Exclusive $J/\psi$ and $\Upsilon$ hadroproduction as a probe of the QCD Odderon

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Abstract
We study the exclusive production of $J/\psi$ or $\Upsilon$ in $pp$ and $\bar{p}p$ collisions, where the meson emerges from the pomeron-odderon and the pomeron-photon fusion. We estimate the cross sections for these processes for the kinematical conditions of the Tevatron and of the LHC.

1 Introduction
The new analysis of experimental data on the exclusive hadroproduction processes by the CDF collaboration [1] shows that these types of processes can be objects of detailed study at the Tevatron and in the near future at the LHC. Up to now, the most intensively studied exclusive hadroproduction processes include the dijet or the $J/\psi$ production in the central rapidity region and the Higgs meson production [2], see Fig. 1. Here, we discuss the exclusive hadroproduction of $J/\psi$ and $\Upsilon$ mesons, i.e.

$$pp (\bar{p}) \to p' V p'' (\bar{p}'') , \quad \text{where} \quad V = J/\psi, \ \Upsilon .$$

(1)

The main motivation of our recent study [3] of the process (1) is that the production of a charmonium $V$, with the quantum numbers $J^{PC} = 1^{--}$, occurs as the result of a pomeron-odderon or pomeron-photon fusion. Such studies can thus probe the dynamics of the odderon [4], i.e. the counterpart with negative charge parity of the pomeron. Odderon escapes experimental verification and until now has remained a mystery, although various ways to detect it through its interference with a pomeron mediated amplitude [5] have been recently proposed (for a review see [6]).

2 The scattering amplitude
In the lowest order of perturbative QCD, the pomeron and the odderon are described by the exchange of two and three non-interacting gluons, respectively. The lowest order contribution to the hadroproduction

$$h(p_A) + h(p_B) \to h(p_{A'}) + V(p) + h(p_{B'})$$

(2)

is illustrated by diagrams of Fig. 2, from which the diagrams (a,b) describe the pomeron-odderon fusion and (c,d) the photon-pomeron fusion. The momenta of particles are parametrized by the Sudakov decompositions

$$p_{A'} = (1 - x_A)p_A + \frac{l^2}{s(1 - x_A)}p_B - l_\perp , \quad p_{B'} = \frac{k^2}{s(1 - x_B)}p_A + (1 - x_B)p_B - k_\perp ,$$

(3)

*Dedicated to the memory of Leszek Łukaszuk, co-father of the odderon, who recently passed away.*
Fig. 1: Kinematics of the exclusive meson production in pp (p̅p) scattering.

Fig. 2: The lowest order diagrams contributing to the pomeron-odderon fusion (a,b) and the pomeron–photon fusion (c,d) for vector meson production.

with $l^2 = -l_+ \cdot l_+ \approx -(p_A - p_A')^2 \equiv -t_A$, $k^2 = -k_+ \cdot k_+ \approx (p_B - p_B')^2 \equiv -t_B$ and

$$p = \alpha p_A + \beta p_B + p_\perp$$

$$\alpha_p = x_A - \frac{k^2}{s(1-x_B)} \approx x_A, \quad \beta_p = x_B - \frac{l^2}{s(1-x_A)} \approx x_B, \quad p_\perp = l_\perp + k_\perp \quad (4)$$

which lead to the mass-shell condition for the vector meson, $V = J/\psi, \Upsilon$,

$m_{V}^{2} = s x_A x_B (l_\perp^2 + k_\perp^2)$.

The scattering amplitude written within the $k_\perp$-factorization approach is a convolution in transverse momenta of $t$-channel fields. For instance, the contribution of Fig. 2a reads:

$$\mathcal{M}_{PO} =$$

$$-is \frac{2 \cdot 3}{2! \cdot 3!} \int \frac{d^2 l_1}{l_1^2} \frac{d^2 l_2}{l_2^2} \delta^2(l_1 + l_2 - l) \frac{d^2 k_1}{k_1^2} \frac{d^2 k_2}{k_2^2} \frac{d^2 k_3}{k_3^2} \delta^2(k_1 + k_2 + k_3 - k) \times \phi_{\psi}^{\lambda_{\psi}}(l_1, l_2) \phi_{J/\psi}^{\lambda_{J/\psi}}(k_2, k_3)$

where $\Phi_{P}^{\lambda_{1} \lambda_{2}}(l_1, l_2)$ and $\Phi_{P}^{\lambda_{1} \lambda_{2} \lambda_{3}}(k_1, k_2, k_3)$ are the impact factors describing the coupling of the pomeron and the odderon to scattered hadrons, respectively, whereas $\Phi_{J/\psi}^{\lambda_{J/\psi}}(l_2, k_1, k_2)$ is the effective $J/\psi$-meson production vertex.

