Rapidity gap survival in the black-disk regime

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Abstract
We summarize how the approach to the black–disk regime (BDR) of strong interactions at TeV energies influences rapidity gap survival in exclusive hard diffraction $pp \rightarrow p + H + p$ ($H =$ dijet, $\bar{Q}Q$, Higgs). Employing a recently developed partonic description of such processes, we discuss (a) the suppression of diffraction at small impact parameters by soft spectator interactions in the BDR; (b) further suppression by inelastic interactions of hard spectator partons in the BDR; (c) effects of correlations between hard and soft interactions, as suggested by various models of proton structure (color fluctuations, spatial correlations of partons). Hard spectator interactions in the BDR substantially reduce the rapidity gap survival probability at LHC energies compared to previously reported estimates.

1 Introduction
At high energies strong interactions enter a regime in which cross sections are comparable to the “geometric size” of the hadrons, and unitarity becomes an essential feature of the dynamics. By analogy with quantum–mechanical scattering from a black disk, in which particles with impact parameters $b < R_{\text{disk}}$ experience inelastic interactions with unit probability, this is known as the black–disk regime (BDR). The approach to the BDR is well–known in soft interactions, where it generally can be attributed to the “complexity” of the hadronic wave functions. It is seen \emph{e.g.} in phenomenological parametrizations of the $pp$ elastic scattering amplitude, whose profile function $\Gamma(b)$ approaches unity at $b = 0$ for energies $\sqrt{s} \gtrsim 2$ TeV. More recently it was realized that the BDR is attained also in hard processes described by QCD, due to the increase of the gluon density in the proton at small $x$. Theoretically, this phenomenon can be studied in the scattering of a small–size color dipole ($d \sim 1/Q$) from the proton. Numerical studies show that at $\sqrt{s} \sim$ few TeV the dipole–proton interaction is close to “black” up to $Q^2 \sim$ several 10 GeV\textsuperscript{2} [1]. This fact has numerous implications for the dynamics of $pp$ collisions at the LHC, where multiple hard

\textsuperscript{1} A preliminary version of this report was published in the proceedings of DIS07, Munich, 16–20 Apr. 2007, arXiv:0708.3106 [hep-ph].
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interactions are commonplace. For example, it predicts dramatic changes in the multiplicities and 
$p_T$ spectra of forward particles in central $pp$ collisions compared to extrapolations of the Tevatron 
data [2]. Absorption and energy loss of leading partons by inelastic interactions in the BDR can 
also account for the pattern of forward pion production in $d$–$Au$ collisions at STAR [3].

Particularly interesting is the question what the approach to the BDR implies for exclu-
sive hard diffractive scattering, $pp \rightarrow p + H + p$. In such processes a high–mass system 
($H = \text{dijet}, QQ, \text{Higgs}$) is produced in a hard process involving exchange of two gluons be-
tween the protons. At the same time, the spectator systems must interact in a way such as not to 
produce additional particles. This restricts the set of possible trajectories in configuration space 
and results in a suppression of the cross section compared to non-diffractive events. For soft 
spectator interactions this suppression is measured by the so–called rapidity gap survival (RGS) 
probability. Important questions are (a) what role the BDR plays in traditional soft–interaction 
RGS; (b) how the physical picture of RGS is modified by hard spectator interactions in the BDR 
at LHC energies; (c) how fluctuations of the strength of the $pp$ interaction related to inelastic 
diffraction influence RGS in hard diffractive processes; (d) how possible correlations between 
hard and soft interactions affect RGS.

These questions can be addressed in a recently proposed partonic description of exclusive 
diffraction [4], based on Gribov’s parton picture of high–energy hadron–hadron scattering. Ques-
tions (a) and (b) can be studied within this framework in a practically model–independent way. 
They require only basic information about the strength of hard and soft interactions and their 
impact parameter dependence, which is either known experimentally or can be obtained from 
reasonably safe extrapolations of existing data to higher energies. Questions (c) and (d) require 
more detailed assumptions about correlations in the partonic wavefunction of the proton, which 
relate to less understood features of the $pp$ interaction at high energies. We can address them 
by implementing within the approach of Ref. [4] specific dynamical models of nucleon structure 
(color fluctuations, transverse correlations between partons). Our studies of these questions are 
of exploratory nature.

