $a \approx 1 \leftrightarrow I\lambda^2 = 1.37\times10^{18}$ W μm^2/cm^2

Critical plasma density $$N_c = \frac{\omega_0^2 m}{4\pi e^2} \sim 10^{21}\ \text{cm}^{-3}$$

S-number $$S = \frac{N}{aN_c}$$

Gordienko & Pukhov
Phys. Plasmas **12**, 043109 (2005)

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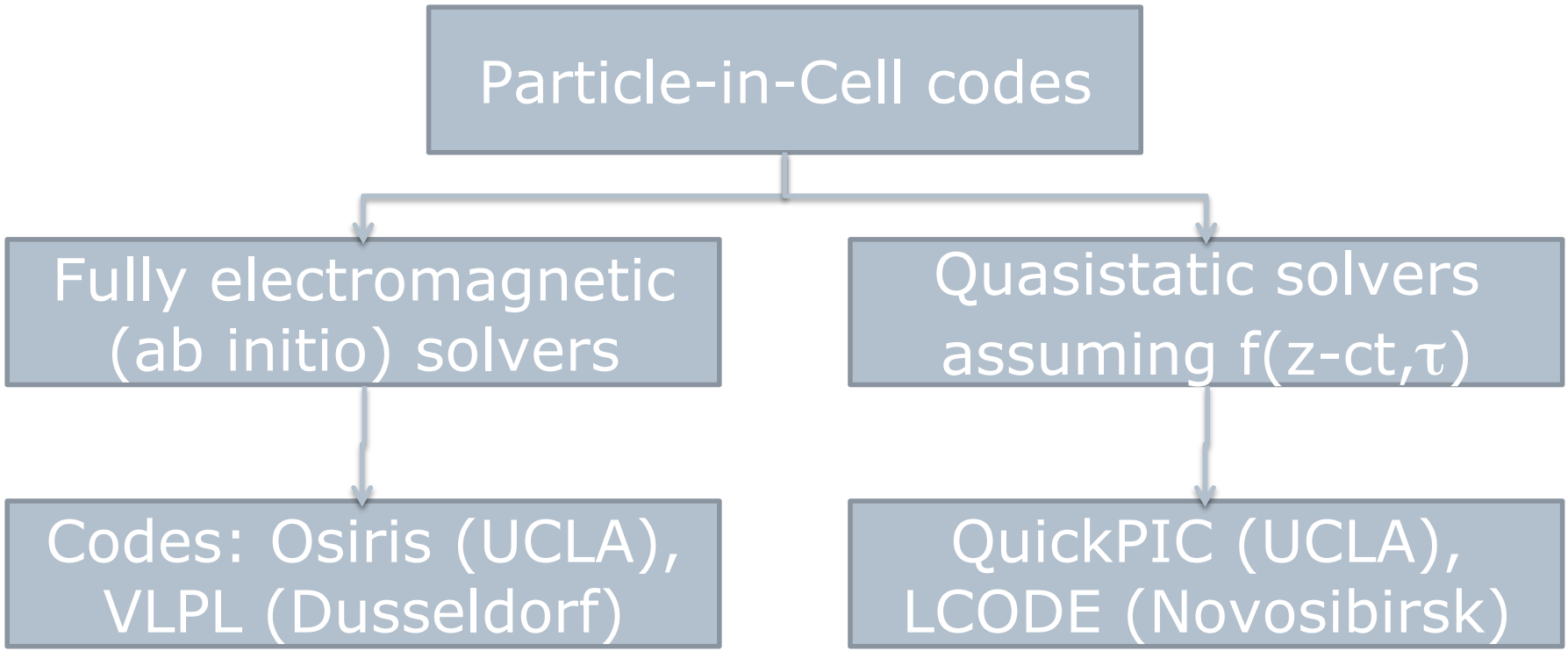
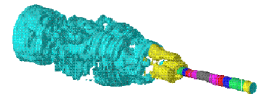
Numerical methods