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The proton impact factors are non-perturbative objects and we describe them within the Fukugita-Kwieciński eikonal model [7]. For the pomeron exchange the impact factor of the proton is the product

$$\Phi_{\psi}^{\lambda_{\psi}}(l_1, l_2) = 3 \Phi_{\psi}^{\lambda_{\psi}}(l_1, l_2) \mathcal{F}(l_1, l_2) \quad (6)$$
of the impact factor of a quark
\[ \Phi_{q}^{\lambda_1 \lambda_2}(l_1, l_2) = -\bar{\alpha}_s \cdot 8\pi^2 \cdot \frac{\delta_{\lambda_1 \lambda_2}}{2 N_c}, \]  
(7) and the phenomenological form-factor \( F_P \) describing the proton internal structure
\[ F_P(l_1, l_2) = F(l_1 + l_2, 0, 0) - F(l_1, l_2, 0), \]  
(8) which vanishes when any of \( l_i \to 0 \), as required by colour gauge invariance. The function \( F(k_1, k_2, k_3) \) is chosen in the form
\[ F(k_1, k_2, k_3) = \frac{A^2}{A^2 + \frac{1}{\pi} \left( (k_1 - k_2)^2 + (k_2 - k_3)^2 + (k_3 - k_1)^2 \right)}, \]  
(9) with \( A \) a phenomenological constant chosen to be half of the \( \rho \) meson mass, \( A = m_\rho/2 \approx 384 \text{ MeV} \). The analogous impact-factor for the odderon exchange reads
\[ \Phi_{P}^{\epsilon_1 \epsilon_2 \epsilon_3}(k_1, k_2, k_3) = 3 \Phi_{q}^{\epsilon_1 \epsilon_2 \epsilon_3}(k_1, k_2, k_3) F_O(k_1, k_2, k_3), \]  
(10) where
\[ \Phi_{q}^{\epsilon_1 \epsilon_2 \epsilon_3}(k_1, k_2, k_3) = i \frac{\alpha_s}{2} 2^5 \pi^2 \frac{d^{\epsilon_3 \epsilon_2 \epsilon_1}}{4 N_c}, \]  
(11) and the form-factor \( F_O \) has a form
\[ F_O(k_1, k_2, k_3) = F(k = k_1 + k_2 + k_3, 0, 0) - \sum_{i=1}^{3} F(k_i, k - k_i, 0) + 2 F(k_1, k_2, k_3). \]  
(12) The derivation of the effective production vertex of \( J/\psi \), \( \Phi_{J/\psi}^{\lambda_2 \epsilon_1 \epsilon_2}(l_2, k_1, k_2) \), in Eq. (5) is one of the main results of our study. The charmonium is treated in the non-relativistic approximation and it is described by the \( \bar{c}c \to J/\psi \) vertex
\[ \langle \bar{c}c | J/\psi \rangle = \frac{g_{J/\psi}}{2} \tilde{\epsilon}^*(p) \left( p \cdot \gamma + m_{J/\psi} \right), \quad m_{J/\psi} = 2m_c, \]  
(13) with the coupling constant \( g_{J/\psi} \) related to the electronic width \( \Gamma_{e^+e^-}^{J/\psi} \) of the \( J/\psi \to e^+ e^- \) decay
\[ g_{J/\psi} = \sqrt{\frac{3 m_{J/\psi} \Gamma_{e^+e^-}^{J/\psi}}{16 \pi \alpha^2 \alpha_m Q_c^2}}, \quad Q_c = \frac{2}{3}. \]  
(14) The effective vertex \( g + 2g \to J/\psi \) is described by the sum of the contributions of the diagrams in Fig. 3 which has the form
\[ \Phi_{J/\psi}^{\lambda_2 \epsilon_1 \epsilon_2}(l_2, k_1, k_2) = \frac{3}{4} \alpha_s^2 8\pi^2 \frac{d^{\epsilon_3 \epsilon_2 \epsilon_1}}{N_c} \left| V_{J/\psi}(l_2, k_1, k_2) \right|, \]  
\[ V_{J/\psi}(l_2, k_1, k_2) = \frac{4\pi m_c g_{J/\psi}}{4\pi m_c g_{J/\psi} \left[ -x_B \tilde{e}^* \cdot p_B + \tilde{e}^* \cdot l_{2\perp} \right] \left( l_2^2 + (k_1 + k_2)^2 + 4m_c^2 \right) + \tilde{e}^* \cdot l_{2\perp} + \tilde{e}^* \cdot p_B \left( x_B - \frac{4k_1 \cdot k_2}{8\pi A} \right) \right]. \]  
(15)
Fig. 3: The six diagrams defining the effective vertex $g + 2g → J/\psi$.