2 Black–disk regime in soft spectator interactions

A simple “geometric” picture of RGS is obtained in the approximation where hard and soft 
interactions are considered to be independent [4]. The hard two–gluon exchange process can 
be regarded as happening locally in space–time on the typical scale of soft interactions. In the 
impact parameter representation (see Fig. 1a) the RGS probability can be expressed as

\[ S^2 = \int d^2b \ P_{\text{hard}}(b) \ |1 - \Gamma(b)|^2. \]  

Here $P_{\text{hard}}(b)$ is the probability for two hard gluons from the protons to collide in the same 
impac parameter point, given by the overlap integral of the squared transverse spatial distributions of 
gluons in the colliding protons, normalized to $\int d^2b \ P_{\text{hard}}(b) = 1$ (see Fig. 1b). The function 
$|1 - \Gamma(b)|^2$ is the probability for the two protons not to interact inelastically in a collision with 
impact parameter $b$. The approach to the BDR in $pp$ scattering at energies $\sqrt{s} \gtrsim 2$ TeV implies 
that this probability is practically zero at small impact parameters, and becomes significant only 
for $b \gtrsim 1$ fm (see Fig. 1b). This eliminates the contribution from small impact parameters in the
Fig. 1: (a) Transverse geometry of hard diffractive $pp$ scattering. (b) Dashed line: Probability for hard scattering process $P_{\text{hard}}(b)$ as function of the $pp$ impact parameter, $b$. Dotted line: Probability for no inelastic interactions between the protons, $|1 - \Gamma(b)|^2$. Solid line: Product $P_{\text{hard}}(b)|1 - \Gamma(b)|^2$. The RGS probability (1) is given by the area under this curve. The results shown are for Higgs production at the LHC ($\sqrt{s} = 14$ TeV, $M_H \sim 100$ GeV). (We point out that the distributions shown in Fig. 8 of Ref. [4] correspond to a gluon $t$-slope $B_g = 4$ GeV$^{-2}$, not $B_g = 3.24$ GeV$^{-2}$ as stated in the caption. The plot here shows the correct distributions for $B_g = 3.24$ GeV$^{-2}$.)

integral (1) (see Fig. 1b) and determines the value of the RGS probability to be $S^2 \ll 1$. One sees that the approach to the BDR in soft interactions plays an essential role in RGS at high energies.

3 Black–disk regime in hard spectator interactions

At LHC energies even highly virtual partons ($k^2 \sim$ few GeV$^2$) with $x \gtrsim 10^{-2}$ experience “black” interactions with the small–$x$ gluons in the other proton. This new effect causes an additional suppression of diffractive scattering which is not included in the traditional RGS probability [4]. One mechanism by which this happens is the absorption of “parent” partons in the QCD evolution leading up to the hard scattering process (see Fig. 2a). Specifically, in Higgs production at the LHC the gluons producing the Higgs have momentum fractions $x_{1,2} \sim M_H/\sqrt{s} \sim 10^{-2}$; their “parent” partons in the evolution (quarks and gluons) typically have momentum fractions of the order $x \sim 10^{-1}$ and transverse momenta $k_T^2 \sim$ few GeV$^2$. Quantitative studies of the BDR in the dipole picture show that at the LHC energy such partons are absorbed with near-unit probability if their impact parameters with the other proton are $\rho_{1,2} \lesssim 1$ fm (see Fig. 2b). For proton–proton impact parameters $b < 1$ fm about 90% of the strength in $P_{\text{hard}}(b)$ comes from parton–proton impact parameters $\rho_{1,2} < 1$ fm (cf. Fig. 1a), so that this effect practically eliminates diffraction at $b < 1$ fm. Since $b < 1$ fm accounts for 2/3 of the cross section [see Eq. (1) and Fig. 1b)], and the remaining contributions at $b > 1$ fm are also reduced by absorption, we estimate that inelastic interactions of hard spectators in the BDR reduce the RGS probability at LHC energies to about 20% of its soft–interaction value.