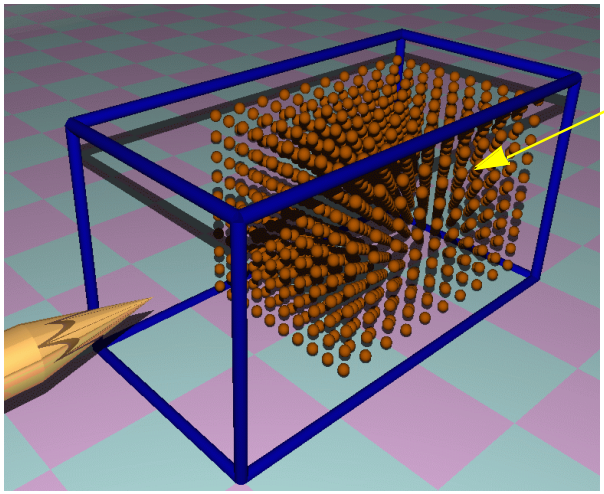


Virtual Laser Plasma Lab
3D PIC Code



VLPL

A. Pukhov, *J. Plasma Physics, vol.* 61, *part* 3, *p.* 425 (1999).



Plasma or neutral gas

Gas of an arbitrary element can be used.

The code VLPL is written in C++,
strongly using Object Oriented Programming
and parallelized using MPI.

Fields:

$$\mathrm{rot}\bar{B} = \frac{1}{c}\frac{\partial \bar{E}}{\partial t} + \frac{4\pi}{c}\bar{j}$$

$$\mathrm{rot}\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t}$$

$$\mathrm{div}\bar{E}4 = \pi\rho$$

$$\mathrm{div}\bar{B}0 =$$

Particles:

$$\frac{d\bar{p}}{dt} = q\bar{E} + \frac{q}{m\gamma}[\bar{p}\times\bar{B}]$$

$$\gamma = \sqrt{1 + \frac{p^2}{m^2c^2}}$$

Up to 10^9 particles and 10^8 mesh cells,
linearly scalable up to thousands of nodes

Laser Wakefield Acceleration

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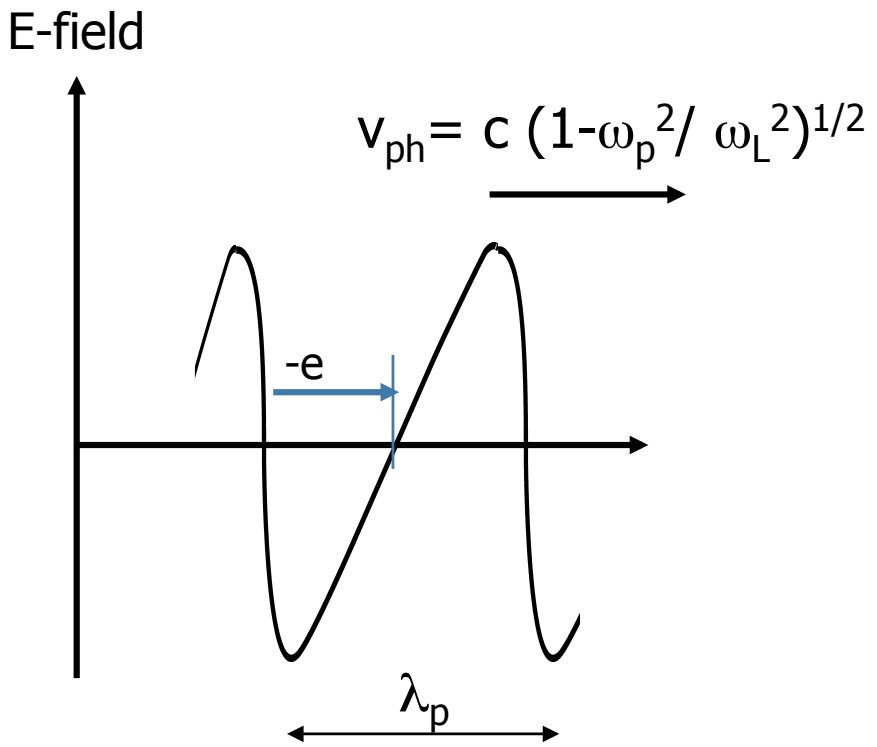
Tajima, Dawson, PRL43, 267 (1979)