In the numerical analysis we set $\alpha_s(m_c) = 0.38$ and $\alpha_s(m_b) = 0.21$.

The analogous formula which describes the photon-pomeron fusion in Fig. 2c has the form

$$M_{\gamma P} =$$

$$- \frac{1}{2!} s \cdot \frac{4}{(2\pi)^4} l^2 \Phi_P^\gamma(l) \int \frac{d^2 k_1}{k_1^2} \frac{d^2 k_2}{k_2^2} \delta^2(k_1 + k_2 - k) \Phi_P^{\sigma_1 \sigma_2} (k_1, k_2) \Phi_{J/\psi}^{\sigma_1 \sigma_2} (l, k_1, k_2),$$

where $\Phi_P^\gamma(l)$ is the phenomenological form-factor of the photon coupling to the proton chosen as $\Phi_P^\gamma(l) = -ie \cdot F(l, 0, 0)$. The pomeron impact factor $\Phi_P^{\sigma_1 \sigma_2} (k_1, k_2)$ is given by Eq. (6) and $\Phi_{J/\psi}^{\sigma_1 \sigma_2} (l, k_1, k_2)$ is the corresponding effective vertex expressed through $V_{J/\psi} (l, k_1, k_2)$ in Eq. (15)

$$\Phi_{J/\psi}^{\sigma_1 \sigma_2} (l, k_1, k_2) = \alpha_s eQ_c \frac{8\pi}{N_c} V_{J/\psi} (l, k_1, k_2).$$

The phases of the scattering amplitudes describing the two mechanisms of $J/\psi$-meson production differ by the factor $i = e^{i\pi}/2$. Consequently, they do not interfere and they contribute to the cross section as a sum of two independent contributions.

### 3 Estimates for the cross sections

We analyse the contributions of pomeron-odderon fusion and the photon-pomeron fusion separately. Denoting $M_{PO}^{\text{tot}} = M_{PO} + M_{\gamma P}$ and $M_{\gamma P}^{\text{tot}} = M_{\gamma P} + M_{PO}$, we calculate the differential cross sections with respect to the rapidity $y ≈ \frac{1}{2} \log(x_A/x_B)$, the squared momentum transfers in the two $t$-channels, $t_A$, $t_B$, and the azimuthal angle $\phi$ between $k$ and $l$

$$\frac{d \sigma_i}{dy dt_A dt_B d\phi} = \frac{1}{512\pi^4 s^2} |M_i^{\text{tot}}|^2 \quad i = PO, \gamma P,$$

and the partially integrated cross sections

$$\frac{d \sigma_i}{dy} = \sum_e \int_{t_{\text{min}}}^{t_{\text{max}}} dt_A \int_{t_{\text{min}}}^{t_{\text{max}}} dt_B \int_0^{2\pi} d\phi \frac{d \sigma_i^{(e)}}{dy dt_A dt_B d\phi},$$

with $t_{\text{min}}^A = 0 = t_{\text{min}}^B$ for the $PO$-fusion and $t_{\text{min}}^A = m_p^A$, $t_{\text{min}}^B = m_p^B$ for the $\gamma P$-fusion, and we set $t_{\text{max}} = 1.44 \text{ GeV}^2$. This leads to the naive predictions shown in the Table 1. More
realistic cross-sections are obtained by taking into account phenomenological improvements, such as related to the BFKL evolution (which is very important for the pomeron exchange and which may be omitted for the odderon exchange [3]), the effects of soft rescatterings of hadrons, and the precise determination of the value of the model parameter $\alpha_s$ in the impact factors. For that we write the corrected cross-sections in the form

$$
\frac{d\sigma_{\gamma P}^\text{corr}}{dy} \bigg|_{y=0} = \bar{\alpha}_s^2 S_{\text{gap}}^2 E(s, m_V) \frac{d\sigma_{\gamma P}}{dy},
$$