In the above argument one must also allow for the possibility of trajectories with no gluon emission. Mathematically, they correspond to the Sudakov form factor–suppressed $\delta(1-x)$–term in the evolution kernel. While such trajectories are not affected by absorption, their contributions
are small both because of the Sudakov suppression, and because they effectively probe the gluon density at the soft input scale, \( Q_0^2 \sim 1 \text{ GeV}^2 \). The probability for a gluon not to emit a gluon when evolving from virtuality \( Q_0^2 \) to \( Q^2 \), is given by the square of the Sudakov form factor,

\[
C = \left[ S_G(Q^2/Q_0^2) \right]^2 = \exp \left( -\frac{3\alpha_s}{\pi} \ln^2 \frac{Q^2}{Q_0^2} \right). \tag{2}
\]

At the same time, each of the parton densities in the trajectory without emissions is suppressed compared to those with emissions by a factor \( g(x, Q^2)/g(x, Q_0^2) \), where \( Q^2 \sim 4 \text{ GeV}^2 \). The overall relative suppression of trajectories without emission is thus by a factor

\[
R = C^2 \left[ \frac{g(x, Q^2)}{g(x, Q_0^2)} \right]^2 \sim \frac{1}{10}. \tag{3}
\]

Although this contribution is suppressed, it is comparable to that of average trajectories with emissions because the latter are strongly suppressed by the absorption effect described above. Combining the two, we find an overall suppression factor of the order \( \sim 0.3 \). In order to make more accurate estimates one obviously would need to take into account fluctuations in the number of emissions more carefully. In particular, trajectories on which only one of the partons did not emit gluons, which come with a suppression factor of \( \sqrt{R} \), may give significant contributions.

The approach to the BDR in hard spectator interactions described here “pushes” diffractive \( pp \) scattering to even larger impact parameters than are allowed by soft–interaction RGS (except for the Sudakov–suppressed contribution discussed in the previous paragraph). This should manifest itself in a shift of the final–state proton transverse momentum distribution to smaller values, which could be observed in \( p_T \)–dependent measurements of diffraction at the LHC.

The estimates reported here are based on the assumption that DGLAP evolution reasonably well describes the gluon density down to \( x \sim 10^{-6} \); the quantitative details (but not the basic
picture) may change if small–$x$ resummation corrections were to significantly modify the gluon density at such values of $x$ (see Ref. [5] and references therein). The effect of hard spectator interactions described here is substantially weaker at the Tevatron energy.

4 Color fluctuations in the colliding protons

In the approximation where hard and soft interactions in the diffractive process are considered to be independent, the RGS probability can be expressed through the $pp$ elastic scattering amplitude, and effects of inelastic diffraction do not enter into consideration, see Ref. [4] and the discussion above. It is important to investigate how accurate this approximation is in practice, and how correlations between hard and soft interactions modify the picture. Such correlations generally arise from correlations between partons in the wave functions of the colliding protons, which can be caused by several physical mechanisms, see Ref. [4] for a discussion. Here we focus on one mechanism which is closely related to the presence of inelastic diffraction channels, namely fluctuations of the size of the interacting configurations (color fluctuations). Our study of this effect here is of exploratory nature; details will be reported in a forthcoming publication.