Plasma wave γ -factor

$$\gamma_{ph}=(1-\,v_{ph}^2/c^2)^{-1/2}=\,\omega_p\,/\omega_L$$

Maximum particle energy

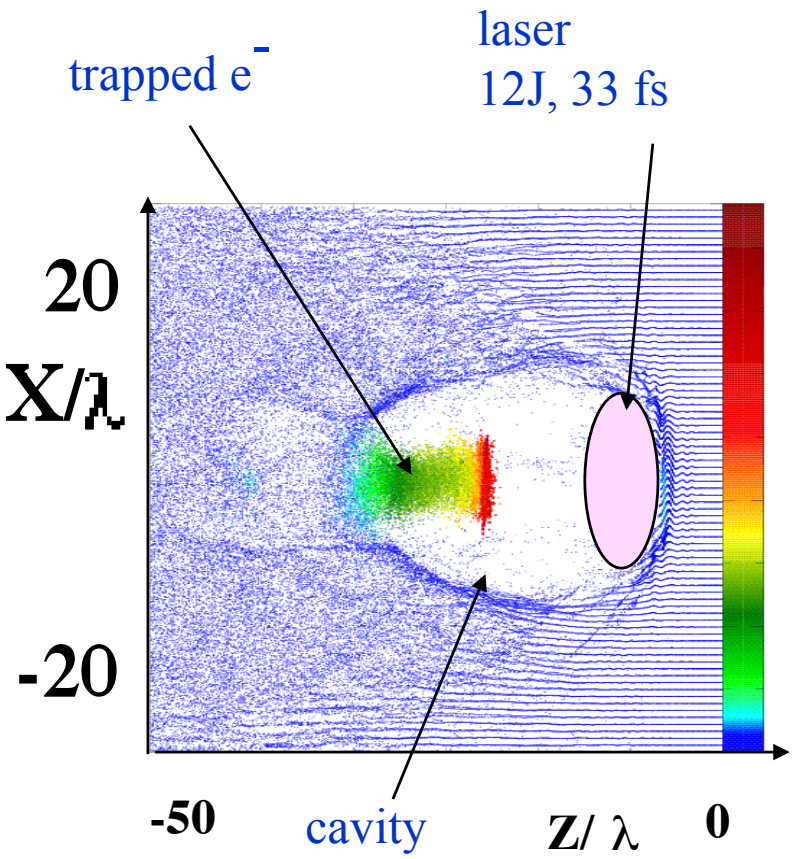
$$W_{max} = eE_{max}\,L_d$$

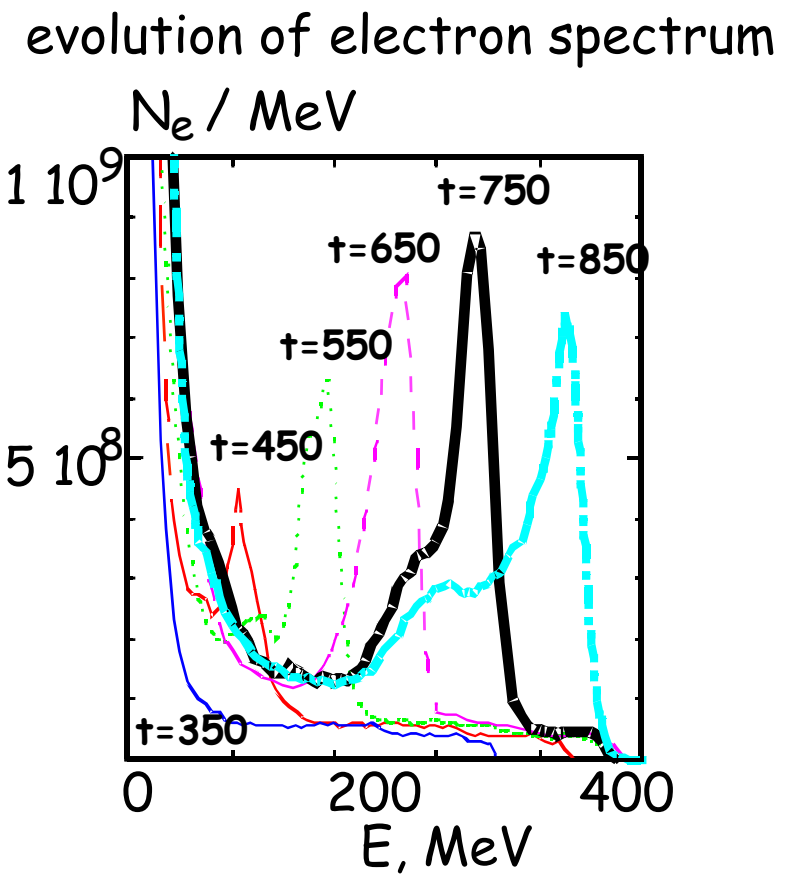
Dephasing length

$$L_d \approx \gamma_{ph}^2\,\lambda_p$$

Laser electron acceleration: the bubble regime

A.Pukhov & J.Meyer-ter-Vehn, *Appl. Phys. B,* **74,** *p.*355 (2002)