$$
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$$

where $d\sigma_{\rho P}/dy$ are the cross sections given by (19) at $\bar{\alpha}_s^2 = 1$. The BFKL evolution for pomeron exchange is taken into account by inclusion of the enhancement factor, which for the central production (i.e. for the rapidity $y = 0$) has the form

$$
E(s, m_V) = (x_0 \sqrt{s}/m_V)^{2\lambda}.
$$

Here, $x_0$ is the maximal fraction of incoming hadron momenta exchanged in the $t$-channels (or the initial condition for the BFKL evolution) and it is set $x_0 = 0.1$. The effective pomeron intercept $\lambda$ is determined by HERA data and it equals $\lambda = 0.2$ ($\lambda = 0.35$) for the $J/\psi$ ($\Upsilon$) production [8].

The gap surviving factor $S_{\text{gap}}^2$ for the exclusive production via the pomeron-odderon fusion is fixed by the results of Durham two channel eikonal model [9]: $S_{\text{gap}}^2 = 0.05$ for the exclusive production at the Tevatron and $S_{\text{gap}}^2 = 0.03$ for LHC. In the case of production from the photon-pomeron fusion, $S_{\text{gap}}^2 = 1$ [10].

The available estimates of the effective strong coupling constant $\bar{\alpha}_s$ in the Fukugita–Kwieciński model yield results with rather large spread: from $\bar{\alpha}_s \approx 1$ [7], through $\bar{\alpha}_s \approx 0.6$–$0.7$ determined from the HERA data [3] to $\bar{\alpha}_s \approx 0.3$ determined from data on elastic $pp$ and $p\bar{p}$ scattering [11]. This led us to introduce three scenarios which differ by the values of $\bar{\alpha}_s$ and of $S_{\text{gap}}^2$.

In the optimistic scenario we use a large value of the coupling, $\bar{\alpha}_s = 1$, combined with the gap survival factors obtained in the Durham two-channel eikonal model. We believe that the best estimates should follow from the central scenario defined by $\bar{\alpha}_s = 0.75$, and Durham group estimates $S_{\text{gap}}^2 = 0.05$ ($S_{\text{gap}}^2 = 0.03$) at the Tevatron (LHC). The pessimistic scenario is defined by $\bar{\alpha}_s = 0.3$ and $S_{\text{gap}}^2 = 1$.

Table 2 shows our predictions for the phenomenologically improved cross sections in all three scenarios. Their magnitudes justify our hope that the process (1) is a subject of experimental study in the near future at the Tevatron and at the LHC [12]. The encouraging feature of our

<table>
<thead>
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<th>$d\sigma/dy$</th>
<th>$J/\psi$</th>
<th>$\Upsilon$</th>
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<tbody>
<tr>
<td>$pp$</td>
<td>20 nb</td>
<td>1.6 nb</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>11 nb</td>
<td>2.3 nb</td>
</tr>
<tr>
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<td>11 nb</td>
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<tr>
<td>$p\bar{p}$</td>
<td>21 nb</td>
<td>1.7 nb</td>
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Table 1: Naïve cross sections $d\sigma/dy$ given by (19) for the exclusive $J/\psi$ and $\Upsilon$ production in $pp$ and $p\bar{p}$ collisions by the odderon-pomeron fusion, assuming $\bar{\alpha}_s = 1$ and analogous cross sections $d\sigma_\gamma/dy$ for the photon contribution.
Table 2: The phenomenologically corrected cross sections $d\sigma^{\text{corr}}/dy|_{y=0}$ for the exclusive $J/\psi$ and $\Upsilon$ production in $pp$ and $p\bar{p}$ collisions by the pomeron–odderon fusion, and analogous cross sections $d\sigma^{\text{corr}}/dy|_{y=0}$ for the photon contribution for the pessimistic–central–optimistic scenarios.

|          | $d\sigma^{\text{corr}}/dy|_{y=0}$ | $J/\psi$ | $\Upsilon$ |
|----------|----------------------------------|----------|------------|
|          | odderon                          | photon   | odderon    | photon |
| Tevatron | 0.3–1.3–5 nb                      | 0.8–5–9 nb| 0.7–4–15 pb| 0.8–5–9 pb|
| LHC      | 0.3–0.9–4 nb                      | 2.4–15–27 nb| 1.7–5–21 pb| 5–31–55 pb|

results is due to the fact, that the measurement of the $t_t$ dependence of the cross section partially permits filtering out the $\gamma P$ contributions and to uncover the $PO$ ones.

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[12] R. Schicker, see talk at this conference.