The basic idea is that in diffractive high–energy scattering the colliding hadrons can be regarded as a superposition of configurations of different size, which are “frozen” during the time of the interaction. In the well–known approach of Good and Walker [6] this is implemented by expanding the incident hadron state in eigenstates of the $T$–matrix of the same quantum numbers. A more general formulation uses the concept of the cross section distribution, $P(\sigma)$, which can be interpreted as the probability for the hadron to scatter in a configuration with given cross section, with $\int d\sigma P(\sigma) = 1$ [7]. It is defined such that its average reproduces the total cross section,

$$\langle \sigma \rangle \equiv \int d\sigma \sigma P(\sigma) = \sigma_{tot},$$

while its dispersion coincides with the ratio of the differential cross sections for inelastic ($pp \rightarrow p + X$) and elastic ($pp \rightarrow p + p$) diffraction at $t = 0$ [8],

$$\omega_\sigma \equiv \frac{\langle \sigma^2 \rangle - \langle \sigma \rangle^2}{\langle \sigma \rangle^2} = \frac{d\sigma_{inel}}{dt} \left| \frac{d\sigma_{el}}{dt} \right|_{t=0}.$$

The dispersion and the third moment of $P(\sigma)$ have been extracted from analysis of the $pp$ and $pd$ data up to $s \approx 8 \times 10^2 \text{GeV}^2$; at higher energies the shape of the distribution is not well known. Extrapolation of a parametrization of the Tevatron data [9] suggests that between the Tevatron and LHC energy $\omega_\sigma$ should drop by a factor $\sim 2$, while at the same time the total cross section is expected to grow, indicating that the relative magnitude of fluctuations decreases with increasing energy (see Figs. 3a and b). Generally, one should expect that the different configurations in diffractive scattering are characterized by a different parton density. In hard diffractive processes $pp \rightarrow p + H + p$ this effect would then lead to a modification of the “independent interaction” result for the RGS probability, Eq. (1).

The theoretical description of the role of cross section fluctuations in hard diffraction is a complex problem, which requires detailed assumptions about the proton’s partonic wave function. Here we aim only for a simple phenomenological estimate, which illustrates the sign and
order-of-magnitude of the effect, as well as its energy dependence. Our basic assumption is that
the strength of interaction in a given configuration is proportional to the transverse area occupied
by color charges. To implement this idea, we start from the cross section distribution \( P(\sigma) \) at
fixed-target energies \( s \approx 8 \times 10^2 \text{ GeV}^2 \), which can be related to the fluctuations of the size of
the basic “valence quark” configuration in the proton wave function and is known well from the
available data [7]. We then assume that

(a) The parton density is correlated with the parameter \( \sigma \) characterizing the size of the interacting
configuration. One simple scenario is to assume that the parton density changes with the size of the configuration only through its dependence on the normalization scale,
\( \mu^2 \propto R_{\text{config}}^2 \propto \sigma \). This is analogous to the model of the EMC effect of Ref. [11], and
leads to a simple scaling relation for the \( \sigma \)-dependent gluon density,

\[
g(x, Q^2 | \sigma) = g(x, \xi Q^2), \quad \xi(Q^2) \equiv (\sigma/\langle \sigma \rangle)^{\alpha_s(Q_0^2)/\alpha_s(Q^2)}, \tag{6}
\]

where \( Q_0^2 \approx 1 \text{ GeV}^2 \). In Higgs boson production one expects \( Q^2 \approx 4 \text{ GeV}^2 \), and \( x = M_H/\sqrt{s} = 0.007 \) (LHC), 0.05 (Tevatron) with \( M_H = 100 \text{ GeV} \). An alternative scenario
— the constituent quark picture — will be discussed below.

(b) The size distribution in soft high-energy interactions is correlated with the parameter \( \sigma \)
characterizing the valence quark configuration. As a minimal model we assume that soft
interactions in a configuration with given \( \sigma \) are described by a profile function of the form

\[
\Gamma(b, s | \sigma) = \exp \left[ -\frac{2\pi b^2}{\sigma_{\text{tot}}(s, \sigma)} \right], \quad \text{with} \quad \sigma_{\text{tot}}(s, \sigma) = \alpha(s) + \beta(s) \frac{\sigma}{\langle \sigma \rangle}, \tag{7}
\]
in which the parameters $\alpha(s)$ and $\beta(s)$ are chosen such as to reproduce the average cross section and dispersion of the high–energy cross section distribution (see Fig. 3a and b) when averaging over the (given) $\sigma$ distribution $P(\sigma)$. Note that the profile in Eq. (7) approaches the black–disk limit at $b \to 0$, and that the average elastic profile $\langle \Gamma(b, s | \sigma) \rangle$ obtained in this way is very close to that found in the standard phenomenological parametrizations of the $pp$ elastic and total cross section data. More sophisticated parametrizations could easily be constructed but would not change our qualitative conclusions.