Scaling Bubble towards TeV

We take the pulse intensity to be 5×10^{19} W/cm^2 and use the pulse duration τ=3 ps, focal spot R=1 mm.

$$W_{\text{laser}} = W_0\left(\frac{1\,\text{TeV}}{E_0}\right)^{3/2} = 9\,\text{J}*\left(\frac{1\,\text{TeV}}{1\,\text{GeV}}\right)^{3/2} = 300\,\text{kJ}$$

the acceleration distance scales as

$$L_{\text{acc}} = L_0\left(\frac{1\,\text{TeV}}{E_0}\right)^{3/2} = 1\,\text{cm}*\left(\frac{1\,\text{TeV}}{1\,\text{GeV}}\right)^{3/2} = 300\,\text{m}$$

Second path to TeV energies: Charged beams as drivers

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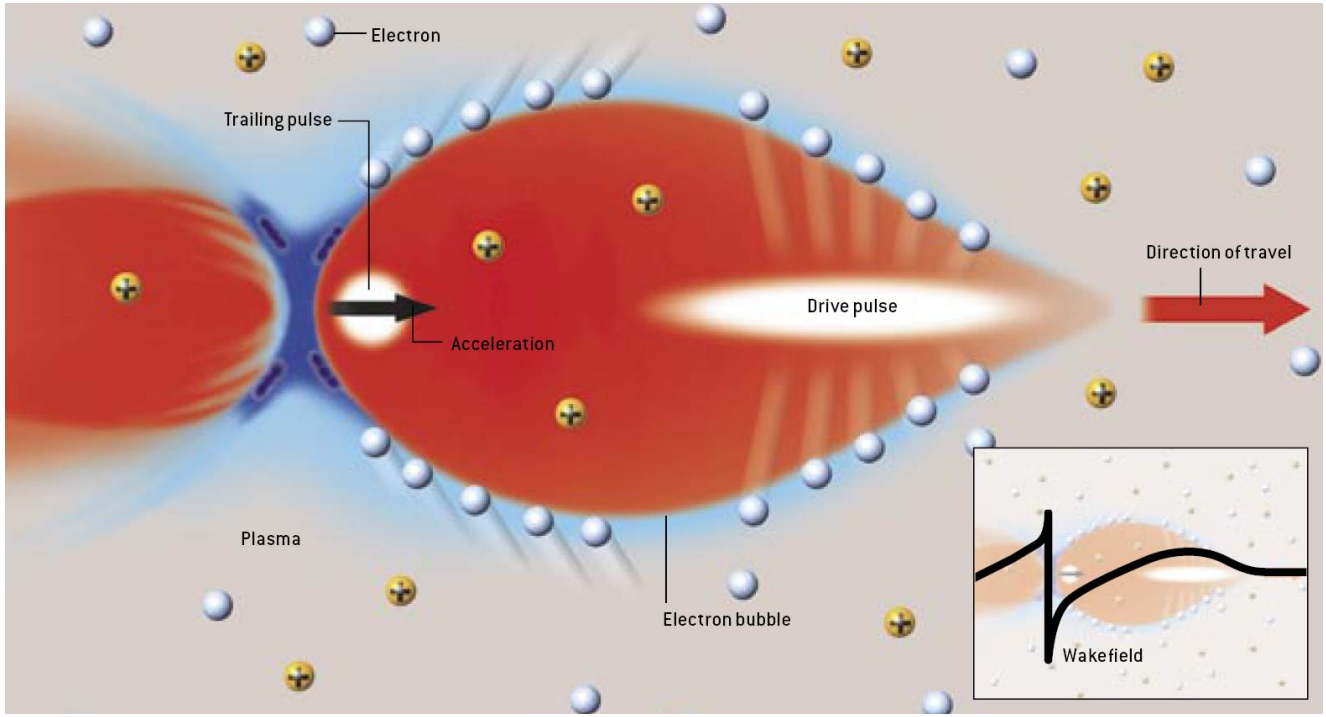
Electron beam driven wake field in the blowout regime



Ch. Joshi, UCLA

Central issue for beam-driven accelerators

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We do have TeV proton beams,
Let us use them as the driver!

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Nonlinear regimes

- Negatively charged driver:

“blowout”

- Positively charged driver:

“suck-in”

Caldwell, Lotov, Pukhov, Simon, Nature Phys. **5**, 363 (2009)

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Proton-driven plasma wake field acceleration

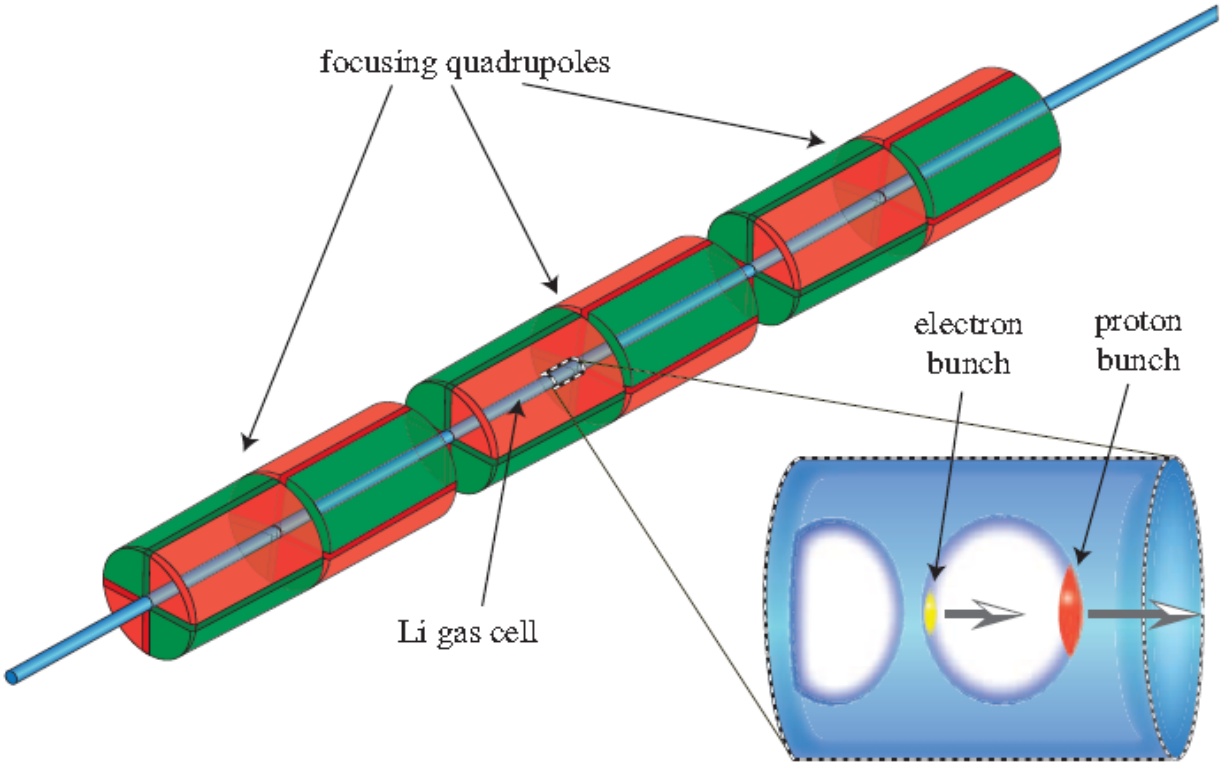

Figure 1: *A schematic description of a section of the plasma wakefield accelerating structure. A thin tube (5 mm diameter) containing Li gas is surrounded by quadrupole magnets with alternating polarity. The quadrupole magnets are 70 cm long. The blow-up shows the plasma bubble created by the proton bunch (red). The electron bunch (yellow) undergoing acceleration is located at the back of the bubble. Note that the dimensions are not to scale.*