With assumptions (a) and (b) we can estimate the effect of color fluctuations in the protons in hard diffraction in a simple way. In the presence of correlations between the parton density and the strength of soft interactions, the RGS probability is now given by

$$ S_{\text{corr}}^2 = \int d^2b \left\langle P_{\text{hard}}(b | \sigma) \left| 1 - \Gamma(b, s | \sigma) \right|^2 \right\rangle, \quad (8) $$

where $P_{\text{hard}}(b | \sigma)$ is the normalized impact parameter distribution for the hard process obtained with the $\sigma$–dependent gluon density Eq. (6), and $\langle \ldots \rangle$ denotes the average over the $\sigma$ distribution. This should be compared to the RGS probability without correlations,

$$ S_{\text{uncorr}}^2 = \int d^2b \left\langle P_{\text{hard}}(b | \sigma) \left| 1 - \Gamma(b, s | \sigma) \right|^2 \right\rangle, \quad (9) $$

which corresponds to the expression obtained previously in the approximation of independent hard and soft interactions, Eq. (1), if we identify the functions there with the average distributions. For a quantitative estimate, we first consider fluctuations of the interacting configurations in only one of the colliding protons, leaving the other protons unchanged. In this case we obtain

$$ \frac{S_{\text{corr}}^2 - S_{\text{uncorr}}^2}{S_{\text{uncorr}}^2} = -0.15 \quad \text{at} \quad \sqrt{s} = 2 \text{TeV} \quad \text{(Tevatron)}. \quad (10) $$

If one could consider the fluctuation effect as a small correction, the total effect would be additive and thus proportional to the number of protons, i.e., Eq. (10) would have to be multiplied by 4, corresponding to the two protons in both the amplitude and the complex-conjugate amplitude in the cross section. While the magnitude of the correction Eq. (10) does not really justify such additive treatment, we can at least to conclude that the overall effect from correlations in this model should be a reduction of the RGS probability by $\sim 1/2$. Note that the sign of the correlation effect simply reflects the fact that smaller configurations, which have higher transparency and thus larger survival probability, have a lower density of small–$x$ partons in model adopted here.

Our treatment of color fluctuations here assumes that the basic picture of independent hard and soft interactions in RGS is still valid, and that the fluctuations can be incorporated by way of an “external” parameter controlling the size of the interacting configurations. As explained above (Sec. 3) and in Ref. [4], this assumption breaks down at the LHC energy, where hard spectator interactions approach the BDR. The correction described here thus should be valid at

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Note that there are small differences between the functional forms of the $\sigma$–averaged distributions in Eq. (9) and the original ($\sigma$–independent) distributions used previously in evaluating Eq. (1). This is only the result of imperfect modeling of the $\sigma$–dependent distributions and immaterial for the physical correlation effect discussed here.
RHIC and Tevatron energies but not at the LHC. In particular, this can be seen in the fact that the correlation effect of Eq. (10) is obtained from modification of the impact parameter distribution of hard diffraction at \( b \lesssim 1 \text{ fm} \), where we expect hard spectator interactions to be “black” at the LHC, see Sec. 3. Thus, corrections from inelastic diffractive channels of the kind discussed here play a minor role at the LHC energy. This is a welcome conclusion, as it means that our predictions for the RGS probability at the LHC are not substantially modified by such corrections.