Short proton driver, $\tau\omega_p \sim 1$: Excellent wakefield generation

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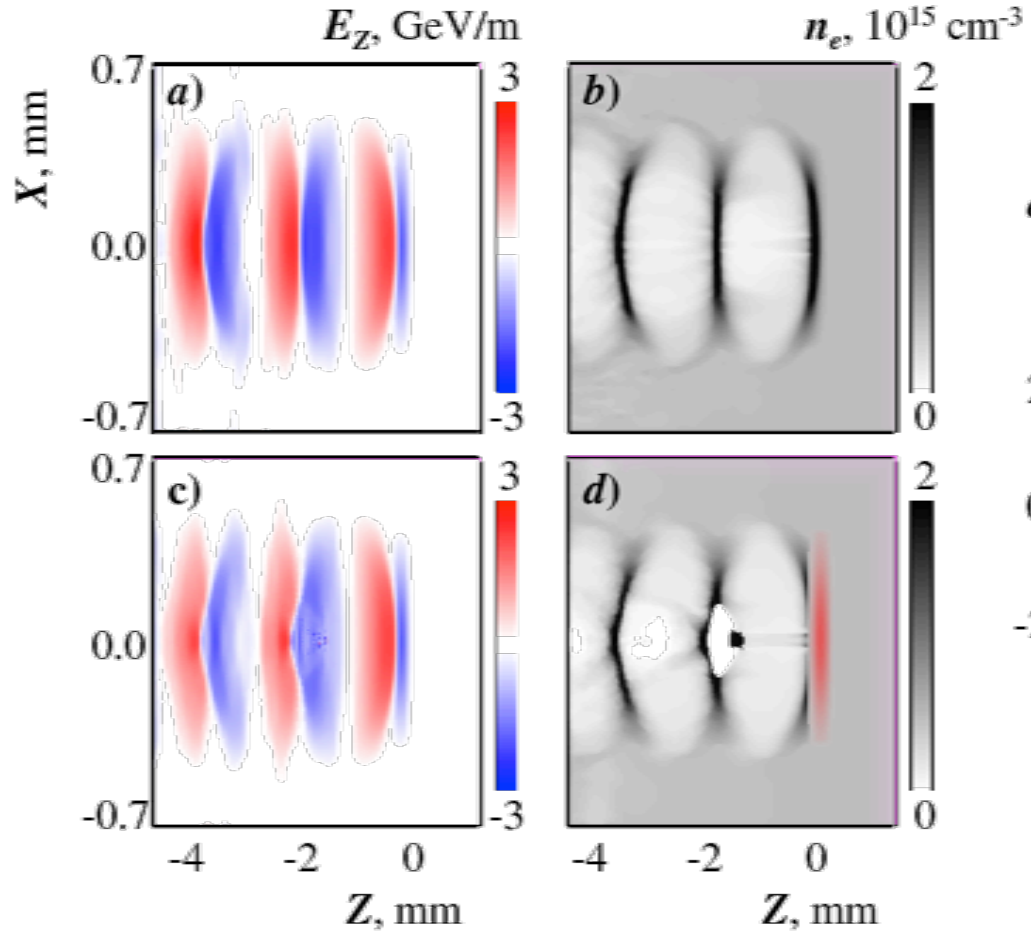
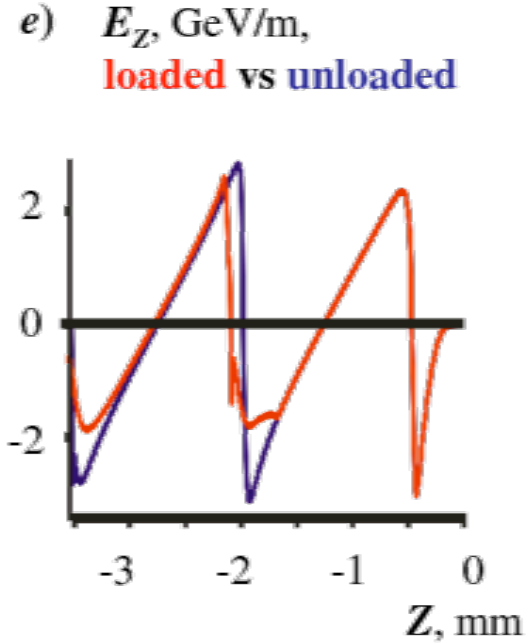
Caldwell, Lotov, Pukhov, Simon, Nature Phys. **5**, 363 (2009)





Problem:
existing proton beams are long

- 100 μm short proton beam assumed
- Realistic beams are ~10 cm long

Solution:
self-modulated regime

Kumar, Pukhov, Lotov, Phys. Rev. Lett. **104**, 255003 (2010) (2010)

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Self-modulation instability

Equation for the beam envelope radius r_b

$$\frac{\partial^2 r_b}{\partial \tau^2} - \frac{L^2}{r_b^3} = -\frac{\omega_b^2}{\gamma_0}\int_0^{\xi} r_b(\xi')I_1(k_p r_b)K_1(k_p r_b)f(\xi')\,k_p \sin k_p(\xi-\xi')d\xi'$$

Positive feedback

$\omega_b^2 = 4\pi e\rho_0/m_i$ is the beam plasma frequency

$L = r_{b0}^2\omega_{\beta 0}^2$ is the beam angular momentum

Instability growth rate

We express the instability growth as the number of exponentiations at time τ and distance ξ from the beam head

$$\Gamma\tau = (\omega_{\beta 0} k_p \xi\tau/2)^{1/2}.$$

where

$$\omega_{\beta 0}{}^2 = 4\pi n_b e^2/m_i\gamma_0.$$

Kumar, Pukhov, Lotov, Phys. Rev. Lett. **104**, 255003 (2010) (2010)

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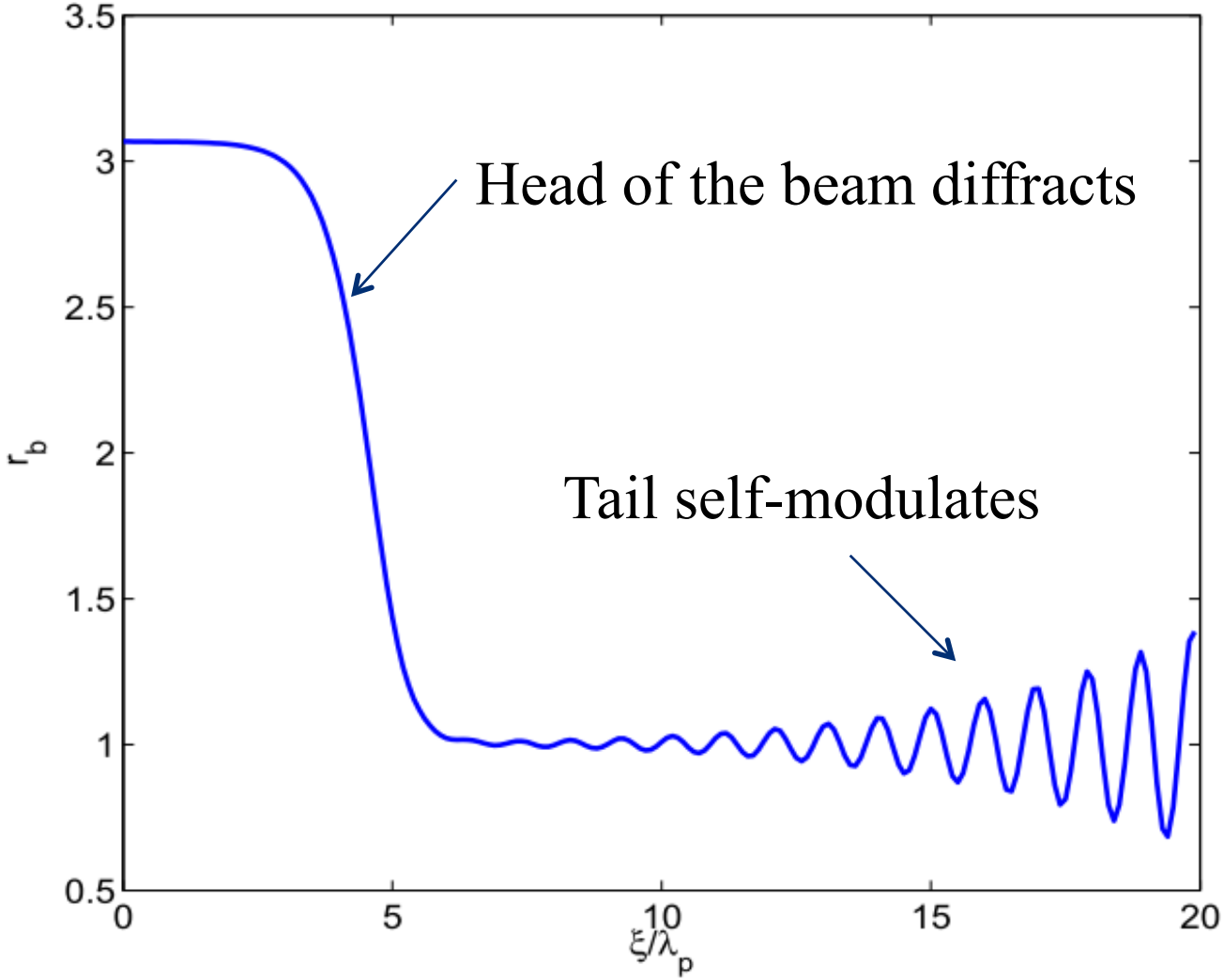
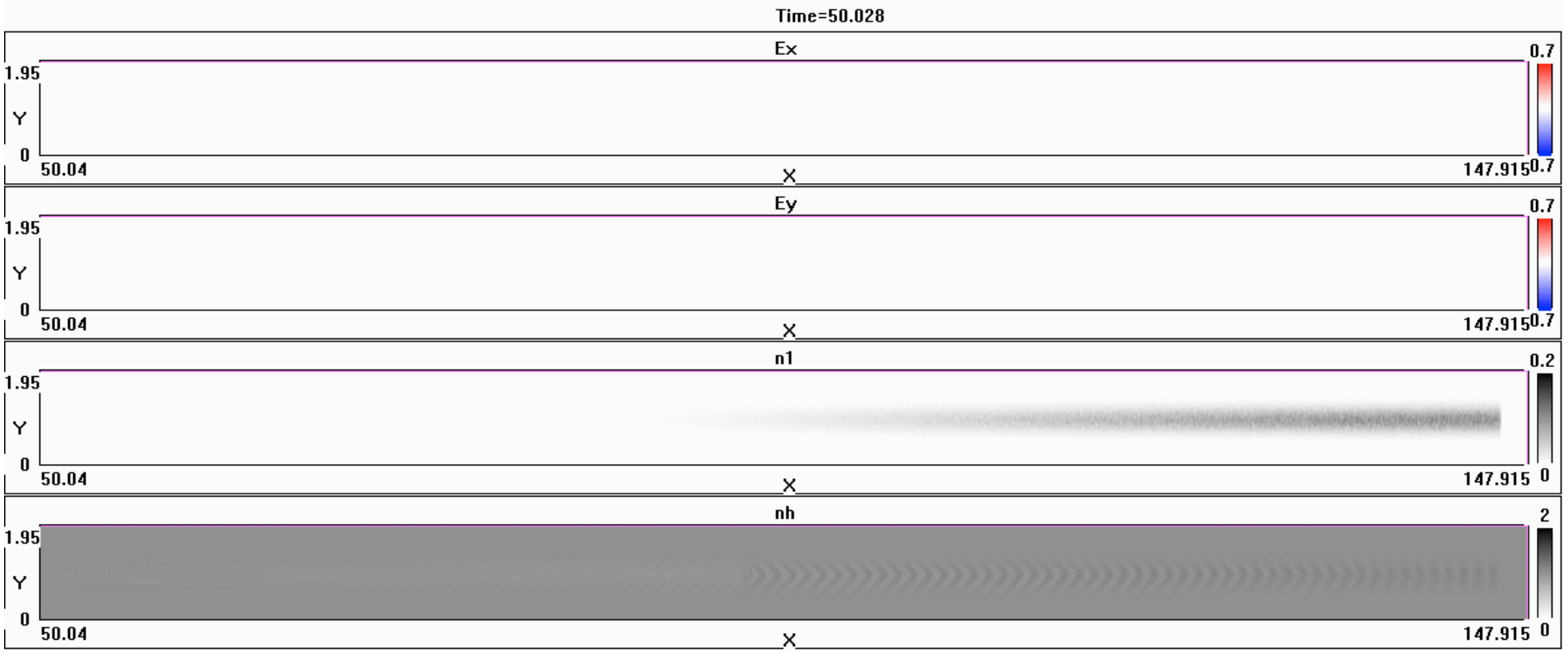
"Analytical" simulation



Beam self-modulation
3D PIC results



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Outlook

- Self-modulation today is the only option to excite wakes with long proton (ion) beams
- Tests with GeV/m fields can be done at CERN
- **Huge simulation effort** is required to find out the optimal plasma-beam parameters

  3D PIC simulation takes ~1 week CPU time for 100 meters of plasma