The numerical estimate of correlation effects reported here was obtained with the assumption that the gluon density in the interacting configurations scales with the size of the configuration as in Eq. (6). Physically, this corresponds to the assumption that the valence quark configuration in the proton acts coherently as source of the gluon field, and that there are no other physical scales in the proton besides the size of that configuration. This is clearly an extreme scenario and does not take into account the physical scales generated by the non-perturbative vacuum structure of QCD. An alternative scenario would be a constituent quark picture, in which the normalization scale of the gluon density is determined by the “size” of the constituent quark (related to the spontaneous breaking of chiral symmetry) and not related to the size of the multi–quark configuration in the nucleon. For this picture the relation between the gluon density and the size of the interacting configuration would be very different from Eq. (6). It leads to a different kind of correlation between hard and soft interactions, see Ref. [4] and Section 5 below.

5 Transverse spatial correlations between partons

The partonic approach to RGS of Ref. [4] also allows one to incorporate effects of correlations in the partonic wavefunction of the protons. They can lead to correlations between hard and soft interactions in diffraction, which substantially modify the picture of RGS compared to the independent interaction approximation. The analysis of the CDF data on \( pp \) collisions with multiple hard processes indicate the presence of substantial transverse correlations between partons with \( x \gtrsim 0.1 \) [1]. Such correlations naturally arise in a constituent quark picture of the nucleon with \( r_q \lesssim R \) (see Fig. 4).

It is interesting that the observed enhancement of the cross section due to correlations seems to require \( r_q/R \sim 1/3 \), which is the ratio suggested by the instanton vacuum model of chiral symmetry breaking (see Ref. [12] for a review). Such correlations modify the picture of RGS in hard diffractive \( pp \) scattering compared to the independent interaction approximation in two ways [4]. On one hand, with correlations inelastic interactions between spectators are much more likely in configurations in which two large–\( x \) partons collide in a hard process than in average configurations, reducing the RGS probability compared to the uncorrelated case. On the other hand, the “lumpiness” implies that there is generally a higher chance for the remaining spectator system not to interact inelastically compared to the mean–field approximation. A quantitative treatment of correlations in RGS, incorporating both effects, remains an outstanding problem.
6 Summary

The approach to the BDR at high energies profoundly influences the physics of RGS in exclusive diffractive scattering. The onset of the BDR in soft spectator interactions at $\sqrt{s} \gtrsim 2$ TeV eliminates diffractive scattering at small impact parameters and determines the basic order-of-magnitude of the RGS probability at the Tevatron and LHC. At LHC energies, the BDR in hard spectator interactions pushes diffractive scattering to even larger impact parameters and further reduces the RGS probability by a factor of 3 (likely more, 4–5), implying that $S^2 < 0.01$, much smaller than initial estimates reported in the literature, see Ref. [4] and references therein. At the same time, this effect reduces the relative importance of color fluctuations related to inelastic diffraction, making our theoretical predictions of the RGS probability more robust. At the Tevatron energy, we have seen that color fluctuations lower the RGS probability compared to the approximation of independent hard and soft interactions. The simple model estimate presented in Sec. 5 suggests reduction by a factor of the order 1/2; however, more refined estimates are certainly needed. Finally, spatial correlations between partons are likely to modify the picture of RGS both at the Tevatron and the LHC energy; a detailed study of this effect would be of principal as well as of considerable practical interest.

The total RGS probability is an “integral” quantity, which combines contributions from very different trajectories of the interacting $pp$ system. It is also difficult to determine experimentally, as its extraction requires precise knowledge of the cross section of the hard scattering process (gluon GPD, effective virtualities, etc.). Much more detailed tests of the diffractive reaction mechanism can be performed by studying the transverse momentum dependence of the diffractive cross section, which can be interpreted without knowledge of the hard scattering process. In particular, the predicted onset of the BDR in hard interactions between the Tevatron and LHC energy (Sec. 3) should cause substantial narrowing of the $p_T$ distribution, which could be observed experimentally. At RHIC and Tevatron energies, the correlation effects described in Sec. 5 imply that the $p_T$ distribution is narrower than predicted by the independent interaction approximation, allowing one to test this picture experimentally. This underscores the importance of planned transverse momentum–dependent measurements of diffraction at RHIC and LHC.

References